Teachers Guide

GRADE 5 – UNIT 1 – MODULE 1

BRIDGES™
IN MATHEMATICS

Module 1

Multiplication & Volume

Teacher Masters

Pages renumber with each module.

Student Book Pages

Page numbers correspond to those in the consumable books.

Home Connections Pages

Page numbers correspond to those in the consumable books.

Module 1
Multiplication & Volume

Overview

Module 1 sets the tone for the year and establishes what a math community looks and sounds like. After playing Work Place 1A The Product Game several times, students discuss strategies and use the game board to make mathematical observations and preview math strands they will encounter during the course of the year. In Session 3, students take the Unit 1 Pre-Assessment and complete Mathographies, reflective writing that gives a glimpse into their understanding and beliefs about mathematics. The first investigation of the year invites them to work with properties of multiplication and volume. In the final session, students reflect on the Unit 1 Pre-Assessment and are introduced to the first problem string of the year.

Planner

Session & Work Places Introduced	P&I	PS	MF	WP	A	HC	DP
Session 1 Building a Community Students discuss what a mathematics community looks and sounds like. The teacher records responses and emphasizes the need for a respectful, focused environment. Students are introduced to their first Work Place. **Work Place 1A** The Product Game Players try to claim four spaces in a row by finding products of given factors. On each turn a player can change one of two factors to try to make the best possible move.	●			●			●
Session 2 More About The Product Game Students play The Product Game and then discuss game strategies and evaluate their work on building a community of learners. Then they examine The Product Game Record Sheet, record mathematical observations, and sort their observations by math topic.	●			●		●	●
Session 3 Unit 1 Pre-Assessment Students take the Unit 1 Pre-Assessment. Then, they fill out individual mathographies that the teacher collects and stores for the rest of the year.	●				●		●
Session 4 Boxing Baseballs Students learn about a business owner who needs to decide how he will box 24 baseballs to ship to customers. Students work in partners to find all of the possible solutions. Then the class reconvenes to share a few of their strategies.	●					●	●
Session 5 More Boxing Baseballs Students review their work on the Unit 1 Pre-Assessment and complete a self-reflection sheet. The teacher leads the first problem string of the year, and students continue to work on the baseball boxing problem for the rest of the session.		●			●		●

P&I – Problems & Investigations, **PS** Problem String, **MF** – Math Forum, **WP** – Work Place, **A** – Assessment, **DP** – Daily Practice, **HC** – Home Connection

Materials Preparation

Each session includes a complete list of the materials you'll need to conduct the session, as well as notes about any preparation you'll need to do in advance. If you would like to prepare materials ahead of time for the entire module, you can use this to-do list.

Task		Done
Copies	Run copies of Teacher Masters T1–T13 according to the instructions at the top of each master.	
	If students do not have their own Student Books, run a class set of Student Book pages 1–6.	
	If students do not have their own Home Connections books, run a class set of the assignments for this module using Home Connections pages 1–4.	
Work Place Preparation	Prepare the materials for Work Place 1A using the lists of materials on the Work Place Guides (Teacher Master T1).	
Charts	Make a T-chart and label one column "Looks Like" and the other column "Sounds Like" for Session 1.	
Paper Cutting	Cut two of each the following arrays out of 1-inch grid paper: 4×3, 4×6, 8×6, 4×12, 2×24, 1×48, 3×16 for Session 5. You will label these with student help during the lesson. You will need 8–10 sheets of grid paper.	
Special Items	Write student names on Student Books and have them ready to pass out in Session 1. Write student names on spiral or composition notebooks to use as math journals and have them ready to pass out in Session 2.	

Additional Resources

Please see this module's Resources section of the Bridges Educator site for a collection of resources you can use with students to supplement your instruction.

Session 1
Building a Community

Summary

In this session, students discuss what a mathematics community looks like and sounds like. The teacher records responses on a chart and emphasizes the need for a respectful, focused environment in which all students feel comfortable and are able to learn. Then, students are introduced to their first Work Place, The Product Game.

Skills & Concepts

- Find all factor pairs for a whole number between 1 and 100 (4.OA.4)
- Demonstrate an understanding that a whole number is a multiple of each of its factors (4.OA.4)
- Determine whether a whole number between 1 and 100 is a multiple of a given 1-digit number (4.OA.4)
- Reason abstractly and quantitatively (5.MP.2)
- Construct viable arguments and critique the reasoning of others (5.MP.3)

Materials

Copies	Kit Materials	Classroom Materials
Problems & Investigations Building a Community		
		• chart paper • markers
Work Places Introducing Work Place 1A The Product Game		
TM T1 Work Place Guide 1A The Product Game **TM T2** 1A The Product Game Record Sheet **SB 1*** Work Place Instructions 1A The Product Game	• 2 game markers, each a different color	
Daily Practice		
SB 2 You Choose		

HC – Home Connection, **SB** – Student Book, **TM** – Teacher Master
Copy instructions are located at the top of each teacher master.
** Run 1 copy of this page for use by the teacher and other adult helpers during Work Place time.*

Vocabulary

An asterisk [*] identifies those terms for which Word Resource Cards are available.

equation*
factor*
multiple*
multiply*
product*

Preparation

- Prepare Student Books for use by writing students' names on them.
- Make a T-chart on a sheet of chart paper. Label one column "Looks Like" and the other column "Sounds Like."
- In today's session, you'll introduce Work Place 1A The Product Game. Before this session, you should review the Work Place Guide and Work Place Instructions. Make copies of 1A The Product Game Record Sheet; you'll need a single copy for use today, 1 copy per student pair for class use in Session 2, and a class set plus extra to store in the Work Place 1A The Product Game tray, along with the materials listed on the guide. The Work Place Guide also includes suggestions for differentiating the game to meet students' needs.

Implementing Work Places

For additional information about setting up Work Places and managing them effectively, see the Implementation section of the Bridges Educator site.

Problems & Investigations

Building a Community

1 Gather students in the discussion area. Set the stage for today's session with a discussion about what makes a community of learners.

Start by talking about the role students will play as mathematicians this year. Then ask them what they think math class should look like and sound like so everyone has a chance to think, ask questions, and learn.

> ***Teacher*** *We are going to do a lot of important work in math this year, and you are all going to help because all of you are mathematicians. A mathematician is someone who thinks about math, talks about math, asks math questions, and solves math problems. Mathematicians often try to find the most efficient way to solve a problem. A mathematician is also someone who makes mistakes and then tries to figure out how to fix them, even if that takes some time. Mathematicians sometimes work together and sometimes they work alone. Sometimes they help someone else learn something, and sometimes someone else helps them. Right now, I want you to start thinking about some of the things that will help you be a mathematician this year. What should our room sound like and what should it look like while we are doing math?*

2 Give students a minute or two of quiet think time to consider your questions, and then invite students to share their ideas one at a time. Record students' suggestions on the prepared T-chart.

ELL To help students understand what you are saying, use gestures, write key words where everyone can see, and if necessary and possible, invite bilingual students to help translate. Your goal is to help all students become better mathematicians, which is more likely to happen if students feel safe, comfortable, and respected.

Encourage students to clarify their ideas and be descriptive about what the room will look and sound like. Ask them to compare the way the classroom will sound during an active lesson, such as when everyone is playing a game, and the way it will sound during a whole-class lesson or independent work time when people are thinking, listening, and sharing ideas. Use this discussion to build a community of learners—model listening and responding to students respectfully and comment when you see students setting examples you want the whole class to follow.

> ***Teacher*** *Let's hear what you think the room should look like and sound like so everyone can be a mathematician this year. Raise your hand when you have an idea to share.*
>
> ***Janna*** *Everyone should be working.*
>
> ***Teacher*** *What does "working" look like?*
>
> ***Janna*** *Well, I guess it means people sitting and solving problems.*
>
> ***Martin*** *Last year in math we played a lot of games, too, though. We didn't just solve problems.*
>
> ***Troy*** *And after we solved problems, we talked about them a lot.*
>
> ***Teacher*** *So, being a mathematician means doing many different things—solving problems, playing games, and talking about math. OK, but what does solving problems look like?*

Kyra *Sometimes we use math tools like rulers or blocks to help us solve problems, but sometimes we just use pencil and paper. It helps me to talk with a partner, too.*

Teacher *I'll write your ideas down here. For "Looks Like," we can say Using tools to solve problems, working with a partner, and playing games. For "Sounds Like," we can say talking about math problems. Who has another idea?*

Craig *We should listen when someone else is sharing.*

Teacher *Why is that important in our math class? Why does it help us to listen to other students' ideas about math?*

Aaron *Sometimes we have a way that works but somebody else has a way that works even better. Maybe their way can make it easier or faster for us.*

Amber *It's hard to talk when people aren't listening.*

Marc *Sometimes someone makes a mistake and other students laugh and then I don't want to share.*

Teacher *So you're saying if someone gets mixed up it's important not to laugh or make fun of them. Do you suppose that asking questions of each other and really trying to understand each other's thinking would help?*

Students *Yes!*

Teacher *I'll add these ideas to our chart. I think you're saying that if people were watching our classroom, they could see people being respectful to others, listening, and asking questions to help understand. Is that right?*

Looks like	Sounds like
Using tools to solve problems	Talking about math
Working with a partner	Talking with a partner
Playing games	1 person talking, everyone else listening and looking
	Asking questions

Make sure students understand that there will be times when the classroom is conversation-filled and busy, such as when everyone is playing a game, and other times when the classroom needs to be quiet for people to think, talk, and listen.

3 Give students a minute to think about how they can help the classroom look and sound this way during math. Then, have students turn and talk to a partner to share their ideas.

Students will frequently turn and talk to a partner during mathematics lessons this year. Taking time to establish expectations and routines during the first lessons will help students learn to use this time productively.

4 Tell students they will have a chance to practice their ideas today as they learn their first Work Place of the year.

Save the T-chart for use in the next session.

Work Places

Introducing Work Place 1A The Product Game

5 Display the Word Resource Cards for *factor* and *product*. Ask students to turn and talk to a partner about the definitions of these words.

6 Ask volunteers to share their definitions with the class, and then reveal the definitions on the backs of the Word Resource Cards.

7 Introduce The Product Game using the 1A The Product Game Record Sheet and two game markers.

- Display the Work Place 1A The Product Game Record Sheet so everyone can see it.
- Explain that the game will help students recognize factors of numbers and practice their multiplication facts.

8 Briefly summarize the game before playing against the class. (You might also invite students to review the Work Place Instructions 1A The Product Game in their Student Books at this time.)

Players try to claim four spaces in a row by finding products of given factors. On each turn, a player can change one of the two factors to try to get four products in a row. As they play, students consider the factor pairs of several products to determine their best move. The winner is the first player to claim four spaces in a row.

9 Use student participation to help model how to play The Product Game, using the Work Place Instructions 1A Student Book page as needed.

- Explain that today you will play against the class. You will go first and mark Os and the class will mark Xs, as in tic-tac-toe.
- Choose a factor by placing a game marker on the factor in the column of numbers on the left.

> ***Teacher*** *I am going to place the first game marker on the number 5. Can I mark a product yet?*
>
> ***Student*** *No, it takes two factors to make a product.*

- Have the class choose a factor and place a game marker on that factor.
- Choose a student to tell you what product the two factors make.

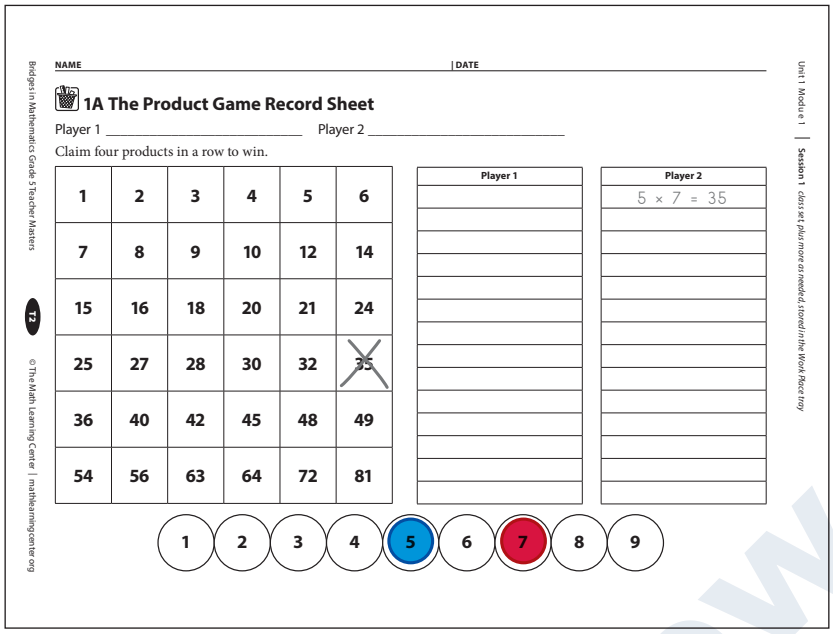
> ***Teacher*** *I chose 5 and you chose 7. What is the product?*
>
> ***Gregory*** *It's 35.*
>
> ***Teacher*** *Is that a problem you just know or did you have to figure it out somehow?*
>
> ***Gregory*** *I have the 5s facts memorized, so it was one I just knew.*

- Write an X on that product.

