Teachers Guide
GRADE 4 – UNIT 2 – MODULE 3

BRIDGES IN MATHEMATICS
Module 3
**Multiplication Stories & Strategies**

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**Teacher Masters**
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- Multiplication Strategies Forum Planner ................................................................................................. T2
- Multiplication Checkpoint ......................................................................................................................... T3
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- 2C Moolah on My Mind Record Sheet ................................................................................................. T6
- 2C Moolah on My Mind Challenge Record Sheet ............................................................................... T7
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**Student Book Pages**
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- Riley’s & Raymond’s Ratio Tables ........................................................................................................65
- Maggie’s Chickens ...................................................................................................................................... 66
- Eggs & Apples ........................................................................................................................................... 67
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**Home Connections Pages**
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Module 3
Multiplication Stories & Strategies

Overview
Module 3 opens with a set of story problems followed by a math forum in which students have opportunities to share and extend their strategies for working with single- and double-digit multiplication as well as solving multi-step story problems. In Session 3, students investigate the effect of doubling and halving factors in multiplication combinations. The use of coins and units of measure in Session 4 provides practice multiplying with landmark numbers, as well as practice converting from larger to smaller units of metric measure. Session 5 serves as a wrap-up of sorts, as students solve more story problems, and review and add to the class list of strategies for solving multi-digit multiplication problems.

Planner

<table>
<thead>
<tr>
<th>Session &amp; Work Places Introduced</th>
<th>P&amp;I</th>
<th>PS</th>
<th>MF</th>
<th>WP</th>
<th>A</th>
<th>HC</th>
<th>DP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Session 1 Multiplication Story Problems</td>
<td>●</td>
<td></td>
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<tr>
<td>Students answer a few questions based on their displayed ratio tables. Then the teacher presents a story problem for the class to discuss and solve together. Students work in pairs to solve more story problems that require them to first estimate, then talk about what the problem is asking, and finally develop strategies to solve the problem. Students will present their work with the story problems in a math forum in the next session.</td>
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| Session 2 Multiplication Story Problems Math Forum | ● | ● | ● | ● |   |    |    |
| Students review their work from the previous session in preparation for a math forum. The teacher guides the students as they present and participate in the forum, which centers on discussion of strategies and models for 2-digit multiplication. Then students spend the rest of the session visiting Work Places. |

| Session 3 Doubling & Halving | ● | ● | ● | ● |   |    |    |
| Today’s session begins with a problem string to prompt thinking about doubling and halving. Then students complete a brief checkpoint on multiplication arrays, equations, and ratio tables. Students spend the rest of the session doing Work Places. |

| Session 4 Multiplying with Money & Units of Metric Measure | ● | ● | ● | ● |   |    |    |
| This session connects students’ work with ratio tables to money and measurement conversions. The teacher introduces a new Work Place called Moolah on My Mind, which uses the landmark numbers in coin values to offer practice with multi-digit multiplication. The class plays several rounds of the game as a whole group, and then students play again in pairs. |

| Work Place 2C Moolah on My Mind | ● | ● | ● | ● |   |    |    |
| Players take turns rolling two dice and spinning a coin spinner. They add the numbers rolled and then multiply the sum of the numbers by the value of the coin. Then they add the running totals of money after each turn. The player who collects the most money after 10 wins. |

| Session 5 Another Look at Multiplication Strategies | ● | ● | ● | ● |   |    |    |
| Students solve a story problem that involves 6 × 26. They share their strategies and add new strategies to the Multiplication Strategies chart begun in Module 2, Session 2. Then students write their own story problems for 8 × 24 and 11 × 22 and solve them in an assignment that can be collected as a work sample. Students who complete the assignment before the end of the session go to Work Places. |

### Materials Preparation

Each session includes a complete list of the materials you’ll need to conduct the session, as well as notes about any preparation you’ll need to do in advance. If you would like to prepare materials ahead of time for the entire module, you can use this to-do list.

<table>
<thead>
<tr>
<th>Task</th>
<th>Done</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Copies</strong></td>
<td></td>
</tr>
<tr>
<td>Run copies of Teacher Masters T1–T9 according to the instructions at the top of each master.</td>
<td></td>
</tr>
<tr>
<td>Run two display copies of Student Book pages 65 &amp; 66.</td>
<td></td>
</tr>
<tr>
<td>If students do not have their own Student Books, run a class set of Student Book pages 65–72.</td>
<td></td>
</tr>
<tr>
<td>If students do not have their own Home Connections books, run a class set of the assignments for this module using pages 35–39 in the Home Connections Book.</td>
<td></td>
</tr>
<tr>
<td><strong>Work Place Preparation</strong></td>
<td></td>
</tr>
<tr>
<td>Prepare the materials for Work Place 2C using the lists of materials on the Work Place Guide (Teacher Master T5).</td>
<td></td>
</tr>
<tr>
<td><strong>Special Items</strong></td>
<td></td>
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<tr>
<td>You might find it helpful to collect a few objects that represent different metric measurements for students to see during Session 4. For example, for units of length a meter stick can show centimeters, meters, and millimeters. For mass, a quarter weighs 5 grams or a small can of soup is about 300 grams. For volume, a 1 liter bottle offers a landmark, and many reusable water bottles are marked with milliliters on the side.</td>
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</tbody>
</table>
Session 1

Multiplication Story Problems

Summary
Students work together to answer a few questions based on their displayed ratio tables. Then the teacher presents a story problem for the class to discuss and solve together. Students work in pairs to solve more story problems that require them to first estimate, then talk about what the problem is asking, and finally develop strategies to solve the problem. They will present their work with the story problems in a math forum in the next session.

Skills & Concepts
• Multiply two 2-digit numbers using strategies based on place value and the properties of operations (4.NBT.5)
• Use equations and rectangular arrays to explain strategies for multiplying with multi-digit numbers (4.NBT.5)
• Make sense of problems and persevere in solving them (4.MP.1)
• Reason abstractly and quantitatively (4.MP.2)
• Use appropriate tools strategically (4.MP.5)

Materials

<table>
<thead>
<tr>
<th>Problems &amp; Investigations</th>
<th>Kit Materials</th>
<th>Classroom Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solving Multiplication Story Problems</td>
<td>- base ten area pieces (class set, plus 1 set for display)</td>
<td>- student-made ratio table strips from Unit 2, Module 2, Session 4 (see Preparation)</td>
</tr>
<tr>
<td></td>
<td>- base ten linear pieces (class set, plus 1 set for display)</td>
<td>- student math journals</td>
</tr>
</tbody>
</table>

Daily Practice
SB 67 Eggs & Apples

Vocabulary
An asterisk [*] identifies those terms for which Word Resource Cards are available.

area model of multiplication array*
equation*
multiple*
ratio table*

Preparation
Be sure all the ratio table strips students made toward the end of the previous module are displayed around the room where they are easily seen. You will need them for the next several sessions, so plan to leave them up for at least another week.

Read Session 2 to see how students might share their work from today's session. Before tomorrow's forum, use the Multiplication Strategies Forum Planner to help select students to share their work.
Problems & Investigations

Solving Multiplication Story Problems

1 Open the session by letting students know that today they will revisit their ratio tables and then work on multiplication story problems for a math forum in the next session.

Have base ten area and linear pieces easily accessible.

2 Display your copy of the Riley’s & Raymond’s Ratio Tables Student Book page and have students find the page in their books.

Ask them to read problem 1a and then discuss it with the class.

   Teacher What can you tell me about Riley’s ratio table? Why is it so short?
   Rich I think we are looking at only a part of it—like it was torn off or something.
   Lin I think that Riley was using the number 22.
   Teacher Why do you think that?
   Lin Because the 10th number there is 220. I know that 10 × 22 is 220.
   DJ I knew Riley’s number was 22 because I looked at 220 to 242 and that’s 22.
   Teacher How does that tell you?
   DJ Well, if you were at 10 and 220 and then you were at 11 and 242, you added one more of something. So the difference between 242 and 220 is 22.

3 Give students a few minutes to work on problems 1b and 1c. Then ask several students to share their strategies for answering the questions.

   • Help students understand that the difference between the multiples is the multiplier (the number they skip-counted by to create the ratio table.)
   • Have your copies of the Riley & Raymond’s Ratio Tables Student Book page handy in case you want to extend the tables, as in the dialog below.

ELL Provide opportunities for students to request clarification or rephrasing. Ask all students to justify their thinking to provide an atmosphere where students are comfortable asking questions and sharing.

