Teachers Guide
GRADE 4 – UNIT 1 – MODULE 1
Module 1
Models for Multiplication & Division

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Teacher Masters
Pages renumber with each module.
School Supplies................................................................. T1
Number Line Puzzle 1 ............................................................ T2
Unit 1 Pre-Assessment .......................................................... T3
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Crayons & Story Problems...................................................... T7
Broken Lid Crayons ................................................................. T8
Camping Story Problems Forum Planner ......................... T9

Student Book Pages
Page numbers correspond to those in the consumable books.
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How Many Pencils? ............................................................... 2
How Many Pens? ................................................................. 3
How Many Erasers? ............................................................. 4
Claudia’s School Supplies .................................................... 5
Crayons & Story Problems Sheet .......................................... 6
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Broken Lid Crayons Sheet ..................................................... 8
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Number Line Puzzles .......................................................... 1
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Modeling Multiplication & Division ...................................... 5
Module 1
Models for Multiplication & Division

Overview
In Module 1, the teacher works to set the tone for the year and to build community within the class. Students brainstorm what the class should look like and sound like in Session 1, extending and refining their list as the module continues. Also in Session 1, the teacher launches a multi-day investigation in which students determine the number of school supplies collected in a fictional fourth grade classroom. This context allows students to review and extend their understanding of multiplication strategies, concepts, and models including open number lines, ratio tables, and arrays. Students take the Unit 1 Pre-Assessment in Session 3 and reflect on their work in Session 4. They participate in the first math forum of the year in Session 6, sharing their work on a variety of story problems from Session 5.

Planner

<table>
<thead>
<tr>
<th>Session</th>
<th>P&amp;I</th>
<th>PS</th>
<th>MF</th>
<th>WP</th>
<th>A</th>
<th>HC</th>
<th>DP</th>
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</thead>
<tbody>
<tr>
<td><strong>Session 1</strong> Setting Our Course for the Year</td>
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<tr>
<td>Session 1 opens with a discussion about how the classroom should look and sound during math period to ensure that everyone has a chance to be successful. Students’ ideas are recorded on a chart to set the tone for the day’s work. Students listen to a story that launches them into an investigation to determine quantities of school supplies to be used in a fictional fourth grade classroom. They solve problems in pairs and share their strategies with the class. To close the session, the class makes a poster about their Community of Learners to refer to for the rest of the year.</td>
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<td><strong>Session 2</strong> Number Lines</td>
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<tr>
<td>This session opens with the first problem string of the year. Students set up their student journals and work through the string to review using an open number line to represent multiplication problems. Then the investigation into Mrs. Carter’s school supplies continues as they use a variety of strategies, including the problem string, to find quantities of supplies in Mrs. Carter’s class. Students examine a Number Line Puzzle and apply the strategies they are practicing.</td>
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<td><strong>Session 3</strong> Unit 1 Pre-Assessment</td>
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<td>This session begins with a short problem string in which students investigate ratio tables. Then students complete the Unit 1 Pre-Assessment. They reflect on the assessment during the next session. This session ends with a brief exploration of the math manipulatives and materials students will use throughout the year.</td>
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<td><strong>Session 4</strong> Methods &amp; Models for Multiplication</td>
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<tr>
<td>After reviewing the pre-assessment given in Session 3, students solve several multiplication story problems and the teacher models their strategies using open number lines, ratio tables, and rectangular arrays. In the back of their math journals, students begin a handbook involving a collection of mathematics vocabulary and strategies to use throughout the year.</td>
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<td><strong>Session 5</strong> Models for Division</td>
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<td>Students investigate the relationship between multiplication and division using open number lines, ratio tables and the area model. After students collaborate to connect the three models to division, they begin a set of multiplication and division story problems.</td>
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<td><strong>Session 6</strong> Math Forum on Multiplication &amp; Division</td>
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<td>Students finish the Camping Story Problems Student Book page from Session 5. Then the teacher leads the first math forum, a purposeful classroom discussion of students’ strategies. Today’s forum focuses on grouping and sharing problems, using the array model to strengthen understanding. The teacher asks intentionally selected students to share their strategies for solving two of the Camping Story Problems. Throughout the forum, the teacher guides students’ understanding of the strategies and how the strategies relate to each other.</td>
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**Materials Preparation**

Each session includes a complete list of the materials you’ll need to conduct the session, as well as notes about any preparation you’ll need to do in advance. If you would like to prepare materials ahead of time for the entire module, you can use this to-do list.

<table>
<thead>
<tr>
<th>Task</th>
<th>Done</th>
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</thead>
<tbody>
<tr>
<td><strong>Copies</strong></td>
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<tr>
<td>Run copies of Teacher Masters T1–T9 according to the instructions at the top of each master.</td>
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<tr>
<td>Run a single display copy of Student Book pages 1 and 3.</td>
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<tr>
<td>If students do not have their own Student Books, run a class set of Student Book pages 1–13.</td>
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<tr>
<td>If students do not have their own Home Connections books, run a class set of the assignments for this module using pages 1–6 in the Home Connections Book.</td>
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<tr>
<td><strong>Charts</strong></td>
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<tr>
<td>Prior to Session 1, create two charts for recording students’ thinking. On the first, make a T-chart with one column labeled <em>Looks Like</em> and the other column labeled <em>Sounds Like</em>. On the second, write <em>Community of Learners</em> at the top</td>
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<tr>
<td><strong>Special Items</strong></td>
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<tr>
<td>Prepare Student Books for use by writing students’ names on them.</td>
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<td>Write each student’s name on a journal.</td>
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<tr>
<td>Have your manipulatives organized and ready for students to preview.</td>
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</table>

**Additional Resources**

Please see this module’s Resources section of the Bridges Educator site for a collection of resources you can use with students to supplement your instruction.
Session 1
Setting Our Course for the Year

Summary
Session 1 opens with a discussion about how the classroom should look and sound during math period to ensure that everyone has a chance to be successful. Students’ ideas are recorded on a chart to set the tone for the day’s work. Then the teacher tells a story to launch an investigation in which students determine quantities of school supplies to be used in a fictional fourth grade classroom. They solve problems in pairs and share their strategies with the class. To close the session, the class makes a poster about their Community of Learners to refer to for the rest of the year.

Skills & Concepts
• Interpret products of whole numbers (3.OA.1)
• Make a comparison statement to match a multiplication equation (4.OA.1)
• Solve story problems involving a multiplicative comparison using multiplication or division (4.OA.2)
• Make sense of problems and persevere in solving them (4.MP.1)
• Construct viable arguments and critique the reasoning of others (4.MP.3)

Materials

<table>
<thead>
<tr>
<th>Problems &amp; Investigations</th>
<th>Kit Materials</th>
<th>Classroom Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Setting Our Course for the Year</strong></td>
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<tr>
<td>TM T1 School Supplies</td>
<td></td>
<td>• 2 pieces of chart paper taped to the wall in an easily accessible location</td>
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<tr>
<td>SB 1* More School Supplies</td>
<td></td>
<td>• marker</td>
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<tr>
<td><strong>Daily Practice</strong></td>
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<tr>
<td>SB 2 How Many Pencils?</td>
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</table>

* Run 1 copy of this page for display.

Preparation
• Prepare Student Books for use by writing students’ names on them.
• Create two charts for recording students’ thinking. On the first, make a T-chart with one column labeled *Looks Like* and the other column labeled *Sounds Like*. On the second, write *Community of Learners* at the top.
Problems & Investigations

Setting Our Course for the Year

1. Gather students in the discussion area and explain that they will be working together this year to learn mathematics. Ask them to think about what the room should look like and sound like when they are learning math.

   *Teacher:* A community is a group of people who share something in common. So when you all work on math in school, you are a community of math learners. You might not have thought of this, but all of you are mathematicians. A mathematician is someone who thinks about math, talks about math, asks math questions, and solves math problems. Let’s take a moment to think about what will help you do well as a mathematician this year. What should our room sound like and look like while we are doing math?

2. Give students a minute or two to think and then invite students to share their ideas, one at a time. Record students’ suggestions on the prepared T-chart.

   *ELL:* Help ELL students understand what you are saying by using gestures, writing key words where everyone can see, and if possible, finding a student who is proficient in both languages to help translate. Your goal is to help all students become better mathematicians, which is more likely to happen if students feel safe, comfortable, and respected. You will begin building a community of learners during this discussion as you model listening and responding to students respectfully and acknowledge students who are setting positive examples. Encourage students to give detailed descriptions as they share. Ask students to compare the way the classroom will sound during an active lesson, such as when everyone is playing a game, and the way it will sound during a whole-class lesson or independent work time when people are thinking, listening, and sharing ideas.

   *Teacher:* Let’s hear what you think the room should look like and sound like so everyone can be a mathematician this year. Raise your hand when you have an idea to share.

   *Kendra:* Everyone should be working.

   *Teacher:* What does “working” look like?

   *Kendra:* Well, I guess it means people at their seats, solving problems.

   *Jamal:* Last year in math we played a lot of games, too, though. We didn’t just solve problems.

   *Isabel:* And, after we solved problems, we talked about them a lot.

   *Teacher:* So, being a mathematician means doing many different things—solving problems, playing games, and talking about math. OK, but what does solving problems look like?

   *Kendra:* Sometimes we use stuff like tiles and cubes to help us solve problems, but sometimes we just use pencil and paper. It helps me to talk with a partner, too.

   *Teacher:* I’ll write your ideas down here. For Looks Like, we can say “using tools to solve problems,” “working with a partner,” and “playing games.” For Sounds Like, we can say “talking about math problems.”
Give students a few moments to think about how they can help the classroom look and sound this way during math. Then have them turn and talk to a partner to share their ideas.

*Students will frequently turn and talk to a partner during mathematics lessons this year. Taking time to establish expectations and routines during the first lessons will help students learn to use this time productively. See the Implementation section of the Bridges Educator site for more information on establishing expectations and routines.*

Tell students they will have a chance to practice their ideas today during their first math investigation, and then introduce the investigation by sharing a multiplication story about a fictional fourth grade classroom.

*My friend, Mrs. Carter, teaches fourth grade. Every fall, Mrs. Carter has her students bring in school supplies for the year. They bring in boxes of pencils, pens, sets of markers and crayons, folders, glue sticks, notebook paper, and boxes of tissues. Then instead of people using just the supplies they brought, everybody shares everything all year long. Can you imagine the pile of school supplies in their classroom? They have over 30 students in the class, so that means more than 30 boxes of pencils and over 30 sets of crayons. Everyone brings in 5 folders so that’s 30-plus sets of 5 folders. That’s a lot of supplies! One of the first things they do when school begins is count and organize all of their supplies for the year.*

Display the School Supplies Teacher Master, revealing only the top half of the page.

- Give students a few moments to study the instructions at the top of the sheet.
- Then read the instructions aloud and invite a student to read problem 1a to the class.

*Lily* There are 32 students in Mrs. Carter’s class. Each student brought in 2 glue sticks. How many glue sticks is that in all?

Ask students to think about the problem privately for a moment, and then have them turn and talk with a partner to generate a solution.

- Have students put their thumb up in front of their chest when they have an answer.
- When most students are ready, invite several volunteers to share their answers.
- Record any and all responses, including answers that are incorrect, without comment.

Invite several students to explain how they got their answers. Record their strategies where everyone can see them.
As students share, look for opportunities to emphasize community building. For example, give positive feedback on respectful listening, note partnerships who worked well together and so on. If students have difficulty working together, ask how they might do something differently next time to build respect and collaboration.