Ongoing Assessment

The Assessment Guide includes a Work Places Differentiation Chart for each unit. If you like, you can use these charts to make notes about which students need support or challenge with the skills featured in each Work Place. Suggestions for differentiating a particular Work Place activity are included on the Work Place Guide.

- Have a student tell you the equation to write in the Player 2 column to represent that product. Write in the equation. Explain that by keeping track of the equations, you can check at the end of the game to make sure that each of the winning spaces was correctly marked.



NAME _____ | DATE _____

1A The Product Game Record Sheet

Player 1 _____ Player 2 _____

Claim four products in a row to win.

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	~~35~~
36	40	42	45	48	49
54	56	63	64	72	81

Player 1	Player 2
	5 × 7 = 35

1 2 3 4 5 6 7 8 9

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Unit 1 Module 1 | Session 1 *class set; plus more as needed; stored in the Work Place tray*

10 Continue to play by taking turns against the class.

Share your thinking as you choose certain factors that will enable you to mark products on the sheet that are advantageous to you.

> ***Teacher*** *I'm going to stop and think for a minute about the products that are still available. Hmm … I want to choose products that are near ones I've already marked so I can get closer to having four in a row. Now, what factors could I use to make this product?*

11 As you play, emphasize the parts of the game that might be challenging for students.

- Invite students to share strategies for solving multiplication combinations.
- Encourage students to think flexibly when choosing where to place their factor marker. Emphasize the fact that they want to place it in such a way that they're able to mark an open multiple on the sheet or one that is likely to help them win the game.

12 Ask a student to turn to a partner and summarize the directions of the game. Tell students they will have an opportunity to play the game in pairs in future sessions.

13 Close the session by reviewing the T-chart about what the class should look like and sound like in order to build a successful math community.

- Have students turn to a partner and talk about one thing they feel the class did well during The Product Game.
- Acknowledge the class for something you noticed they did well.

Daily Practice

The optional You Choose Student Book page provides additional opportunities to apply the following skills:

- Recall from memory all products of two 1-digit numbers (3.OA.7)
- Multiply a 2-digit whole number by a 1-digit whole number using strategies based on place value and the properties of operations (4.NBT.5)

Session 2

More About The Product Game

Summary

Students review how to play The Product Game, and then play with a partner. The class reconvenes to discuss game strategies and to evaluate their work toward building a community of learners. Students take time to examine The Product Game Record Sheet, record mathematical observations, and sort their observations by math topic. Finally, the teacher assigns the What's the Problem? Home Connection.

Skills & Concepts

- Find all factor pairs for a whole number between 1 and 100 (4.OA.4)
- Demonstrate an understanding that a whole number is a multiple of each of its factors (4.OA.4)
- Determine whether a whole number between 1 and 100 is a multiple of a given 1-digit number (4.OA.4)
- Interpret numerical expressions without evaluating them (5.OA.2)
- Make sense of problems and perservere in solving them (5.MP.1)
- Look for and express regularity in repeated reasoning (5.MP.8)

Materials

Copies	Kit Materials	Classroom Materials
Work Places More Product Game		
	• 2 game markers, each a different color, per pair of students	• Community of Learners T-chart (from Session 1) • 1A The Product Game Record Sheet (from Session 1, half-class set) • Work Place Instructions 1A The Product Game page in Student Books plus a teacher copy (from Session 1)
Problems & Investigations Making Mathematical Observations		
TM 3 Math Topics		• student math journals (see Preparation)
Home Connection		
HC 1–2 What's the Problem?		
Daily Practice		
SB 3 Product Game Problems		

HC – Home Connection, **SB** – Student Book, **TM** – Teacher Master
Copy instructions are located at the top of each teacher master.

Vocabulary

An asterisk [*] identifies those terms for which Word Resource Cards are available.

equation*
expression*
factor*
multiply*
product*

Preparation

- In this session, students will play The Product Game with a partner for the first time. If you did not run copies for this in Session 1, do so for this session.
- Students will record observations in their math journals in this session.
- Also in this session, students will work with partners for the first time.

Work Places

The Product Game

1 Open the session by reviewing the T-chart created in Session 1 for what the classroom should look like and sound like during math time.

- Tell students that today they will have an opportunity to play The Product Game with a partner.
- Ask students to keep the Community of Learners T-chart in mind as they play. At the end of the game, they will reflect on how well they followed the class guidelines.

2 Review the directions for playing the game as described in the Work Place Instructions 1A The Product Game Student Book page. Have students get a partner and gather materials to play.

SUPPORT/ELL Pull a small group of students to play together with you until they feel comfortable playing independently, and you are certain they understand how to play.

CHALLENGE Choose students to play against each other at a higher level rather than having them choose their own partners in order to create more of a challenge and allow strategy building.

3 As students play, circulate to make observations, answer questions, and provide differentiated instruction as suggested in the Work Place Guide.

Make note of students' strategies, strengths, and struggles while playing to help you guide discussion later in the session.

4 After about 20 minutes, gather students for a class discussion about the first Work Place of the year.

Be sure to address game strategy and to comment on how students worked together for the first time.

> ***Teacher*** *This is the first game you have played with a partner this year. Let's take a minute to talk about how things went. Look over our chart for a moment. What did you see that went well or needed work?*
>
> ***Students*** *My partner and I stayed on task the whole time. We only talked about the game.*
>
> *There were some players next to us who were pretty loud.*
>
> *I had a hard time trying to figure out where to move my marker a few times, but my partner helped me.*
>
> ***Teacher*** *I saw a lot of the things that you are describing. When there are so many pairs playing, it's easy to be distracted or get loud, and we need to focus on the math that we are trying to do. When we stay on task, we all learn more.*
>
> *When I walked around, a few pairs told me they thought that there had to be a good way to get four products in a row quickly, but they hadn't figured it out yet. Would someone be willing to share a strategy that you used today?*

Elicit strategies such as choosing factors to yield products positioned in the middle of the board to yield more possible moves later; weighing the number of products possible at each turn with each of the two existing markers; and choosing factors the student is confident multiplying.

Math Practices in Action 5.MP.1

Games of strategy are, in some sense, extended problems. When students spend time developing strategies for winning these games, they are becoming accustomed to making sense of problems and persevering in solving them. In the case of The Product Game, their efforts also contribute to multiplication and division fact fluency and greater sophistication with multiples and factors.

> ***Teacher*** *I'd like you encourage all of you to take the time to make good game moves while you are playing and not to feel pressured to rush. Playing this game will improve your math ability if you challenge yourself to try combinations you need to think about, or explore several different possibilities rather than picking the first and easiest.*

5 Discuss writing expressions to represent student moves in the game.

- Model what to do if the markers were on 2 and 6, and a player moved a marker from 6 to 7. Write the expression to represent the new multiplication problem: 2×7.
- Discuss the difference between an expression and an equation, and touch on how mathematicians use them to keep track of their work.

Note A mathematical expression, such as 6×7, represents a quantity. An equation, such as $6 \times 7 = 42$, which includes an equality (or inequality) sign, asserts that two quantities have (or do not have) the same value.

Problems & Investigations

Making Mathematical Observations

6 After the class discussion, display a copy of The Product Game Record Sheet. Tell students that two important elements of mathematics are observation and description, but don't elaborate at this time.

7 Prepare students to use their math journals.

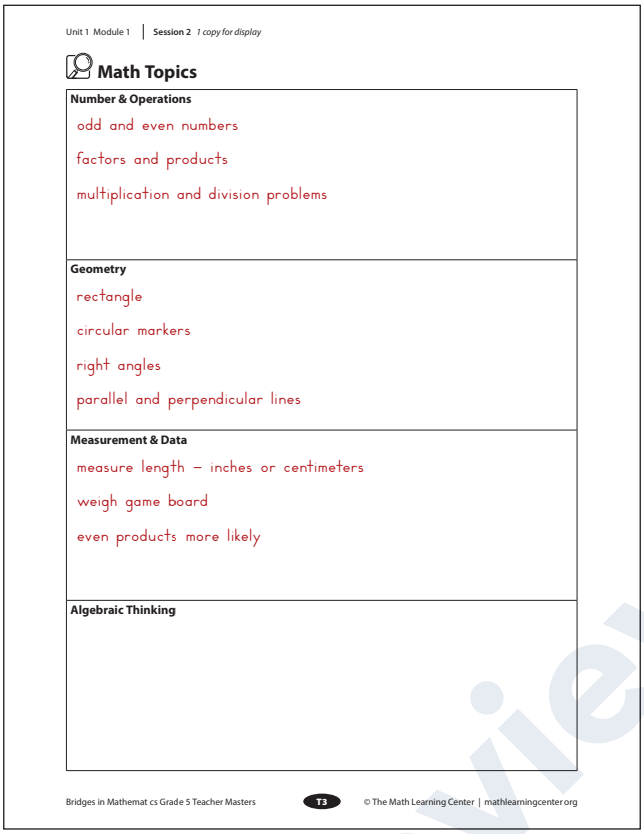
- Hold up a math journal and tell students they will each use one for the rest of the year.
- Explain that every time they use their journals, students will write a heading and the date. This way, it will be easier to find work that might help them later on.
- Pass out journals. Have students turn to the first page and write the title "Math Observations" and the date.

8 Ask students to record as many mathematical observations as they can about The Product Game Record Sheet. Invite them to use words, numbers, or labeled sketches as they record their observations.

9 After a few minutes, ask students to turn and talk about their observations with a person sitting next to them. Then, invite students to share their thinking with the class.

> ***Students*** *There are odd and even numbers on the board, but not all numbers from 1 to 81.*
>
> *There are factors and products.*
>
> *The sheet of paper is a rectangle.*
>
> *The markers are circular.*
>
> *You can make multiplication and division problems with the numbers on the record sheet.*
>
> *The record sheet has right angles.*
>
> *The game board has parallel lines.*
>
> *And perpendicular lines, too.*
>
> *There are more even numbers on the game board.*
>
> *We could measure the game board in inches or centimeters.*
>
> *Or see how much it weighs!*

10 After the first wave of observations have been shared, display the Math Topics Teacher Master. With suggestions from the class, record students' previous observations and any new ones inspired by the topics on the sheet.



Unit 1 Module 1 | Session 2 *1 copy for display*

Math Topics

Number & Operations
odd and even numbers
factors and products
multiplication and division problems
Geometry
rectangle
circular markers
right angles
parallel and perpendicular lines
Measurement & Data
measure length – inches or centimeters
weigh game board
even products more likely
Algebraic Thinking

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11 If one or more categories remain empty, ask students to generate additional observations related to those particular categories. Record students' ideas as they volunteer them.

It is fine if your class does not generate ideas for all the mathematical areas on the sheet. The goal of this activity is to alert students that math is much more than arithmetic. Let them know they will be working with all four topics throughout the year and sharing observations on a regular basis.

12 Close the session by asking students if they have any suggested additions to make for the class chart about what math looks and sounds like.

- Record any suggestions students might have along with one or two of your own ideas.
- Recognize the class for one area they modeled particularly well during today's lesson.

Home Connection

13 Introduce and assign the What's the Problem? Home Connection, which provides more practice with the following skills:

- Write numerical expressions with parenthesis (5.OA.1)
- Write a simple expression to record calculations with numbers (5.OA.2)
- Interpret numerical expressions without evaluating them (5.OA.2)

Extension

Find a place in the room to hang a blank copy of 1A The Product Game Record sheet, along with the Math Topics Teacher Master you started with the class today. Encourage students to continue to brainstorm mathematical observations to add to the list.

Daily Practice

The optional Product Game Problems Student Book page provides additional opportunities to apply the following skills:

- Recall from memory all products of two 1-digit numbers (3.OA.7)
- Find all factor pairs for a whole number between 1 and 100 (4.OA.4)
- Demonstrate an understanding that a whole number is a multiple of each of its factors (4.OA.4)
- Determine whether a whole number between 1 and 100 is a multiple of a given 1-digit number (4.OA.4)
- Write a simple expression to record calculations with numbers (5.OA.2)

Session 3

Unit 1 Pre-Assessment

Summary

Students take the Unit 1 Pre-Assessment, which offers a preview of what is to come in Unit 1 while providing information about where students are with Unit 1 skills and concepts. Then students fill out individual mathographies in which they describe their feelings as well as their strengths and difficulties with mathematics, and list some of their goals for the year. The teacher collects the mathographies to review, and saves them to return to students at the end of the school year.

