Roles of Students & Teachers

Chapter 2

Mathematics instruction often is approached in terms of stating and exemplifying rules — the "tell, show, and do" model. Based on the assumption that information can be presented by telling and that understanding will result from being told, such an approach does not work because it frequently overlooks two crucial developmental components: the process of assimilation and the issue of readiness. Essentially, in this approach, students are "ready" intellectually when the teacher is ready for them to receive the information. Learning through such an approach often fails to promote a transfer of mathematical information to new situations.

Teachers need to employ alternative forms of instruction that permit students to build their repertoire of mathematical knowledge and their abilities for posing, constructing, exploring, solving, and justifying mathematical problems and concepts. Promising models for such instruction are all highly interactive. In such models, teachers both model and elicit mathematical discourse by asking questions, following leads, and conjecturing rather than presenting faultless products (Ball 1990; Noddings 1990).

NCTM Professional Standards

ROLES OF STUDENTS

The role of students in *Visual Mathematics* is that of active and attentive explorers and observers, who construct and report on their perceptions and understandings. Thus, the traditional passive student role is changed to one which includes actions such as those shown below. (Full-sized copies are in the Blackline section and are useful for sharing with parents.) See *Course I*, Lesson 6, and *Course II*, Lesson 1, as examples of activities that involve students in discussions of their roles.

Evidence I'm Growing I

Here are some ways that I show I think mathematically and understand the math we do:

a) I support my conclusions with valid mathematical reasoning.

b) I give more than one solution and/or more than one approach to problems.

c) My explanations show careful reasoning and deep thinking.

d) I use models and methods from class to solve problems.

e) I make connections among different ideas in math and between math and other subjects.

f) My explanations show that I am constructing deeper and broader understandings of math concepts.

g) I make and test conjectures, and I make generalizations about ideas and methods.

Evidence I'm Growing II

Here are some ways that I show I want to learn and grow in math:

a) I listen actively to others and I respect their views.

b) I volunteer my ideas in group discussions.

c) I ask thoughtful questions of my classmates, my teacher, and myself.

d) I willingly struggle with ideas or problems that seem challenging.

e) I share my difficulties with problems and ideas.

f) I show joy with my AHA!'s in math and honor my disequilibrium.

g) I extend problems and explore my own "what-if" questions.

h) I pay attention to my thought processes and to ways my understanding is developing.

We notice that, although much of the language in the above statements may be new to our students at the beginning of a *Visual Mathematics* course, they enjoy learning words such as "conjecture," "generalizations," "AHA!", "connections," and "disequilibrium" to describe their actions and feelings. The development of a language associated with their classroom experiences fosters a sense of community among students. As long as new terminology is connected to experiences that give meaning to the language, it isn't necessary to adjust or simplify the mathematical vocabulary that we use. Although the following quote refers to E.B. White's views of children's capabilities as readers, it also captures our belief about students' abilities to deal with challenging language and ideas in mathematics.

> *Anyone who writes down to children is simply wasting his time. You have to write up, not down. Children are demanding. They are the most attentive, curious, eager, sensitive, quick and generally congenial readers on earth. They accept, almost without question, anything you present them with, as long as it is presented honestly, fearlessly and clearly. Some writers for children deliberately avoid using words they think a child doesn't know.*

> *This emasculates the prose and, I suspect, bores the reader. Children are game for anything. I throw them hard words, and they backhand them over the net. They love words that give them a hard time, provided they are in a context that absorbs their attention.*
>
> E.B. White

ROLES OF TEACHERS

> *So what is the role of teaching, if knowledge must be constructed by each individual? In my view, there are two aspects to teaching. The first is to put students into contact with phenomena related to the area to be studied — the real thing, not books or lectures about it — and to help them notice what is interesting; to engage them so they will continue to think and wonder about it. The second is to have the students try to explain the sense they are making, and, instead of explaining things to students, to try to understand their sense. These two aspects are, of course, interdependent: When people are engaged in the matter, they try to explain it and in order to explain it they seek out more phenomena that will shed light on it.*
>
> Eleanor Duckworth

In *Visual Mathematics*, the teacher's role is that of an educator in the root meaning of the word: one that educes — one that draws out — the mathematician that exists within every student. This is done by engaging the students in activities and experiences in which they can explore mathematical concepts and procedures in sensory-rich settings, report on their findings, and have them accepted as valid and valued. The teacher nurtures student confidence and autonomy and nourishes each developing mathematician by:

- Facilitating the use of manipulatives, models, sketches, and diagrams to explore and represent math concepts and solve problems.
- Posing engaging activities that reveal mathematics as an interconnected whole and as relevant to the students' world.
- Applying effective questioning strategies aimed at drawing out and challenging students' thinking about mathematical concepts and procedures.
- Focusing on concept understanding.
- Listening closely to student thinking in order to understand the sense students make of the mathematics at hand.
- Encouraging students to be the source of authority regarding the correctness or worth of their own ideas.
- Using a variety of instructional formats: small group, whole class, and individual.
- Creating a nonthreatening environment that builds confidence and encourages student risk-taking and active participation.
- Fostering student collaboration and discourse about mathematics.
- Assessing learning and promoting reflection as integral parts of instruction.
- Encouraging multiple views of concepts, problems, and procedures.
- Respecting students' right to struggle with an idea, and hence, stepping back, confident that students can resolve their questions.
- Allowing time for students to make observations and construct meaning from their interactions with classmates, materials, and models.