**Chin** We think it’s 64. We did 32 plus 32 because all 32 students brought in 2 glue sticks, and that is 2 groups of 32.

**Rico** We doubled 32 to get 64 for the same reason.

**Claire** We started by counting by 2s, 32 times, but that was taking too long. Then we figured out that we could count by 32s instead and that was 64.

**Teacher** You realized that your strategy wasn’t very efficient and you changed your thinking. That’s really working like a mathematician!

8 Work with input from students to record one of the equations they proposed (or a different one) and the answer labeled with the correct units, on the lines provided.

Take the opportunity to remind students how important it is to show the solution clearly when they’re solving problems, and to label the answer with the correct units—glue sticks, in this case.

9 Invite a different student to read problem 1b aloud. Give pairs a few moments to talk over their ideas, and then call on several volunteers to share and explain their answers.

**Nia** If every kid brought 2 glue sticks, there must be twice as many glue sticks as kids.

**Dustin** There are 64 glue sticks, because we know 32 and 32 is 64.

**Tomas** If you double 32, it’s 64.

10 Confirm students’ thinking by filling in the answer to problem 1b, and explaining that there are, indeed, twice or 2 times as many glue sticks as students in Mrs. Carter’s room.

Then write the equation 64 = 2 × 32 in the space below the problem. Ask students to read the equation aloud and show thumbs up if they agree that it’s true.
Students 64 equals 2 times 32.
Yep, that’s right, because 32 and 32 is 64, and that’s the same as $2 \times 32$.

Remind students that this equation might also be read as 64 is the same as 2 groups of 32. Then read the equation to the class as a statement of multiplicative comparison (64 is 2 times as many as 32), and ask students to comment.

Teacher OK! I’ll bet some of you learned in third grade that this equation can be read as “Sixty-four is the same as 2 groups of 32.” But there’s another way to read and think about multiplication equations—one that we’re going to be talking about a lot this year. Listen as I read the equation to you again, and show thumbs up if you agree. Ready? Sixty-four is 2 times as many as 32. I see lots of thumbs up. Who’d like to explain why they agree?

Arthur Because it’s true. Sixty-four is twice as much as 32.

Amira Like, if you had 32 of something, and I had 64, I could say I had 2 times as many as you.

Teacher Does it work the other way? Is 64 also 32 times as many as 2?

Ryan Yes! That’s why we stopped counting by 2s, because we would have had to go 32 times to get to 64.

Don’t worry if some students seem puzzled or confused at this point. Thinking of multiplication as a form of comparison (how many times as much or how many times as many) may be a new idea for most; one that involves new concepts and vocabulary. Multiplicative comparisons will be revisited informally several times over the next few sessions and formally addressed in Module 3 of this unit. They will also be addressed throughout the year in Number Corner.

Reveal the second problem and invite a student to read it aloud. Ask students how many sets of 8 markers there are, [4, because there are 4 students at a table] Write the equation $4 \times 8 = m$ and talk about it briefly.

- Remind students that the multiplication symbol (x) that we read aloud as “times” can mean “groups of” or “sets of.”
- Emphasize that students are finding out how many markers there are in 4 groups of 8.

Ask students to think about the problem privately for a few moments and then have them turn and talk with a partner to generate a solution.

- Have them put their thumb up in front of their chest when they have an answer.
- When most students are ready, invite several volunteers to share their answers.
- Record any and all responses, including answers that are incorrect, without comment.

Invite several students to share how they solved this problem. As they share, write an equation for each new strategy and sketch models when it’s appropriate. Students will explore various models in future sessions, including number lines, ratio tables, and arrays. They will have access to physical models later, as well. You do not need to take time during this lesson for lengthy explanations of models. Instead, use the opportunity to refresh their memories of familiar models and show effective ways to communicate mathematical thinking. For example, if students skip-count to find the product, you can record their strategy on a number line.

Invite one student to read problem 2b aloud. Give pairs a few moments to share ideas, and then call on several volunteers to share and explain their answers.
Eli If every kid at that table brought 8 markers, I think there are 8 times as many markers as kids.

Owen I kind of don’t get it.

Dolores I think it’s right because 32 is 8 times bigger than 4.

16 Confirm students’ thinking by filling in the answer to problem 2b, and explaining that there are, indeed, 8 times as many markers as students at table G.

Then write the equation $32 = 8 \times 4$ in the space below the problem, and read it as a multiplicative comparison (32 is 8 times as many as 4). Have students read it again with you as you write it out in word form.

17 Display a copy of the More School Supplies Student Book page, and give students a minute to read it. Explain that they will work with a partner to solve the problems on this sheet in a few moments.

- Review what the room should look and sound like as students work.
- Introduce Student Books, ask if there are any questions, and then have them begin work with their partners.

**ELL** Confirm that ELL students understand the directions. If possible, have a student explain the directions, or work with ELL students once the class begins the assignment.

- As students work, walk around the room to make observations, listen to their conversations, and offer support as needed.

18 When there are about ten minutes left in class, have students join you in the discussion area.

19 Ask a few students to describe how the room looked and sounded as they worked. Emphasize what went well and, if necessary, ask them what could be improved.

20 Post the second piece of chart paper titled “A Community of Learners” near the T-chart where everyone can see it.

- Ask the class what they need to do to make a classroom community where everyone can learn, where everyone can make mistakes and ask for help, and everyone can feel comfortable.
- Give students a minute to think about the question and then call on several students.
- Record their ideas on chart paper.

*If some of the ideas are nearly the same, record them as a single idea on the chart. While you want to make sure students understand what makes a community of learners, it is also helpful to keep the list relatively short. You might want to make revisions to the chart as you go and copy a final draft after the lesson ends.*

Nora We should listen when someone else is talking.
Teacher  Why does it help us to listen to other students' ideas about math?

Pedro  Sometimes we have a way that works, but somebody else has a way that is easier or faster.

Nate  I don't like it if someone makes fun of me when I get something wrong.

Teacher  So you're saying if someone gets mixed up it's important not to laugh or make fun of them. Do you suppose that asking questions and really trying to understand each other's thinking would help?

Students  Yes!

Teacher  I'll add these ideas to our chart. I think you're saying that if people were watching our classroom, they could see people being respectful to others, listening, and asking questions to help understand. Is that right?

21 Ask students to study the chart and think about what they can do to help make this classroom a community of learners. Tell them they will have opportunities to practice building their community over the next few days.

Keep the Community of Learners chart on display to refer to throughout the year, if possible. Learning to work together is an ongoing process and the chart will be more effective if the class revisits it from time to time. Helpful ideas may be added or original ideas may be revised.

22 Close the session by recognizing students for starting off the year in math on a positive note.

Daily Practice

The optional How Many Pencils? Student Book page provides additional opportunities to apply the following skills:

- Solve one-step story problems using multiplication (supports 3.OA)
- Fluently multiply with products to 100 using strategies (3.OA.7)
- Use strategies based on place value, properties of operations, or the relationship between addition and subtraction to add fluently with sums to 1,000 (3.NBT.2)
Session 2
Number Lines

Summary
This session opens with the first problem string of the year. Students set up their student journals and work through the string to review using an open number line to represent multiplication problems. Then the investigation into Mrs. Carter's school supplies continues as students use a variety of strategies, including the problem string, to find quantities of supplies in Mrs. Carter's class. Students examine a Number Line Puzzle and apply the strategies they are practicing. Finally the teacher introduces and assigns the Number Line Puzzles Home Connection.

Skills & Concepts
• Interpret products of whole numbers (3.OA.1)
• Solve multiplication story problems with products to 100 involving situations of equal groups (3.OA.3)
• Solve for the unknown in a multiplication equation involving 3 whole numbers (3.OA.4)
• Fluently multiply with products to 100 using strategies (3.OA.7)
• Solve story problems involving a multiplicative comparison using multiplication or division (4.OA.2)
• Reason abstractly and quantitatively (4.MP.2)
• Construct viable arguments and critique the reasoning of others (4.MP.3)
• Look for and make use of structure (4.MP.7)

Materials

<table>
<thead>
<tr>
<th>Copies</th>
<th>Kit Materials</th>
<th>Classroom Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem String</td>
<td>Number Lines</td>
<td></td>
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<tr>
<td>TM T2</td>
<td></td>
<td>· Looks Like/Sounds Like and Community of Learners charts from Session 1, posted on the wall</td>
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<tr>
<td>Number Line Puzzle 1</td>
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<td>· piece of scratch paper</td>
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<tr>
<td>SB 3*</td>
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<td>· student math journals</td>
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<td>How Many Pens?</td>
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<td>Home Connection</td>
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<td>HC 1–2</td>
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<td>Number Line Puzzles</td>
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<tr>
<td>Daily Practice</td>
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<td>SB 4</td>
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<td>How Many Erasers?</td>
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Preparation
• Before this session, see the Implementation section of the Bridges Educator site for more information on teaching problem strings.
• Write each student’s name on a journal. See the Implementation section of the Bridges Educator site for more information on student math journals.
• Find an area in your classroom for the problem string. You will need a place where everyone can see and you have plenty of space to write. This can be on a whiteboard, document camera or projector, or on chart paper. The lesson presumes you will do this in the discussion area to promote student discourse. If this is not possible in your classroom, you may have students remain at their seats or find an alternative space.
• Post the Looks Like/Sounds Like and Community of Learners charts from the previous session where everyone can see them.

Vocabulary
An asterisk [*] identifies those terms for which Word Resource Cards are available.
commutative property of multiplication*
efficient
product*
strategy
Problem String

Number Lines

1. Have students bring a pencil and join you in the discussion area.

2. Show students where you posted the Looks Like/Sounds Like and Community of Learners charts from the first session, and give them a moment to review them.

Ask students to think privately about one or two ideas they believe they personally modeled well yesterday.

3. Introduce problems strings and student journals.

Tell students:
- A problem string is a series of problems that students will solve and discuss one at a time.
- Strings often start out with an easier problem, and then the problems get harder as the string continues.
- The problems at the beginning of the string often help solve the problems toward the end of the string.
- Solving the problems in a string involves thinking like a mathematician because students want to find clever and efficient ways to solve the problem. Efficient strategies are quick and can be explained clearly.
- During a problem string, the students will solve each problem, share strategies and answers, and discuss each other’s thinking.
- Students will do their work in their journals. When they talk about their work, the teacher will usually represent their work for everyone to see.

You may want to invite students who have participated in problems strings before to comment on what they remember to help other students get more of a sense of how strings go. You can also assure students that what you are explaining will make much more sense when they are doing a string.

4. Tell students that Mrs. Carter needs some more help. Ask students to work with the person next to them to figure out how many rolls of tape are in Mrs. Carter’s classroom if each of her 32 students brought in three rolls of tape.

While students are discussing the problem, pass out student journals.

5. Model writing “Number Lines Problem String” and the date as students do the same on the first page of their journals.

- Discuss how students will use their journals this year.
- Emphasize that each time students work in their journals, they will need to include the date and a heading that describes their work.

   Teacher: This year, you’re going to fill your journals with treasures—ideas, strategies, information, new words, and more. Your journal will be a tool that will help you learn. So please take care of your journals and keep them neat and organized. Every time you write in your journal, you need to write the date and a heading. This way you and I can go back and find your work easily.