Skills & Concepts

- Solve multi-step story problems involving division with remainders (4.OA.3)
- Multiply two 2-digit numbers using strategies based on place value and the properties of operations (4.NBT.5)
- Divide a 2-digit number by a 1-digit number, with a remainder, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division (4.NBT.6)
- Write and evaluate numerical expressions with parentheses (5.OA.1)
- Write a simple expression to record calculations with numbers, and Interpret numerical expressions without evaluating them (5.OA.2)
- Demonstrate an understanding that a solid figure that can be packed without gaps or overlaps by *n* unit cubes has a volume of *n* cubic units (5.MD.3b)
- Show that the volume of a right rectangular prism with whole number edge lengths can be found by multiplying the edge lengths, or by multiplying the area of the base by the height (5.MD.5a)
- Make sense of problems and persevere in solving them (5.MP.1)
- Attend to precision (5.MP.6)

Materials

Copies	Kit Materials	Classroom Materials
Assessment Unit 1 Pre-Assessment		
TM T4–T6 Unit 1 Pre-Assessment **TM T7** Base Ten Grid Paper (see Preparation)	• base ten area and linear pieces (see Preparation)	• scratch paper (see Preparation)
Problems & Investigations Writing Mathographies		
TM T8–T9 Mathography		
Daily Practice		
SB 4 More Product Game Problems		

HC – Home Connection, **SB** – Student Book, **TM** – Teacher Master
Copy instructions are located at the top of each teacher master.

Vocabulary

An asterisk [*] identifies those terms for which Word Resource Cards are available.

dimension*
double
equation*
evaluate
expression*
halve
multiply*
number relationship
parentheses*
rectangular prism*
volume*

Preparation

- Prepare sets of base ten area and linear pieces as well as scratch paper and half-sheets of the Base Ten Grid Paper Teacher Master, so these materials are easily accessible to students who want to use them during the assessment.
- Note that you will need to score the Unit 1 Pre-Assessment before Module 1, Session 5. If you cannot mark the Pre-Assessment by Session 5, make room for reflection time in Module 2.

Assessment

Unit 1 Pre-Assessment

1 Set the stage for the Unit 1 Pre-Assessment.

- Tell students that today they will take the Unit 1 Pre-Assessment to help everyone, teacher and students alike, know where they are with the concepts and skills in the unit. With the results, everyone will be better able to prepare for the unit.
- Tell students how much time they have. We recommend you allow 40–45 minutes. Ask students who finish earlier to check their work carefully and then quietly read to themselves or visit the The Product Game Work Place until you call time.
- Encourage students to do their best but to not get bogged down on any one problem. Tell students that if they get really stuck on one problem, they can skip it for now, work on other problems, and then return to it if they have time.

2 Place the Unit 1 Pre-Assessment Teacher Master on display as helpers give a copy of the assessment to each student.

- Have students write their names and the date on the first page, and give them a minute to look over the assessment.
- Let students know how and where to access base ten area and linear pieces, scratch paper, or half-sheets of Base Ten Grid Paper if they want to use any of these materials to help with some of the problems on the assessment.
- Remind students to wait for further instructions from you before they start.

3 Talk with students about some of the strategies they can use to make the best use of their time during an assessment.

Model the following strategies as you describe them to students.

- Read the whole assessment before you begin, to get a sense of what you need to do.
- Notice which problems might be easier or more difficult for you. You may put a small star by easier problems and a question mark by more challenging ones.
- Think about how to use your time during the test so you have time and energy to finish all the problems.
- If you get really stuck on one problem, skip it for now, work on other problems, and then go back to it later, if you have time.
- Pay special attention to math words like those on Word Resource Cards. You may want to underline them, especially if you are having a hard time understanding a question.

4 When students understand what to do, let them begin.

Remind students to raise their hands if they need help reading a problem; this is not meant to be a reading test.

SUPPORT If some students are unable to complete the assessment in the allotted time, allow them additional time later in the day or early the next day to finish their work.

5 As students finish, have them turn in their assessment and then quietly read until everyone else finishes.

6 When all the students are finished, let them know they will get their Unit 1 Pre-Assessments back soon so they can see the results and set their own goals for the unit accordingly.

You will have students reflect on the Unit 1 Pre-Assessment in Session 5 of this module. If it is impossible to score the Unit 1 Pre-Assessments before Session 5, mark them when you can and then make time for students to reflect on their own work and set goals as described in Session 5.

Assessment Guide

See the Grade 5 Assessment Guide for scoring and intervention suggestions.

Problems & Investigations

Writing Mathographies

7 Display a copy of the Mathography Teacher Master and explain what a "Mathography" is.

- Ask a volunteer to explain what a biography is.
- Tell students that their mathographies will give information about their lives as mathematicians.

8 Review the questions together.

- As you review the questions, take the opportunity to share a few of your own school math experiences with the class.
- Let students know you are very interested in reading what they have to say, and assure them that their writing will not be shared with other students.
- Ask students to be as specific as possible when answering the questions.

9 Once students understand what to do, give them about 15 minutes to write their responses.

ELL To make this activity more accessible to the English-language learners in your room, pair them with a peer who can translate the questions. If necessary, allow them to respond in their native language.

10 After students have spent some time writing, collect their papers.

Explain that you will read their mathographies, save them, and return them at the end of the year so students can see how they've grown, if any of their ideas have changed, and if they've reached their goals for the year.

SUPPORT If some students have more to say than they can manage in the time allotted at the end of this session, give them additional time to complete their mathographies within the next day or so.

You might want to keep the mathographies in one big envelope or place them in students' math portfolios. Some time after the session, take time to read students' mathographies. The information you glean from them will give you insight into your students' abilities and needs.

11 Close the session by inviting several volunteers to share their math goals for the year. Tell students again how much you are looking forward to reading what they have written.

Daily Practice

The optional More Product Game Problems Student Book page provides additional opportunities to apply the following skills:

- Find all factor pairs for a whole number between 1 and 100 (4.OA.4)
- Demonstrate an understanding that a whole number is a multiple of each of its factors (4.OA.4)
- Determine whether a whole number between 1 and 100 is a multiple of a given 1-digit number (4.OA.4)

Session 4
Boxing Baseballs

Summary

Students learn about a business owner who needs to decide how he will box 24 baseballs to ship to customers. After introducing the problem, the teacher sends students to work with partners to find all possible configurations. Near the end of the session, the class reconvenes to share a few of the strategies they've used to create as many different configurations of boxes as possible. Finally, the teacher assigns the Multiplication Connections Home Connection.

Skills & Concepts

- Find all factor pairs for a whole number between 1 and 100 (4.OA.4)
- Write numerical expressions with parentheses (5.OA.1)
- Write a simple expression to record calculations with numbers, and interpret numerical expressions without evaluating them (5.OA.2)
- Demonstrate an understanding that a solid figure that can be packed without gaps or overlaps by *n* unit cubes has a volume of *n* cubic units (5.MD.3b)
- Make sense of problems and persevere in solving them (5.MP.1)
- Look for and express regularity in repeated reasoning (5.MP.8)

Materials

Copies	Kit Materials	Classroom Materials
Problems & Investigations Boxing Baseballs		
TM T10 Boxing Baseballs **TM T11** More About Brad's Boxes	• Omnifix cubes, class set (see Preparation) • Word Resource Cards: *dimension, rectangular prism*	• 12″ × 18″ sheets of paper (half-class set; newsprint is fine) • student math journals • a piece of copy paper to mask portions of the teacher master
Home Connection		
HC 3–4 Multiplication Connections		
Daily Practice		
SB 5 Facts & Boxes		

HC – Home Connection, **SB** – Student Book, **TM** – Teacher Master
Copy instructions are located at the top of each teacher master.

Vocabulary

An asterisk [*] identifies those terms for which Word Resource Cards are available.

base*
dimension*
expression*
height
length
rectangular prism*
width

Preparation

Prepare the Omnifix cubes (2,000 in all) for easy access and reasonably equitable distribution. If you have a class of 30, each student pair should get about 130 cubes, or enough to build about five different rectangular prisms with a volume of 24 cubes before some have to be taken apart to make others.

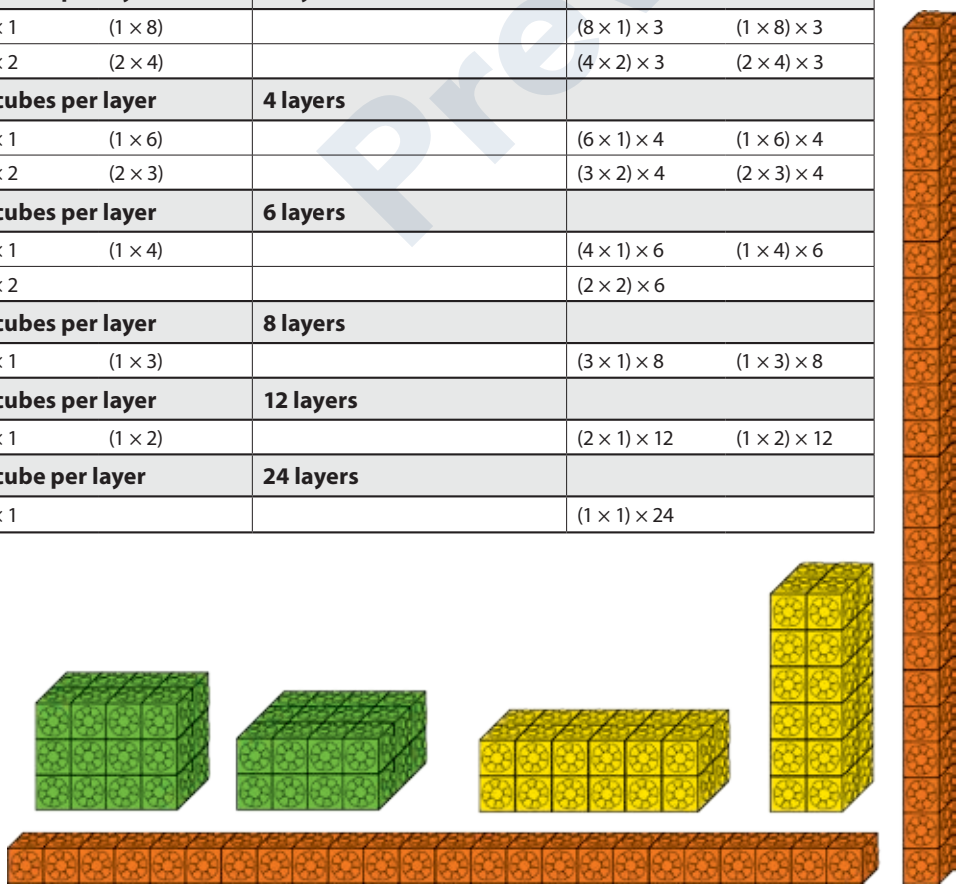
Mathematical Background

In this session, students are challenged to figure out all of the different ways they can arrange 24 cubes in the form of a rectangular prism. The problem involves a business owner who needs to figure out how to package groups of 24 baseballs, each of which is contained in a small cubic box, so that they can be shipped in a single, large box. He can arrange them in 1 layer of 24, 2 layers of 12, 3 layers of 8, and so on. The layers can be arranged in rectangular formations with dimensions that are factors of the total number of cubes in each layer. (See chart below.)

Students will conduct their initial exploration of the problem today and will return to it in Session 5 and in the following module. Sometime in Session 5, you'll need to press them to keep track of their work in a systematic way so that they can be certain they are identifying all of the possible arrangements of 24 cubes. Essentially, the problem requires that students use what they understand about factoring to identify the dimensions of all possible rectangular prisms with a volume of 24 cubes. By emphasizing the dimensions of each layer and the number of equal layers, we are moving students toward the formulas for calculating the volume of a rectangular prism ($V = l \times w \times h$ or $V = b \times h$), which they will address more explicitly later in the year.

Dimensions of Layers (Base)		**Number of Layers** (Height)	**Expressions** (Base) × Height	
24 cubes per layer		**1 layer**		
24 × 1	(1 × 24)		(24 × 1) × 1	(1 × 24) × 1
12 × 2	(2 × 12)		(12 × 2) × 1	(2 × 12) × 1
8 × 3	(3 × 8)		(8 × 3) × 1	(3 × 8) × 1
6 × 4	(4 × 6)		(6 × 4) × 1	(4 × 6) × 1
12 cubes per layer		**2 layers**		
12 × 1	(1 × 12)		(12 × 1) × 2	(1 × 12) × 2
6 × 2	(2 × 6)		(6 × 2) × 2	(2 × 6) × 2
4 × 3	(3 × 4)		(4 × 3) × 2	(3 × 4) × 2
8 cubes per layer		**3 layers**		
8 × 1	(1 × 8)		(8 × 1) × 3	(1 × 8) × 3
4 × 2	(2 × 4)		(4 × 2) × 3	(2 × 4) × 3
6 cubes per layer		**4 layers**		
6 × 1	(1 × 6)		(6 × 1) × 4	(1 × 6) × 4
3 × 2	(2 × 3)		(3 × 2) × 4	(2 × 3) × 4
4 cubes per layer		**6 layers**		
4 × 1	(1 × 4)		(4 × 1) × 6	(1 × 4) × 6
2 × 2			(2 × 2) × 6	
3 cubes per layer		**8 layers**		
3 × 1	(1 × 3)		(3 × 1) × 8	(1 × 3) × 8
2 cubes per layer		**12 layers**		
2 × 1	(1 × 2)		(2 × 1) × 12	(1 × 2) × 12
1 cube per layer		**24 layers**		
1 × 1			(1 × 1) × 24	

The table at left shows 16 distinct arrangements of 24 cubes because it treats rotations of the same cube differently. For example, 24 layers with 1 cube in each layer (a tower of 24 single cubes) is treated as a different arrangement than 1 layer with 24 cubes in it (a row of 24 single cubes lying flat). These rectangular prisms are congruent, and students might discuss that. If not, that is fine, too.



