NUMBER CORNER®

SECOND EDITION

TEACHERS GUIDE

SEPTEMBER

GRADE 5

Published by The MATH LEARNING CENTER Salem, Oregon

Number Corner September

Teacher Masters

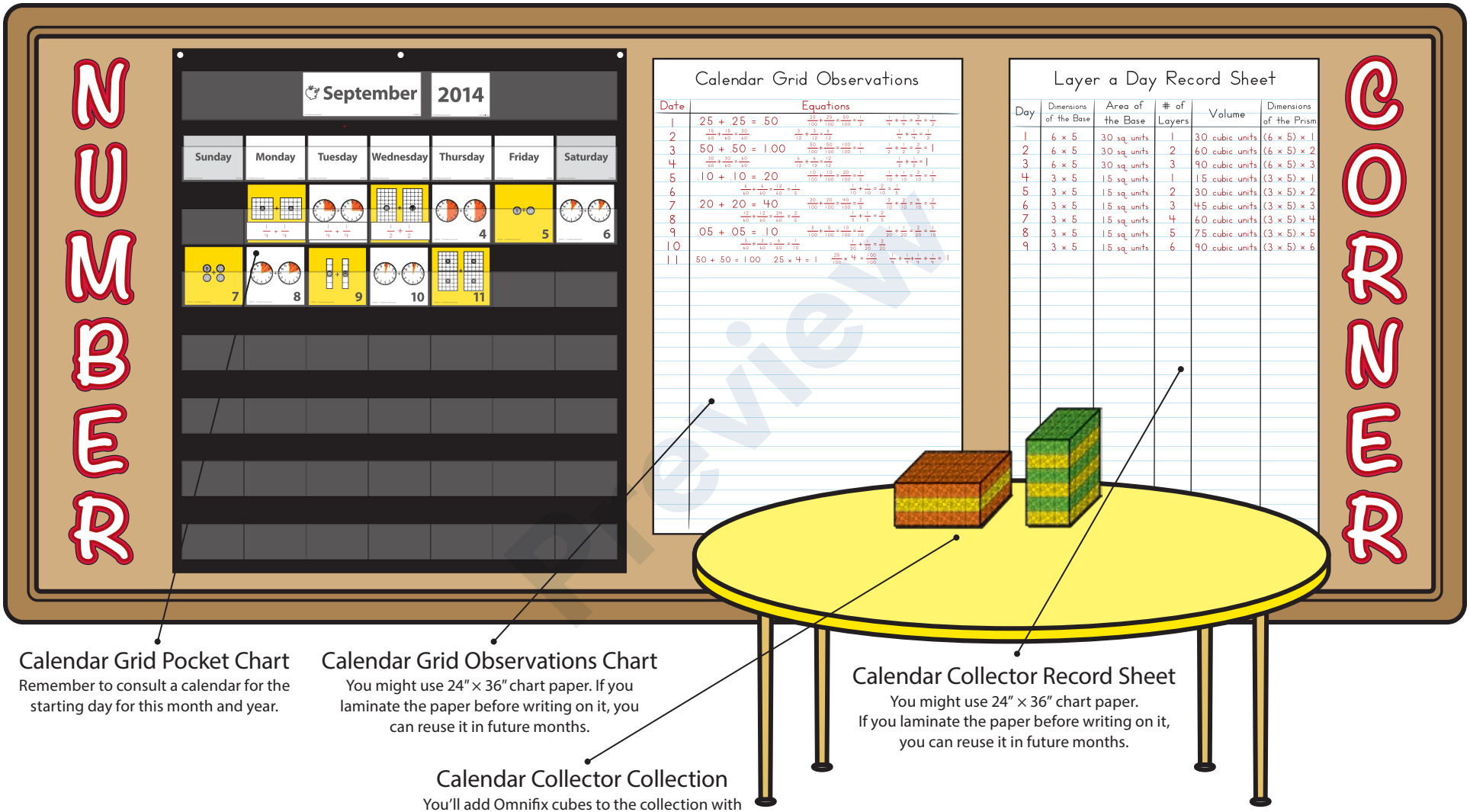
Pages renumber each month.

Number Corner Student Book Pages

Page numbers correspond to those in the consumable books.