   Teacher How did you find the 13th row?
   Anna I added on 22 to 242 and then did it again. That’s 264 and then 286.
   Teacher Why did you add 22 twice?
   Anna Because the last number, 242, is the 11th one. So to get the 13th, you need to add two more 22s.
   Teacher What about the 5th row? How did you think about that?
   Ayisha I started with 198 and subtracted 22 to get 176. That’s the 8th one. I kept doing that until I got the 5th one, which is 110.
   Teacher I could model that on the ratio table like this.
Rosalia  I remembered that the 5s are Half-Tens facts, so since $10 \times 22$ is 220, then $5 \times 22$ is half of that, 110.

Teacher  How did you find half of 220?

Rosalia  Half of 200 is 100. Half of 20 is 10. So 110.

Teacher  I am going to model that on the ratio table like this.

Teacher  You were all using 22 a lot. Why?

Brian  It’s a pattern. Riley’s table was built by adding 22 over and over again. So we can find missing numbers by doing that, too.

Note: If you feel students would benefit from additional practice with the ratio tables, assign the rest of the Riley’s & Raymond’s Ratio Tables Student Book page at a later time.

4  Next, display your copy of the Maggie’s Chickens Student Book page, showing only the first row. Give students a moment to look it over, and invite a student to read the problem aloud.

5  Discuss the first problem.
   - Have students turn to a partner and talk about what the problem is asking them to do. Invite a few students to share and record their thoughts.
   - Then ask students to estimate the answer. Ask a few students to share their estimates and explain their reasoning. Record at least one of the estimates on your copy of the sheet.

6  When students understand what to do, have them get out their journals and go to work on the first problem.
   - Display a copy of the Base Ten Grid Paper Teacher Master. Let students know they can use the paper, base ten area pieces, or any other manipulatives that are helpful to them.
• Remind students that they are seeking efficient strategies and they can use any of the strategies they have learned this year.

7 As students work, walk around the room, listen to conversations, and take note of student pairs you want to have present their work.

Look for students who use base ten pieces, ratio tables, or arrays on the base ten grid paper to share their work. Ideally, all three models will be represented when students share.

**SUPPORT** If some students are not using any of these tools or models, look to see if their work could be clarified or elevated by using them. Talk with them about their approach to the problem and show them how their work would look with base ten area pieces or on a ratio table or on the grid paper.

8 After most pairs have solved the first problem, reconvene the class and invite selected students to share how they found the answer. As they present, have them first explain their estimate and then their strategy.

Invite students who used base ten area pieces, base ten grid paper, or ratio tables to share their work.

*You might also consider inviting a student who solved the problem without modeling to share, then follow up with a student whose work models the same thinking. Make explicit connections between the examples with the class.*

9 Have students take out their math journals and write about the strategies that were shared, using the heading “Multiplication Strategies.”

*Teacher* It’s so interesting how you used these different strategies and all came up with the same answer. I’d like you to consider which of these strategies makes the most sense to you and which seems the most efficient.

Which one will you try the next time you find a problem like this?

10 Ask students to find the Maggie’s Chickens Student Book page, and let them know that they will work with a partner to solve the rest of the problems. Explain the directions and have students complete the assignment.

• Tell students that they need to fill in the first two columns on the table before solving the problem.

• Explain that students can continue to use their journals for their work, and then fill in the answer in the last column when they are done.

• Remind students that even though they are working with a partner, both partners’ pages should be filled out.

• Encourage students to use the most efficient strategy they can at this time. Encourage them to try out a strategy that was just shared.

*As students work through the rest of the sheet, they may notice that the answers to problems 1 and 2 are the same, as are the answers to problems 3 and 4. This is intentional. The answers are the same because one factor doubles while the other halves. In problem 1, the multiplication equation is 12 × 13. In problem 2, the equation is 6 × 26. The 12 was halved and the 13 doubled. When one factor doubles and the other halves, the product stays the same. While this sounds straightforward, it will take a while for fourth graders to grasp the idea and understand why it works. In your conversations with students today and in the math forum tomorrow, encourage speculation about the answers. Encourage students to think about why they might be the same, but don’t rush their thinking. As the unit progresses, students will have more opportunities to solve similar problems and will explore models to help them come to their own understandings of why the product stays the same.*
11 As students work, walk around the room to provide challenge and support as needed. As you circulate, use your copy of the Multiplication Strategies Forum Planner Teacher Master to make notes about the strategies students are using.

The Forum Planner indicates what kinds of strategies you’re likely to see.

**SUPPORT** If students are using repeated addition or skip-counting, model showing that in a ratio table, just like they created in Session 5. When they have done a few entries, ask them if they could skip straight to the \( \times 10 \) entry and go from there.

**SUPPORT** If students using repeated addition or skip-counting need more support to develop multiplicative strategies, model their work on a number line so they can see opportunities for more efficient grouping.

**SUPPORT** If students are frustrated by the larger numbers or not sure how to start, help them outline the coop using base ten area pieces or grid paper to model the problem. Then help them find the total of the pieces or boxes.

**SUPPORT** If students are using base ten area pieces but are forgetting to use the units in the bottom right corner of the array, encourage them to use linear pieces and help them make sure their array has all of the parts.

**CHALLENGE** If students can find the answers accurately and easily using strategies they learned this year, encourage them to analyze the problems to see if they can discover a new strategy or new understanding of multiplication. For example, if they wonder why some of the answers are the same, ask questions to help them think about why that might be without telling them.

**CHALLENGE** If students are comfortable using base ten area pieces and base ten grid paper, talk to them about how to set up a ratio table to find the answer. Show them how to set up a basic T-chart and how to figure out what goes in each column. Each row in a ratio table corresponds with a row or column in an array. Help them find more efficient ways to use a ratio table, such as do \( 1 \times n \), \( 2 \times n \), and then jump to \( 10 \times n \). Show how \( 1 \times n \) in a ratio table corresponds to a row in an array while the \( 10 \times n \) corresponds to 10 rows in an array (and a mat in the base ten area pieces).

**CHALLENGE** If students are using base ten area pieces, help them move from the concrete to the representative or abstract models. If they are sketching on grid paper, show them how to sketch their work using open arrays. You will work more on this transition in the next few sessions.

**CHALLENGE** If students are using grid paper and dividing 2-digit \( \times \) 2-digit products into all 4 pieces, encourage them to look for fewer, bigger chunks. Can they leave one factor whole and just split the other?

12 Close the session.

- Let students know they will share their work in a math forum during the next session.
- Ask students to raise their hands if they:
  - Tried a new strategy today
  - Worked through confusion to make sense of something hard
  - Had an estimate that was close to their actual answer
- Have students clean up and put away their materials.

*If students did not finish at least two story problems, give them some time to keep working at the beginning of the next session.*

### Daily Practice

The optional Eggs & Apples Student Book page provides additional opportunities to apply the following skills:

- Multiply two 2-digit numbers using strategies based on place value and the properties of operations (4.NBT.5)
- Use equations, area models, and rectangular arrays to explain strategies for multiplying with multi-digit numbers (4.NBT.5)
Session 2
Multiplication Story Problems Math Forum

Summary
Students review their work from the previous session in preparation for a math forum. The teacher guides the students as they present and participate in the forum, which centers on discussion of strategies and models for 2-digit multiplication. Then students spend the rest of the session visiting Work Places. To end the session, the teacher assigns the Multiplying by Multiples of Ten Home Connection.

Skills & Concepts
- Multiply two 2-digit numbers using strategies based on place value and the properties of operations (4.NBT.5)
- Use equations and rectangular arrays to explain strategies for multiplying with multi-digit numbers (4.NBT.5)
- Construct viable arguments and critique the reasoning of others (4.MP.3)
- Attend to precision (4.MP.6)

Materials

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<tr>
<th>Copies</th>
<th>Kit Materials</th>
<th>Classroom Materials</th>
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<tbody>
<tr>
<td>Math Forum</td>
<td>Multiplication Story Problems Math Forum</td>
<td></td>
</tr>
<tr>
<td>TM T1</td>
<td>Base Ten Grid Paper</td>
<td>- base ten area pieces (1 set for display) - base ten linear pieces (1 set for display) - student math journals - Maggie's Chickens Student Book page (SB 66, completed in Session 1) - Multiplication Strategies Forum Planner (TM T2, with notes from Session 1)</td>
</tr>
</tbody>
</table>

Work Places in Use
1C The Multiple Wheel (introduced in Unit 1, Module 2, Session 5)
1D Spinning Around Multiplication (introduced in Unit 1, Module 2, Session 6)
1E Products Four in a Row (introduced in Unit 1, Module 3, Session 1)
1F Dragon's Gold (introduced in Unit 1, Module 3, Session 4)
2A What's Missing? Bingo (introduced in Unit 2, Module 1, Session 4)
2B Division Capture (introduced in Unit 2, Module 2, Session 1)

Home Connection
HC 35–36
Multiplying by Multiples of Ten

Daily Practice
SB 68
Mario's Marbles

Vocabulary
An asterisk [*] identifies those terms for which Word Resource Cards are available.
array* area model equation* multiple* ratio table*

Preparation
Write the list of Work Places from which students can choose today. You can just write the numbers (1C - 2B) or write out the full names if you have time. (See the list in the Work Places in Use row of the Materials Chart for the complete list of Work Places used today.)
Math Forum

Multiplication Story Problems Math Forum

1. Open the session by letting students know they will have a few minutes to look over their work from yesterday to prepare for today’s math forum on multiplication story problems.