Each pair of rectangular prisms shown here is congruent, though they have different numbers of layers and each layer's dimensions are different. As a result, students will treat them as different arrangements for the purposes of this investigation.

Problems & Investigations

Boxing Baseballs

1 Open the session by telling students they will begin an investigation today that will continue over several days.

- Have students pair up.
- Distribute Omnifix cubes to each student pair.

Remind students that during math class the interlocking cubes are tools, not toys, and to not begin handling them until invited to do so.

2 Display the Boxing Baseballs Teacher Master and introduce the context of the day's work.

> *Brad always loved playing baseball. He turned his passion into an online business that sells personalized baseballs with hand-stitching in the colors of the teams that are ordering. His business is doing well, and he needs to decide how to package his baseballs because people are starting to order a dozen, two dozen, or more at a time.*

3 Discuss the packaging for a dozen of Brad's Baseballs. Ask students how many different ways Brad could arrange the baseballs if each ball was packaged in a small box that was a perfect cube and then 12 of those boxes were arranged together in a single layer that would fit in a large rectangular box.

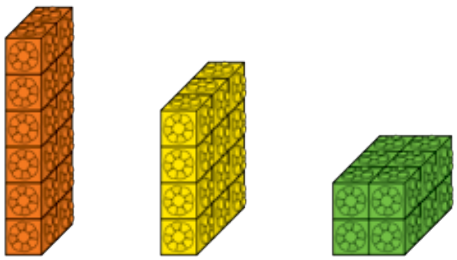
- Record the arrangements as students suggest them. *(1 × 12, 2 × 6, and 3 × 4)*
- Ask students to justify how they know they have found all the ways the 12 cubes could be arranged in a single layer.

SUPPORT/ELL Have a student use 12 cubes to model the arrangements for the class as students suggest them.

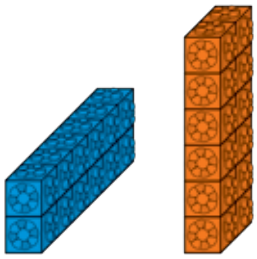
4 Tell students that Brad is also considering packing the cubic boxes in multiple layers. Ask students to work with their partners to create some arrangements of 12 cubes that have more than 1 layer.

- Give students several minutes to work.
- When most pairs have two or more constructions in front of them, have them share and compare with another pair sitting nearby. Do they have any that match?

> ***VJ*** *We made these three boxes. Do you have any like ours?*



Dinah *We made one that's kind of like your first one, but just turned on its side—see?*



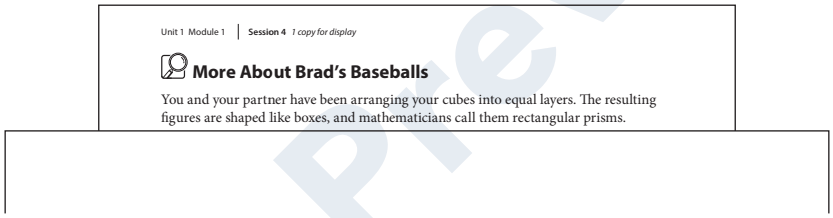
Allie *Oh, yeah. Those two are the same, just one laying down almost flat, and the other one standing up. So one has 6 layers of 2 cubes and the other has 2 layers of 6 cubes.*

5 When students have had a minute or two to share and discuss their arrangements, reconvene the class and explain that they're going to do quite a bit more work with boxes in the coming days. In order to talk with each other like this, everyone in class is going to need some common terms and understandings.

Have student pairs pull all of their materials close and line up the boxes they've made so they can see them easily. They will be holding some of them up in just a few minutes. Ask pairs to talk briefly with each other. How many layers does each arrangement have? How big is each layer?

6 Display the top portion of the More About Brad's Boxes Teacher Master, keeping most of the sheet covered with a piece of copy paper for now.

Read the text at the top of the sheet to the students and encourage them to look and listen very carefully, as they'll need this information when they start working on the day's main problem.



Unit 1 Module 1 | **Session 4** *1 copy for display*

More About Brad's Baseballs

You and your partner have been arranging your cubes into equal layers. The resulting figures are shaped like boxes, and mathematicians call them rectangular prisms.

7 Move the copy paper down the sheet, item by item, and use the text and illustrations to do the following:

- Define and briefly discuss the attributes of rectangular prisms.

If students comment that the fourth prism pictured toward the top of the teacher master is a cube, confirm their thinking, and then let them know that a cube is a special kind of rectangular prism—one with 6 congruent square faces—just as a square is a special kind of rectangle (one with 4 equal side lengths).

- Define the term *dimensions*, and explain that all rectangular prisms have 3 dimensions—*length*, *width*, and *height*.
- Explain that each box or rectangular prism they've built can be described by its *base*—the bottom layer—and the number of layers. Furthermore, this can be done in the form of a numerical expression.
  - » A prism with a base of 1 by 4 cubes with 3 layers is represented by the expression $(1 \times 4) \times 3$.
  - » A prism with a base of 2 by 3 cubes with 2 layers would be represented with the expression $(2 \times 3) \times 2$. (Work with input from the class to label this box on the sheet and write the expression needed to represent it.)

Using parentheses to group the dimensions of the base, as well as talking about the dimensions of and number of layers, helps students begin to think about the volume of rectangular prisms in ways that build conceptual understanding of the formulas $V = l \times w \times h$ and $V = b \times h$, which they will explore later in the year.

- Have students practice reading and interpreting expressions by holding up the box(es) they built that match the three expressions toward the bottom of the sheet: $(1 \times 1) \times 12$, $(1 \times 6) \times 2$, and $(2 \times 2) \times 3$.

The last three prompts are meant to help students see that changing the orientation of the prism does not change the total volume. However, it can change the size and number of layers, which, for the purposes of this investigation, makes it a different arrangement of baseball boxes.

8 Now introduce today's central problem, and review expectations.

- Tell students that Brad's customers most often order two dozen baseballs. The students' task is to use the Omnifix cubes to build models of all the different ways 24 cubic boxes could be arranged in layers so that Brad can think about what kinds of boxes he wants to use to package orders of 24 baseballs.
- They will record all the arrangements they model on a large sheet of paper. They can use numbers, words or labeled sketches.
- Encourage them to organize their work in such a way that they can be certain they have found all the possible arrangements.

In Module 2, students will create posters to present their work to their classmates. You might let students know that today's work is a first draft for that poster.

9 By way of quick review before they start building, show students the Word Resource Cards for *dimension* and *rectangular prism*.

- Display and briefly discuss both sides of each card—the definition and the illustrations.
- Explain that students will need to keep a record of the dimensions of all the rectangular prisms they build.
- They don't have enough cubes to build all the models, so it is important that they accurately record the dimensions of their models before they take them apart to build new ones.

After sharing the Word Resource Cards, post them where students will be able to refer to them. Leave the notes on the board, as well as the More About Brad's Boxes Teacher Master on display as well, if possible.

10 Give each student pair a 12″ by 18″ sheet of paper to share, and let them begin.

Circulate and ask scaffolding and clarifying questions where needed. Note the strategies students are using so you can call on a few of them to share later in the session.

- How many layers are in this arrangement?
- What are the dimensions of each layer (of the base)?
- What expression can you write to show the dimensions of the base and the height?
- How can you organize your work to help you know when you have found all of the arrangements?

SUPPORT Help students record the dimensions of the arrangements they have built. Emphasize the use of parentheses to represent the base of each arrangement. Suggest that students keep the various arrangements built for as long as possible for visual reference instead of deconstructing each time and using the same 24 cubes over and over.

ELL Build one arrangement together and record its dimensions using parentheses. Show a nonexample by writing the dimensions so they do not represent the arrangement, and draw an X through the nonexample so students can work from both a pictorial example and nonexample.

CHALLENGE Encourage students to justify how they know they are finding all of the arrangements.

11 About ten minutes before the end of class, collect students' papers and gather the class for discussion.

Let students know they will have more time to work on this investigation in the next session, and remind them to write their names on their paper.

12 Formalize the routine of having students turn and talk with a partner during math discussions.

- Explain that you will often pause during discussion to allow partners time to talk about what is happening. State the guidelines for turning and talking with a partner:
  - » Turn to face the person next to you.
  - » Discuss the question or topic in a quiet voice.
  - » Make sure both partners get a chance to talk, and be respectful to your partner's ideas and work.
  - » Be prepared to share part of your conversation with the whole class.
  - » If you have a question, be prepared to ask it during the whole class discussion.

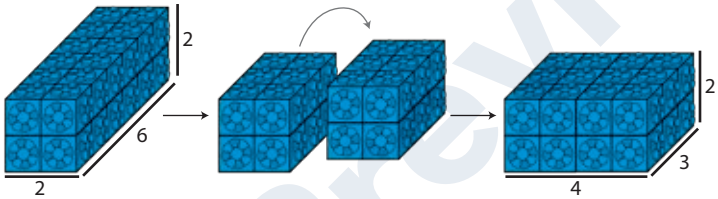
You might write your expectations on chart paper for students to refer to throughout the year.

13 Invite two or three students to share their strategies for finding the different ways Brad can arrange the 24 cubic boxes.

Ask these students to use the cubes to model their strategies so their classmates can see and better understand their thinking.

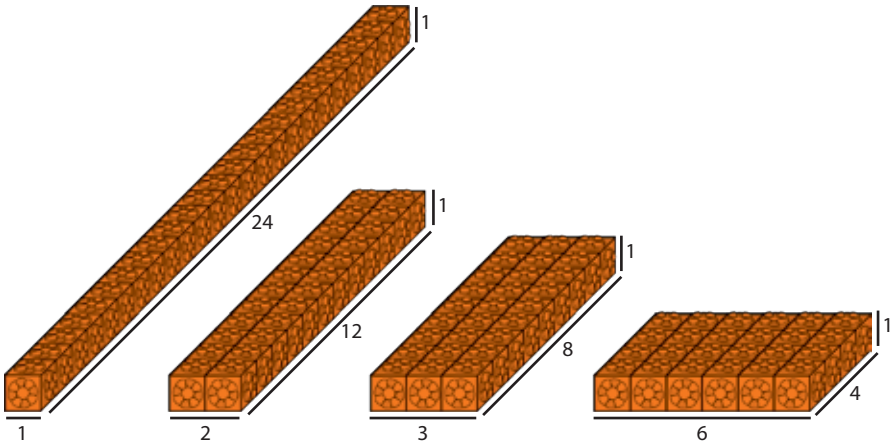
The two strategies shown here are by no means the only possibilities. If you do not have a student who used the strategy of separating boxes into halves and recombining them to form different boxes, consider modeling it yourself.

Halving & Doubling Strategy



Sergio *See, if you make an arrangement, you can get another by breaking it in half and putting the halves together in a different way.*

Using the Factors of 24



Hannah *I thought about just making an arrangement with 1 layer first. If there's 1 layer, there has to be 24 cubes in that layer, so I thought about the different factor pairs for 24. You have 1 and 24, 2 and 12, 3 and 8, and 4 and 6. These are all really skinny and flat, but now I'm going to see how I can do it in 2 layers and then 3 layers and so on.*

14 Then have students practice turning and talking to a neighbor.

- Ask students to turn and talk about one of the strategies that has just been shared.
- Then, call on a few students to share from their conversations.

> ***Teacher*** *That was a really interesting strategy Sergio just shared—the one about building an arrangement, and then splitting it in half and putting the halves together in a new way to make an arrangement with a different base and height. Please turn and talk with your partner about this strategy. Did you understand it? Can you explain it? Could you use it? Do you have a question about it? Practice the steps of turning and talking with your partner that we just talked about. Then, I'll ask a few of you to share with the whole group.*

15 After students turn and talk, revisit the guidelines and reflect on how the class did. Remind students that they will be doing this regularly in class and they need to know exactly what to do.

> ***Teacher*** *I noticed most of you turned so you were looking at your partner. The noise level was reasonable—everyone was talking, but no one was too loud. Throughout the year, I will ask you to turn and talk with a partner. This is an important part of our math class that will help you learn more about the math we are doing.*

16 Close the session.

- Review the terms *rectangular prism* and *dimension* one last time.
  - » Remind students that all the models they built today are called rectangular prisms because all six faces are rectangles.
  - » Have the students name the three dimensions of a rectangular prism (length, width, and height).
- Assist students in finding places to store their intact models and the loose cubes so they can efficiently continue the investigation in the next session.