6. Establish the context for the problem string and write the first problem,

   \[ 4 \times 2 = p. \]
Deliver the string shown in the chart. After the chart, there is a sample progression of dialog that illustrates how the first set of problems in this string might play out in a classroom.

- Pose each problem one at a time by writing it on the board, and give students time to work (see sample dialog that follows).
- After students have had adequate time to record and solve the problem, ask the class for the answer, and then invite two or three students to explain how they solved the problem.
- Represent the strategies on an open number line to show students’ thinking.
- Focus on choosing strategies that employ doubling and partial products strategies.

**CHALLENGE** Encourage students to use the most efficient or sophisticated strategy they can think of. Then encourage them to look back at their work and see if they can see an even more efficient strategy that they could have used.

### Problem String  Number Lines

<table>
<thead>
<tr>
<th>Problems</th>
<th>Sample Strategies &amp; Recording</th>
<th>Connections</th>
</tr>
</thead>
</table>
| $4 \times 2 = p$ | ![Number Line](4x2.png) | $4 + 4 = 8$
| | | $4 \times 2 = 8$
| $8 \times 2 = p$ | ![Number Line](8x2.png) | $8 + 8 = 16$
| | | $8 \times 2 = 16$
| | | $4 + 4 + 4 + 4 = 16$
| $12 \times 2 = p$ | ![Number Line](12x2.png) | Students might combine the answers to the first 2 problems to get the answer to the 3rd problem (i.e., using partial products: $(4 \times 2) + (8 \times 2) = 12 \times 2$)
| $16 \times 2 = p$ | ![Number Line](16x2.png) | Students may use partial products as they notice earlier problems combine to solve later problems (e.g., $(8 \times 2) + (16 \times 2) = 24 \times 2$). They might find doubling more efficient for $16 \times 2$ and $32 \times 2$.
| $24 \times 2 = p$ | ![Number Line](24x2.png) | |
| $32 \times 2 = p$ | ![Number Line](32x2.png) | |

### Sample Dialog

**Teacher** I talked to my friend, Mrs. Carter, yesterday. She had a great first day of school, and so did I. Her only problem was that she was feeling really overwhelmed by all of her classroom supplies! She was really excited to hear that you’ve been figuring out how many of each supply her class has. She was wondering how many colored pencils they have, since she asked each student to bring in 2 packs of colored pencils. So, if there are 4 students at one table, how many sets of colored pencils does that table have?
Hold up 2 packs of colored pencils to help ELL students understand the context.

Sara: I got 8. I did 4 plus 4, and that’s 8.

Teacher: Thank you. Usually I want you to share your thinking, but when we’re doing a problem string, I’m just going to ask for the answer first. Did anyone get an answer that was different than 8? OK, now, who can tell me how you got 8?

Raul: I just knew it.

Teacher: I bet a lot of people “just knew it.” Put your thumb up if you just knew 4 × 2. The first problem is one you may have memorized, and it’s OK to say you just know it. What if you didn’t know it? How could you figure it out?

Willie: I doubled 4 to get 8.

Sara: I added 4 and 4 to get 8.

Teacher: Let me put your thinking on a number line. OK, here’s a jump to 4 and another jump to 8. That shows both 4 doubled and 4 plus 4.

4 x 2 = 8
4 + 4 = 8

Teacher: Tell us how you solved 12 × 2.

Keiko: I just doubled 12 to get 24.

Teacher: OK, I’m going to show that on the number line like this. Who solved it differently?

12 x 2 = 24

Eric: I noticed a pattern and that helped me. The number of students is going up by 4 and the answer is going up by 8. So, I added 8 to the last answer and that gave me 24.

Teacher: Hmm. That’s interesting, but I think I’m a little confused. Who can explain Eric’s thinking to me?

Pedro: I think I can explain. We keep having more students, right? So, every time we add 4 more students, we’re really adding 8 more sets of pencils because each student brought in 2 sets. The number on the left goes up by 4 and the answer goes up by 8.

Teacher: Let me add that to our first number line. Does anyone still have a question about that? Does anyone have another strategy?
Monica: Well, mine is sort of like the last one, but different. I noticed that the first two problems equal the third problem. The 4 sets of 2 plus the 8 sets of 2 is 12 sets of 2.

8 Conclude this part of the lesson by asking students to summarize the steps of a problem string. 

   Help students fill in any missing steps.
   • Students solve a problem independently in journals.
   • Some students share their answers.
   • Some of those students share how they got the answer.
   • The class thinks about the each student’s strategy and asks questions to make sure they understand.
   • Sometimes students summarize the strategy with a partner.
   • Students think about how to use the strategy to solve a different problem.

9 Use the Looks Like/Sounds Like chart to discuss how the classroom should look and sound during a problem string.

10 Have students return to their seats. Introduce the How Many Pens? Student Book page.
   • Cover the bottom of the page so students only see the directions and the first problem.
   • Give them a moment to read the page silently and then invite a student to read them aloud.
   • Have students think quietly for a moment about the problem and then ask them to turn and talk to a partner about how they would solve it.
   • Invite several students to share their thinking. Elicit or suggest the idea of using a number line.
   • Ask students if they have any questions.
   • Tell them they will solve the remaining problems independently and talk with them about how the classroom should look and sound.

11 As students share, walk around the room to look for students who are using efficient strategies, identify areas of concern, and provide support.

   SUPPORT: Help students set up a number line on their papers and walk them through how to use it. Encourage them to look for connections between problems that might help with larger numbers later in the string.

   CHALLENGE: Encourage students to analyze the numbers to find connections and relationships. For example, ask them if they see a relationship between the numbers in problems 1, 2, and 3. Challenge them by giving them connected problems with bigger numbers. For example, build on $7 \times 4$ with $7 \times 8$, $7 \times 12$, and $7 \times 16$.

   When students finish the assignment, have them share their strategies and solutions with a partner.

12 After most students have completed the task, display the Number Line Puzzle 1 Teacher Master so everyone can see it.
• Ask students what they notice about Number Line Puzzle 1, and invite several to share their observations.

• Ask students how they can figure out the missing information on the number line. Use questions like the following:
  » How do you know what numbers to use?
  » What strategy are you using?
  » Do you see any relationships between the numbers that might help you?

• Fill in the numbers as they share.

13 When the Number Line Puzzle is complete, close the session by summarizing the key points of today’s lesson.

  **Teacher**  We did a lot of work today! You learned the process for doing problem strings and finished the first one of the year. We will see a lot of these this year, so it is important that you know how they work. You showed different strategies and asked each other questions in the problem string that showed you’re listening to and learning from each other. Then you put your multiplication strategies to work and solved a Number Line Puzzle. We’ll do more of these this year, too. You are off to a great start in math this year!

**Home Connection**

14 Introduce and assign the Number Line Puzzles Home Connection, which provides more practice with the following skills:

• Interpret products of whole numbers (3.OA.1)
• Solve for the unknown in a multiplication equation involving 3 whole numbers (3.OA.4)
• Fluently multiply with products to 100 using strategies (3.OA.7)
• Make a comparison statement to match a multiplication equation (4.OA.1)
• Solve multi-step story problems involving only whole numbers, using addition and multiplication (4.OA.3)

**Daily Practice**

The optional How Many Erasers? Student Book page provides additional opportunities to apply the following skills:

• Solve one-step story problems using multiplication (supports 3.OA)
• Solve for the unknown in a multiplication equation involving 3 whole numbers (3.OA.4)
• Fluently multiply with products to 100 using strategies (3.OA.7)
• Solve story problems involving a multiplicative comparison using multiplication or division (4.OA.2)
Session 3

Unit 1 Pre-Assessment

Summary
This session begins with a short problem string in which students investigate ratio tables. Then students complete the Unit 1 Pre-Assessment. They reflect on the assessment during the next session. This session ends with a brief exploration of the math manipulatives and materials students will use throughout the year.

Skills & Concepts
• Solve division problems by finding an unknown factor (3.OA.6)
• Fluently multiply and divide with products and dividends to 100 using strategies (3.OA.7)
• Multiply whole numbers from 1–9 by multiples of 10 from 10–90 using strategies based on place value and properties of operations (3.NBT.3)
• Identify a multiplication equation to match a comparison statement and vice versa (4.OA.1)
• Solve story problems involving a multiplicative comparison using multiplication or division (4.OA.2)
• Solve multi-step story problems involving only whole numbers, using addition and multiplication (4.OA.3)
• Find all factor pairs for a whole number between 1 and 100; demonstrate an understanding that a whole number is a multiple of each of its factors (4.OA.4)
• Determine whether a whole number between 1 and 100 is prime or composite (4.OA.4)
• Multiply a 2-digit whole number by a 1-digit whole number using strategies based on place value and the properties of operations (4.NBT.5)
• Make sense of problems and persevere in solving them (4.MP.1)
• Reason abstractly and quantitatively (4.MP.2)

Materials

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<td>SB 5</td>
<td>Claudia’s School Supplies</td>
<td></td>
</tr>
</tbody>
</table>

Vocabulary
An asterisk [*] identifies those terms for which Word Resource Cards are available.

ratio table*  
factor*  
product*

Preparation
• Review the Unit 1 Pre-Assessment and read about assessment in the Assessment Guide.
• Note that you will need to evaluate the pre-assessment before the next session (Session 4). If you cannot evaluate the pre-assessment before the next session, do so before the end of this module, and make time for students to reflect on it as described in Session 4.
• Have your manipulatives organized and ready for students to preview. See the Implementation section of the Bridges Educator site for more information on organizing materials.
Unit 1  Module 1  Session 3

Problem String

Ratio Table

1. Gather students in the discussion area with their journals and a pencil to introduce this session’s activities. Tell students that first they will work through a short problem string and then they will take the Unit 1 Pre-Assessment.

2. Ask students to think silently for a moment about how they would describe a problem string, and invite a few students to share their thoughts with the class.

3. Then have students complete the following problem string.
   - Ask students to write today’s date and the title “Ratio Table Problem String” on a fresh page in their math journals.
   - Present each problem one at a time by writing it on the board. Present the problems in order from top to bottom.
   - After writing a problem on the board, verbally present it as a story problem (see sample dialog in the Resources section of the Bridges Educator site).

   **Teacher**  My friend Mrs. Carter says she and her class are making good progress in figuring out how many of each school supply they have, and they thank you for your help. One of the supplies we haven’t thought about yet are folders. Mrs. Carter asked each student to bring in 5 folders, one for each academic subject. So, if there are 4 students at 1 table, how many folders does that table need?

   **ELL**  Hold up 5 folders to help ELL students understand the context.
   - Give students time to solve each problem on their own.
   - Invite 2 or 3 students to share their strategies for solving each problem, one at a time, and record their strategies in a ratio table.
   - A ratio table can be set up like a vertical T-chart with columns of numbers or as a horizontal table with two rows of numbers, as shown in the chart. During the problem string, introduce students to both ways of setting up ratio tables.
   - For the final string, write 33 × 5 = f and explain that on the third day of school, Mrs. Carter got a new student, and now she has 33 students in her class. Ask how many folders are in the classroom now.