• Providing experiences that cast an idea in a new, and perhaps contradictory, light, and allowing student disequilibrium.

It is difficult to break away from such traditional teaching habits as lecturing, stressing memorization, "comforting" puzzled students by showing them methods and solutions, and asking questions only when we want the student to guess at what we are thinking. However, current research suggests that these changes are critical.

> *In many classrooms, learning is conceived of as a process in which students passively absorb information, storing it in easily retrievable fragments as a result of repeated practice and reinforcement. Research findings from psychology indicate that learning does not occur by passive absorption alone (Resnick 1987). Furthermore, ideas are not isolated in memory but are organized and associated with the natural language that one uses and the situations one has encountered in the past. This constructive, active view of the learning process must be reflected in the way much of mathematics is taught.*
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> *…A variety of instructional methods should be used in classrooms in order to cultivate students' abilities to investigate, to make sense of, and to construct meanings from new situations; to make and provide arguments for conjectures; and to use a flexible set of strategies to solve problems from both within and outside mathematics. … greater opportunities should be provided for small-group work, individual explorations, peer instructions, and whole-class discussions in which the teacher serves as a moderator. These alternative methods of instruction will require the teacher's role to shift from dispensing information to facilitating learning, from that of director to that of catalyst and coach.*
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> NCTM Curriculum Standards

Decision-making role of teachers

A very important role of the teacher in *Visual Mathematics* is that of decision-maker. The lessons are designed to be starting points of investigations; the sequence of actions provide a direction one could pursue and be likely to evoke meaningful mathematical discussions and observations related to the big idea of the lesson. However, in reality the direction a lesson takes is determined by the student thinking that surfaces. The teacher poses a situation or question from an activity, listens to student thinking, and then must make decisions, such as the following:

- I wonder what that student means. What shall I do to further elicit her thinking?
- Will pursuing this student idea be mathematically productive?
- We could do this next or do that next; which direction will be most fruitful?
- Will exploring this student idea, even though I know it won't work, be worthwhile mathematical activity for the students?
- I don't know how/whether this idea works; am I willing to take the risk and explore with the class?
- Will following this student's path lose other students?
- Shall I sacrifice overall student interest for a brief period in order to build confidence in this student who wants to share and rarely volunteers?
- Would having additional students share ideas be mathematically productive?
- Will sharing with the large group benefit the majority of students or has sufficient sharing gone on in small groups?

- Shall I have each student work independently on this activity sheet or shall I have each group work on one together?
- The mathematical idea that just came up isn't related to the big idea of the lesson; shall we pursue it now and extend the time of the lesson or shall we come back to this idea later?
- Do I need to pose a different twist on this question in order to better find out how students are thinking?
- Students are struggling. Shall I let them decide when it's time to move on? Shall I give them new information? Shall I move on knowing a more complex question or a different context may cause them to reconsider and reformulate their ideas? Or, are there important conceptual underpinnings that need revisiting?
- Have the big ideas come forward yet, and if so, do I need to continue the lesson?
- Am I letting old beliefs about mastery get in the way, or do students legitimately need more time on this lesson?
- I wanted this idea to come up here and it didn't — how critical is it to the big idea of the lesson? Will it come up later if I wait? Will sharing my idea now move things along or will it roadblock students' thinking?
- That student's line of reasoning is mathematically unsound. Will another action I have planned raise a contradiction? Do I need to create a new situation that will raise a contradiction, and if so, how long should I wait before posing the new situation?
- Am I providing enough/too much wait time? enough/too much time for individual reflection? Are my actions in general drawing out/shutting down student thinking?
- Is my line of questioning genuine, or am I trying to lead students to guess what I am thinking?

These are only a few of the myriad questions that float through a teacher's mind and require on-the-spot decisions during a *Visual Mathematics* lesson.

We have noticed many commonalties in students' methods of approaching situations posed in the lessons, and familiarity with some of these commonalties can be assuring and provide a basis for deciding what to do next, particularly for the teacher who is new to *Visual Mathematics*. The Comments column of each lesson activity provides a sampling of possible student responses along with a brief discussion of the mathematics related to or implied by such responses. However, it is important to note that these examples are provided to give you a feel for what might come up — not what should come up. No activity ever looks exactly the same twice because student responses and student needs are never identical. We have found this feature to be what keeps the lessons alive, challenging, and fascinating for us, no matter how many times we explore them with students.

Implementing the instructional roles suggested by documents such as the Standards or a curriculum such as *Visual Mathematics* can create tensions for teachers, particularly for those of us whose only models of instruction focused on the teacher as the meaning-maker and sole source of authority in the classroom. Adopting an investigative spirit about teaching mathematics — a willingness to explore with the students — implies not always having answers. Honoring the constructivist view of learning implies letting students' mathematical thinking determine the direction of an activity, and hence, this means not always knowing the direction or amount of time a lesson will take. Engaging students in group work, hands-on activities, discussions, and debate implies a noisy, active classroom. Subscribing to the belief that within every student exists an accessible and capable mathematician implies allowing students to struggle with ideas and resolve their own questions. Students, parents, and colleagues accustomed to a traditional approach may resist and suggest a teacher's job is to give answers. When faced with such tensions, perhaps the most heartening thought to keep in mind is that disequilibrium is a sign of new learning for us, too! Creating meaningful changes in our practice takes time. (See Chapter 7 for a discussion of ways other teachers have found to support themselves and each other as they make changes in their teaching practice.)