Home Connection

17 Introduce and assign the Multiplication Connections Home Connection, which provides more practice with the following skills:

- Multiply a 2-digit whole number by a 1-digit whole number using strategies based on place value and the properties of operations (4.NBT.5)
- Multiply two 2-digit numbers using strategies based on place value and the properties of operations (4.NBT.5)
- Write and evaluate numerical expressions with parenthesis (5.OA.1)
- Write a simple expression to record calculations with numbers (5.OA.2)
- Interpret numerical expressions without evaluating them (5.OA.2)

Daily Practice

The optional Facts & Boxes Student Book page provides additional opportunities to apply the following skills:

- Write and evaluate numerical expressions with parentheses (5.OA.1)
- Interpret numerical expressions without evaluating them (5.OA.2)

Session 5

More Baseball Boxes

Summary

Students review their work on the Unit 1 Pre-Assessment and complete a self-reflection sheet. Then, after establishing norms for the new routine, the teacher leads the students in the first problem string of the year. Students spend the rest of the session continuing their work with the baseball problem.

Skills & Concepts

- Find all factor pairs for a whole number between 1 and 100 (4.OA.4)
- Write numerical expressions with parentheses (5.OA.1)
- Write a simple expression to record calculations with numbers, and interpret numerical expressions without evaluating them (5.OA.2)
- Compare the size of a product to the size of one of its factors on the basis of the size of the other factor, without performing the indicated multiplication (5.NF.5a)
- Demonstrate an understanding that a solid figure that can be packed without gaps or overlaps by *n* unit cubes has a volume of *n* cubic units (5.MD.3b)
- Construct viable arguments and critique the reasoning of others (5.MP.3)
- Look for and express regularity in repeated reasoning (5.MP.8)

Materials

Copies	Kit Materials	Classroom Materials
Assessment Reflecting on the Unit 1 Pre-Assessment		
TM T12 Unit 1 Pre-Assessment Student Reflection Sheet		• Unit 1 Pre-Assessments, scored (from Session 3)
Problem String Double a Dimension, Double the Area		
TM T13 Two-Centimeter Grid Paper		• precut arrays (see Preparation) • student math journals • chart paper (optional, see Preparation) • Boxing Baseballs student work and models (from Session 4) • More About Brad's Boxes (from Session 4)
Problems & Investigations Continuing Work on the Baseball Problem		
TM T10 Boxing Baseballs **TM T11** More About Brad's Baseballs	• Omnifix cubes, class set (see Preparation) • Word Resource Cards: *dimension, rectangular prism*	• 12″ × 18″ sheets of paper (half-class set; newsprint is fine) • student math journals • a piece of copy paper to mask portions of the teacher master
Daily Practice		
SB 6 Fact Connections		

HC – Home Connection, **SB** – Student Book, **TM** – Teacher Master
Copy instructions are located at the top of each teacher master.

Vocabulary

An asterisk [*] identifies those terms for which Word Resource Cards are available.

area*
array*
base*
dimension*
double/doubling
expression*
factor*
half*
halving
height
multiple*
value

Preparation

- Have students' Unit 1 Pre-Assessments marked and ready to hand back to them.
- Find an area in your classroom to conduct the problem string. You will need a place where everyone can see and you have plenty of space to write. This can be on a whiteboard, a document camera or projector, or on chart paper. The lesson presumes you will do this in the discussion area to promote student discourse. If this is not possible in your classroom, you may have students remain at their seats or find an alternative space.
- Using copies of the Two-Centimeter Grid Paper Teacher Master, cut out two of each the following arrays: 4×3, 4×6, 8×6, 4×12, 2×24, 1×48, 3×16. The four largest arrays will require cutting and taping. You will label these with student help during the lesson.

Assessment

Reflecting on the Unit 1 Pre-Assessment.

1 Let students know they will have a chance to reflect on the pre-assessment and set goals for Unit 1. Then, they will do a problem string together and continue to work on the Boxing Baseballs problem.

2 Hand students their scored Unit 1 Pre-Assessments and give them a minute or so to look over the results.

Looking at the pre-assessment results can help students recognize the learning expectations for the unit, identify which skills and concepts they currently understand, and focus their efforts more effectively on those things they need to learn.

- Review with students how they can use the results of the Unit 1 Pre-Assessment to help them throughout the rest of the unit.
- Encourage students to ask questions, but do not explain how to do problems at this time. Similar problems will be introduced throughout the unit.
- Advise students to not be discouraged if their results were disappointing. They have several weeks to develop their skills, and they will take a similar assessment at the end of the unit.
- Advise students to not be complacent if their results were excellent. The pre-assessment is just a quick snapshot to guide your teaching, and students will have opportunities to improve their mathematical understanding during the unit.

3 Then display a copy of the Unit 1 Pre-Assessment Student Reflection Teacher Master. Give students each a copy and work with them to fill it in.

- Go over the sheet, one row at a time, with the class.
- For each row, read the skill and make sure students understand it. Ask volunteers to explain, or use one of the associated items on the pre-assessment to explain the skill to the class.
- Have students look at the assessment item(s) associated with that particular skill, talk in pairs about how they did with the skill, and then mark their reflection sheets accordingly.

> ***Teacher*** *The first question on the reflection sheet asks, "Can you find the value of an expression with parentheses?" What does that mean? Turn to your neighbor and explain what the question is asking.*
>
> ***Students*** *I think it's like when there are parentheses in a problem, like on the first one, where it says $(9 \times 2) \times 5$.*
>
> *I put 90 for that one, and I got it right.*
>
> *I don't get it. I think they use parentheses when you're supposed to do something first, but when you multiply numbers, you can do them in any order.*

Yeah, but look at the one under that, where it says (50 × 64) – (1 × 64). That one would be kind of confusing without the parentheses.

Teacher *One of the things we'll study in Unit 1 is how to write and solve equations and expressions. Mathematicians use parentheses as a way to help write their equations more clearly, and often as a way to signal to others that they need to do the operations in the parentheses first.*

Ana *I had a hard time with all of the parentheses ones. I got some of them right because I just ignored the parentheses and tried my best, but some of them came out wrong.*

Teacher *Remember that any time you take a pre-assessment, there may be words you don't yet know or problems you can't yet solve. Look over your pre-assessment to see how you did with the items in problem 1. Then talk with the person next to you, and work together to decide how each of you should mark yourselves on that skill. After you've talked, mark your own sheet and write yourself a reminder note if you like.*

Miguel *I got all of them right because we did stuff with parentheses in expressions and equations last year, so I can put a check where it says, "I can do this well already."*

DJ *I got the first two right, but I was confused about the others, so I guess I should put a check where it says, "I can do this sometimes."*

Bridges in Mathematics Grade 5 Teacher Masters

NAME | DATE

Unit 1 Pre-Assessment Student Reflection Sheet

Skill	Look at these problems.	I can do this well already.	I can do this sometimes.	I need to learn to do this.	Notes
Can you find the value of an expression with parentheses?	1		√		
Can you fill in the blanks to make a set of equations true?	2	√			
Can you use number relationships to make one side of an equation equal to the other side without finding the answer?	3			√	

Unit 1 Module 1 | Session 5 *classroom copy*

4 When you and the students have finished working through all the skills listed on the reflection sheet, have students star the two they feel they need to work on most in the next few weeks.

You might do a quick survey so you and the students can see which skills they and their classmates identified as areas of particular need. Quickly read down the list of skills and have students show thumbs up as you read the skills they starred.

5 Finally, give students a couple of minutes to describe in writing any other goals, needs, requests, or questions at the bottom of the sheet.

When students are finished, collect the Unit 1 Pre-Assessment and Student Reflection sheets. Staple them together and file them so that you can combine them with the Unit 1 Post-Assessment at the end of the unit. You might also use them partway through the unit to discuss with individual students their progress in terms of the skills they needed to focus on.

Problem String

Double a Dimension, Double the Area

6 Introduce problem strings by sharing the following information with the class.

- A problem string is a series of problems that students will solve and discuss one at a time.
- Strings often start out with an easier problem and get harder as the string continues.
- The problems at the beginning of the string often help solve the problems toward the end of the string.
- Solving the problems in a string involves thinking like a mathematician because students want to find clever and efficient ways to solve the problem. Efficient strategies are quick and can be explained clearly.
- During a problem string, the students will solve each problem, share strategies and answers, and discuss each other's thinking.
- Students will do their work in their journals. When they talk about their work, the teacher will usually represent their work for everyone to see.

You may want to invite students who have participated in problems strings before to comment on what they remember to help other students get more of a sense of how strings go. You can also assure students that what you are explaining will make much more sense when they are doing a string.

7 Model writing "Double a Dimension, Double the Area Problem String" and the date as students do the same on the first page of their journals.

- Discuss how students will use their journals this year.
- Emphasize that each time students work in their journals, they will need to include the date and a heading that describes their work.

8 Establish the context for the problem string and write the first problem: 4×3.

One of the purposes today's problem string is to acquaint or re-acquaint students with the problem string classroom routine, which was also used in Bridges Grades 3 and 4. The problem string itself is pitched at a relatively easy level so students can review the process while solving a series of related problems.

9 Deliver the string shown in the chart. After the chart, there is a sample progression of dialog that illustrates how the first set of problems in this string might play out in a classroom.

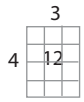
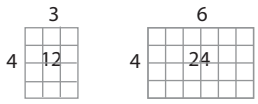
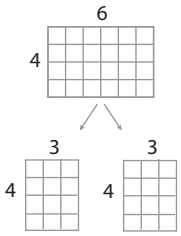
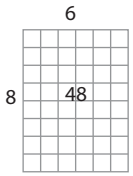
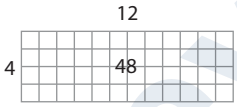
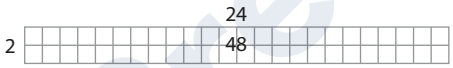
- Pose each problem one at a time by writing it on the board, and give students time to work (see sample dialog that follows).
- After students have had adequate time to record and solve the problem, ask the class for the answer. Write the answer to complete the equation.
- Invite two or three students to explain how they solved the problem.
- Represent the strategies using an array model to show students' thinking.
- Focus on choosing strategies that employ doubling and halving.

CHALLENGE Encourage students to use the most efficient or sophisticated strategy they can think of. Then encourage them to look back at their work and see if they can see an even more efficient strategy that they could have used.

Math Practices in Action 5.MP.8

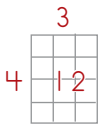
Many of the problem strings this year are designed to help students look for and express regularity in repeated reasoning. By repeatedly doubling and halving, students are able to make generalizations about the properties of multiplication and develop strategies that contribute to computational fluency.

Problem String Double a Dimension, Double the Area

Problems	Sample Strategies & Recording		Connections
4 × 3		4 × 3 = 12	**Big Idea** The goal of this problem string is to help students who have not automatized multiplication facts internalize relationships among facts. Students explore the doubling/halving strategy, an application of the associative property and an important step toward multiplicative thinking.
4 × 6	It is the same on the 4 side but twice as long on the 3 side.  If you cut the 4 by 6 in half, you have two 4 by 3s. 	4 × 6 = 24	
8 × 6		8 × 6 = 48	
4 × 12		4 × 12 = 48	
2 × 24		2 × 24 = 48	
1 × 48		1 × 48 = 48	
3 × 16		3 × 16 = 48	Bring out the doubling and halving connection to the earlier 8 × 6 problem.

Sample Dialog

Teacher *I am going to model this problem* (4 × 3) *with a 4 by 3 rectangular array. Where can you see the 4 in this array? What about the 3?*



Students *There are 4 rows and 3 columns.*

4 squares going down and 3 squares going across.

Teacher *So what is the 12 in this array?*

Student *It's the area of the rectangle. If you count the little squares, there are 12 of them.*

Teacher *What should the 4 × 6 array look like compared to the 4 by 3 already up here?*

Monica *It should be the same on the 4 side but twice as long on the 3 side.*

Teacher *How do the areas compare?*

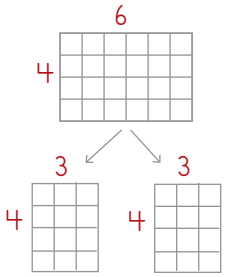
Barry *Since it's twice as wide, the area is twice as big.*

Teacher *Does that make sense to everyone? Could someone please restate that in their own words?*

Luis *The area of the small rectangle is 12. We doubled one side so now the bigger rectangle is twice as wide, so it has twice the area and 2 times 12 is 24.*

Daria *If you take the 4 by 3 and lay it on top of the 4 by 6, you can see that it fits twice.*

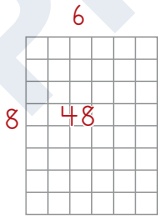
Hector *If you cut the 4 by 6 in half, you have two 4 by 3s.*



10 Continue the same procedure with the next four problems.

As you have students share, be very explicit about your process. Explain that the way you are asking students to share is very deliberate and what you will expect them to do every time they do a string.

Teacher *What should the 8 × 6 array look like compared to the 4 by 6 and 4 by 3 already up here?*



11 Finish the problem string with 3 × 16.

The last problem of a string is often the most difficult. Give students ample time to find the answer. In many strings, the last problem is less obviously connected to the other problems.

