



GRADE 4 SUPPLEMENT

Set A8 Number & Operations: Whole Number Properties

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Skills & Concepts

- ★ identify applications of commutative, associative, and distributive properties with whole numbers

Bridges in Mathematics Grade 4 Supplement

Set A8 Number & Operations: Whole Number Properties

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Bridges in Mathematics is a standards-based K–5 curriculum that provides a unique blend of concept development and skills practice in the context of problem solving. It incorporates the Number Corner, a collection of daily skill-building activities for students.

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Set A8 ★ Activity 1



ACTIVITY

The Commutative, Associative, & Distributive Properties

Overview

Although students often use the commutative, associative, and distributive properties to solve problems, they may not know the properties by name. This activity introduces the properties in the context of mental math.

Skills & Concepts

- ★ identify applications of commutative, associative, and distributive properties with whole numbers

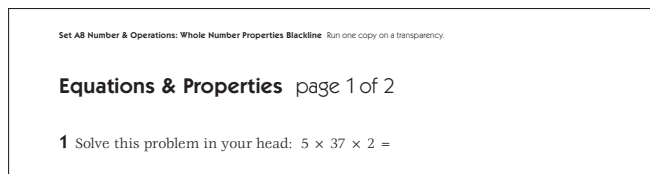
You'll need

- ★ Equations & Properties (pages A8.5 and A8.6, run a copy of each sheet on a transparency)
- ★ Defining & Using the Properties (page A8.7, run a class set)
- ★ paper to mask parts of the overhead
- ★ overhead pens

Advance Preparation Before you conduct this activity, write the terms Commutative Property, Associative Property, and Distributive Property on the board.

Instructions for The Commutative, Associative, & Distributive Properties

1. Draw students' attention to the terms you have written on the board. Read them together. Ask volunteers to share anything they know about these terms.
2. Explain that even if these terms don't sound very familiar, students may know more about them than they think. Each of the properties you have listed has to do with the way numbers behave, and today, you are going to investigate these behaviors together.
3. Place Equations and Properties on the overhead. Before you turn on the projector, cover the entire transparency with a piece of paper. Explain that when you turn on the projector, students will each think privately about the problem they see and raise their hand when they have the answer. Then turn on the projector and move the paper to reveal problem 1.



4. When most hands are up, ask students to pair-share their solutions and strategies. Then call on several volunteers to share their thinking with the class.

Students *It's 390. At first I thought it was hard. Then I realized I could just multiply 5×2 . That's 10, and 10×39 is 390. I got the same thing. I was going to round 37 up to 40, but then I saw 5×2 .*

Activity 1 The Commutative, Associative, & Distributive Properties (cont.)

5. Chances are, at least some students will report that they switched the order of the numbers, perhaps mentally reversing the position of 37 and 2 to make the problem read $5 \times 2 \times 37$. Is this okay? Is it true that 37×2 is the same as 2×37 ? Ask students to talk with one another about this issue, and then invite volunteers to defend the move.

Students *Sure it's okay. You can switch numbers when you multiply. Like 3×4 is the same as 4×3 , right?*

You can do that when you add, too. $7 + 3$ is 10, but so is $3 + 7$.

6. Reveal the rest of the overhead one section at a time. Read and discuss the steps with your students to explain that when reformulating $5 \times 37 \times 2$ to make it easier to solve mentally, you actually need to use both the associative and the commutative properties. Together, these two properties let you add or multiply a string of numbers in any order.

Set A8 Number & Operations: Whole Number Properties Blackline Run one copy on a transparency.