2. Have students find their Maggie’s Chickens Student Book page from the previous session and then review the questions and their work with their partner. If necessary, give students time to finish solving the problems. It is OK if a few pairs do not finish problem 4.

3. As students work, check in with the students you have selected to share. Talk with them about which part of their work you would like them to present.

4. Gather students in the discussion area. Have students leave their work at their seats where it is still easily accessible.

5. Review the procedure for participating in a math forum, and then conduct the forum using the Multiplication Strategies Forum Planner you filled in during Session 1.

   - Only some student pairs will present their work, but everyone will participate by adding to what the presenters share, asking questions, talking with a partner, and thinking about how they can use what is shared to solve other problems.

   - Forums are times for learning. As students talk and think, they should be learning something new or understanding something more clearly.

   - All students need to be respectful at all times.

   SUPPORT As you work through the forum, encourage students to ask questions when they don’t understand. Ask all students to justify their answers so that struggling students will know that it is the norm, not an indication that they are wrong, when they are questioned.

   CHALLENGE Encourage students to search for elegant, clever solutions. Ask students to compare their first choice of strategy with other strategies and discuss which is the most efficient for these numbers. Encourage them to generalize what characteristics of certain numbers lend the problem to particular strategies.

   ELL Provide opportunities for students to request clarifications and rephrasing. As the forum becomes a platform for sharing, the classroom atmosphere will help students feel they can ask for help. Offer visual models, and provide a same-language peer if available.

6. Collect students’ Maggie’s Chickens Teacher Master and wrap up the forum.

   - Tell students you look forward to seeing them try some of the strategies that were explored.

   - File the pages to keep as work samples to use in parent conferences, portfolios, etc.

Work Places

7. Invite students to choose a Work Place activity to play with a partner and go to that Work Place.

   - Have students pick up their Work Place folders and a pencil. Remind them to fill out their Work Place Logs as they finish each one. If necessary, review the procedure for going to Work Places.
As students visit Work Places, walk around to observe students and answer questions. Use the Differentiation & Assessment Chart in the Work Place Guide to help you work with students.

Close the session.
- Have students put away Work Place materials.
- Ask students to turn and talk to a partner about their current favorite strategy for multiplication and how they would teach that strategy to their parents.

### Home Connection

Introduce and assign the Multiplying by Multiples of Ten Home Connection, which provides more practice with the following skills:
- Solve multi-step story problems involving only whole numbers, using addition, subtraction, and multiplication (4.OA.3)
- Multiply a 2- or 3-digit whole number by a 1-digit whole number using strategies based on place value and the properties of operations (4.NBT.5)
- Multiply two 2-digit numbers using strategies based on place value and the properties of operations (4.NBT.5)

### Daily Practice

The optional Mario’s Marbles Student Book page provides additional opportunities to apply the following skills:
- Multiply a 2-digit whole number by a 1-digit whole number using strategies based on place value and the properties of operations (4.NBT.5)
- Multiply two 2-digit numbers using strategies based on place value and the properties of operations (4.NBT.5)
- Determine whether a whole number between 1 and 100 is prime or composite (4.OA.4)
Session 3
Doubling & Halving

Summary
Today’s session begins with a problem string to prompt thinking about doubling and halving. Then students complete a brief checkpoint on multiplication arrays, equations, and ratio tables. Students spend the rest of the session doing Work Places.

Skills & Concepts
- Solve multi-step story problems involving only whole numbers, using multiplication (4.OA.3)
- Multiply a 2-digit whole number by a 1-digit whole number using strategies based on place value and the properties of operations (4.NBT.5)
- Multiply two 2-digit numbers using strategies based on place value and the properties of operations (4.NBT.5)
- Use equations and area models to explain strategies for multiplying with multi-digit numbers (4.NBT.5)
- Divide a 2-digit number by a 1-digit number, using strategies based on place value, the properties of operations, or the relationship between multiplication and division (4.NBT.6)
- Apply the area formula for a rectangle to solve a problem (4.MD.3)
- Construct viable arguments and critique the reasoning of others (4.MP.3)
- Model with mathematics (4.MP.4)
- Look for and express regularity in repeated reasoning (4.MP.8)

Materials

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<tr>
<th>Copies</th>
<th>Kit Materials</th>
<th>Classroom Materials</th>
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</thead>
<tbody>
<tr>
<td><strong>Problem String</strong> Doubling &amp; Halving</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TM T1 Base Ten Grid Paper</td>
<td>• student math journals</td>
<td></td>
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<tr>
<td><strong>Assessment</strong> Multiplication Checkpoint</td>
<td></td>
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<tr>
<td>TM T3–T4 Multiplication Checkpoint</td>
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<table>
<thead>
<tr>
<th>Work Places in Use</th>
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<tbody>
<tr>
<td>1C The Multiple Wheel (introduced in Unit 1, Module 2, Session 5)</td>
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<td>1D Spinning Around Multiplication (introduced in Unit 1, Module 2, Session 6)</td>
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<td>1E Products Four in a Row (introduced in Unit 1, Module 3, Session 1)</td>
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<tr>
<td>1F Dragon’s Gold (introduced in Unit 1, Module 3, Session 4)</td>
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<tr>
<td>2A What’s Missing? Bingo (introduced in Unit 2, Module 1, Session 4)</td>
</tr>
<tr>
<td>2B Division Capture (introduced in Unit 2, Module 2, Session 1)</td>
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<table>
<thead>
<tr>
<th>Daily Practice</th>
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<tbody>
<tr>
<td>SB 69 How Can You Solve…?</td>
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</tbody>
</table>

Vocabulary
An asterisk [*] identifies those terms for which Word Resource Cards are available.
- area*
- dimension*
- double/doubling factor*
- half*
- product*
- relationship

Preparation
- Set up the area where you do your problem strings with a document camera or projector and a few sheets of base ten grid paper. You might want extra grid paper for students who need additional structure for sketching arrays.
- Write the list of Work Places from which students can choose today. You can just write the numbers (1C–2B) or write out the full names if you have time. (See the list in the Work Places in Use row of the Materials Chart for the complete list of Work Places used today.)
**Problem String**

**Doubling & Halving**

1. Let students know that today they will begin with a multiplication problem string and then complete a brief checkpoint. As they finish, they will go to Work Places.

2. Have students join you in the discussion area with their student math journals and a pencil. Then complete the following problem string.
   - Ask them to write today’s date and the title “Doubling & Halving Problem String” on a fresh page in their math journals.
   - Present each problem one at a time by writing it on the board. Present the problems in order from top to bottom.
   - Give students time to solve each one on their own.
   - Invite 2 or 3 students to share their strategies for solving each problem, one at a time, and record their strategies by drawing arrays on base ten grid paper.

**CHALLENGE** Encourage students to use the most efficient or sophisticated strategy they can think of. Then encourage them to look back at their work and see if they can think of a strategy that is even more effective.

**SUPPORT** Some students may benefit from using graph paper or base ten grid paper for this problem string.

**ELL** Model the word “half” and “halving” by cutting a piece of paper in half, or by sketching an array on paper, then cutting it in half to show how one factor changes.

**Problem String** Doubling & Halving, Part 1

<table>
<thead>
<tr>
<th>Problems</th>
<th>Sample Strategies &amp; Recording</th>
<th>Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 × 6</td>
<td><img src="image1" alt="Array" /></td>
<td>This string begins with problems that double or half and then moves to a series that double and half, resulting in the same product.</td>
</tr>
<tr>
<td>8 × 6</td>
<td><img src="image2" alt="Array" /></td>
<td><strong>Big Idea</strong> When one factor doubles, so does the product. When one factor is cut in half, so is the product.</td>
</tr>
<tr>
<td>8 × 3</td>
<td><img src="image3" alt="Array" /></td>
<td></td>
</tr>
</tbody>
</table>

**Math Practices in Action 4.MP.8**

Many problem strings, including this one, involve repeated reasoning. When students solve problems that involve repeating the same kind of reasoning, they begin to recognize patterns that help them develop generalized methods for completing certain kinds of calculations.
Make sure students are comfortable with the doubling and halving that happens in the first three problems before moving to the last four problems, which involve both doubling and halving in the same problem.