   **CHALLENGE**  Encourage students to use the most efficient or sophisticated strategy they can think of. Then encourage them to look back at their work and see if they can think of an even more efficient strategy.
**Problem String** Ratio Table

<table>
<thead>
<tr>
<th>Problems</th>
<th>Sample Strategies &amp; Recording</th>
<th>Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4 \times 5 = f$</td>
<td>Students</td>
<td>Folders</td>
</tr>
<tr>
<td>$8 \times 5 = f$</td>
<td>Students</td>
<td>Folders</td>
</tr>
<tr>
<td>$16 \times 5 = f$</td>
<td>Students</td>
<td>Folders</td>
</tr>
<tr>
<td>$32 \times 5 = f$</td>
<td>Students</td>
<td>Folders</td>
</tr>
<tr>
<td>$33 \times 5 = f$</td>
<td>Students</td>
<td>Folders</td>
</tr>
</tbody>
</table>

In these problems, students use ratio tables to help see what happens when one of the factors in a multiplication problem doubles. Use the words factor and product as you continue the string. Remind students that the product is the answer to a multiplication problem, and factors are the numbers that are multiplied.

Draw students’ attention to the relationship between the 4 and the 8 and the 20 and the 40: when the number of students doubles from 4 to 8, the number of folders doubles from 20 to 40. (i.e., If you have twice as many students, you need twice as many folders).

**Big Idea**
When one factor in a multiplication problem doubles, the product also doubles.

In this problem, the number of students is not doubled. Instead, one more student is added, so that instead of 32 students, we are finding the number of folders 33 students will need. When we add 1 more student to the group ($32 + 1 = 33$), we add the 5 folders that student needs to the total ($160 + 5 = 165$).

**Big Idea**
As long as the ratio remains constant (5 folders for every 1 student), the same ratio table can be used to model strategies for finding the total number of folders required for any number of students.

**SUPPORT**
To support students’ understanding of what is shown in the ratio table, use words and equations as needed to elaborate on their strategies. For example, beside the first entry on the ratio table, you might write: 4 students x 5 folders per student = 20 folders.

To make the application of the associative property more explicit in the second problem, for example, you might write: $8 \times 5 = (2 \times 4) \times 5 = 2 \times (4 \times 5) = 2 \times 20 = 40$. By expressing the 8 as 4 doubled and then applying the associative property, you make it clear why the properties of multiplication make it possible to apply this doubling strategy.

Referring back to the context of the problem helps too: if we double the number of students, we must also double the number of folders.

4. Have students look over their entries in their math journals and make any additions or corrections. Ask them to generalize what they have learned during the string, emphasizing how ratio tables help illustrate the relationship between the problems.

5. Then have them return to their seats and put their journals away.

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Bridges in Mathematics Grade 4 Teachers Guide

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Assessment

Unit 1 Pre-Assessment

When students are seated, introduce the Unit 1 Pre-Assessment.

- Let students know that the pre-assessment will help you and them see how familiar they are with the concepts and skills that will be taught in this unit.
- Remind them there may be problems they can’t solve because they haven’t learned how to do them yet. Assure them you are going to work on everything on the assessment in the coming weeks. For now, they may write, “I don’t know yet,” if they are unable to solve a problem.
- Tell students to raise their hands if they would like you to read a problem aloud to them. If you have a number of students who will need problems read aloud, invite them to sit with you in a small group so you can read for them.
- Have a helper place a container of colored tiles on each table or near each cluster of desks and let students know that if they want, they can use the tiles to help solve some of the problems on the assessment.
- Let them know that they have 25–30 minutes, but the test is not timed and they may have additional time later if they need it.
- Ask students who finish ahead of others to check their work carefully and then read quietly to themselves until you call time.

Display the first page of the Unit 1 Pre-Assessment Teacher Master as helpers give a copy of the assessment to each student.

- Have students write their names and the date on the first page.
- Remind students to wait to begin working on the assessment.

Discuss strategies students can use that will help them during an assessment.

Some students might feel nervous before taking an assessment, while others might look forward to working independently and showing their understanding. Showing strategies students can use to approach an assessment may be helpful to everyone.

Model the following strategies as you describe them.

- Read the whole assessment before you begin to get a sense of what you need to do.
- Notice which problems might be easier or more difficult for you. You might put a small star by easier problems and a question mark by more challenging ones.
- Think about how to use your time during the assessment so that you have time and energy to finish all the problems.

Ask students what the room should look like and sound like as they work on the assessment and then have them begin work.

SUPPORT If you notice students who are consistently answering only the first part of a two-step problem, teach them to circle the word “and” as a reminder.

ELL Read the questions aloud to students and help them identify what the questions are asking them to do.

Collect the pre-assessments when students finish. Let them know they will set their own learning goals for the unit when they look over their assessments during the next session.
Then take a few minutes to introduce students to the math manipulatives they will use this year.

- If you have prepared toolkits, pass out one toolkit to every two students. Give them a few minutes to explore the materials and examine how they are organized.
- If you are not using toolkits, show and explain the organizational system you are using for manipulatives.
- Discuss how the manipulatives should look when students put them away, and ask students what they can do to help manipulatives stay organized this year.
- If you have time, invite students to spend a few minutes exploring the manipulatives. Then have them practice putting them away neatly.

Close the session with a few final challenges: $7 \times \____ = 28$, $\____ \times 6 = 36$, and $7 \times 9 = \____$.

Write the equations one at a time. Give students a moment to think about each. Then invite a few students to explain their answer and how they found it.

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### Daily Practice

The optional Claudia's School Supplies Student Book page provides additional opportunities to apply the following skills:

- Solve multiplication story problems with products to 100 involving situations of equal groups (3.OA.3)
- Solve story problems involving a multiplicative comparison using multiplication or division (4.OA.2)
- Multiply a 2-digit whole number by a 1-digit whole number using strategies based on place value and the properties of operations (4.NBT.5)
Session 4
Methods & Models for Multiplication

Summary
After reviewing the pre-assessment given in Session 3, students solve several multiplication story problems and the teacher models their strategies using open number lines, ratio tables, and rectangular arrays. In the back of their student journals, students begin a handbook involving a collection of mathematics vocabulary and strategies which they will use throughout the year. At the end of the session, the teacher introduces and assigns the Models for Multiplication Home Connection.

Skills & Concepts
- Solve multiplication story problems with products to 100 involving situations of equal groups and arrays (3.OA.3)
- Fluently multiply with products to 100 using strategies (3.OA.7)
- Write a multiplication equation to represent a verbal statement of a multiplicative comparison (4.OA.1)
- Solve story problems involving a multiplicative comparison using multiplication or division (4.OA.2)
- Model with mathematics (4.MP.4)
- Use appropriate tools strategically (4.MP.5)

Materials

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<th>Classroom Materials</th>
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<tbody>
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<td>students’ scored Unit 1 Pre-Assessments</td>
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<td>Unit 1 Pre-Assessment Student Reflection Sheet</td>
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<td>SB 6</td>
<td>Crayons &amp; Story Problems Sheet</td>
<td>red linear pieces (class set, plus 1 set for display)</td>
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<td>Home Connection</td>
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<td>student math journals</td>
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<tr>
<td>SB 7</td>
<td>More Crayons</td>
<td></td>
</tr>
</tbody>
</table>

HC – Home Connection, SB – Student Book, TM – Teacher Master
Copy instructions are located at the top of each teacher master.

Vocabulary
An asterisk [*] identifies those terms for which Word Resource Cards are available.
area model of multiplication* array* model multiply* multiplication open number line ratio table* rectangular array tile array
Assessment

Reflecting on the Unit 1 Pre-Assessment

1. Let students know they will have a chance to reflect on the Unit 1 Pre-Assessment and set goals for Unit 1. Then, they will continue with the school supply investigation.

2. Hand students their scored Unit 1 Pre-Assessments and give them a minute or so to look over them.

Looking at the pre-assessment results can help students recognize the learning expectations for the unit, identify which skills and concepts they currently understand, and focus their efforts in the lessons to come.

- Review with students how they can use the results of the Unit 1 Pre-Assessment to help them throughout the rest of the unit.
- Encourage them to ask questions, but do not explain how to do problems at this time. Similar problems will be introduced throughout the unit.
  - Advise students to not be discouraged if their results were disappointing. They have several weeks to develop their skills, and they will take a similar assessment at the end of the unit.
  - Advise students to not be complacent if their results were excellent. The pre-assessment is just a quick snapshot to guide your teaching, and students will have opportunities to improve their mathematical understanding during the unit.

3. Then display a copy of the Unit 3 Pre-Assessment Student Reflection Teacher Master. Give each student a copy and work with them to fill it in.

- Go over the sheet, one row at a time, with the class.
- For each row, read the skill and make sure students understand it. Ask volunteers to explain, or use one of the associated items on the assessment to explain, the skill to the class.
- Have students look at the assessment item(s) associated with that particular skill, talk in pairs about how they did with the skill, and then mark their reflection sheets accordingly.

Teacher  The first question on the reflection sheet asks, “Can you name all the factors for a number?” What does that mean? Turn to your neighbor and explain what the question is asking.

Students  I think it means the different numbers you can multiply to get the number.

So for 12, I put 1 and 12 because you can multiply those together to get 12.

It said to write all the factors, so I put 1, 2, 3, 4, 6, and 12, because you can go 1 × 12, 2 × 6, and 3 × 4.

Teacher  Well, one way to think about factors is numbers that can be multiplied to get a product. As Derrick mentioned, 2 and 6 are factors of 12 because 2 × 6 is 12.

Sherwin  I don't think it’s a fair question. I don't even know what factors are.

Teacher  Remember that when you take a pre-assessment, there may be words you don't yet know or problems you can't yet solve. Please look over your pre-assessment carefully to see how you did with problem 1a. Then talk with the person next to you, and work together to decide how each of you should mark yourselves on that skill. After
you’ve talked, take a few moments to mark your own sheet and write
yourself a reminder note if you like.

Maria Jose I got both of the rows for 1a right—I remember learning
about factors last year, so I can put a check where it says, “I can do
this well already.”

Callie I didn’t even remember the word factor at all, so I have to
check where it says, “I need to learn to do this.”

When you and the students have finished working through all the skills
listed on the reflection sheet, have students draw a star next to the two
skills they feel they need to work on most in the next few weeks.

You might do a quick survey so you and the students can see which skills they and their
classmates identified as areas of particular need. Quickly read down the list of skills and
have students show thumbs up as you read the skills they marked.

Finally, give students a couple of minutes to describe in writing any other
goals, needs, requests, or questions at the bottom of the sheet.

When students are finished, collect the Unit 1 Pre-Assessment and Student Reflection
sheets. Staple them together and file them so that you can combine them with the Unit 1
Post-Assessment at the end of the unit. You might also use them partway through the unit to
discuss with individual students their progress in terms of the skills they needed to focus on.

Problems & Investigations

Methods & Models for Multiplication

Display the first box of crayons on the Crayons & Story Problems Teacher
Master and introduce the crayon scenario.

Teacher Sometimes students bring in supplies with different numbers
of items. Today we are going to look at some crayon boxes that students
brought in one year for Mrs. Carter’s class. How many crayons are in
this box? Talk with the person next to you to figure out the answer, and
be prepared to share the answer and explain how you got it.
When students have had a minute to share solutions and strategies in pairs, solicit and record the answer(s) without comment. Then explain that in a few moments, you will call on volunteers to explain how they got their answers.

- Let the students know that you will model each strategy they share using an open number line, a ratio table, a tile array, or an area model.
- List these four models on the board for students’ reference.
- Call on several students to share their thinking with the class. Select and draw (or build) the model that seems to best fit the strategy shared by each student (see chart below for examples).

### Student Strategies

**Skip-Counting**

*Kendra* It’s 24. I saw 3 rows of 8, so I just did skip-counting—8, 16, 24.