Equations & Properties page 1 of 2

1 Solve this problem in your head: $5 \times 37 \times 2 =$

Order of operations says we multiply the numbers in order from left to right:

$$(5 \times 37) \times 2$$

We can use two properties to make this problem easier to solve:

Associative Property
Changing the way you group three numbers or numerical expressions when you add or multiply does not change the answer. $(2 + 3) + 4 = 2 + (3 + 4)$ and $(2 \times 3) \times 4 = 2 \times (3 \times 4)$

$$(5 \times 37) \times 2 = 5 \times (37 \times 2)$$

Commutative Property
Changing the order of two numbers or numerical expressions when you add or multiply does not change the answer. $5 + 3 = 3 + 5$ and $3 \times 4 = 4 \times 3$

$$5 \times (37 \times 2) = 5 \times (2 \times 37)$$

If we use the associative property again, we can make the problem even easier to solve.

$$5 \times (2 \times 37) = (5 \times 2) \times 37$$

$$10 \times 37 =$$

What's the answer?

7. Use the second overhead to introduce the distributive property. When you have reviewed all three properties, ask students to identify which one(s) is/are most helpful in solving each of the four problems at the bottom of the second overhead.

Activity 1 The Commutative, Associative, & Distributive Properties (cont.)

Set A8 Number & Operations: Whole Number Properties Blackline Run one copy on a transparency.

Equations & Properties page 2 of 2

2 Solve this problem in your head: $6 \times 17 =$

Did you multiply 6×10 and then 6×7 , and then add the two products? If you did, you used the

Distributive Property
When you multiply a number you can break that number apart. Multiply each part separately, and then add the products. You will still get the same answer.

$$6 \times 17 = 6 \times (10 + 7)$$

$$6 \times (10 + 7) = (6 \times 10) + (6 \times 7)$$

$$(6 \times 10) + (6 \times 7) = 60 + 42$$

$$60 + 42 =$$

What's the answer?

3 Solve the problems below in your head. Use one or more of the properties to help?

Commutative Property *Switch the order of two numbers.*
Associative Property *Group it differently.*
Distributive Property *Break the number apart and multiply one part at a time.*

a $(28 \times 50) \times 2 =$	b $(40 \times 267) \times 60 =$
c $5 \times 37 =$	d $4 \times (25 \times 298) =$

8. Next, give students each a copy of Defining and Using the Properties. Review the sheet with the class and provide clarification as needed. When students understand what to do, let them go to work. Encourage them to share their thinking with others nearby as they work.

Set A8 Number & Operations: Whole Number Properties Blackline Run a class set.

NAME _____ DATE _____

Defining & Using the Properties

1 Write your own definition of each property, and give an example so you can remember how it works.

Property	Definition	Example
a Commutative		
b Associative		
c Distributive		

2 For each problem below:

- Use one or more of the above properties to rewrite the equation in a form that's easier to solve.
- Solve it and write the answer.
- Circle all the properties you used to rewrite the equation (C for commutative, A for Associative, and D for Distributive.)

Problem	Rewrite	Answer	Property
ex. $(70 + 469) + 30$	$(70 + 30) + 469$	569	C A D
a $(5 \times 39) \times 2$			C A D
b $(27 \times 25) \times 4$			C A D
c 4×27			C A D
d $(40 + 579) + 60$			C A D
e 6×28			C A D
f $(16 \times 50) \times 2$			C A D

Activity 1 The Commutative, Associative, & Distributive Properties (cont.)**Extension**

- Consider doing the Factor Puzzles activity described below with some or all of your students. Write $12 \times 25 = 75 \times 4$ on the overhead and ask students to help you prove this statement is true by using factoring and the commutative and associative properties. Can they start with 12×25 and help you take whatever steps are necessary to transform 12×25 into 75×4 ? List the steps students advise you to take on the overhead. After they are finished you could also go back and label each step taken with F (factoring), C (using the commutative property) or A (using the associative property), again, as advised by students.

Students *I see what to do... Turn 12 into 3×4 and write $3 \times 4 \times 25$.*

Then have the 4 and the 25 switch places.