**Problem String** Doubling & Halving, Part 2

<table>
<thead>
<tr>
<th>Problems</th>
<th>Sample Strategies &amp; Recording</th>
<th>Connections</th>
</tr>
</thead>
</table>
| 8 × 6    | See above.                    | It might help students if you cut one array into 2 pieces and move the pieces around to show that although the dimensions (factors) change, the area (product) stays the same. For example, if you cut the 8 × 6 array into two 4-by-6 arrays and then put them back together as a 4-by-12 array, students will see that 8 × 6 = 4 × 12.
| 4 × 12   |                               | Most likely, not all students will see the double-half relationship when you get to 4 × 12, and that’s OK. By the time they see what happens with 2 × 24 and 1 × 48 on the arrays and the ratio table, they will begin to understand. Over time, students will see why understanding this relationship is useful—it creates efficient strategies for harder problems. For example, 3 ½ × 8 does not look very friendly, but doubling the 3 ½ and halving the 8 makes 7 × 4, a much simpler problem to solve. |
| 2 × 24   |                               |             |
| 1 × 48   |                               |             |

3 Write the equation 4 × 12 = 2 × 24, and ask students to turn and talk about the equation with a partner.
   - Ask whether the equation is true or false.
   - Invite a few students to share their thinking.

4 Write the equations 3 × 14 = 6 × 7 and 5 × 12 = 10 × 24. Ask students to turn and talk with a partner about whether the equations are true or false.
   - Invite a few students to share their thinking.
   - Ask students how they could change the equation 5 × 12 = 10 × 24 to make it true.

5 Wrap up the string by asking students to work in pairs to summarize the big idea from the string.
   - As students talk, listen for those who can explain what is happening with doubling and halving the most clearly. Try to call on someone who can explain that when one factor doubles and the other factor is halved, the product stays the same. Have them use a visual model, or model their thinking as they explain, for other students to see.

*It is important that this part of the session ends with a clear summary of the big idea. You may need to give a clear, concise summary if one is not presented by students.*

   - Assure students who may still be confused that there will be more time to process the big ideas discussed today.
Assessment

Multiplication Checkpoint

6 Tell students that today they will show what they have learned about multiplication over the past couple of weeks on a quick checkpoint.

7 Display the Multiplication Checkpoint and give students a moment to look it over.
   • Encourage students to read each question carefully. If necessary, read through the checkpoint with students.
   • Let students know they can ask you for help in reading any of the questions.

8 Have students begin the checkpoint.
   • Tell them they need to work quietly by themselves.
   • While they work, circulate around the room to make observations and answer questions.
   • Give them about 15 minutes or so to do the checkpoint. This is not a timed test, so if some students do not finish the checkpoint in 20 minutes, give them a chance to finish it later.
   • If some students finish much earlier than others, ask them to begin Work Places quietly so they don’t disturb those who are still working.

9 Have students turn in their checkpoints as they finish.

Work Places

10 Have students spend the rest of the session doing Work Places.
   Have them pick up their Work Place folders and a pencil. Remind them to fill out their Work Place Logs as they finish each one.
   • As students do Work Places, circulate to observe, differentiate, and answer questions.

11 Close the session. If you have time, ask students to consider what factor they might change (double or halve) in the expression 5 × 16 to make it easier to solve, and why.
   • Have students put away Work Place materials.
   • Recognize students for their effort on the checkpoint. Remind them that the checkpoint will help identify areas in multiplication where students are strong or need more work.

Daily Practice

The optional How Can You Solve …? Student Book page provides additional opportunities to apply the following skills:
   • Multiply a 2-digit whole number by a 1-digit whole number using strategies based on place value and the properties of operations (4.NBT.5)
Session 4
Multiplying with Money & Units of Metric Measure

Summary
This session begins by connecting students’ work with ratio tables to money and measurement conversions. Then the teacher introduces a new Work Place called Moolah on My Mind, which uses the landmark numbers of coin values to provide more practice with multi-digit multiplication. The class plays several rounds of the game as a whole group, and then students play again in pairs. To end the session, the teacher introduces and assigns the Design a Floor Plan Home Connection.

Skills & Concepts
• Multiply 2 and 3-digit whole numbers by a 1-digit whole number and two 2-digit numbers using strategies based on place value and the properties of operations (4.NBT.5)
• Use equations and rectangular arrays to explain strategies for multiplying with multi-digit numbers (4.NBT.5)
• Identify the relative sizes of centimeters, meters, and kilometers; grams and kilograms; and milliliters and liters (4.MD.1)
• Express a measurement in a larger unit in terms of a smaller unit within the same system of measurement (4.MD.1)
• Record equivalent measurements in different units from the same system of measurement using a 2-column table (4.MD.1)
• Reason abstractly and quantitatively (4.MP.2)
• Look for and express regularity in repeated reasoning (4.MP.8)

Materials

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<th>Classroom Materials</th>
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<tbody>
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<td>Problems &amp; Investigations Multiplying with Money &amp; Metric Measurements</td>
<td>money value pieces (optional, for support)</td>
<td>student math journals items that represent metric measurements (optional, see Preparation)</td>
</tr>
<tr>
<td>Work Places Introducing Work Place 2C Moolah on My Mind</td>
<td></td>
<td></td>
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<tr>
<td>TM T5 Work Place Guide 2C Moolah on My Mind</td>
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<tr>
<td>TM T6 2C Moolah on My Mind Record Sheet</td>
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<tr>
<td>TM T7 2C Moolah on My Mind Challenge Record Sheet</td>
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<tr>
<td>SB 70* Work Place Instructions 2C Moolah on My Mind</td>
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</tr>
<tr>
<td></td>
<td>spinner overlay, half-class set, plus 1 for display</td>
<td>marker</td>
</tr>
<tr>
<td></td>
<td>dice numbered 1–6, class set</td>
<td></td>
</tr>
<tr>
<td></td>
<td>money value pieces (optional, for support)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>plastic coins (optional, for support)</td>
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<tr>
<td>Home Connection</td>
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<td>HC 37–39 Design a Floor Pattern</td>
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<tr>
<td>Daily Practice</td>
<td></td>
<td></td>
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<tr>
<td>SB 71 How Much Money?</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Vocabulary
An asterisk [*] identifies those terms for which Word Resource Cards are available.
centimeter (cm)*
coin
dime
gram (g)*
gram (kg)*
landmark number
liter (l)*
meter (m)*
milliliter (ml)*
multiply*
nickel
penny
quarter

* Run 1 copy of this page for use by the teacher and other adult helpers during Work Place time.
**Preparation**

- You might find it helpful to collect a few objects that represent different metric measurements for students to see. For example, for units of length, a meter stick can show centimeters, meters, and millimeters. For mass, a quarter weighs 5 grams, or a small can of soup is about 300 grams. For volume, a 1 liter bottle offers a landmark, and many reusable water bottles are marked with milliliters on the side.
- In today’s session, you’ll introduce Work Place 2C Moolah on My Mind, which takes the place of Work Place 1C the Multiple Wheel. Before this session, you should review the Work Place Instructions as well as the Work Place Guide, which includes suggestions for differentiating the game to meet students’ needs. Make copies of the Moolah on My Mind Record Sheet for use today, and store the rest in the Work Place 2C Moolah on My Mind tray.

**Problems & Investigations**

**Multiplying with Money & Metric Measurements**

1. Set the stage for today’s activities.
   
   Let students know that you’re going to start by making and discussing some ratio tables that concern multiplying coin values and metric measurements. Then you’re going to introduce a new Work Place that will give them practice multiplying with coin values. When they understand how the game works, they’ll play it in pairs.

2. Open the session by asking students to discuss in pairs and then report to the class how many dimes there are in a dollar, how many nickels, and how many quarters.
   
   As students respond, record the information in the form of ratio tables like those shown here.

   - **Dimes**: $10 \times 10 = 100$ cents
   - **Nickels**: $20 \times 10 = 200$ cents
   - **Quarters**: $4 \times 1 = 4$ quarters

3. Have students recall the metric units for measuring length. Then discuss these terms and their relationships.
   
   Supply the terms *meter* and *centimeter* if students don’t mention them.