**Teacher Models**

**Open Number Line**

Teacher Seems to me that the open number line is a good way to model your skip-counting strategy, Kendra. I’ll start at 0, and take hops of 8, like this. What equation should I write to represent these hops?

*Kendra* Three times 8 is 24?

*Sasha* I think that’s right because you took 3 hops of 8, and you landed on 24.

### Repeated Addition

*Sasha* I said it was 24. I said, OK, there are 8 crayons in the first row, so 2 rows is 16 and 3 rows is 24.

**Ratio Table**

Teacher So, you added 8 crayons for each row. I think a ratio table is a good way to model that strategy, Sasha.

<table>
<thead>
<tr>
<th>Rows of crayons</th>
<th>Number of crayons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
</tr>
</tbody>
</table>

### Multiplication

*Miguel* It’s 3 rows of 8, and I know 3 times 8 is 24.

**Tile Array**

Teacher I’m going to build a tile array at the document camera to model your strategy, Miguel. First I’ll set out linear pieces to show the dimensions, 3 and 8. Then I’ll fill in the array with tiles. How many tiles are there in this array, everyone?

Students Twenty-four!

Teacher How do you know?

*Miguel* Because it’s like the crayons in that box. It’s 8 and 8 and 8, and that’s 24.

### Multiplication

*LaVonne* There’s 8 in every row, so it’s 3 x 8, and that’s 24.

**Area Model**

Teacher I can make a drawing of a rectangle, labeled with the dimensions and area to model your multiplication strategy, LaVonne. First, I’m going to draw a rectangle and label the dimensions, 3 and 8. What’s the area of this rectangle?

Students Twenty-four!

Teacher How do you know?

*LaVonne* Because 3 times 8 is 24.
8 Display the second box of crayons on the Crayons & Story Problems Teacher Master, and note with students that there are twice as many crayons as in the first box. Ask students to work in pairs to determine the number of crayons in this box, and show thumbs up when they have an answer.

9 When most students are ready, solicit their answer(s), and invite a couple of volunteers to share their thinking with the class.

_Teacher_: I see lots of thumbs up. What did you all get?

_Students_: 48!

_Teacher_: Did anyone get a different answer? No? Who’d like to share how they figured it out?

_Lucia_: It’s just double 24—that’s 48.

_Rashad_: This time, it’s 6 rows of 8 instead of 3 rows of 8. Six times 8 is 48.


10 Work with input from the students to expand the models you drew and built for the first box of crayons.

As you work with advice from the class, emphasize the doubling relationship between the two problems, as you did during the previous session when you presented the problem string.

---

11 Introduce the first story problem on the Crayons & Story Problems Teacher Master.

- Read the instructions to the class and then reveal problem 3a.
- Ask the students to turn to the Crayons & Story Problems Sheet in their Student Books as helpers give each table a set of tiles and red linear pieces.
• Have students work on their Student Book page to represent the problem using at least one of the four models; more than one if they have time.

• Encourage students to try all four models, and ask them to have at least one person at their table build an array with linear pieces and tiles to represent the situation.

• While students work at their seats, ask a volunteer to build the tile array where everyone can see while other volunteers draw and label on the board, an open number line, a ratio table, and an area model to represent the problem.

12 Discuss each model with the class.
Here are some questions you might use to spark student thinking:
• How are each of these models alike?
• How are they different?
• Which of these models seems to you to be the best way to represent and solve this problem? Why?

13 Before you reveal the second story problem on the Crayons & Story Problems Teacher Master, show the students how to set up a handbook in the back of their journals.
• Explain that students will add vocabulary words and strategies to their handbook throughout the year to create a collection of helpful tools they can refer to when they need help.
• Ask the students to take out their journals and turn them over so they are looking at the back covers.
• Then have students turn their journals upside down, so they are still looking at the back cover, but it is upside down. The binding should be on the left so the journal opens right to left like a book.
• Have students trace one of their hands on the cover and write “handbook” inside the drawing.
14 Then reveal the second story problem on the teacher master, and have students solve it on the first page of the handbook they just set up.

- Read the problem with the class and explain that everyone needs to represent and solve the problem using all four models: an open number line, a ratio table, a tile array, and an area model.
- Have students label the top of the first page in their handbook with the words “Multiplication Models,” and go to work.
- Circulate as students are working to observe and provide support as needed. Encourage them to share and compare their work.
- Have students clearly label each model they’ve sketched and labeled in their handbook.

- Remind students that they will use these models for multiplicative situations throughout the year.

**Home Connection**

15 Introduce and assign the Models for Multiplication Home Connection, which provides more practice with the following skills:

- Interpret products of whole numbers (3.OA.1)
- Write story problems or describe problem situations to match a multiplication expression or equation (3.OA.1)
- Fluently multiply with products to 100 using strategies (3.OA.7)
- Solve multi-step story problems involving only whole numbers, using addition, subtraction, multiplication (4.OA.3)
Daily Practice

The optional More Crayons Student Book page provides additional opportunities to apply the following skills:

- Solve multiplication story problems with products to 100 involving situations of equal groups and arrays (3.OA.3)
- Fluently multiply with products to 100 using strategies (3.OA.7)
- Write a multiplication equation to represent a verbal statement of a multiplicative comparison (4.OA.1)
- Solve story problems involving a multiplicative comparison using multiplication or division (4.OA.2)
- Multiply a 3-digit whole number by a 1-digit whole number using strategies based on place value and the properties of operations (4.NBT.5)
Session 5
Models for Division

Summary
Students investigate the relationship between multiplication and division using open number lines, ratio tables and the area model. After they collaborate to connect the three models to division, they begin a set of multiplication and division story problems.

Skills & Concepts
• Solve multiplication and division story problems with products and dividends to 100 involving situations of equal groups and arrays (3.OA.1)
• Divide a 2-digit number by a 1-digit number, using strategies based on the relationship between multiplication and division (4.NBT.6)
• Use an area model to explain strategies for dividing a multi-digit number by a 1-digit number (4.NBT.6)
• Reason abstractly and quantitatively (4.MP.2)
• Look for and make use of structure (4.MP.7)

Materials

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<th>Problems &amp; Investigations</th>
<th>Models for Division</th>
</tr>
</thead>
<tbody>
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<td>TM T8 Broken Lid Crayons</td>
<td>• colored tiles (class set, plus 1 set for display)</td>
</tr>
<tr>
<td>SB 8 Broken Lid Crayons Sheet</td>
<td>• red linear pieces (class set, plus 1 set for display)</td>
</tr>
<tr>
<td>SB 9–10 Camping Story Problems</td>
<td></td>
</tr>
<tr>
<td>TM T9 Camping Story Problems Forum Planner</td>
<td>• piece of scratch paper</td>
</tr>
<tr>
<td>Daily Practice</td>
<td>• red colored pencils (class set, plus 1 for display)</td>
</tr>
<tr>
<td>SB 11 Sandwiches &amp; Pizza</td>
<td></td>
</tr>
</tbody>
</table>

Vocabulary
An asterisk [*] identifies those terms for which Word Resource Cards are available.
area*
dimension*
divide*
factor*
quotient*

Preparation
Read Session 6 to see how students might share their work from today’s session. Before tomorrow’s forum, use the Camping Story Problems Forum Planner to help select students to share their work.
Problems & Investigations

Models for Division

1. Open the session by asking students to name the four models for multiplication they used yesterday (open number line, ratio table, tile array or area model) and passing out colored tiles and red linear pieces.

2. Show students the crayon box illustration at the top of the Broken Lid Crayons Teacher Master and ask them to find how many crayons are in each row.

   Teacher: Have you ever tried to open a box and have it accidentally rip? Here is an example of that very thing. How many crayons are in each row? How do you know? Begin working, and I’ll circulate and see what you’re doing.

As students work, circulate and take note of which are using each strategy. This will allow you to intentionally select students for sharing.

3. Ask students to share their strategies, beginning with a student who used tiles or a sketch of an array to represent the problem. Create a labeled sketch of the students’ strategy.

   Student: It looks like an array to me. It’s 4 rows by some columns. So, I took tiles and made columns of 4 and kept going until I had 32. When I finished, I had a 4 by 8 array. So each row has 8 crayons.
Then write the equations that represent the problem. If none of the students used tiles or a sketch of an array, introduce the model yourself.

**Teacher**  What equation would represent this problem?

**Student**  Four times something is 32.

(The teacher writes \(4 \times \_\_ = 32\).)

**Student**  Or it could be 32 divided by 4.

(The teacher writes \(32 \div 4 = \_\_\).)

**Teacher**  And what is the answer? How many crayons are in each row?

(The teacher writes \(4 \times 8 = 32\) and \(32 \div 4 = 8\).)

4  Next, invite a student who used a skip-counting strategy to share while you model using an open number line and ratio table.

**Student**  I skip-counted by 4s until I got to 32 and kept track on my fingers. So, 4, 8, 12, 16, 20, 24, 28, 32. There were 8 columns, so that means there are 8 crayons in each row.

<table>
<thead>
<tr>
<th>Rows of crayons</th>
<th>Number of crayons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
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<td>2</td>
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<td>7</td>
<td>28</td>
</tr>
<tr>
<td>8</td>
<td>32</td>
</tr>
</tbody>
</table>

5  Last, have a student share who used a doubling strategy, and model using an open number line and ratio table.

*Chances are good that at least one of the students will have used a doubling strategy to solve this problem. If none of them have, however, introduce the strategy yourself, as described here.*

**Student**  I thought of 1 column, 4 crayons. So 2 columns, 8 crayons.

Double that for 4 columns, 16 crayons. Double that for 8 columns, 32 crayons. Since there are 8 columns, there are 8 crayons in each row.
Ask students to think quietly for a moment about how the three strategies are the same and different, and then turn to a partner and talk about which strategy—modeling with an array, skip-counting, or doubling—makes the most sense to them and which they might try next time.

Show students the second crayon problem and ask them to think for a minute about how they would write an equation to represent the problem.

Students Each column has 3 crayons. So 2 columns have 6 crayons. Then 3 columns have 9 crayons, 4 have 12 crayons, and 5 have 15 crayons.

You could have doubled from 2 columns with 6 crayons to get the 4 columns with 12 and then just added 1 more column of 3 crayons to get the total of 12 + 3 = 15.
9 Display the open number line on the Broken Lid Crayons Teacher Master. Ask students to look for connections between the open number line model, the original crayon question, and the equation $15 \div 3 = 5$.

Complete the number line as students explain.

*Student* Each jump is a jump of 3, so 3 more than 6 is 9, 3 more than 9 is 12, and 3 more than that is 15.

10 Have student pairs use their linear pieces and tiles to model $3 \times 5 = c$ with a rectangular array.

Remind students to frame the rectangle first with the linear pieces and then fill in the tiles.

11 While students are working, have a volunteer build the array with colored tiles and red linear pieces, label it, and record two multiplication equations that it represents.

12 Ask students to turn to a partner and tell how they could use the first array on the Broken Lid Crayons Teacher Master to represent the division equation $15 \div 3 = 5$.

- After a minute or so, call on volunteers to share their thinking with the class.
- Label the first array as needed to illustrate students’ ideas.

*Students* The 15 is how many crayons are in the box or how many tiles are inside the rectangle.

That’s the area of the rectangle.
The 3 red lines on the side show the 3 rows of crayons in the box and 3 rows of tiles in that rectangle.
The 5 red lines on the top tell how many crayons or tiles are in each row.