September Sample Display

Of the items shown below, some are ready-made and included in your kit; you'll prepare others from classroom materials and the included teacher masters. Refer to the Preparation section in each workout for details about preparing the items shown. The display layout shown fits on a 10′ × 4′ bulletin board or on two 6′ × 4′ bulletin boards. Other configurations can be used according to classroom needs. If you have extra space to work with, a Number Corner header may be made from bulletin board letters, student-drawn letters, or other materials.



Calendar Grid Pocket Chart

Remember to consult a calendar for the starting day for this month and year.

Calendar Grid Observations Chart

You might use 24″ × 36″ chart paper. If you laminate the paper before writing on it, you can reuse it in future months.

Calendar Collector Collection

You'll add Omnifix cubes to the collection with each update. Keep the cubes on display near the record sheet.

Calendar Collector Record Sheet

You might use 24″ × 36″ chart paper. If you laminate the paper before writing on it, you can reuse it in future months.

September Daily Planner

Day	Date	Calendar Grid	Calendar Collector	Computational Fluency	Solving Problems	Problem Strings	Assessment
1						**Activity 1** Problem String 1 (p. 51)	
2				**Activity 1** Introducing the Multiple Game (p. 32)			
3		**Activity 1** Introducing the Calendar Grid (p. 9)					
4		*Update*	**Activity 1** Introducing the Calendar Collector (p. 19)				
5		*Update*	*Update*		**Activity 1** Introducing Solving Problems (p. 40)		
6		**Activity 2** Equations & Equivalencies (p. 14)	*Update*				
7		*Update*	*Update*				Baseline Assessment, Part 1 (p. 62)
8		*Update*	*Update*				Baseline Assessment, Part 2 (p. 64)
9		*Update*	**Activity 2** Revisiting the Collection (p. 22)				
10		*Update*	*Update*		**Activity 2** Discussing Rock Hopping (p. 42)		
11		*Update*	*Update*			**Activity 2** Problem String 2 (p. 55)	
12		*Update*	*Update*	**Activity 2** Partner Multiple Game (p. 36)			
13		**Activity 2** Equations & Equivalencies (p. 14)	*Update*				
14		*Update*	**Activity 3** What's Missing? (p. 26)				
15		*Update*	*Update*	**Activity 3** Maddy's Multiples (p. 37)			
16		*Update*	*Update*		**Activity 3** Solving Another Problem (p. 45)		
17		*Update*	*Update*			**Activity 2** Problem String 3 (p. 55)	
18		**Activity 3** Discussing Predictions & Patterns (p. 17)	*Update*				
19		*Update*	**Activity 4** Analyzing Layers (p. 28)				
20		*Update*	*Update*		**Activity 4** Discussing Field Trip Snacks (p. 46)		

Note *Calendar Grid and Calendar Collector workouts are updated by a student helper outside of Number Corner time, except when the workout is the subject of an activity. Computational Fluency, Problem Strings, Solving Problems, and Assessments do not have updates. Update routines are explained in detail in the workout text. Summaries of the update routines appear below.*

Calendar Grid – The student helper turns one or more calendar markers so that the Calendar Grid is complete up to the current date, then updates the information on the Calendar Grid Observations Chart.
Calendar Collector – The student helper constructs a new layer to the rectangular solid already formed, then fills in the information on the Calendar Collector Record Sheet.

Number Corner
September

Overview

September's workouts focus on addition and subtraction of whole numbers, decimals and fractions, multiples and factors, and volume. Over the course of the month, students will review, revisit, and extend fourth grade skills and concepts as they begin to move into fifth grade content.