   Ask some questions about meters and centimeters and use ratio tables to represent the relationship.
   
   - Money has dollars and cents. Length has meters and centimeters. How does “cent” relate to both contexts? (There are 100 cents in a dollar and 100 centimeters in a meter. “Cent” is from the Latin word “centum” meaning hundred.)
   - How many centimeters are in a meter? (100)
   - If a string is 2 meters long, how many centimeters is it? (200)
   - If a car is 4 meters long, how many centimeters is it? (400)
**Support/ELL** Show a meter stick. Point to the centimeters when referring to centimeters and to the whole stick when referring to meters. Similarly, in the following steps show objects that have the approximate mass and volume as you bring up the terms.

**Challenge** Ask students how many centimeters are in 2 ½ meters and in ¼ of a meter.

4 Ask students to recall the metric units for measuring mass. Then discuss the terms and their relationships.

Supply the units gram and kilogram if students don’t mention them.

Ask some questions about grams and kilograms and use ratio tables to represent the relationship.

- What does “milli” mean? [1,000]
- How many grams are in a kilogram? [1,000]
- If a book is a mass of 2 kilograms, what is its mass in grams? [2,000]
- If a puppy has a mass that is 4 times as much as the mass of the book, what is its mass in kilograms? [8] What is its mass in grams? [8,000]

![Ratio Table for Grams and Kilograms](image)

5 Now have students recall the metric units for measuring volume and discuss the terms and their relationships.

Supply the units liter and milliliter if students don’t mention them.

Ask some questions about liters and milliliters, and use ratio tables to represent the relationship.

- How many milliliters are in a liter? [1,000]
- If a bottle has a volume of 2 liters, how many milliliters does it hold? [2,000]
- If a pitcher has a volume of that is 3 times as much as the bottle, how many liters does it hold? [6] What is its volume in milliliters? [6,000]

![Ratio Table for Liters and Milliliters](image)

6 Ask students to generalize what all of these situations have in common.

*Students* When you double one, you double the other.

Or if you multiply something, like the liters, by 3 or any other number, you multiply the milliliters by the same number.

There are the same number of centimeters in a meter as cents in a dollar.

There are the same number of milliliters in a liter as grams in a kilogram.

You can use a ratio table to find the answers.

7 Have students write the heading “Metric Prefixes” to a new page in their handbooks and record centi-, milli-, and kilo- along with their meanings.
You might also have students add an illustration to help them remember the meaning of the prefixes, such as penny for centi-, a few millimeter marks for milli-, or a grapefruit to show a kilogram.

- “Centi-” means hundredth (1/100).
- “Milli-” means thousandth (1/1000).
- “Kilo-” means thousand (1000).

## Work Places

### Introducing Work Place 2C Moolah on My Mind

8. Then display the 2C Moolah on My Mind Record Sheet. Explain that the class will play a few rounds of the new Work Place game as a team against you, and then they will play the game with a partner.

9. Summarize the game:
   Players take turns rolling two dice and spinning a coin spinner. They add the numbers rolled and then multiply the sum of the numbers by the value of the coin. Then they add the running totals of money after each turn. The player who collects the most money after 10 turns wins the game.

10. Play three rounds of Moolah on My Mind with the class.

Model strategies for students when it is your turn.

- Roll the two dice and write the sum of the dice in parentheses. Then spin the spinner and add the coin value to the expression.
- Before calculating the total, reinforce that you always perform the computation in parentheses first.
- Talk through how you mentally calculate the total using what you know about money (i.e., calculating the value of 8 dimes is the same as multiplying 8 × 10).
- Add the running totals as you play each round.

You will need to decide, based on your students’ current understandings, whether to record money amounts that are under a dollar using a cents sign or a dollar sign and decimal point.
Encourage students’ active participation throughout the game.
- Ask for students’ input to write the multiplication expression.
- To find the total:
  - First ask students to mentally calculate the total of the roll and spin.
  - Then ask them to discuss the strategies they used.
  - Encourage alternative strategies to skip-counting and repeated addition.

**SUPPORT/ELL** Consider offering money value pieces as a tool to clarify the proportional relationships between coins and their values in relationship to a dollar. After this session, put the money value pieces into the Work Place tray for students who need them during Work Places.

11 At the conclusion of the game, ask students to find the difference between the scores.
Ask students how much the winner won and encourage mental calculations for finding the difference.

**SUPPORT** If you have students who are still struggling to make sense of multi-digit subtraction, you may want to record the difference in scores in the form of a subtraction combination and work through the process of finding the difference more explicitly, using a number line if necessary.

12 Have student pairs play Moolah on My Mind.
- Give each pair 2 copies of the 2C Moolah on My Mind Record Sheet, a die numbered 1–6, and a clear spinner overlay.
- See the Moolah on My Mind Work Place Guide for suggestions on how to differentiate.

For example, you might pull together a small group of students who need help developing efficient strategies to multiply by 1, 5, 10, and 25, and play the game together. Model and share your own thinking, and encourage students to help one another as you play.

13 Close the session by having students share some strategies they used for keeping track of their totals.

---

**Home Connection**

14 Introduce and assign the Design a Floor Pattern Home Connection, which provides more practice with the following skills:
- Multiply two 2-digit numbers using strategies based on place value and the properties of operations (4.NBT.5)
- Solve story problems involving money using addition and multiplication of whole numbers (4.MD.2)

---

**Daily Practice**

The optional How Much Money? Student Book page provides additional opportunities to apply the following skills:
- Solve story problems involving a multiplicative comparison using multiplication or division (4.OA.2)
- Multiply a 2-digit whole number by a 1-digit whole number using strategies based on place value and the properties of operations (4.NBT.5)
- Solve story problems involving money using addition and multiplication of whole numbers (4.MD.2)
Session 5
Another Look at Multiplication Strategies

Summary
Students solve a story problem that involves $6 \times 26$. They share their strategies and add new strategies to the Multiplication Strategies chart that they began in Module 2, Session 2. Then they write their own story problems for $8 \times 24$ and $11 \times 22$ and solve them in an assignment that can be collected as a work sample. Students who complete the assignment before the end of the session go to Work Places.

Skills & Concepts
- Multiply 2-digit by 1-digit numbers or two 2-digit numbers using strategies based on place value and the properties of operations (4.NBT.5)
- Use equations and rectangular arrays to explain strategies for multiplying with multi-digit numbers (4.NBT.5)
- Solve story problems involving distance and money, using multiplication of whole numbers (4.MD.2)
- Make sense of problems and persevere in solving them (4.MP.1)
- Construct viable arguments and critique the reasoning of others (4.MP.3)

Materials

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<tr>
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<tbody>
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<td>Another Look at Multiplication Strategies</td>
<td></td>
</tr>
<tr>
<td>TM T8</td>
<td>Maribel's Problem</td>
<td>• base ten area pieces (class set, plus 1 set for display)</td>
</tr>
<tr>
<td>TM T1</td>
<td>Base Ten Grid Paper</td>
<td>• base ten linear pieces (class set, plus 1 set for display)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• student math journals</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Multiplication Strategies chart from Session 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• markers</td>
</tr>
<tr>
<td>Assessment</td>
<td>Multiplication Strategies Work Sample</td>
<td></td>
</tr>
<tr>
<td>TM T1</td>
<td>Base Ten Grid Paper</td>
<td></td>
</tr>
<tr>
<td>TM T9</td>
<td>Multiplication Strategies Work Sample</td>
<td></td>
</tr>
</tbody>
</table>

Work Places in Use
1D Spinning Around Multiplication (introduced in Unit 1, Module 2, Session 6)
1E Products Four in a Row (introduced in Unit 1, Module 3, Session 1)
1F Dragon's Gold (introduced in Unit 1, Module 3, Session 4)
2A What's Missing? Bingo (introduced in Unit 2, Module 1, Session 4)
2B Division Capture (introduced in Unit 2, Module 2, Session 1)
2C Moolah on My Mind (introduced in Unit 2, Module 3, Session 4)

Daily Practice
SB 72 Money Multiple Wheel

Vocabulary
An asterisk [*] identifies those terms for which Word Resource Cards are available.
area*
area model
array*
kilometer (km)*
multiply*
ratio table*
strategies

Preparation
Write the list of Work Places from which students can choose today. You can just write the numbers (1D–2C) or write out the full names if you have time. (See the list in the Work Places in Use row of the Materials Chart for the complete list of Work Places used today.)
Problems & Investigations

Another Look at Multiplication Strategies

1. Start the session with a quick problem string: $0.25 \times 8$, $0.50 \times 8$, $0.50 \times 4$, $1.00 \times 2$.
   - Give students each problem, one at a time, with time to work each problem in their journals and model one or two strategies.
   - Center the discussion on these relationships:
     » Twice $0.25$ is $0.50$.
     » When you double one factor, the product doubles.
     » When you halve one factor, the product halves.