Students familiar with the box notation for division might point out how the area model parallels this notation. This is something you can demonstrate if it doesn’t come from the class.

13 Have students explain how the second array on the Broken Lid Crayons Teacher Master shows division.
Label the second array as needed to illustrate their explanations.

14 Ask students to find the Broken Lid Crayons Sheet in their Student Books. Invite them to choose one of the equations \(15 \div 3 = 5 \) or \(15 \div 5 = 3\) and show it with a labeled sketch of an array.
Encourage students to outline the array using the lines on the paper, rather than tracing around each of the 15 squares, as some might want to do.
- Have students use red colored pencils to show the linear pieces that define the dimensions of their arrays.
- If needed, model how to make a quick sketch on grid paper and how to organize work on the page.

15 Next, read over the directions for the Camping Story Problems with students and then have them spend the remainder of the time working on it independently.
Make sure students understand that they should finish problems 1 through 5 first, and then spend any extra time on the remaining questions. Students will have time to work on this sheet tomorrow if they do not finish it today.
As students work, circulate and offer support.
SUPPORT If students struggle to understand a problem and find an entry point, ask scaffolding questions.
» What is the question asking?
» What might be a close estimate for the answer? Why do you think that?
» What could you draw or write that might help you decide what to do?

**SUPPORT** If students have difficulty explaining their thinking, have them first share their thinking with you orally, and then help them show their thinking on paper using an appropriate model.

16 As you circulate, use your copy of the Camping Story Problems Forum Planner Teacher Master to make notes about the strategies students are using to solve questions 1 and 2. These are the problems you’ll feature in tomorrow’s math forum.

_The forum planner indicates what kinds of strategies you’re likely to see. Notes in Session 6 will explain more about how you might structure the forum._

17 Close the session by asking students to share their observations about connections between the models, the numbers, and the symbols that they have been using.

**Extensions**

To give students more direct practice with the three models and division, you can do the following:

- Ask students to use linear pieces and tiles or base ten area pieces to model 24 ÷ 4 with a rectangular array and find the quotient.

  Have students pause for discussion after they have been working for a few moments. What steps would they use to solve this problem? What are the differences between how the area model shows multiplication and how it shows division?

  ![Area Model](image)

  - Lay down 4 linear units.
  - Fill in the array with 24 tiles.
  - Lay down linear units and count them to find the quotient.

- Have students record the area model for 24 ÷ 4 = 6 on their grid paper.

  Remind them to do the following:
  - Outline only the perimeter of the area.
  - Use a colored pencil to draw the linear pieces on the top and left sides.
  - Label the model with a division equation.

- Have students choose two of the expressions below and represent them using all three models on a sheet of grid paper.

  - 24 ÷ 8
  - 32 ÷ 4
  - 36 ÷ 9
  - 36 ÷ 6

- **CHALLENGE** Have students work in pairs and combine their tiles to model some of the problems below:

  - 36 ÷ 3
  - 56 ÷ 8
  - 60 ÷ 6
  - 48 ÷ 6
  - 60 ÷ 4
  - 63 ÷ 7
Daily Practice

The optional Sandwiches & Pizza Student Book page provides additional opportunities to apply the following skills:

- Fluently multiply with products to 100 using strategies (3.OA.7)
- Solve multi-step story problems involving only whole numbers, using addition, multiplication, division (4.OA.3)
- Multiply a 2-digit whole number by a 1-digit whole number using strategies based on place value and the properties of operations (4.NBT.5)
Session 6

Math Forum on Multiplication & Division

Summary
Students finish the Camping Story Problems Student Book page from Session 5. Then the teacher leads them in the first math forum, a purposeful classroom discussion of students’ strategies. Today’s forum focuses on grouping and sharing division problems, using the array model to strengthen understanding. The teacher asks selected students to share their strategies for solving two of the Camping Story Problems. Throughout the Forum, the teacher guides students’ understanding of the strategies and how the strategies relate to each other. At the end of the session, the teacher introduces and assigns the Modeling Multiplication & Division Home Connection.

Skills & Concepts
- Solve multiplication and division story problems with products and dividends to 100 involving situations of equal groups or arrays (3.OA.3)
- Use a rectangular array to explain strategies for dividing a 2-digit number by a 1-digit number (4.NBT.6)
- Divide a 2-digit number by a 1-digit number, using strategies based on the relationship between multiplication and division (4.NBT.6)
- Construct viable arguments and critique the reasoning of others (4.MP.3)
- Model with mathematics (4.MP.4)

Materials

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<th>Kit Materials</th>
<th>Classroom Materials</th>
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<tr>
<td>SB 9–10</td>
<td>Camping Story Problems (from Session 5)</td>
<td>Word Resource Cards for division and quotient</td>
</tr>
<tr>
<td>TM 9</td>
<td>Camping Story Problems Forum Planner (from Session 5)</td>
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<tr>
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<td></td>
</tr>
<tr>
<td>SB 12–13</td>
<td>Division Models</td>
<td></td>
</tr>
</tbody>
</table>

**Vocabulary**
An asterisk [*] identifies those terms for which Word Resource Cards are available.
division
quotient*

Preparation
- Today you will guide students through their first forum of the year. See the Implementation section of the Bridges Educator site for more information on math forums.
- Post the Word Resource Cards for division and quotient where everyone in class can see them.
Math Forum

Grouping & Sharing

1. Set the stage for today’s lesson.
   - Tell students they will have time to work on the Camping Story Problems Student Book page they started yesterday.
   - Remind them to work on the additional problems and challenge problems on the second page if they finish the first five problems.

2. Have students continue their work on the Camping Story Problems page. Circulate, asking scaffolding questions when necessary and continuing to look for students to share in today’s forum.
   As you circulate, use your copy of the Camping Story Problems Forum Planner Teacher Master from Session 5 to make notes about the strategies students are using to solve questions 1 and 2.

3. Gather the students in the class discussion area. Have them bring their journals, their Student Books, and a pencil.

4. Introduce the math forum.
   Today’s forum has two goals. First, you will teach the class the protocol for a forum. Second, you will use the forum to highlight both the connections between multiplication and division and the possibility of using division strategies to solve sharing problems as well as grouping problems.
   - Tell students that a math forum is an active learning time when some class members will share their thinking and the class will discuss problems and their solution strategies.
   - Let students know that only a few people will share today, and others will share in future forums.
   - Emphasize that the role of the audience is just as important as the role of the presenters. The audience is responsible for listening respectfully, asking questions, and learning from others.
   - Tell students the procedure.
     » You will ask a student to briefly describe his strategy.
     » While the student describes the strategy, you will model it on the board.
     » Students in the audience will listen to understand and ask questions when they don’t understand.
   - Let students know that the way you model a student’s strategy may look different than it does on the student’s paper because you are trying to help the class extend their learning about division, but you will still show how the student used the numbers to solve the problem.

   Future forums will have students present their work in a more formal way.

5. Give students a minute to quietly look over the first two problems on their pages to remember the situations and how they thought about them.

6. Ask a student who used the sharing strategy for problem 1 to share how he was thinking. Model the strategy with an array, a written sentence, and an equation.
   By modeling students’ thinking, you are trying to help students see how the model represents their mental processes. At the same time, your model might differ somewhat from the student’s. This might be so you can more easily compare one strategy with another, so you can build students’ understanding of a model, or so students can generalize a strategy to apply it to other problems.
Teacher  You solved the first problem by dealing out the 24 campers into 6 tents, right? Can you tell us a little about that?

Mario  First I got 24 tiles. Then I put 6 tiles out—they stand for the first camper in each tent. Then I kept putting students in each group.

Teacher  (With tiles at a projector) So, something like this?

Teacher  I am going to move the tiles just a little so that they are in an array. So I have 6 columns for 6 tents, and I’m going to draw loops around them so we can see the groups clearly. So, what do each of these loops represent?

Mario  One tent with 4 campers.

Teacher  So where do you see the answer?

Sasha  The number of campers in each group is 4 because there are 4 tiles.

Teacher  What sentence can we write about this situation? What equation can we write?

6 tents, 4 campers each
24 campers divided into 6 tents is 4 campers in each tent
24 ÷ 6 = 4

7  Ask a student who solved the second problem by adding groups of 6 to share how she was thinking. Model the strategy with a ratio table, a written sentence, and an equation.

Teacher  Now, some of you noticed that the numbers in problems 1 and 2 are the same, but the problems are different. Let’s talk about number 2, and then let’s compare. Lee, will you please tell us how you solved problem 2?

Lee  I knew there were 6 kids for each table, so I wrote down a 6. Then 2 tables for 12 campers, 3 tables for 18, and 4 tables for 24.

(As the student talks, the teacher records the numbers in a ratio table.)

<table>
<thead>
<tr>
<th>Tables</th>
<th>Campers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
</tr>
</tbody>
</table>
Teacher: Does this represent your thinking? Great. Now, I have a question for you. How did you know when to stop? Why didn’t you keep going to 5 tables?
Lee: Because there were 24 campers, so I have enough tables.

8 Re-examine the array you built to model the first problem with the students to see if it’s possible to find the answer to problem 2 with the same array. Leave the information you recorded about the array intact, and slide the array down so you and the students can take another look at it and make notations about the second problem.

Teacher: Let’s see if we can find the tables and campers in this array that we have from the campers and tents. Can you find the tables and campers from problem 2 in this array from problem 1?
Lee: This time, each row is like a table.
(Suppose the teacher separates the tiles slightly and draws loops around the rows.)
Sasha: There are 6 campers in each row. The loops, the rows are the tables.
Teacher: What sentence can we write about this situation? What equation can we write?

7 tables campers
1 6
2 12
3 18
4 24
6 campers can sit at each table.
24 campers divided into tables of 6 campers each is 4 tables.
$24 \div 6 = 4$

9 Compare the notes and equations you have recorded for both problems. Note with students that the numbers and the equations for both problems are the same, but the answers have different meanings.

Teacher: Let’s slide the tiles away and look at what we wrote about both of these problems. Talk with the person sitting next to you. What do you notice, what do you see? (Suppose the teacher gives the students a minute to talk, and then calls on volunteers.)
Meredith: They both have the same equation.
Darius: It’s $24 \div 6$ for both of those problems.
Julia: It’s the same answer for both, too. They’re both 4.
Teacher: What does the 4 mean in the first equation?
Luis: It’s kids. It’s how many kids there can be in each tent.
Teacher: What about the 4 in the second equation. Does it mean kids?
Amber: Yes! Oh, I mean, no. It’s about the tables.
Ramona: It’s how many tables. If 6 kids can be at each table, you need 4 tables to hold them all.
Teacher: Interesting!
6 tents, 4 campers each
24 campers divided into 6 tents is 4 campers in each tent.
24 ÷ 6 = 4

6 campers can sit at each table.
24 campers divided into tables of 6 campers each is 4 tables.
24 ÷ 6 = 4

The first problem is an example of division as sharing, while the second is an example of division as grouping. These terms and the related concepts will resurface in later units. For now, it's enough that students have opportunities to solve both types of problems.

10 Ask a student who used the known fact that 6 × 4 = 24 to solve either problem 1 or 2 to share how she was thinking.
Reinforce the idea that even though the problems are different, the same equation, 24 ÷ 6 = 4, can be used to represent both.