Activities

Workouts	Day	Activities	D	G	SB
Calendar Grid Fraction & Decimals This month's Calendar Grid focuses on fraction and decimal equivalencies, as well as adding fractions with like denominators and adding decimals (tenths and hundredths). Each day, a student helper reveals the next marker on the Calendar Grid and records information on the Calendar Grid Record Sheet. On days when the Calendar Grid is featured, students share observations about the markers, generate equivalent expressions to match the visuals on the markers, search for and describe emerging patterns in the sequence, and make predictions about future markers based on their observations.	3	**1** Introducing the Calendar Grid	●		
	6, 13	**2** Equations & Equivalencies	●		
	18	**3** Discussing Predictions & Patterns	●		
Calendar Collector Layer a Day This month, students collect data about rectangular prisms as they are built one layer at a time. Students explore the relationships between dimensions, area, and volume, and apply the associative and commutative properties of multiplication.	4	**1** Introducing the Calendar Collector	●		
	9	**2** Revisiting the Collection	●		
	14	**3** What's Missing?	●		●
	19	**4** Analyzing Layers	●		
Computational Fluency Multiple Game To open the first Computational Fluency workout this month, the teacher introduces the Multiple Game. Students play with the teacher in Activity 1, then play with a partner in Activity 2. In Activity 3, students complete a Student Book page about the game, multiples, products, primes, and composite numbers.	2	**1** Introducing the Multiple Game	●	●	
	12	**2** Partner Multiple Game	●	●	●
	15	**3** Maddy's Multiples	●		●
Solving Problems Solving Problems Using Multiples & Factors The Solving Problems workout this month features two sets of problems. Students are given time to solve the problems and then discuss their solutions and strategies as a class. The mathematical content focuses mainly on using multiples and factors, particularly least common multiples and greatest common factors. In addition, the problems themselves will almost certainly help students identify diagrams and organized lists as useful problem-solving tools.	5	**1** Introducing Solving Problems	●		●
	10	**2** Discussing Rock Hopping	●		
	16	**3** Solving Another Problem	●		●
	20	**4** Discussing Field Trip Snacks	●		
Problem Strings Addition & Subtraction Strategies This month, students solve and discuss problem strings designed to elicit efficient strategies for adding and subtracting whole numbers and decimal numbers.	1	**1** Problem String 1	●		●
	11	**2** Problem String 2	●		●
	17	**3** Problem String 3	●		●
Assessment Baseline Assessment Students complete a written assessment of fourth grade skills. They complete the first part of the assessment during one Number Corner session and the second part the following day.	7	**Baseline Assessment, Part 1** Completing Pages 1–3			●
	8	**Baseline Assessment, Part 2** Completing Pages 4–6			●

D – Discussion, **G** – Game, **SB** – Number Corner Student Book

Teaching Tips

Use the first month of Number Corner to establish routines that students will use for the rest of the year. For example, if students are coming to a discussion area or space designated for Number Corner, help them learn how to get there quickly and quietly and make sure they know what materials to bring. Be very explicit about the expectations for these routines and transitions, and get students to reflect on how they are doing and what they could be doing better.

Set up your Number Corner materials before the start of the school year. This will help you familiarize yourself with the workouts and will make organization easier once the school year starts.

Don't worry too much if students are not getting all of the math in this month's Number Corner (or if it seems too easy). Use September as an opportunity to get to know your students. Number Corner provides ongoing opportunities for informal assessment as students share, explain, and justify their thinking.

Number Corner should take about 20 minutes a day. It's great if you have more time to spend on Number Corner activities, but don't worry if you feel that you are not getting everything done in each activity this month. As you and your students adjust to the rhythms and routines of Number Corner, the activities will begin to go more quickly.

Number Corner Student Book pages accompany some of the workouts. Ideally, these will be done and discussed in class. However, if you are running out of time, you can assign them as homework. Use these Student Book pages as another means of informal assessment.

Work on getting as much student participation as you can during Number Corner. Students will be asked to explain their thinking and to share their strategies. Try to refrain from explaining for them or to them. When students have the opportunity to talk through their thinking, their learning experience is more positive and meaningful. If a student makes a mistake, refrain from identifying it right away. Usually, the student or a classmate will catch it. Encourage students to ask questions, summarize each other's ideas, and make connections to the conversation. These steps will contribute to powerful learning in your classroom.

Target Skills

The table below shows the major skills and concepts addressed this month. It is meant to provide a quick snapshot of the expectations for students' learning during this month of Number Corner.