Then you can put the 3 and the 25 together because they'll multiply to make 75.

$$\begin{aligned}
 &12 \times 25 \\
 &(3 \times 4) \times 25 \quad \text{F (factoring)} \\
 &3 \times (4 \times 25) \quad \text{A (associative factor)} \\
 &3 \times (25 \times 4) \quad \text{C (commutative property)} \\
 &(3 \times 25) \times 4 \quad \text{A (associative property)} \\
 &75 \times 4
 \end{aligned}$$

Next, have students advise you on the steps required to prove that $5 \times 100 = 10 \times 50$.

$$\begin{aligned}
 &5 \times 100 \\
 &5 \times (10 \times 10) \quad \text{F (factoring)} \\
 &(5 \times 10) \times 10 \quad \text{A (associative)} \\
 &(10 \times 5) \times 10 \quad \text{C (commutative)} \\
 &10 \times (5 \times 10) \quad \text{A (associative)} \\
 &10 \times 50
 \end{aligned}$$

Now write the following statements on the whiteboard or overhead:

$$4 \times 50 = 5 \times 40$$

$$12 \times 25 = 6 \times 50$$

$$25 \times 16 = 20 \times 20$$

$$30 \times 20 = 60 \times 10$$

Ask student pairs to choose any two of these statements, write them in their journals and prove they are true by using factoring and the commutative and associative properties of multiplication. (All the expressions require factoring but some might involve only one of the properties, depending on how students order the factors.) If your students enjoy these puzzles, you might make up some more or ask interested children to create additional puzzles for their classmates.

**INDEPENDENT WORKSHEET**

Use Set A8 Independent Worksheets 1 and 2 to provide students with more practice identifying applications of the commutative, associative, and distributive properties with whole numbers.

Equations & Properties page 1 of 2

1 Solve this problem in your head: $5 \times 37 \times 2 =$

Order of operations says we multiply the numbers in order from left to right:

$$(5 \times 37) \times 2$$

We can use two properties to make this problem easier to solve:

Associative Property

Changing the way you group three numbers or numerical expressions when you add or multiply does not change the answer. $(2 + 3) + 4 = 2 + (3 + 4)$ and $(2 \times 3) \times 4 = 2 \times (3 \times 4)$

$$(5 \times 37) \times 2 = 5 \times (37 \times 2)$$

Commutative Property

Changing the order of two numbers or numerical expressions when you add or multiply does not change the answer. $5 + 3 = 3 + 5$ and $3 \times 4 = 4 \times 3$

$$5 \times (37 \times 2) = 5 \times (2 \times 37)$$

If we use the associative property again, we can make the problem even easier to solve.

$$5 \times (2 \times 37) = (5 \times 2) \times 37$$

$$10 \times 37 =$$

What's the answer?

Equations & Properties page 2 of 2

2 Solve this problem in your head: $6 \times 17 =$

Did you multiply 6×10 and then 6×7 , and then add the two products? If you did, you used the

Distributive Property

When you multiply a number you can break that number apart. Multiply each part separately, and then add the products. You will still get the same answer.

$$\begin{aligned} 6 \times 17 &= 6 \times (10 + 7) \\ 6 \times (10 + 7) &= (6 \times 10) + (6 \times 7) \\ (6 \times 10) + (6 \times 7) &= 60 + 42 \\ 60 + 42 &= \end{aligned}$$

What's the answer?

3 Solve the problems below in your head. Use one or more of the properties to help?

Commutative Property *Switch the order of two numbers.*

Associative Property *Group it differently.*

Distributive Property *Break the number apart and multiply one part at a time.*

a $(28 \times 50) \times 2 =$	b $(40 \times 267) \times 60 =$
c $5 \times 37 =$	d $4 \times (25 \times 298) =$

NAME _____

DATE _____

Defining & Using the Properties

1 Write your own definition of each property, and give an example so you can remember how it works.

Property	Definition	Example
a Commutative		
b Associative		
c Distributive		

2 For each problem below:

- Use one or more of the above properties to rewrite the equation in a form that's easier to solve.
- Solve it and write the answer.
- Circle all the properties you used to rewrite the equation (C for commutative, A for Associative, and D for Distributive.)