Noah When I read both problem 1 and problem 2, I knew that they were both 24 ÷ 6. I remembered that 6 × 4 is 24, so 24 ÷ 6 = 4. Then I figured out how that fit each problem.

11 Review the word quotient and have students add it to the handbook in the back of their journals.
- Draw students’ attention to the Word Resource Card you have posted for quotient.
- Have students record the word, give an example, and make a sketch in their handbooks.
- Remind students that the unknown dimension in a division problem is called the quotient.

12 Have students turn to a partner and talk about which model (open number line, array, or ratio table) they are most comfortable with right now.

13 Close the session by calling on several students to share examples of respectful behavior and participation during today’s math forum.

Home Connection

14 Introduce and assign the Modeling Multiplication & Division Home Connection, which provides more practice with the following skills:
- Write story problems or describe problem situations to match a multiplication expression or equation (3.OA.1)
- Solve story problems involving a multiplicative comparison using multiplication (4.OA.2)
- Divide a 2-digit number by a 1-digit number, using strategies based on the relationship between multiplication and division (4.NBT.6)
- Solve story problems involving intervals of time using addition, subtraction, and multiplication of whole numbers (4.MD.2)
Daily Practice

The optional Division Models Student Book page provides additional opportunities to apply the following skills:

- Write story problems or describe problem situations to match a multiplication expression or equation (3.OA.1)
- Solve division problems by finding an unknown factor (3.OA.6)
- Fluently multiply and divide with products and dividends to 100 using strategies (3.OA.7)
- Solve story problems involving a multiplicative comparison using multiplication (4.OA.2)
- Divide a 2-digit number by a 1-digit number, using strategies based on the relationship between multiplication and division (4.NBT.6)
School Supplies

The students in Mrs. Carter’s class brought in their school supplies for the year. They brought in paper and pencils, markers and crayons, glue, folders, and more. Help them figure out how many of each item they have. For each problem, show your thinking with numbers, sketches, or words. Then write an equation that represents your work.

1. There are 32 students in Mrs. Carter’s class. Each student brought in 2 glue sticks.
   a. How many glue sticks is that in all?
   
   Equation答: ____________ Answer, labeled with correct units

   b. If each student brought in 2 glue sticks, there are ______ times as many glue sticks as students in Mrs. Carter’s room.

2. In Mrs. Carter’s classroom, students sit in table groups. There are 4 students at each table. Each of the 4 students at table G brought in 8 markers.
   a. How many markers total did the students at table G bring?
   
   Equation答: ____________ Answer, labeled with correct units

   b. If each of the 4 students at table G brought in 8 markers, there are ____ times as many markers as students at table G.
**Number Line Puzzle 1**

Study the number line puzzle. What do you notice? How can you figure out what numbers are missing?

![Number Line Puzzle 1](image-url)
1. Complete the chart.

<table>
<thead>
<tr>
<th></th>
<th>a List all the factors of the number.</th>
<th>b List two multiples of the number.</th>
<th>c Write P if the number is prime or C if the number is composite.</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

2. Write a prime number in the space below and tell how you know it is prime.

3. Write a composite number in the space below and tell how you know it is composite.

4. The equation $5 \times 7 = 35$ can mean:
   - 35 rulers are 5 rulers and 7 rulers put together
   - 35 pencils are 7 times as many as the 5 pencils at the green table
   - 35 markers are 5 markers less than 7 markers
   - 5 erasers split into 7 groups is 35 erasers

5. Fill in the bubbles beside the two equations that best represent this situation: Marcus has 15 toy cars. That is 3 times as many as his brother Craig has. How many toy cars does Craig have? (In the equations below, $c$ stands for Craig’s toy cars.)
   - $15 = 3 \times c$
   - $15 \times 3 = c$
   - $15 - 3 = c$
   - $15 \div c = 3$

*continued on next page*
Unit 1 Pre-Assessment page 2 of 3

6 Write and solve a multiplication equation for each of these problems.

a Eric is 11 years old. Eric’s dad is 3 times older than Eric. How old is Eric’s dad?

b Amber bought a pair of pants and a pair of shoes. The shoes cost 3 times as much as the pants. The pants cost $15. How much did the shoes cost?

c Jamal bought a book and a CD. The book cost $14. The CD cost $7. How many times more than the CD did the book cost?

7 Fill in the blanks to complete this ratio table.

<table>
<thead>
<tr>
<th>Number of Students</th>
<th>Number of Folders</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>60</td>
</tr>
</tbody>
</table>

8 Find and write in the missing dimension on each of the rectangles below.

9 Fill in the blanks.

a

\[
\begin{array}{cccccc}
7 & 5 & 6 & \boxed{3} & 50 \\
\times 8 & \times & \boxed{9} & \times 8 & \times 22 & \times 7 \\
& & & \boxed{40} & 32 & \boxed{200}
\end{array}
\]

b

\[
\begin{array}{cccccc}
9 & \times \boxed{10} = 90 \\
4 \times 20 = \boxed{80} \\
7 \times \boxed{4} = 28 \\
\boxed{6} \times 10 = 60
\end{array}
\]

(continued on next page)
10 Solve each of the story problems below. Show your thinking with numbers, sketches, or words. Then write an equation that represents your work, and record the answer, labeled with the correct units.

a Each of the 4 students at the red table has 8 markers. The class has 5 times as many markers as the entire red table. How many markers total does the whole class have?

b Abby saw 3 rows of crayons in her 24-count crayon box. How many crayons are in each row?

11 The green table has 5 students and each student brought 6 folders. The red table group has 6 students and each student brought 8 folders. How many folders do both groups have together?

a Solve the problem above. Show your thinking with numbers, sketches, or words. You do not need to write an equation for this problem.

b Which equation best represents this story problem? (The letter \( f \) stands for the number of folders both groups have together.)

- \( (5 \times 6) + (6 \times 8) = f \)
- \( 5 + 6 + 6 + 8 = f \)
- \( ((5 \times 6) + (6 \times 8)) \div 11 = f \)
- \( (6 \times 8) - (5 \times 6) = f \)
## Unit 1 Pre-Assessment Student Reflection Sheet

<table>
<thead>
<tr>
<th>Skill</th>
<th>Look at these problems.</th>
<th>I can do this well already.</th>
<th>I can do this sometimes.</th>
<th>I need to learn to do this.</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can you name all the factors for a number?</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1a</td>
</tr>
<tr>
<td>Can you list two multiples for a number?</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1b</td>
</tr>
<tr>
<td>Can you tell whether a number is prime or composite and explain how you know?</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1c, 2, 3</td>
</tr>
<tr>
<td>Do you understand how multiplication and division can be used to compare quantities?</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4, 5, 6a, 6b, 6c</td>
</tr>
<tr>
<td>Can you write and solve a multiplication equation to represent a story problem?</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6a, 6b, 6c</td>
</tr>
<tr>
<td>Can you multiply and divide to fill in the blanks on a ratio table?</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>If you know the area and one dimension of a rectangle, can you use that information to find the other dimension?</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>Can you multiply and divide to fill in the missing numbers in multiplication combinations?</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9a, 9b</td>
</tr>
<tr>
<td>Can you solve a multiplication or division story problem and then choose or write an equation to represent the problems?</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10a, 10b, 11a, 11b</td>
</tr>
<tr>
<td>Can you solve a story problem that requires more than one step and more than one operation?</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10a, 11a</td>
</tr>
</tbody>
</table>

- After you have made a mark and some notes about the skills above, draw a star next to the two skills that you need to work on the most during this unit.
- Write other ideas about what you want or need to learn how to do during this unit.
Crayons & Story Problems

How many crayons?

1

2

3 Use at least one of the models listed here to represent and solve each of the problems below.

- an open number line
- a ratio table
- a tile array
- an area model

a The kids at table E have 9 pencils in the supply box they share. The kids at table F have 3 times that many pencils in their supply box. How many pencils do the kids at table F have in their supply box?

b Mrs. Carter has 4 pens in the cup on her desk. She has 5 times that many pens in her desk drawer. How many pens does Mrs. Carter have in her desk drawer?
## Broken Lid Crayons

<table>
<thead>
<tr>
<th>Number of Columns</th>
<th>Number of Crayons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>15</td>
</tr>
</tbody>
</table>

1 × 3  2 × 3  3 × 3

3  6  15
## Camping Story Problems Forum Planner

Use this planner to make a record of the strategies you see students using to solve problems during Session 5. Use the third column to indicate the order in which you plan to have students share during the forum in Session 6.

### Strategies for solving $24 \div 6$

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Student Names and Notes</th>
<th>Order of Sharing in Forum</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sharing</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The student deals out 24 campers into 6 tents.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Grouping</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The student adds groups of 6.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Known fact that $6 \times 4 = 24$</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The student knows that $6 \times 4$ is 24, and uses that knowledge to determine that $24 \div 6 = 4$.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
More School Supplies

Help Mrs. Carter’s students figure out how many of each supply they have. For each problem, show your thinking with numbers, sketches, or words. Then write an equation that represents your work.

1  The 4 students at table G each brought in 6 pencils. How many pencils are at table G?

2  The 4 students at table E each brought in a box of 12 crayons. How many crayons are at table E?

3  The 4 students at table B and the 4 students at table D each brought in 12 pens. How many pens do tables B and D have in all?

4  **CHALLENGE** The students at table C brought in a total of twice as many pens as the students at tables B and D put together. How many pens did the students at table C bring?
How Many Pencils?

Not all of Mrs. Carter’s students brought in the same number of pencils to use for the school year. Help the students figure out how many pencils the class has. For each problem, show your thinking with numbers, sketches, or words. Then write an equation that represents your work.

1. Seven students brought in 6 pencils each. How many pencils did they bring in all?

   Equation Answer, labeled with correct units

2. Eight students each brought in 9 pencils. How many pencils did they bring in all?

   Equation Answer, labeled with correct units

3. Six students brought in 12 pencils each. How many pencils did they bring in all?

   Equation Answer, labeled with correct units

4. How many pencils did the students in problems 1, 2, and 3 bring in all together?

   Equation Answer, labeled with correct units

5. Fill in the blanks.

   \[ 7 \times 8 = \underline{\hspace{1cm}} \quad 7 \times \underline{\hspace{1cm}} = 63 \quad \underline{\hspace{1cm}} = 4 \times 8 \quad \underline{\hspace{1cm}} \times 6 = 30 \]
How Many Pens?

All of the students in Mrs. Carter’s class brought in packs of pens, but the packs do not all have the same number of pens. Help Mrs. Carter figure out how many pens the class has. For each problem, show your thinking with numbers, sketches, or words. Then write an equation that represents your work.

1. Four students brought packs of pens with 3 pens in each pack. How many pens in all did these students bring to class?

   Equation   Answer, labeled with correct units

2. Four students brought packs of pens with 4 pens in each pack. How many pens in all did these students bring to class?

   Equation   Answer, labeled with correct units

3. Four students brought packs of pens with 7 pens in each pack. How many pens in all did these students bring to class?

   Equation   Answer, labeled with correct units

4. **CHALLENGE** Four students brought packs with 3 times as many pens as the students in problem 2. How many pens did these students give to the collection of class supplies?

   Equation   Answer, labeled with correct units
How Many Erasers?

Mrs. Carter’s fourth grade students brought lots of erasers to use for the school year. Help the students figure out how many erasers they have. For each problem, show your thinking with numbers, sketches, or words. Then write an equation that represents your work.