2. Display the Maribel’s Problem Teacher Master and read the story problem out loud. Ask students to write answers to the first two questions in their journals and then share their ideas with a partner.

3. Select a few volunteers to share their thinking with the class.
   - Write a sentence or two on the display copy to summarize the group’s thinking.
   - Write the multiplication expression that would be used to represent the situation $(6 \times 26 \text{ km})$.
   - Review the importance of including the unit of measurement, as well as the abbreviation for kilometers.

4. Then ask students to share some of the strategies they might use to solve the problem.
   Although some students will compute and share the answer quickly, take time during the discussion for students to share a wide variety of strategies for finding the solution. Ask students who “just know it” to backtrack and figure out what went on in their heads prior to knowing it.
   - As each strategy is volunteered, record it on the teacher master or the board, and work it through with student input to see the results.
   - Throughout the discussion, ask students to consider how some strategies are similar and to identify those strategies that seem to be most efficient.
   - If appropriate, add new strategies to the Multiplication Strategies chart. Use models that fit the students’ strategies.

The table below features strategies and models fourth graders are likely to propose.

<table>
<thead>
<tr>
<th>Name</th>
<th>Date</th>
</tr>
</thead>
</table>

Maribel’s Problem
Maribel and her 5 friends have a running club. Next week, they will all run a metric marathon. (A metric marathon is a race of 26 kilometers.) The club keeps a record of the total number of kilometers run by all members of the club combined. How many kilometers will they add to the record after they all run the metric marathon?

1. What is this problem asking us to do?
   Find out how many kilometers Maribel and her 5 friends ran in all.

2. What multiplication expression can we write to represent this problem?
   $6 \times 26 \text{ km}$
## Multiplication Strategies

<table>
<thead>
<tr>
<th>Description</th>
<th>Visual Model or Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Break the number into place-value parts and use any one of the next four models:</td>
<td></td>
</tr>
<tr>
<td><strong>Make an Array with Base Ten Pieces</strong>&lt;br&gt;Use the linear pieces to frame a 6-by-26 array and then fill it in with strips and units to find the area.</td>
<td><img src="image" alt="Array Diagram" /></td>
</tr>
<tr>
<td><strong>Make a Sketch on Base Ten Grid Paper</strong>&lt;br&gt;Mark the frame for a 6-by-26 array on the grid paper and then fill it in to find the area.</td>
<td><img src="image" alt="Base Ten Grid Diagram" /></td>
</tr>
<tr>
<td><strong>Make a Quick Sketch</strong>&lt;br&gt;Show the linear dimensions and then fill in the rectangles with the partial products to find the total area.</td>
<td><img src="image" alt="Quick Sketch Diagram" /></td>
</tr>
<tr>
<td><strong>Make a Quick Sketch</strong>&lt;br&gt;Sketch a 6-by-20 rectangle and a 6-by-6 rectangle, then fill in the rectangles with the partial products to find the total area.</td>
<td><img src="image" alt="Quick Sketch Diagram" /></td>
</tr>
<tr>
<td><strong>Use a Friendly Number</strong>&lt;br&gt;Multiply 25 groups of 6 and then add one more group of 6 to find the total. (This strategy can be modeled on a ratio table.)</td>
<td><img src="image" alt="Ratio Table" /></td>
</tr>
<tr>
<td><strong>Use Money as a Benchmark</strong>&lt;br&gt;Think about quarters, since 26 is very close to 25.</td>
<td><img src="image" alt="Money Benchmark Diagram" /></td>
</tr>
<tr>
<td><strong>Skip-count</strong>&lt;br&gt;Make 6 jumps of 26 or 6 jumps of 25 plus one of 6. Model the strategy on a number line.</td>
<td><img src="image" alt="Number Line Diagram" /></td>
</tr>
<tr>
<td><strong>Decompose the Numbers</strong>&lt;br&gt;Break 26 into 20 and 6 and then multiply by 6, applying the distributive property. Add the two products to get the total.</td>
<td><img src="image" alt="Decomposition Diagram" /></td>
</tr>
<tr>
<td><strong>Use the Standard Algorithm</strong></td>
<td><img src="image" alt="Algorithm Diagram" /></td>
</tr>
</tbody>
</table>
If any of the students proposes using the standard algorithm, invite students to make sense of the strategy by pair-sharing and then sharing their insights as a class. Have them model how the process works using base ten area pieces or sketches, and ask them to show where they see the numbers in the model.

Alejandro What I do on these—my mom showed me—is first multiply 6 × 6. That’s 36. But 36 is too much to put in the ones place. You have to carry the 3 and put it over in the tens column, above the 2.

Teacher So you start with the ones, and you multiply 6 × 6 to get 36. Where can you see the 36 in our 6-by-26 array?

Alejandro It’s the 6-by-6 array over on the end.

Teacher And then what do you do next?

Alejandro Well, I just go 6 times 2 is 12, but I guess it’s really 6 times 20. I can see that with the base ten area pieces. There’s 60 and then 60 more. That’s the same as 120.

Ariel What about the 3?

Alejandro That’s really 30, and I just add it in. It’s like I grab 30 from the 36 and trade them in for tens, so I have 12 tens and I just add on 3 more tens. So the answer is 156.

If no one mentions the standard algorithm, you might want to share the method yourself, taking care that students understand that it’s only one of many viable options (and not necessarily the most efficient in some cases). Regardless of whether you or a student proposes the algorithm, be sure that students see how the process connects to the array model.

Students may also find the strategy easier to follow if you demonstrate the following method of recording that shows each partial product rather than “carried” tens:

\[
\begin{array}{r}
26 \\
\times 6 \\
\hline
120 \\
+ 36 \\
\hline
156
\end{array}
\]

\[
\begin{array}{r}
26 \\
\times 6 \\
\hline
36 \\
+120 \\
\hline
156
\end{array}
\]
**Assessment**

**Multiplication Strategies Work Sample**

5 Display your copy of the Multiplication Strategies Work Sample Teacher Master, as helpers give each student a copy of the sheet.

- Review the page with the class, and clarify as needed. If you plan to collect and score this assignment as a work sample, let students know.
- When students understand what to do, give them the remainder of the session to complete the sheet independently.
- Students can use any of the math manipulatives they find useful, as well as base ten grid paper.
- Remind students that they can refer to the class Multiplication Strategies chart or the strategies you’ve recorded for the problem the class just solved.

**CHALLENGE** Invite students who finish the assignment quickly with ease and accuracy to solve the challenge problem at the bottom of the sheet. If you’re going to collect and score the page as a work sample, you might let them know that the challenge item will not be scored.

6 Have students turn in the assignment as they finish and go to Work Places.

**Work Places**

7 Invite students who finish early to choose Work Places quietly so they don’t disturb those who are still working.

8 Close the session.

- If time allows, run through a quick problem string orally: 2 × 8, 4 × 8, 8 × 8, 10 × 8, 9 × 8, 90 × 8. As you ask each fact, call on students to tell how it relates to the previous fact.
- Collect students’ completed Multiplication Strategies sheets as a work sample if desired.

**Daily Practice**

The optional Money Multiple Wheel Student Book page provides additional opportunities to apply the following skills:

- Solve story problems involving a multiplicative comparison using multiplication or division (4.OA.2)
- Determine whether a whole number between 1 and 100 is prime or composite (4.OA.4)
- Multiply a 2-digit whole number by 10 or 100 (supports 4.NBT)
- Multiply a 2- or 3-digit whole number by a 1-digit whole number using strategies based on place value and the properties of operations (4.NBT.5)
- Multiply two 2-digit numbers using strategies based on place value and the properties of operations (4.NBT.5)
- Solve story problems involving money using addition, subtraction, multiplication, and division of whole numbers and decimals (4.MD.2)
Base Ten Grid Paper

(continued on next page)
### Multiplication Strategies Forum Planner

Use this planner to make a record of the strategies you see students using to solve problems during Session 1. Prior to Session 2, use the third column to indicate the order in which you plan to have students share during the forum.