- If students notice the quartering/multiplying by 4 relationship between 3 × 16 and 12 × 4, acknowledge it, but do not spend a lot of time discussing it. This concept will return later.

If students have not worked with the multiplication strategy of doubling and halving before, do not expect them to formalize it from just this string. They will continue to work with this strategy throughout the unit.

12 Have students look over their math journal entries and make any additions or corrections. Then have them put their journals away while you return their baseball boxes work from Session 4.

Have student pairs pick up the models they built in Session 4, along with their loose Omnifix cubes.

Problems & Investigations

Continuing Work on the Baseball Problem

13 Review the baseball problem with students and then let them know that they will spend the rest of the session working on the problem in pairs.

- Display the More about Brad's Boxes Teacher Master, and review with students how to write an expression to represent the base and height of a rectangular prism.
- Ask students to make sure that they are recording their arrangements of 24 cubes in a careful and organized way so that they can be sure they have found all of the possibilities. (See the chart in the Mathematical Background section of Session 4 for an example.)

14 Circulate to observe and assist students as needed while they work in pairs.

ELL/SUPPORT Ask students to restate the problem. As they do, help them clarify the meaning to make sure they understand the task.

CHALLENGE Encourage students to devise a way to prove that they have found all of the variations.

15 Close the session.

- Collect the Boxing Baseballs papers and save them for use in the next module.
- Keep all existing baseball box models intact for reference in upcoming sessions.
- Tell students that in the next session they will create posters to present their work to their classmates.

Daily Practice

Use the Fact Connections Student Book page to provide students more practice with the following skills:

- Write numerical expressions with parentheses (5.OA.1)
- Interpret numerical expressions without evaluating them (5.OA.2)

Teacher Masters

GRADE 5 – UNIT 1 – MODULE 1

BRIDGES®
IN MATHEMATICS

Work Place Guide 1A The Product Game

Summary

Players try to claim four spaces in a row by finding products of given factors. On each turn, a player changes one of two factors to try to get four products in a row. As they play, students consider the factor pairs of several products to determine their best move. The winner is the first player to claim four spaces in a row.

Skills & Concepts

- Fluently multiply with products to 100 using strategies (3.OA.7)
- Find all factor pairs for a whole number between 1 and 100 (4.OA.4)
- Demonstrate an understanding that a whole number is a multiple of each of its factors (4.OA.4)
- Determine whether a whole number between 1 and 100 is a multiple of a given 1-digit number (4.OA.4)

Materials

Copies	Kit Materials	Classroom Materials
TM T1 Work Place Guide 1A The Product Game **TM T2** 1A The Product Game Record Sheet **SB 1** Work Place Instructions 1A The Product Game	• 2 game markers	

Assessment & Differentiation

Here are some quick observational assessments you can make as students begin to play this game on their own. Use the results to differentiate as needed.

If you see that…	Differentiate	Example
One or more students randomly chooses a factor and then sees if the product is available	SUPPORT Encourage students to look at the available products first. Is it possible to cover the factor pairs for any of those products by moving one of the game markers?	"You have X's and O's all over the board! It looks a bit random. If you could choose a product to cover to try to get four in a row, which product would you pick? Now, is it possible to move one of the factor markers to get a factor pair for that product?"
Students are quickly choosing factors and covering products	CHALLENGE Encourage students to consider all the possible moves and choose the best one.	"Is that the best possible move? Do you think it's better to get a product on the edge or in the middle? Is it possible to block your opponent and help yourself get closer to winning at the same time?"
Students are strategically choosing factors to produce products that either block their opponent, get themselves in a winning position, or both	CHALLENGE Encourage students to examine the game board and share observations. Have students play the game variation for 5 in a row.	"Are there products missing from the board? Which ones? Why? Could you create a board for only the factors 1–8? Or 1–7? Or 1–6? What products would those boards contain? Why?"

English-Language Learners Use the following adaptations to support the ELL students in your classroom.

- Write the word *factor* above the line of factors. Write the word *product* above the grid of products. Make sure these Word Resource Cards are prominently posted.
- Provide a same-language peer, if one is available.
- Play a sample game in a small group and allow opportunities for students to request clarification and rephrasing,

NAME | DATE

1A The Product Game Record Sheet

Player 1 _____ Player 2 _____

Claim four products in a row to win.

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

Player 1

Player 2

1 2 3 4 5 6 7 8 9

Math Topics

Number & Operations
Geometry
Measurement & Data
Algebraic Thinking

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Unit 1 Pre-Assessment page 1 of 3

1 Evaluate (solve) each expression below.

a $(9 \times 2) \times 5 =$

b $5 \times (13 \times 2) =$

c $50 \times 64 - (1 \times 64) =$

d $2 \times (6 \times 4) + 2 \times (3 \times 4) + 2 \times (6 \times 3) =$

2 Fill in the blanks to make each set of equations true.

a $(99 \times 497) + (1 \times 497) =$ _____ $\times 497 =$ _____

b $98 \times 36 =$ (_____ $\times 36) - (2 \times 36) =$ _____

3 Fill in the missing number in the equation below. Then, describe how you can use number relationships to fill in the blank without having to find the answer to 16×14.

_____ $\times 28 = 16 \times 14$

4 Write true or false next to each equation. Then, describe below each one how you can use number relationships to tell whether the equation is true or false without having to find the answer on both sides.

a $18 \times 120 = 9 \times 240$ _____

b $(3 \times 4) \times 6 = 3 \times (4 \times 6)$ _____

c $58 \times 17 = (60 \times 17) + (2 \times 17)$ _____

5 Write a numerical expression to represent each statement below. Include any grouping symbols (such as parentheses) you need to make the expression as clear as possible.

a To find 15×18, I double and halve.

b To find 9×26, I find 10 times 26 and subtract 1 group of 26.

c How can I find the volume of a box that has a 18-by-21 base and 37 layers?

(continued on next page)

NAME | **DATE**

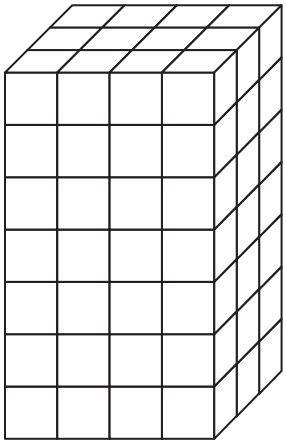
Unit 1 Pre-Assessment page 2 of 3

6 Write and solve an equation to represent each statement below.

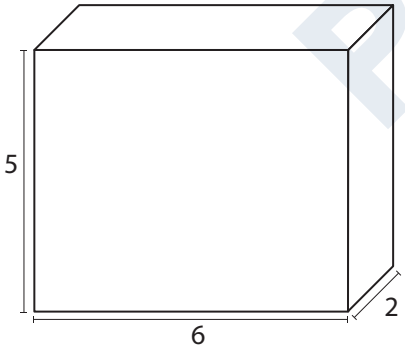
a To find 15 times 32, I multiply 10 times 32 and add it to 5 times 32.

b To find 42 times 25, I double and halve.

7 How many 1 × 1 × 1 cubes are in the rectangular prisms shown below? Write and solve equations to show.



a Equation for number of cubes: _____



b Equation for number of cubes: _____

8 Brad counted the number of balls in one layer of a box, and found out there were 24. The box has 9 layers. How many balls can the box hold in all? Show your work.

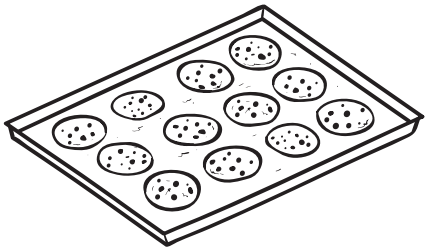
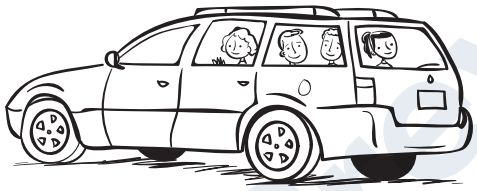
(continued on next page)