Problem	Rewrite	Answer	Property
ex. $(70 + 469) + 30$	$(70 + 30) + 469$	569	(C) (A) D
a $(5 \times 39) \times 2$			C A D
b $(27 \times 25) \times 4$			C A D
c 4×27			C A D
d $(40 + 579) + 60$			C A D
e 6×28			C A D
f $(16 \times 50) \times 2$			C A D

NAME _____

DATE _____

Set A8 ★ Independent Worksheet 1



INDEPENDENT WORKSHEET

Properties Make It Easier

Here are two properties that make it easier to do mental math.

Commutative Property *Changing the order of two numbers or numerical expressions when you add or multiply does not change the answer.*

$$5 + 3 = 3 + 5 \quad \text{and} \quad 3 \times 4 = 4 \times 3$$

Associative Property *Changing the way you group three numbers or numerical expressions when you add or multiply does not change the answer.*

$$(2 \times 3) \times 4 = 2 \times (3 \times 4)$$

2 For each problem below:

- Use one or both of the above properties to rewrite the equation in a form that's easier to solve.
- Solve it and write the answer.
- Circle all the properties you used to rewrite the equation (C for commutative, A for Associative, or C and A if you used both properties.)

Problem	Rewrite	Answer	Property
ex. $(70 + 469) + 30$	$(70 + 30) + 469$	569	C A
a $(5 \times 68) \times 2$			C A
b $(24 \times 25) \times 4$			C A
c $(32 \times 50) \times 2$			C A
d $(40 + 579) + 60$			C A
e $50 \times (20 \times 16)$			C A

(Continued on back.)

Independent Worksheet 1 Properties Make It Easier (cont.)

Distributive Property When you multiply a number you can break that number apart. Multiply each part separately, and then add the products. You will still get the same answer.

2 Use the distributive property to make the problems below easier. Fill in the missing steps to get the answers.

example ↓ 6×13
$6 \times (10 + 3)$ ↓
$(6 \times 10) + (6 \times 3)$ ↓
$60 + 18 = 78$

a 7×23 ↓
$7 \times (20 + 3)$ ↓
↓
↓

b 8×24 ↓
$8 \times (20 + 4)$ ↓
↓
↓

c 5×45 ↓
↓
$(5 \times 40) + (5 \times 5)$ ↓
↓

d 3×28 ↓
↓
$(3 \times 20) + (3 \times 8)$ ↓
↓

e 5×63 ↓
↓
↓
↓

f 6×35 ↓
↓
↓
↓

g 9×24 ↓
$9 \times (20 + 4)$ ↓
↓
↓

h 7×39 ↓
↓
↓
↓

NAME _____

DATE _____

Set A8 ★ Independent Worksheet 2



INDEPENDENT WORKSHEET

C is for Commutative, A is for Associative, D is for Distributive

1 Draw a line from each property to the correct definition on the right. Write two more examples below each property to show how it works.

Commutative Property	Changing the way you group three numbers when you add or multiply does not change the answer. $(2 \times 3) \times 4 = 2 \times (3 \times 4)$
Associative Property	You can break a number apart, multiply each part separately, and then add the products. You will still get the same answer. $6 \times 15 =$ $6 \times (10 + 5) =$ $(6 \times 10) + (6 \times 5) =$ $60 + 30 = 90$
Distributive Property	Changing the order of two numbers when you add or multiply does not change the answer. $5 + 3 = 3 + 5$ and $3 \times 4 = 4 \times 3$

2 For each problem below:

- Write it a different way so it's easier to solve in your head.
- Solve it and write the answer.
- Circle the letter(s) that show(s) the property or properties you used.

Problem	Rewrite	Answer	Property
a 8×32			C A D
b $(4 \times 48) \times 25$			C A D
c $(60 + 6,532) + 40$			C A D
d $(5 \times 456) \times 2$			C A D
e 6×85			C A D