#### Strategies for solving $16 \times 12$

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Student Names and Notes</th>
<th>Order of Sharing in Forum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skip-count by 12s or 16s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Students began to use skip-counting or repeated addition and then realized it was more efficient to show their work in a ratio table</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Ratio Table</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Students started with the ratio table and were able to use it efficiently. (For example, they realized they could double or use 10 times instead of doing $1 \times a$, $2 \times a$, $3 \times a$ and so on.) These students might have found $10 \times 16$ and added it to $2 \times 16$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Base Ten Area Pieces or Area Model</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Students used base ten area pieces or the area model on base ten grid paper. These students might divide the $16 \times 12$ array into 4 pieces.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OR</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Students might divide the $16 \times 12$ array into a $16 \times 10$ and add it to $16 \times 2$ Ask the class to find connections between what they see on the ratio table and what they see in the base ten pieces or area model.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Strategies for solving $24 \times 8$

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Student Names and Notes</th>
<th>Order of Sharing in Forum</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Area Model</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Students use an array to find $24 \times 8$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Multiplication Checkpoint  page 1 of 2

1  Fill in the blanks and complete the equations for each array.

\[
\begin{array}{c|c}
6 & 60 \\
\hline
6 & \_ \_ \_ \\
\end{array}
\]

\[
\begin{array}{c|c|c}
10 & 100 \\
\hline
50 & 5 \\
\end{array}
\]

\[
6 \times \_ \_ \_ = \_ \_ \_
\]

\[
\_ \_ \_ \times \_ \_ \_ = \_ \_ \_
\]

2  Fill in the blanks in the ratio table below.

<table>
<thead>
<tr>
<th>Bags of Marbles</th>
<th>Marbles in Bags</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>36</td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
</tr>
</tbody>
</table>

3  True or False?

a  \(20 \times 8 = 8 \times (2 \times 10)\) ______

b  \(15 \times 12 = 15 \times (10 \times 2)\) ______

4  Fill in the blanks.

a  \(8 \times 14 = (\_ \_ \_ \times 10) + (8 \times 4)\)

b  \((16 \times 40) = (10 \times 40) + (\_ \_ \_ \times 40)\)

(continued on next page)
Max says he can solve $12 \times 14$ by solving $10 \times 14$ and $2 \times 14$. Do you agree or disagree? Why?

Ashley’s room is 11 by 16 feet. Her brother’s room is 13 by 14 feet. Whose room has more square feet? Show your thinking using numbers, labeled sketches, or words.
Work Place Guide 2C Moolah on My Mind

Summary
Players take turns rolling two dice and spinning a coin spinner. They add the numbers rolled and then multiply the sum of the numbers by the value of the coin. Then they add the running totals of money after each turn. The player who collects the most money after 10 turns wins the game.

Skills & Concepts
- Multiply 2 and 3-digit whole numbers by a 1-digit whole number using strategies based on place value and the properties of operations (4.NBT.5)
- Multiply two 2-digit numbers using strategies based on place value and the properties of operations (4.NBT.5)
- Use equations and rectangular arrays to explain strategies for multiplying with multi-digit numbers (4.NBT.5)
- Solve story problems involving money, using multiplication (4.MD.2)

Materials

<table>
<thead>
<tr>
<th>Copies</th>
<th>Kit Materials</th>
<th>Classroom Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>TM T5</td>
<td>• 6 dice numbered 1–6</td>
<td></td>
</tr>
<tr>
<td>TM T6</td>
<td>• 3 spinner overlays</td>
<td>• plastic coins (optional, for support suggestions)</td>
</tr>
<tr>
<td>TM T7</td>
<td>• money value pieces (optional, for ELL suggestion)</td>
<td></td>
</tr>
<tr>
<td>SB 70</td>
<td>Work Place Instructions 2C Moolah on My Mind</td>
<td></td>
</tr>
</tbody>
</table>

Assessment & Differentiation

<table>
<thead>
<tr>
<th>If you see that…</th>
<th>Differentiate</th>
</tr>
</thead>
<tbody>
<tr>
<td>A student is struggling to get started, struggling with the directions, or unsure of how to do the math.</td>
<td>SUPPORT: Pull together small groups of students and play the game together.</td>
</tr>
<tr>
<td>A student is struggling to add or multiply with money.</td>
<td>SUPPORT: Play a round or two of the game with these students. Provide plastic coins and help students group them into amounts they can easily add. Then have them play on their own with the coins.</td>
</tr>
<tr>
<td>A student is skip-counting or using repeated addition.</td>
<td>SUPPORT: Help students find more efficient ways to multiply. Help students use the landmark numbers of the coins to multiply more efficiently. Encourage them to use doubling, start with a 10 ×, or use what they know about money to help them, as appropriate to the problem. Examples</td>
</tr>
<tr>
<td>A student is very proficient at multiplying.</td>
<td>CHALLENGE: Encourage them to read and implement Variation A, B, C, or D.</td>
</tr>
<tr>
<td>A student can multiply 1- and 2-digit numbers easily.</td>
<td>CHALLENGE: Encourage students to come up with tips or suggestions that would help other students play the game. What shortcuts or strategies can they use? Or, have them find the most efficient strategy for any given problem. Have them race a partner to see who can correctly solve a problem the fastest.</td>
</tr>
</tbody>
</table>

English-Language Learners Use the following adaptations to support the ELL students in your classroom.

- Make sure ELL students are familiar with the coins, their names, and their values. Provide money value pieces, if you have them, or real coins. Give them time to explore the coins and time to practice adding and multiplying quantities of coins.
- Partner ELL students with bilingual students so they can play the game in 2 languages. Encourage the bilingual students to help ELL students learn key vocabulary for the game (coin, penny, nickel, dime, quarter, add, sum, multiply, product, etc.)
**2C Moolah on My Mind Record Sheet**

<table>
<thead>
<tr>
<th>Multiplication Expression</th>
<th>Amount of Money You Got This Turn</th>
<th>Total So Far</th>
</tr>
</thead>
<tbody>
<tr>
<td>sum of the 2 numbers times the coin value</td>
<td>(   +   ) × ¢</td>
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<td></td>
</tr>
</tbody>
</table>
### 2C Moolah on My Mind Challenge Record Sheet

**Multiplication Expression**
- sum, difference, or product of the 2 numbers times the coin value
- Pick one: 
  - \(( + ) \times \$\)
  - \(( - ) \times \$\)
  - \(( \times ) \times \$\)

<table>
<thead>
<tr>
<th>Amount of Money You Got This Turn</th>
<th>Total So Far</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
Maribel's Problem

Maribel and her 5 friends have a running club. Next week, they will all run a metric marathon. (A metric marathon is a race of 26 kilometers.) The club keeps a record of the total number of kilometers run by all members of the club combined. How many kilometers will they add to the record after they all run the metric marathon?

1. What is this problem asking us to do?

2. What multiplication expression can we write to represent this problem?

3. What are some of the different strategies we can use to solve this multiplication problem?
Multiplication Strategies Work Sample

Here are two ways to write the same multiplication problem.

| 8 \times 24 = _____ | 8 \times 24 | 11 \times 22 = _____ | 11 \times 22 |

1. Write a story problem to go with each multiplication combination.

   a

   b

2. Solve each problem and show your thinking. Label your answers with the correct units.

   a

   b

3. **CHALLENGE** A running club added 338 kilometers to their record after each member ran the metric marathon, a distance of 26 kilometers. How many members are in the club? Show your work in your math journal or on another sheet of paper.

   There are ___________ members in the club.
Riley’s & Raymond’s Ratio Tables

1. Riley made a ratio table, but you can only see this part of it.

<table>
<thead>
<tr>
<th>9</th>
<th>198</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>220</td>
</tr>
<tr>
<td>11</td>
<td>242</td>
</tr>
</tbody>
</table>

a. What number did Riley use to make her ratio table? _______
b. What is the 13th row? _______
c. What is the 5th row? _______

2. Raymond made a ratio table, but you can only see this part of it.

<table>
<thead>
<tr>
<th>17</th>
<th>306</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>324</td>
</tr>
<tr>
<td>19</td>
<td>342</td>
</tr>
<tr>
<td>20</td>
<td>360</td>
</tr>
</tbody>
</table>

a. What number did Raymond use to make his ratio table? _______
b. What is the 10th row? _______
c. What is the 22nd row? _______

3. **CHALLENGE** The area of a rectangle is 280 and one dimension is 14. What is the other dimension?
## Maggie's Chickens

<table>
<thead>
<tr>
<th>Problem</th>
<th>What is the problem asking us to do?</th>
<th>Reasonable estimates</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Maggie’s chickens live in a coop in her backyard. The coop is 12 feet by 13 feet. What is the area of the coop?</td>
<td>sq. ft.</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Maggie is getting several more chickens. She built a new coop in her backyard for the new chickens. This coop is 6 feet by 26 feet. What is the area of the new coop?</td>
<td>sq. ft.</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Maggie’s pig just had a litter of piglets. Maggie’s mother built a pen for the piglets. The pen is 16 feet by 12 feet. What is the area of the pen?</td>
<td>sq. ft.</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>As the piglets grow they will need more space. Maggie’s dad is building another pen for half of the pigs. This pen is 24 feet by 8 feet. What is the area of this pen?</td>
<td>sq. ft.</td>
<td></td>
</tr>
</tbody>
</table>
### Eggs & Apples

Read each problem and estimate the answer. Think about familiar strategies that will help you solve the problems. Show your work using numbers, labeled sketches, or words.