NAME | **DATE**

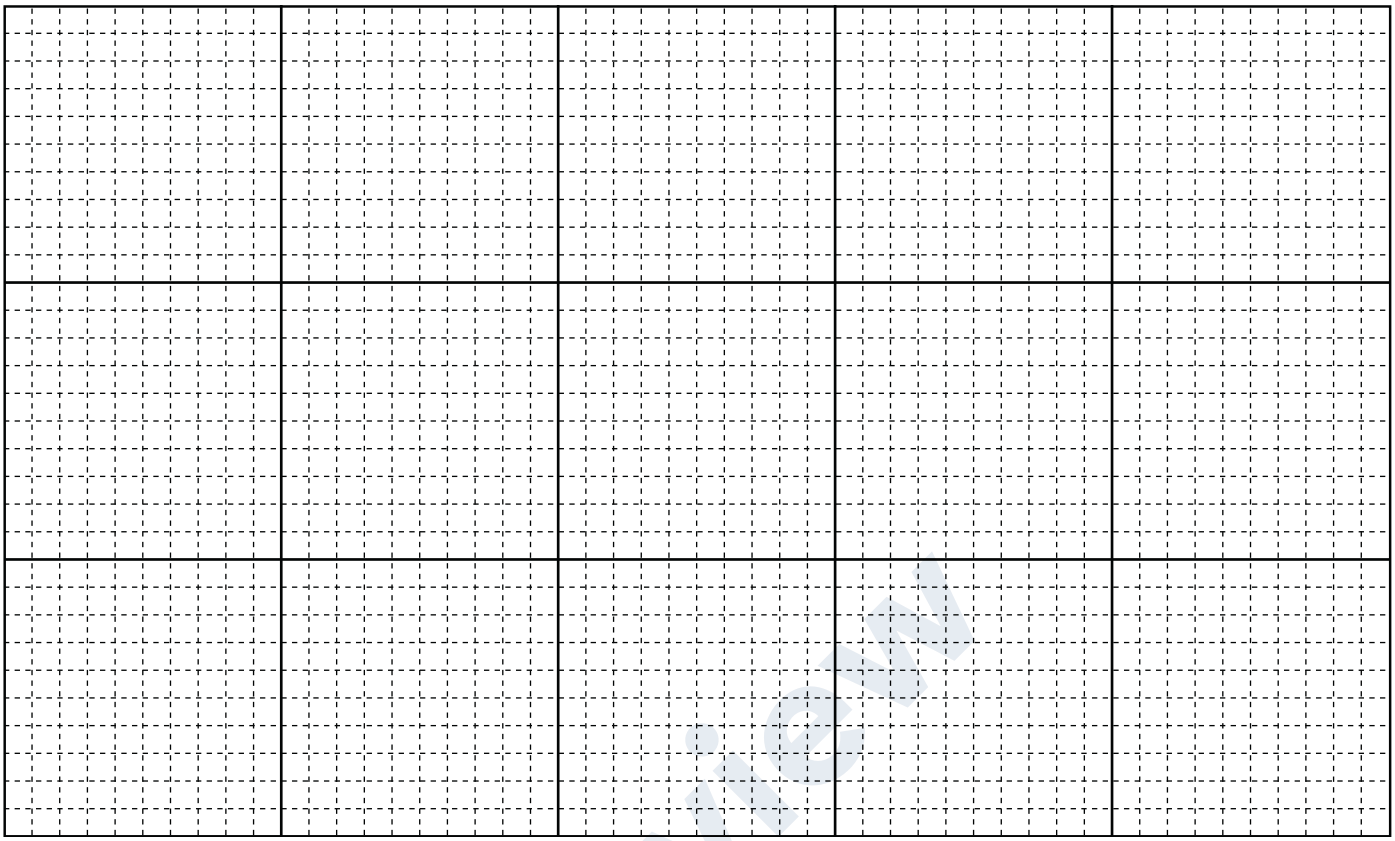
Unit 1 Pre-Assessment page 3 of 3

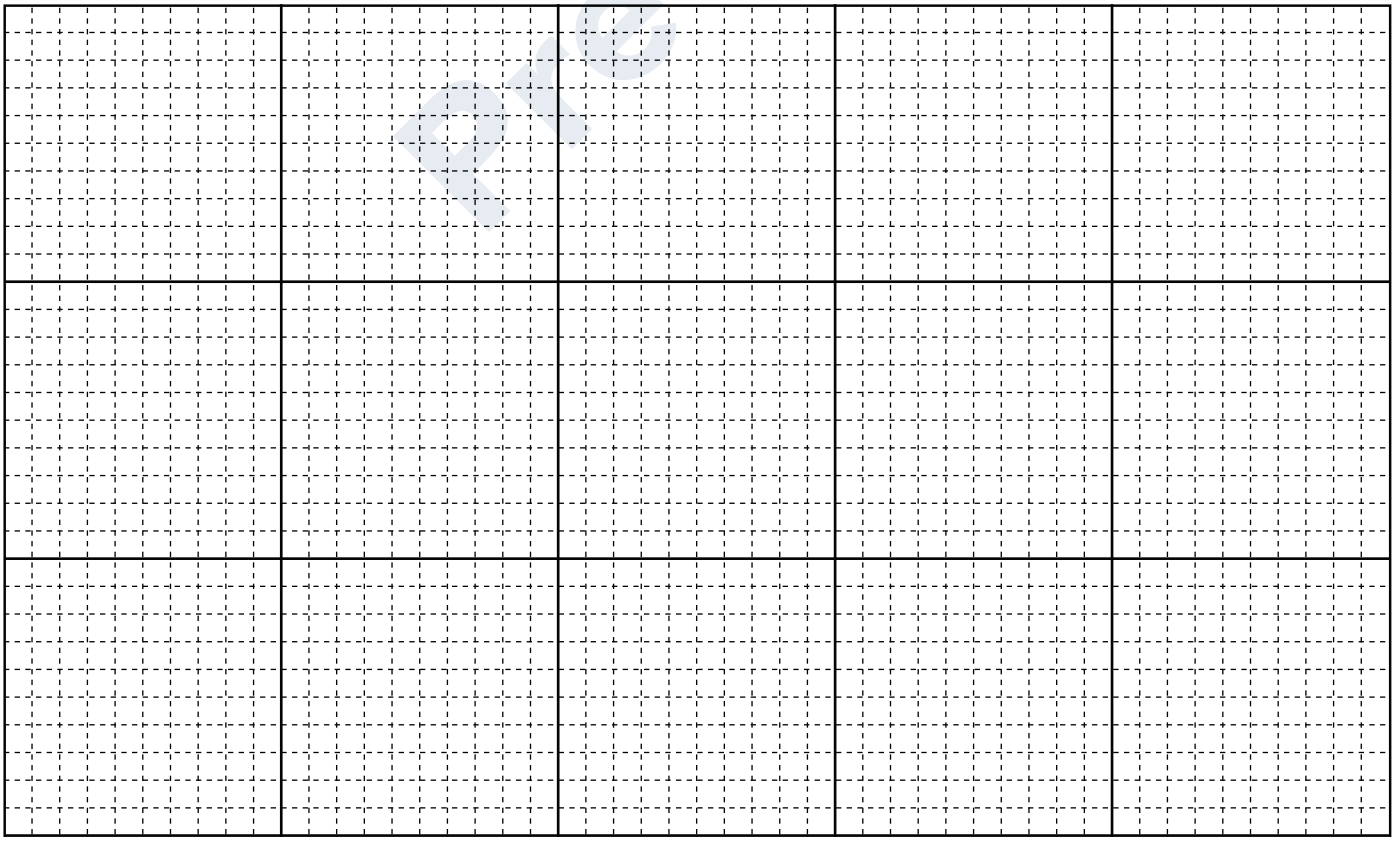
9 For each problem below:

- Use numbers, words, or labeled sketches to solve the problem. Show your work.
- Handle the remainder, if there is one, in the way that best fits the situation.
- Write an equation to show each problem and the answer.

Story Problem	Your Work
a  Four friends made 57 cookies and shared them equally. How many cookies did each friend get?	**Equation**
b  There are 57 kids in the After-School Club. Tomorrow they are going to the park. If each car can carry 4 kids, how many cars will they need to get to the park?	**Equation**
c  Andrew paid $57.00 for 4 super-size pizzas. How much did each pizza cost?	**Equation**

Base Ten Grid Paper





NAME | **DATE**

Mathography page 1 of 2

Answer the following questions as best you can.

1 What is mathematics?

2 How do you feel about math?

3 Think back over your years in school. Tell about a few of your strongest math memories.

(continued on next page)

NAME | **DATE**

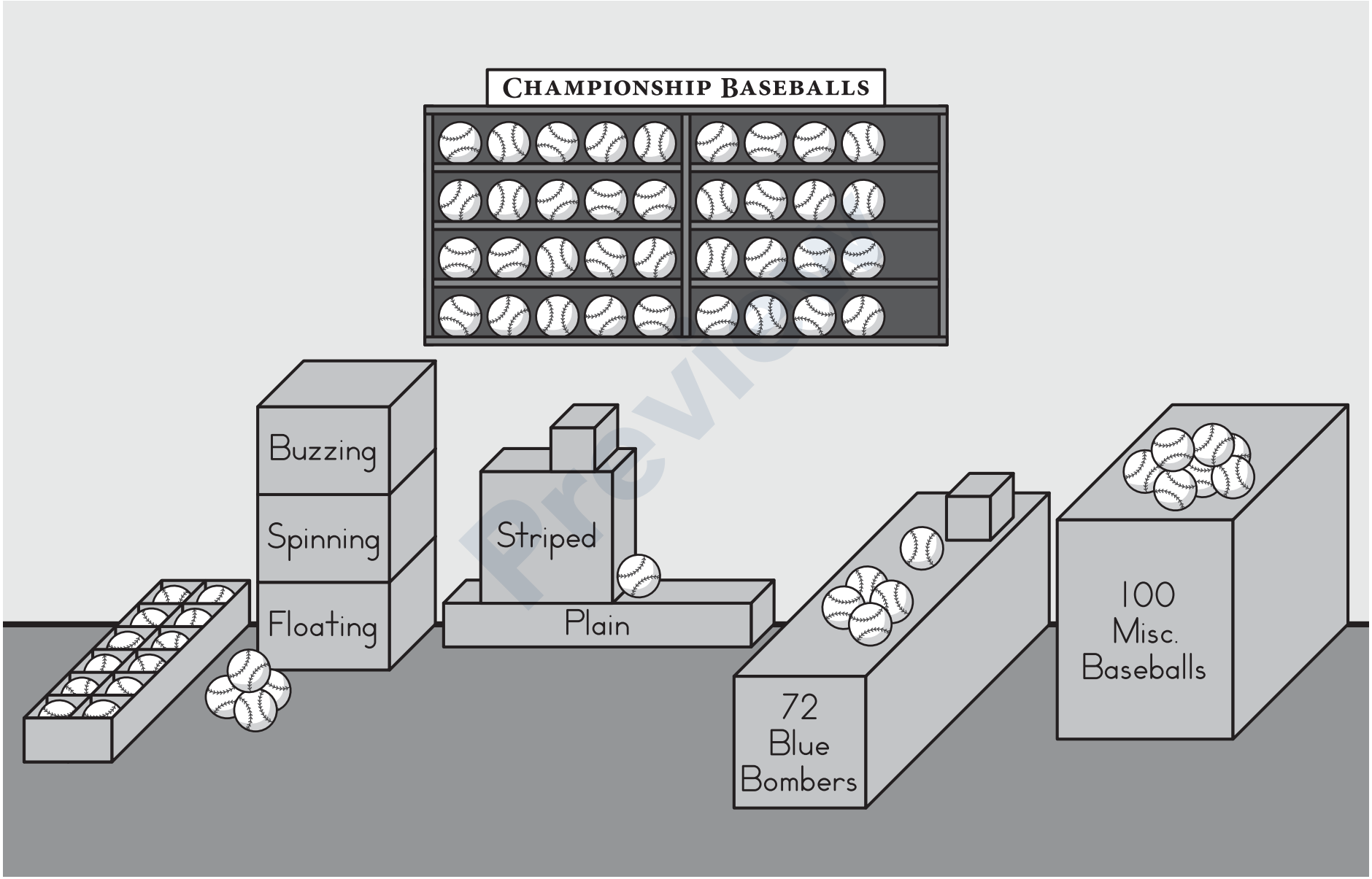
Mathography page 2 of 2

4 What are some of the things that are easy for you in math?

5 What are some of the things that are more difficult for you right now?

6 What are your goals for the year in math? What would you like to get better at?

Boxing Baseballs

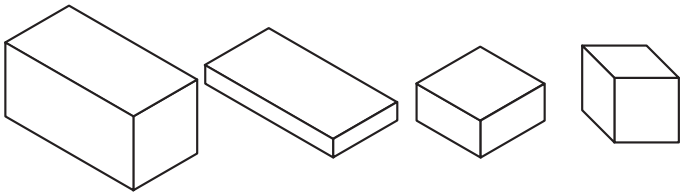


More About Brad's Baseballs

You and your partner have been arranging your cubes into equal layers. The resulting figures are shaped like boxes, and mathematicians call them rectangular prisms.

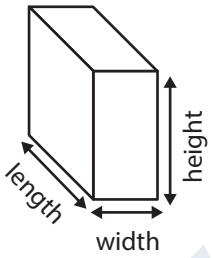
Rectangular Prism

Here are several examples.



Dimensions

All rectangular prisms have 3 dimensions: length, width, and height.

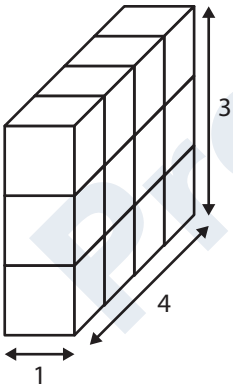


Using Numbers to Describe Rectangular Prisms

The prism at left below is made up of 3 layers, and each layer is 1 cube wide and 4 cubes long. We call this bottom layer the base. We use the following expression to show the dimensions of the base (1 × 4) and the height (3).

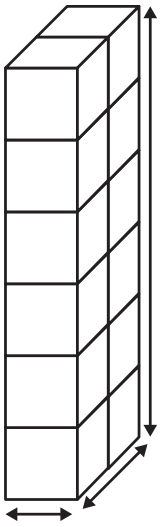
(1 × 4) × 3

When you multiply all three dimensions, you get the volume (the total number of cubes) of the rectangular prism.



What are the dimensions of *this* rectangular prism? Let's label it.

What expression would we use to show the dimensions of the base and the height?



Practice

- If you built a rectangular prism that matches one of these expressions, hold it up:

(1 × 1) × 12 **(1 × 6) × 2** **(2 × 2) × 3**

- Hold up the (1 × 4) × 3 prism you built, or build one right now.
- Now build a (4 × 1) × 3 prism. What do you notice?
- Now build a (3 × 1) × 3 prism. What do you notice?

NAME | **DATE**

Unit 1 Pre-Assessment Student Reflection Sheet

Skill	Look at these problems.	I can do this well already.	I can do this sometimes.	I need to learn to do this.	Notes
Can you find the value of an expression with parentheses?	1				
Can you fill in the blanks to make a set of equations true?	2				
Can you use number relationships to make one side of an equation equal to the other side without finding the answer?	3				
Can you use number relationships to tell if an equation is true or false without having to do the multiplication?	4				
Can you write an expression that includes grouping symbols to represent calculations?	5				
Can you write and solve an equation to represent computations that someone described in words?	6				
Can you write and solve an equation to show how many cubes it takes to build a rectangular prism?	7				
If you know how many baseballs there are in one layer of a box and how many layers the box has, can you find the total number of balls?	8				
Can you solve division story problems with remainders and handle the remainder for each problem depending on the situation?	9				

- After you have made a mark and some notes about each skill above, draw a star next to the two skills that you need to work on the most during this unit.
- Write other ideas about what you want or need to learn how to do during this unit.

NAME | **DATE**

Two-Centimeter Grid Paper

Student Book

GRADE 5 – UNIT 1 – MODULE 1

BRIDGES®
IN MATHEMATICS

Work Place Instructions 1A The Product Game

Each pair of players needs:

- a 1A The Product Game Record Sheet to share
- 2 game markers
- pencils

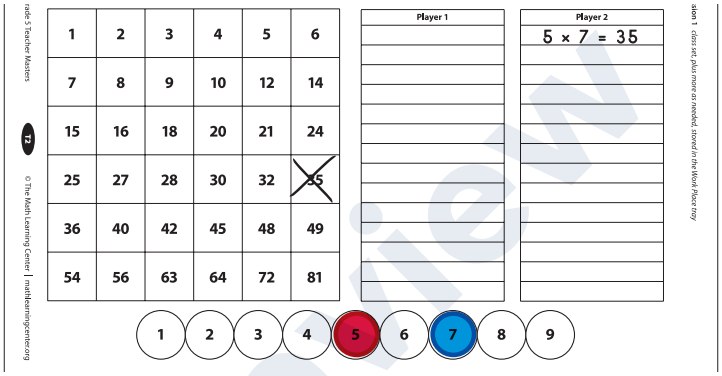
1 Players decide who is going first. Player 1 is O and Player 2 is X.

2 Player 1 places one of the game markers on any factor.

3 Player 2 places the other game marker on a factor. Then, he multiplies the two factors, draws an X on the product, and writes an equation to match the combination.

