Basic Operations

OVERVIEW

Students use tile to express their perceptions of the four basic arithmetic operations. Models for these operations which are used in subsequent activities are introduced. Part I deals with multiplication and division and Part II discusses addition and subtraction.

Prerequisite Activity

None

Materials

At least 15 tile per student.

Actions

Comments

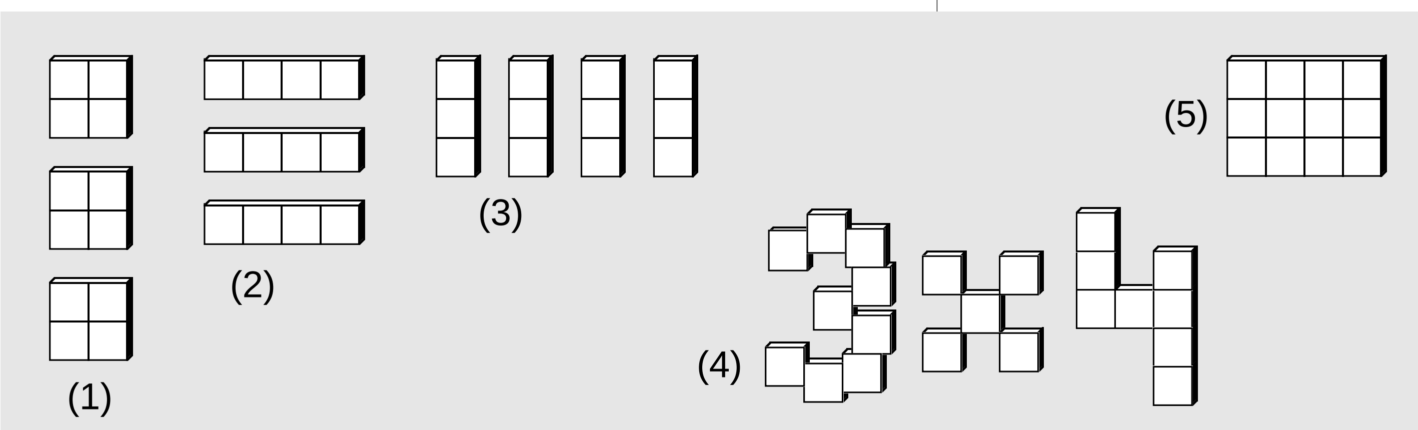
Part I **Multiplication and Division**

1. Distribute 15 to 20 tile to each student. Write "3×4" on the blackboard. Ask each student to use tile to represent the expression on the board. Emphasize that there is no *right* way to do this and that you expect a variety of representations.

1. Writing "3×4" on the blackboard rather than saying "three times four", "the product of three and four", "three multiplied by four", or some other phrase, forces the students to focus on the symbols. The intention is that students determine the meaning of the symbols for themselves.

2. (a) Call attention to the various representations. Ask students to talk about theirs. Accept all without judgment.

2. The representations will give you some insight about the ways in which your students view multiplication. Some possible representations are shown below.



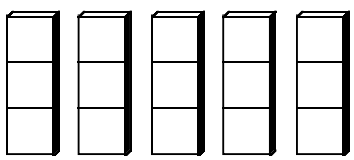
(b) Draw attention to the rectangular array model (5). If no one makes this model, make it yourself and add it to the collection of representations of 3×4.

Some students may think of 3×4 as 3 sets of 4 as in sketches (1) and (2). Others will think of it as 4 sets of 3 as in (3). Those who form symbols, as in (4), may think of arithmetic predominantly as symbols and symbol manipulation without other meaning. The rectangular array (5) is a particularly useful model of multiplication. Note that models (2) and (3) are easily converted to rectangles by pushing the rows or columns together. *(Continued)*

Actions

3. Write “15 ÷ 3” on the board. Ask your students to use tile to represent this expression. Call attention to the various representations and discuss them with your students. Again, accept all representations without judgment.

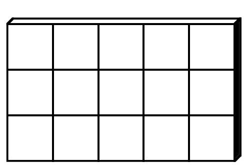
4. Discuss the *grouping* and *sharing* methods of division. Give examples, or ask your students to give examples of both uses of division.



15 ÷ 3 : grouping method

15 ÷ 3 : sharing method

(Note that both tile arrangements become the same rectangular array when rows or columns are pushed together:

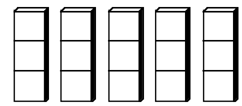


The array contains 15 tiles in 3 rows and 5 columns.)

Comments

2. *(Continued)* Thus, a 3×4 rectangle may be viewed as either 3 sets of 4 or as 4 sets of 3. This illustrates the commutative nature of multiplication.

3. Both symbolic and non-symbolic representations may occur. Two common non-symbolic models are five groups of three,



or three groups of five.

4. Representing 15 ÷ 3 by 5 groups of 3 is an example of the *grouping* model of division. In this case, 15 ÷ 3 is thought of as the number of groups of 3 into which 15 can be divided, as in the question: “If each student is to receive 3 pencils, how many students will 15 pencils supply?”