1. Maggie has 15 chickens. Each chicken lays 24 eggs a month. How many eggs do Maggie’s chickens lay in a month in all?
   
   a. My estimate: _______
   
   b. My work:
   
   Maggie’s chickens lay _______ eggs in a month altogether.
   
   c. What strategy did you use? Why did you choose this strategy?

2. Maggie is picking apples for her chickens. She has 12 baskets. She puts 30 apples in each basket. How many apples does Maggie pick in all?
   
   a. My estimate: _______
   
   b. My work:
   
   Maggie picks _______ apples in all.
   
   c. What strategy did you use? Why did you choose this strategy?
Mario’s Marbles

Mario loves marbles and is always adding new marbles to his collection. Help Mario keep track of his marbles in the following problems. Show your work using numbers, labeled sketches, or words. Write a multiplication equation for problems 1 and 2.

1 Mario organized some of his marbles. He used lots of marbles to make 11 piles. Each pile had 14 marbles. How many marbles did he use in all?

Equation: _______________________________________________

2 Mario organized the rest of his marbles. He made 7 piles and put 22 marbles in each pile. How many marbles did he use in all?

Equation: _______________________________________________

3 Mario saw bags of marbles for sale at the store. One bag had 49 marbles in it. Mario wondered if he could organize the 49 marbles into equal piles.

a Is 49 prime or composite?

b Can Mario organize his marbles into equal piles? Explain your answer.

4 Another bag of marbles has 61 marbles in it. Mario wondered if he could organize the 61 marbles into equal piles.

a Is 61 prime or composite?

b Can Mario organize his marbles into equal piles? Explain your answer.
How Can You Solve …?

Corinne is in fourth grade and she has to solve a lot of math problems. Help Corinne solve the following problems.

1. Corinne has to solve $10 \times 8$ and $5 \times 16$.
   
   a. What observations can you share with Corinne about these problems to help her solve them?

   b. How would you tell or show Corinne how to solve these problems?

   c. Solve $10 \times 8$ and $16 \times 5$.

2. Corinne has to solve $12 \times 9$ and $18 \times 6$.

   a. What observations can you share with Corinne about these problems to help her solve them?

   b. How would you tell or show Corinne how to solve these problems?

   c. Solve $12 \times 9$ and $18 \times 6$. 
Work Place Instructions 2C Moolah on My Mind

Each pair of players needs:
- 2 dice numbered 1–6
- spinner overlay
- a 2C Moolah on My Mind Record Sheet for each player

1. Players take turns rolling one die. The player with the higher number goes first.
2. Player 1 rolls both dice, adds the two numbers, and then spins the coin spinner.
3. Player 1 writes an expression in the first column to show the results of the rolls and spin.
4. Player 1 multiplies to find out how much money she collected and writes that amount in the second column. Then Player 1 writes it again in the last column so she can keep a running total of her money.
5. Then it is Player 2’s turn. Player 2 repeats Steps 3, 4, and 5.
6. Players help each other make sure they are adding their money accurately and that each other’s running totals are correct.

7. When both players have taken 10 turns, the game is over and the player with the most money wins.

Game Variations

A. The player with the least amount of money at the end of the game wins.
B. Pick two numbers and call them “take-aways.” For example, a player’s take-aways could be 4 and 7. If two numbers are rolled that add up to one of the “take-aways” (such as 3 + 4 = 7), the player spins the coin spinner, multiplies the coin value by the take-away number, and takes that amount of money away from his total, instead of adding it.
C. Replace one of the 1–6 dice with a 4–9 die, or use two 4–9 dice instead. Players will multiply and add larger numbers.
D. Use the Challenge version of the Moolah on My Mind Record Sheet. Instead of adding the sum of the two dice, players choose whether to add, subtract or multiply the numbers. The winner is the player who finishes the game closest to $6.00.
How Much Money?

1 Nadia has 2 nickels and 3 pennies. Taj has four times as much money as Nadia.
   a How much money does Taj have? Show your work using numbers, labeled
      sketches or words.

   b Fill in the bubble beside the equation that best represents this situation ($m$
      stands for Taj’s money).
      - $(2 + 3) \times 4 = m$
      - $(2 + 3) - 4 = m$
      - $4 \times ((2 \times 5) + (3 \times 1)) = m$

2 Zafar has 1 dime, 1 nickel, and 2 pennies. Petra has three times as much money as Zafar.
   a How much money does Petra have? Show your work using numbers, labeled
      sketches, or words.

   b Write an equation to represent this problem.

3 Challenge Sara has only nickels in her hand, and David has exactly the same
   number of dimes and no other coins. Together, they have a total of $1.05. How
   many nickels does Sara have? How many dimes does David have? Show your work
   using numbers, labeled sketches, or words.
Money Multiple Wheel

1 Fill in the Multiple Wheel.

2 Fill in the blanks.

3 Fill in the bubble beside every item below that is not correct.
   - 10 × 39 is 39 tens
   - 10 × 39 has to be less than 300 because 10 × 30 = 300
   - 39 is prime
   - 10 × 39 is 39 hundreds

4 CHALLENGE Tera counted the coins in her bank and found 25 dimes, 12 quarters, and 15 nickels. If she saves twice as much next month, how much money will she have in all? Show your work.
1 Solve each problem below:
   a  \( 2 \times 16 = \) _______
   b  \( 20 \times 16 = \) _______
   c  \( 4 \times 21 = \) _______
   d  \( 40 \times 21 = \) _______
   e  \( 8 \times 15 = \) _______
   f  \( 80 \times 15 = \) _______

2 Fill in the blanks
   a  \( 6 \times 20 = 6 \times 2 \times \) _______
   b  \( 30 \times 8 = 3 \times \) _______ \( \times 8 \)
   c  \( 5 \times 100 = \) _______ \( \times 10 \times 10 \)
   d  \( 40 \times 7 = \) _______ \( \times 10 \times 7 \)

4 Fill in the Multiple Wheel below.
5  Kyra is putting up streamers for a party. She uses 75 feet of streamers to decorate one wall. Two more walls also each use 75 feet of streamers. Kyra cuts 75 feet of streamers for the last wall, but this wall has a large poster on it, and she only needs to use 68 feet of streamers. How many feet of streamers did Kyra use in all?

6  **CHALLENGE**  Luis and Kyra are getting ready for the party. Luis makes 6 sheets of cookies. Each sheet has 13 cookies on it. He also makes 4 trays of brownies. Each tray has 16 brownies on it. How many cookies and brownies did Luis make in all?
Design a Floor Pattern  

Note to Families
This Home Connection combines math and design. Students use their creativity to design a pattern and then practice computation to determine how much it would cost to make that pattern in tiles.

1. Choose one of the two Floor Plans: Floor Plan 1 below or Floor Plan 2 on the back of this page. (If you really enjoy this project, you can do both.)

2. Draw one of the following 3 tile designs in each square on your floor plan. Do not use the same design for every square.

![Tile A](image1)
![Tile B](image2)
![Tile C](image3)

3. Answer questions 1–6 on the worksheet.

Floor Plan 1

(continued on next page)
Floor Plan 2
Calculating the Costs of Your Floor Pattern(s)

Here is the cost of each tile.

**Tile A**: 25 cents
**Tile B**: 50 cents
**Tile C**: 1 dollar

Use your floor plan(s) and the information above to answer the questions below (Remember, you only have to do one of the floor plans, not both.)

<table>
<thead>
<tr>
<th>Question</th>
<th>Floor Plan 1</th>
<th>Floor Plan 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1  How many tiles did you need for the floor plan you chose?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2  How many of each tile did you use in your design of the floor plan?</td>
<td>a</td>
<td>a</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>b</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>c</td>
</tr>
<tr>
<td>3  How much money did all the A tiles cost?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4  How much money did all the B tiles cost?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5  How much money did all the C tiles cost?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6  How much money did the entire floor pattern cost?</td>
<td></td>
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</tr>
</tbody>
</table>