1. Four students each brought 5 erasers. How many erasers did these 4 students bring?

   Equation

   Answer, labeled with correct units

2. Four students each brought 6 erasers. How many erasers did these 4 students bring?

   Equation

   Answer, labeled with correct units

3. Eight students each brought 5 erasers. How many erasers did these 8 students bring?

   Equation

   Answer, labeled with correct units

4. **CHALLENGE** Eight students each brought twice as many erasers as the students in problem 3. How many erasers did these 8 students bring?

   Equation

   Answer, labeled with correct units

5. Fill in the blanks in the number line puzzle below.

   2 \times 6 \quad 3 \times 6 \quad \square \times 6 \quad 8 \times 6 \quad \square \times 6

   30 \quad \square \quad 48 \quad 60
Claudia’s School Supplies

Solve each problem below. Use numbers, sketches, or words to show your work.

1. Claudia bought school supplies in August. She bought 4 packages of pencils. Each package had 12 pencils in it. How many pencils did Claudia buy?

   Equation: 
   Answer, labeled with correct units: 

2. Claudia bought 8 packages of pens. Each package had 6 pens in it. How many pens did Claudia buy?

   Equation: 
   Answer, labeled with correct units: 

3. Claudia bought extra packages of crayons. Each package had 8 crayons in it. Fill out the ratio table below to find out more about how many crayons Claudia bought.

<table>
<thead>
<tr>
<th>Packages</th>
<th>Crayons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>80</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>

4. While Claudia was at the store, she saw a box of crayons that had 8 times the number of crayons as the little boxes she bought to bring to class. How many crayons were in the box?

   Equation: 
   Answer, labeled with correct units: 
Crayons & Story Problems Sheet
More Crayons

1. Each of the models below represents a student’s strategy for finding the number of crayons in a box.

   a. The first box has 4 rows of 6 crayons. How many crayons are there? Show your thinking and write an equation to show your answer.

      ____________________________________________________________________

   b. The second box has 6 rows of crayons. How many crayons are there? Complete the ratio table and write an equation to show your answer.

      | Rows of Crayons | Number of Crayons |
      |-----------------|------------------|
      | 1               | 6                |
      | 2               | 12               |
      | 3               | 18               |
      | 6               | 6                |

   c. The third box has 5 rows of crayons. How many crayons are there? Fill in the blank and write an equation.

      24          18       12       6

2. Mark has twice as many crayons as the box modeled on the number line in the problem above. Write an equation to show how many crayons Mark has.

   ____________________________________________________

3. Fill in the blanks:

   \[
   \begin{align*}
   10 & + 450 = 890 \\
   89 & + 29 = 918 \\
   100 & - 25 = 975 \\
   900 & - 500 = 400 \\
   \end{align*}
   \]
Broken Lid Crayons Sheet
Camping Story Problems page 1 of 2

Model each problem with a labeled sketch of an open number line, a ratio table, or an array. Then, write an equation to show your answer.

1. At Camp Mosquito, there are 24 campers and 6 tents. If each tent has the same number of campers, how many campers are in each tent?

2. During dinner at Camp Mosquito, 6 campers can sit at each table. How many tables are needed for 24 campers?

3. Tennis balls come 3 to a can. Coach Brammer has 27 tennis balls. How many cans does she have?

4. The campers are getting ready for lunch. There are 8 groups of campers and 56 brownies. How many brownies will each group get if they share evenly?

5. If 8 glasses can be filled from one carton of orange juice, how many glasses can be filled from 7 cartons?

(continued on next page)
If you have time, choose some of the problems below to solve.

6  Rico gave his sister 6 boxes of crayons. She was thrilled to get 48 new crayons. How many crayons were in each box if each box had the same number of crayons?

7  There are 36 kids in the class. They need to make 4 equal teams. How many kids will be on each team?

8  **CHALLENGE** It’s time for a picnic! There are 8 hot dog buns in a package. Sixteen people are coming to the picnic, and you know each person is going to be hungry enough to eat 2 hot dogs. How many packages of hot dog buns should you buy?

9  **CHALLENGE** Sports equipment bags that usually cost $9.97 each are on sale for $7.97 each. If the coach buys 9 bags on sale, how much money does she save?

10 **CHALLENGE** Colorful binders that usually cost $4.49 are on sale for $2.99 each. How much money will you save if you buy 8 binders at the sale price?
Sandwiches & Pizza

1. Rodney had a friend over on Saturday. His dad took them out for sandwiches. Rodney’s dad and the boys each got a sandwich for $6 and a drink for $2. They shared one large cookie that cost $3. How much did they spend in all?

2. Jasmine had a pizza party with 3 of her friends. They ordered 2 pizzas. Each pizza had 8 slices. They all ate the same amount of pizza and finished both pizzas. How many did each person eat? Show all your work.

3. Complete the equations.

\[
\begin{align*}
1 \times 8 & = 0 \\
73 \times 2 & = 146 \\
10 \times 10 & = 100 \\
49 \times 7 & = 343
\end{align*}
\]
1 Each table of 4 students in Mrs Thornton’s class brought 9 glue sticks.

a Fill in the blanks in the ratio table.

<table>
<thead>
<tr>
<th>Number of Tables</th>
<th>1</th>
<th>2</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Glue Sticks</td>
<td>9</td>
<td>27</td>
<td>90</td>
<td></td>
</tr>
</tbody>
</table>

b Write a story problem that matches one of the entries in the glue stick ratio table.

c One of the tables in Mr. Still’s class brought in 3 times as many glue sticks as one of the tables in Mrs. Thornton’s class. How many glue sticks did that table group in Mr. Still’s class bring? Write and solve an equation to show.
2 Fill in the missing dimensions in the arrays.

\[
\begin{array}{c}
\square \quad 6 \\
\square \quad 42 \\
\end{array} \quad \begin{array}{c}
\square \quad 6 \\
\square \quad 48 \\
\end{array} \quad \begin{array}{c}
\square \quad 6 \\
\square \quad 54 \\
\end{array}
\]

3 Write at least two equations to match one of the arrays in problem 2.

4 Fill in the blanks on the number lines.

\[
\begin{array}{cccccccc}
2 \times 3 & \square \times 3 & 4 \times 3 & 6 \times 3 & 8 \times 3 & 9 \times 3 & \square \times 3 & 30 \\
2 \times 6 & \square \times 6 & 4 \times 6 & 8 \times 6 & \square \times 6 & 10 \times 6 & \square & 18 & 54 & \square
\end{array}
\]

5 Fill in the blanks to make the equations true.

\[
10 \times 4 = 5 \times \square \quad 10 \times 3 = 5 \times \square \quad 10 \times 5 = 5 \times \square \quad 10 \times 2 = 5 \times \square
\]

\[
10 \times 10 = 5 \times \square \quad 5 \times 8 = 10 \times \square \quad 5 \times 4 = 10 \times \square
\]
Number Line Puzzles  page 1 of 2

Note to Families
Students can use number lines to review the multiplication facts they learned in third grade. Number lines can help students use facts they know to help them figure facts they don’t remember. Talk together about relationships between facts that you see in the two number lines below, such as numbers that double.

1  Fill in the blanks in the number lines.

a  

\[2 \times 4 \quad 3 \times 4 \quad 4 \times 4 \quad 8 \times 4 \quad 9 \times 4 \quad \text{?} \times 4\]

b  

\[2 \times 8 \quad \text{?} \times 8 \quad 4 \times 8 \quad 8 \times 8 \quad \text{?} \times 8 \quad 10 \times 8\]

2  Complete the facts.

\[
\begin{array}{cccccccc}
8 & 8 & 8 & 8 & 8 & 7 & 7 \\
\times 2 & \times 4 & \times 8 & \times 10 & \times 9 & \times 10 & \times 9 \\
\end{array}
\]

3  Roger’s little brother, Saul, wants to know if \(5 \times 7 = 7 \times 5\). If you were Roger, how would you explain to Saul whether the equation is true?

(continued on next page)
4 Each of the 29 students in Mr. Brown’s fourth grade brought 2 notebooks to class the first day of school. How many notebooks was that in all? Show your thinking with numbers, sketches, or words. Then write an equation that represents your work.

Equation Answer, labeled with correct units

5 Each of the students in Mr. Smith’s class also brought in 3 pocket folders. Mr. Smith wrote a multiplication equation to compare the number of students to the number of pocket folders they brought in. Fill in the bubble to show what this equation means.

\[ 87 = 3 \times 29 \]

○ 87 is 3 more than 29  ○ 87 is 3 times as many as 29  ○ 29 is 3 times as many as 87

6 **CHALLENGE** If 5 students each brought in 8 boxes with 10 pencils per box, and 10 students each brought in 8 boxes with 5 pencils per box, how many total pencils did the students bring in? Show your thinking with numbers, sketches, or words.

Equation Answer, labeled with correct units
## Models for Multiplication  page 1 of 2

Write a story situation to go with each multiplication model.

<table>
<thead>
<tr>
<th>Multiplication Model</th>
<th>Story</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>exo</strong></td>
<td>Keith’s dog, Spot, ate 2 cans of dog food every day for 3 days in a row. Spot ate 6 cans of dog food in 3 days.</td>
</tr>
<tr>
<td><strong>1</strong></td>
<td>7 × 6 = 42</td>
</tr>
<tr>
<td><strong>2</strong></td>
<td>4 × 4 = 16</td>
</tr>
<tr>
<td><strong>3</strong></td>
<td>4 × 6 = 24</td>
</tr>
<tr>
<td><strong>4</strong></td>
<td>Number of _____</td>
</tr>
<tr>
<td>Number of _____</td>
<td>6</td>
</tr>
</tbody>
</table>

(continued on next page)
### Multiplication Model

<table>
<thead>
<tr>
<th>Multiplication Model</th>
<th>Story</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>![Grid with 5 rows and 6 columns]</td>
</tr>
<tr>
<td>$5 \times 6 = 30$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6</th>
<th>![Crayons with 8 each row]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8 \times 9 = 72$</td>
<td></td>
</tr>
</tbody>
</table>

7. There are 4 rows of crayons in this box. Each row has the same number of crayons. How many crayons are in the box? Show your thinking.

8. Teachers collected $5 from each of the 130 fourth grade students at the beginning of the year for field trips. The first field trip cost $120. The second field trip cost $250. How much can they spend on the last field trip if they need to have $25 left over to wash the bus? Show your thinking using words, numbers, or pictures.
Modeling Multiplication & Division page 1 of 2

For problems 1 and 2, complete the sketches and write the equations.

1

\[ \underline{\quad} \times \underline{\quad} = \underline{\quad} \]

2

\[ \underline{\quad} \div \underline{\quad} = 3 \]

3 Copy one equation from above and write a story problem to go with it.

**ex** I bought 5 packs of pencils. Each pack had 4 pencils in it. How many pencils did I get? \((5 \times 4 = 20)\)

Complete the number line and ratio table.

4

\[2 \times 5\] 15
\[4 \times 5\] 25
\[6 \times 5\]

5

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>6</td>
<td>9</td>
<td>15</td>
<td>18</td>
<td>21</td>
</tr>
</tbody>
</table>

(continued on next page)
6 Mr. Still’s class has music for 50 minutes and then independent reading for 20 minutes. Music starts at 8:30. What time does Mr. Still’s class finish independent reading?

7 Ms. Ford’s class starts art at 9:30 and finishes at 10:15. They spend twice as much time in math class. If they start math at 1:10, what time do they finish math?