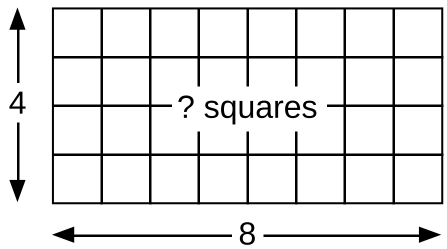
Representing 15 ÷ 3 by 3 sets of 5 is an example of the *sharing* method of division. Here, 15 ÷ 3 is thought of as the number of objects in each of 3 sets when 15 objects are shared equally among them, as in the question: “If 15 pencils are divided equally among 3 students, how many will each student get?”

The grouping method of division is also called the *subtractive* or *measurement* method since groups are subtracted away or measured off. The sharing method is also called the *dealing* or *partitive* method since objects are dealt or partitioned into sets.

Actions | **Comments**

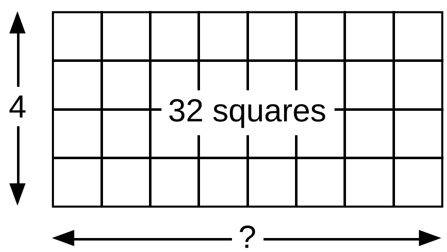
5. Show how multiplication and division are related in the rectangular array model.

Figure A



4 × 8 is the number of squares

Figure B



32 ÷ 4 is the missing dimension

5. The process of multiplication may be thought of as determining the number of tile in a rectangular array when the number of tile along each edge is known (or, equivalently, finding the area of a rectangle when the dimensions are given. See Figure A).

The process of division may be thought of as determining the number of tile on one side of the array when the number of tile on the other side and the total number in the array are known (or, finding one dimension of a rectangle given the other dimension and the area. See Figure B.).

Part II **Addition and Subtraction**

6. Ask your students to form two groups of tile, one containing 9 tile and the other 5. Write “9 + 5” on the blackboard and ask your students to use their tile to represent the expression on the board. Discuss what happens.

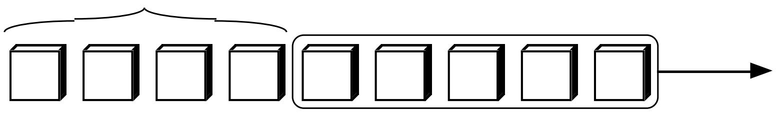
6. Most students will combine the groups of 9 and 5.

7. Write “9 – 5” on the board and ask your students to represent this expression with their tile. Discuss what happens.

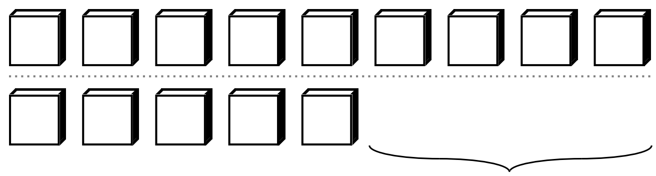
7. Many students will form a group of 9 tile and take 5 tile from this group. Other students may determine how many tile must be added to a group of 5 tile to match the group of 9 tile.

Actions

8. Discuss the *take-away* and *difference* methods of subtraction. Give examples, or ask your students to supply examples, of both uses of subtraction.



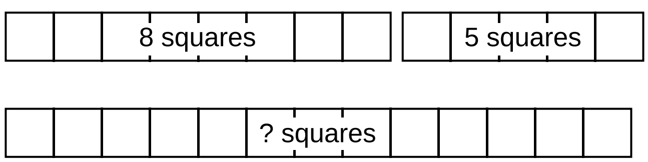
9 – 5 : take-away method



9 – 5 : difference method

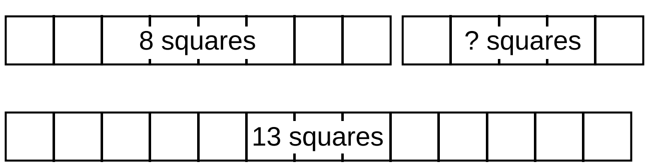
9. Show how the operations of addition and subtraction are related.

Figure C



? = 8 + 5

Figure D



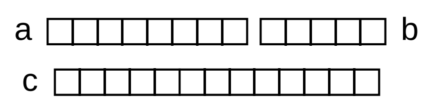
? = 13 – 8

Comments

8. Carrying out 9 – 5 by taking 5 tile from 9 tile is an example of the *take-away* method of subtraction. This method is appropriate in responding to the question: “If a student has 9 pencils and gives 5 of them away, how many pencils does the student left?”

Determining 9 – 5 by finding how many tile must be added to 5 tile to match a group of 9 tile is an example of the *difference* method of subtraction. This method is appropriate for the question: “If a student has 9 pencils and another has 5, how many more pencils will the first student have than the second.”

9. Suppose there are 3 groups of squares *a*, *b* and *c* such that the number of squares in *a* and *b* together is the number of squares in *c*.



In the process of addition, one knows the number of squares in *a* and *b* and seeks the number of squares in *c* (Figure C).

In the process of subtraction, one knows the number of squares in *c* and one of the groups *a* or *b* and seeks the number of squares in the other group (Figure D).