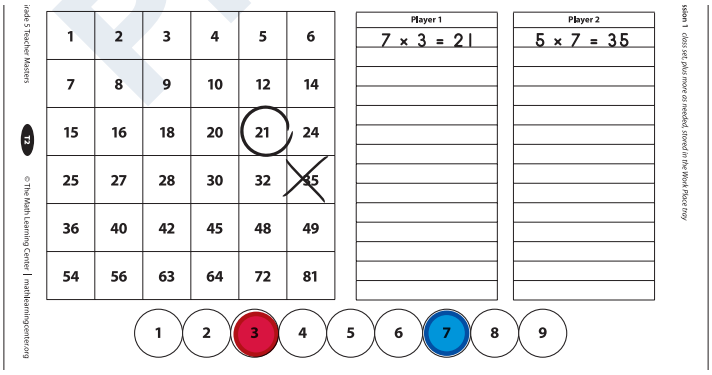
Player 1 *I choose 5.*

Player 2 *I choose 7. Let's see, 5 × 7 is 35, and I'm X, so I'll put my X on 35.*



4 Player 1 moves one game marker to get a new product. She can move either of the markers.

Player 1 *I'll move the factor marker from the 5 to the 3. Since 7 × 3 is 21, I get to put an O on 21.*



5 Play continues until a player gets four products in a row across, up and down, or diagonally.

- Only one factor marker can be moved during a player's turn.
- Players can move a game marker so that both are on the same factor. For example, both markers can be on 3. The player would mark the product 9 because 3 × 3 = 9.
- If the product a player chooses is already covered, the player loses that turn.

Game Variation

A Players play for five in a row.

NAME | **DATE**

You Choose

1 Choose 15 of the problems below to solve.

$8 \times 5 =$	$7 \times 7 =$	$4 \times 6 =$	$3 \times 8 =$	$4 \times 7 =$
$4 \times 9 =$	$6 \times 7 =$	$6 \times 8 =$	$8 \times 4 =$	$3 \times 6 =$
$10 \times 4 =$	$8 \times 10 =$	$8 \times 9 =$	$6 \times 11 =$	$12 \times 10 =$
$15 \times 4 =$	$40 \times 6 =$	$50 \times 8 =$	$10 \times 9 =$	$14 \times 9 =$
$25 \times 4 =$	$11 \times 9 =$	$6 \times 12 =$	$12 \times 9 =$	$7 \times 60 =$
$30 \times 6 =$	$13 \times 8 =$	$11 \times 5 =$	$25 \times 8 =$	$12 \times 8 =$

2 Explain how you decided which problems to solve.

NAME | DATE

Product Game Problems

ex Chloe and Ava were playing The Product Game. Their factor markers were on 4 and 5. Ava decided to move the marker from 5 to 7. Write a numerical expression to represent her move.

4×7

1 Chris and Katie were playing The Product Game. Their factor markers were on 9 and 2. Chris decided to move the marker from 2 to 6. Write a numerical expression to represent his move.

2 Eric and William were playing The Product Game together. William put an X on 42. One factor marker was on 6. The other factor marker was on _____.

3 Cindy placed an X on the product 36. What are all the possible locations of the two factor markers?

4 Eli placed an O on the product 24. What are all the possible locations of the two factor markers?

5 Hannah and Sean were playing The Product Game. Hannah needed to land on the product 18 to win the game. The markers were on 4 and 6.

a Which factor marker should Hannah move?

b Where should she place it?

6 Solve the following problems.

8	8	4	8	6		
× 10	× 5	× 6	× 6	× 12	× 12	× 8
					96	32

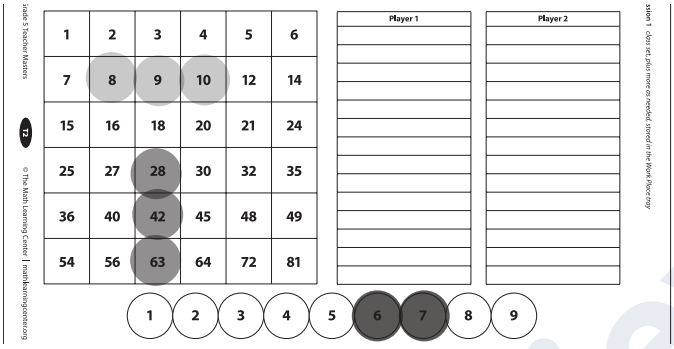
NAME | DATE

More Product Game Problems

1 Jack and Connor are playing The Product Game. They are using light and dark markers instead of X's and O's to cover their products on the game board.

a Jack is using the light markers. What move should he make next? Tell why.

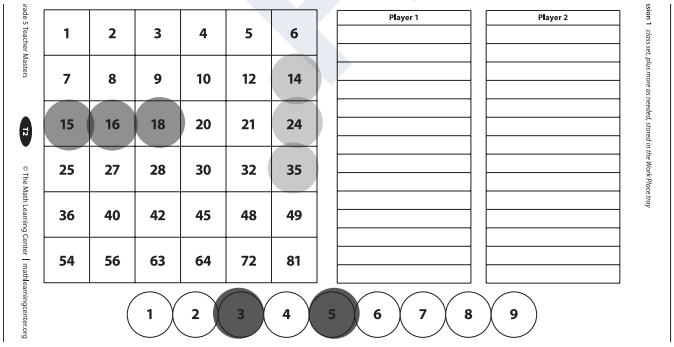
b Connor is using the dark markers. What move should he make next? Tell why.



2 Melanie and Jasmine are also playing The Product Game.

a Melanie is using the light markers. What move should she make next? Tell why.

b Jasmine is using the dark markers. What move should she make next? Tell why.



3 Solve the following.

11	11			9	9	9
× 8	× ☐	× 3	× 6	× 4	× ☐	× 9
☐	44	21	42	☐	45	☐

NAME | DATE

Facts & Boxes

1 To multiply numbers by 5, Kaylee first multiplies by 10 and then finds half the product.

a Write an expression with parentheses to show how Kaylee would solve 9×5.

b What is 9×5?

c Marshall says he would rather use 10×5 to find 9×5.
Write an expression with parentheses that uses 10×5 to find 9×5.

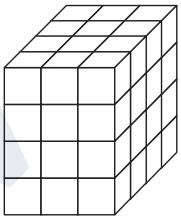
Match each expression with the correct box.

2 4 layers of 3-by-5 cubes $(3 \times 5) \times 4$

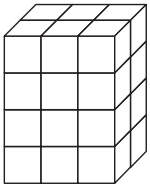
3 4 layers of 3-by-2 cubes $(3 \times 2) \times 4$

4 4 layers of 3-by-4 cubes $(3 \times 4) \times 4$

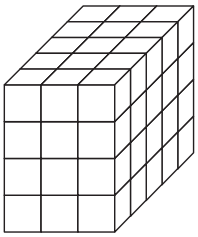
a



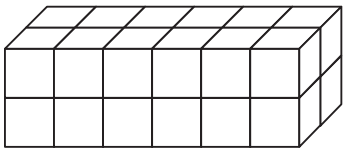
b



c



5 Fill in the dimensions of this box: (_____ × _____) × _____
dimensions of each layer — number of layers



6 Solve the following problems.

8	8	12	12	3	7	
× 4	× 8	× 10	× 5	× 7	× 6	× 6
						42

NAME | DATE

Fact Connections

1 Fill in the facts. Look for relationships.

3	3	3	6	6	6
× 2	× 4	× 8	× 2	× 4	× 8

2 Use the above information to help you fill in the blanks.

a $3 \times 4 =$ _____ $\times (3 \times 2) =$ _____

b $3 \times 8 =$ _____ $\times (3 \times 4) =$ _____

c $6 \times 2 = (3 \times 2) \times$ _____ $=$ _____

d $6 \times 4 = 2 \times (6 \times$ _____$) =$ _____

e $2 \times (6 \times 4) =$ _____ $\times 8 =$ _____

3 Fill in the facts. Look for relationships.

4	4	4	8	8	8
× 2	× 4	× 8	× 2	× 4	× 8

4 Use the above information to help you write an equation that includes parentheses.

ex $8 \times 4 = 2 \times (8 \times 2)$ "To find 8×4, I can double 8×2."

a $4 \times 6 =$

b $4 \times 12 =$

c $8 \times 8 =$

5 **CHALLENGE** Complete the following equations.

a $4 \times 67 =$ _____ $\times (2 \times 67)$

b $8 \times 198 = 2 \times ($_____ $\times 198)$

c _____ $\times 3{,}794 = 2 \times (4 \times 3{,}794)$

Home Connections

GRADE 5 – UNIT 1 – MODULE 1

BRIDGES®
IN MATHEMATICS

NAME | DATE

What's the Problem? page 1 of 2

ex To find 3 times any number, Maria doubles the number, then adds the number again.

a Write an expression with parentheses to show how Maria would solve 3 × 6.
(2 × 6) + 6

b What is 3 × 6? 18

c What is another way to think about 3 × 6?
You could do 3 × 5, which is really easy, and then add 3 more, like this (3 × 5) + 3

1 To find 4 times any number, Susan uses the Double-Double strategy (multiply by 2, then by 2 again). Susan wrote (2 × 9) × 2 to record how she would solve 4 × 9.

a What is 4 × 9?

b What is another way to solve 4 × 9?

2 To find 5 times any number, Kaylee first multiplies by 10 and then finds half the product.

a Write an expression with parentheses to show how Kaylee would solve 7 × 5.

b What is 7 × 5?

c What is another way to solve 7 × 5?

3 When given any number times 9, Jasper multiplies the number by 10 and then removes one group of the number.

a Write an expression with parentheses to show how Jasper would solve 3 × 9.

b What is 3 × 9?

c What is another way to think about 3 × 9?

(continued on next page)

NAME | **DATE**

What's the Problem? page 2 of 2

4 Braden loves multiplying by 8 because he can double-double-double.

a Write an expression with parentheses to show how Braden would solve 8×7.

b What is 8×7?

c What is another way to think about 8×7?

5 Jonah was asked to add 4 and 7 then multiply the sum by 9. Which expression shows Jonah's problem? (The *sum* is the answer to an addition problem.)

a $(4 + 7) \times 9$ **b** $(7 - 4) \times 9$ **c** $4 + (7 \times 9)$

6 Patrick needed to multiply 4 and 6 then subtract 12 from the product. Write an expression with parentheses to show the problem. (The *product* is the answer to a multiplication problem.)

7 Violet divided 81 by 9 then multiplied the quotient by 3. Write an expression with parentheses to show the problem. (The *quotient* is the answer to a division problem.)

8 Solve.

a $54 - (3 \times 8)$ **b** $(28 \div 7) \times 4$

9 **CHALLENGE** Rafael was given the problem 44×9. Write an expression to show how you would solve the problem.

NAME | DATE

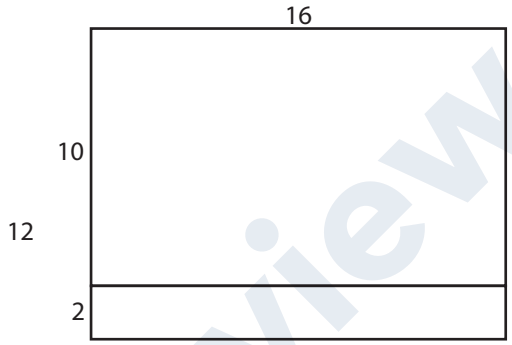
Multiplication Connections page 1 of 2

ex To multiply a number by 5, Marissa first multiplies by 10 and then finds half the product.

ex Write an expression with parentheses to show how Marissa would solve 24 × 5.
(24 × 10) ÷ 2

ex What is 24 × 5?
120

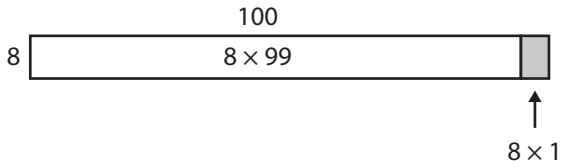
1 To multiply a number by 12, Carter likes to multiply the number by 10 and then multiply it by 2 and add the products. Here is a picture of his thinking.



a Write an expression with parentheses to show how Carter would solve 12 × 16.

b What is 12 × 16? _____

2 To multiply a number by 99, Sofia likes to multiply by 100 and then subtract 1 group of the factor. Here is a picture of her thinking.



a Write an expression with parentheses to show how Sofia would solve 8 × 99.

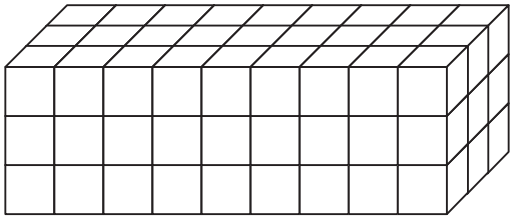
b What is 8 × 99? _____

(continued on next page)

NAME | **DATE**

Multiplication Connections page 2 of 2

3 Fill in the dimensions of this box: _____ × _____ × _____



4 Solve the following problems.

2	4	8	10	28	28	13
× 13	× 13	× 13	× 28	× 5	× 15	× ☐
☐	☐	☐	☐	☐	☐	52

5 Find the products.

a (2 × 5) × 8 = _____ **b** (2 × 8) × 5 = _____ **c** (5 × 8) × 2 = _____

6 Which of the problems in item 5 is the easiest for you to solve? In other words, in which order would you prefer to multiply the three factors? Why?

7 Find the products.

a (6 × 7) × 10 = _____ **b** (6 × 10) × 7 = _____ **c** (7 × 10) × 6 = _____

8 Which of the problems in item 7 is the easiest for you to solve? In other words, in which order would you prefer to multiply the three factors? Why?