Major Skills/Concepts Addressed	CG	CC	CF	SP	PS
4.OA.4 Find all factor pairs for a whole number between 1 and 100			●	●	
4.OA.4 Demonstrate an understanding that a whole number is a multiple of each of its factors			●	●	
4.NF.1 Recognize equivalent fractions	●				
4.NF.4a Demonstrate an understanding that a fraction a/b is a multiple of the unit fraction 1/*b*	●				
4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100	●				
4.NF.6 Write fractions with denominators 10 or 100 in decimal notation	●				
5.OA.1 Write and evaluate numerical expressions with parentheses		●			
5.OA.2 Interpret numerical expressions without evaluating them		●			
5.NBT.7 Add and subtract decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and the relationship between addition and subtraction	●				●
5.MD.4 Measure the volume of a solid figure by counting the number of cubic units that fill it, with no gaps or overlaps		●			
5.MD.5a Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes		●			
5.MD.5a Show that the volume of a right rectangular prism with whole number edge lengths can be found by multiplying the edge lengths or by multiplying the area of the base by the height		●			

Major Skills/Concepts Addressed	CG	CC	CF	SP	PS
5.MD.5a Represent the product of three whole numbers as the volume of a right rectangular prism whose edge lengths are equal to those three whole numbers		●			
5.MP.1 Make sense of problems and persevere in solving them				●	
5.MP.2 Reason abstractly and quantitatively	●	●			
5.MP.3 Construct viable arguments and critique the reasoning of others	●			●	●
5.MP.4 Model with mathematics		●		●	●
5.MP.5 Use appropriate tools strategically			●		
5.MP.7 Look for and make use of structure		●		●	
5.MP.8 Look for and express regularity in repeated reasoning			●		●

CG – Calendar Grid, **CC** – Calendar Collector, **CF** – Computational Fluency,
SP – Solving Problems, **PS** – Problem Strings

Assessments

This month, students complete a written Baseline Assessment. This assessment serves as an early warning system that makes it possible to quickly identify individuals who may need extra support or special services if they haven't been identified for such in earlier grades. The table below lists the skills assessed in each part of the Baseline Assessment. Note that these are all skills students should have mastered in fourth grade.

The Baseline Assessment is a one-time tool, designed to inform your instruction rather than gauge students' growth over time. Quarterly checkups that appear in October, January, March, and May serve a similar purpose: each provides a snapshot of individual students at that particular time of year, with regard to the skills that have been emphasized in the couple of months prior to the checkup. If you want to gauge students' growth and progress over time with regard to the Common Core State Standards, you can use the optional Comprehensive Growth Assessment, located in the Grade 5 Number Corner Assessment Guide.

Skills/Concepts Assessed in the Baseline Assessment

- Solve story problems involving a multiplicative comparison using multiplication or division (4.OA.2)
- Generate a shape pattern that follows a given rule (4.OA.5)
- Divide a 2- or 3-digit number by a 1-digit number, with and without remainders using strategies based on place value, the properties of operations, or the relationship between multiplication and division (4.NBT.6)
- Use equations or arrays to explain strategies for dividing a multi-digit number by a 1-digit number (4.NBT.6)
- Compare two fractions with different numerators and different denominators (4.NF.2)
- Add and subtract mixed numbers with like denominators (4.NF.3c)
- Solve story problems involving subtraction of fractions referring to the same whole and with like denominators (4.NF.3d)
- Multiply a fraction by a whole number (4.NF.4b)
- Compare two decimal numbers with digits to the hundredths place (4.NF.7)
- Identify the relative sizes of centimeters, meters, and kilometers; grams and kilograms; and milliliters and liters (4.MD.1)
- Record equivalent measurements in different units from the same system of measurement using a 2-column table (4.MD.1)
- Solve story problems involving distance, liquid, and mass, using addition, subtraction, multiplication, and division of whole numbers (4.MD.2)
- Solve story problems that involve expressing measurements given in a larger unit in terms of a smaller unit within the same system of measurement (4.MD.2)
- Use diagrams to represent measurement quantities (4.MD.2)
- Apply the area and perimeter formulas for a rectangle to solve a problem (4.MD.3)

- Make a line plot to display a data set comprised of measurements taken in halves, fourths, eighths of a unit (4.MD.4)
- Solve problems involving subtraction of fractions shown on a line plot (4.MD.4)
- Identify and draw lines of symmetry; identify figures with line symmetry (4.G.3)

Materials Preparation

Each workout includes a list of required materials by activity. You can use the table below to prepare materials ahead of time for the entire month.

Materials		**Done**
Copies	Run copies of Teacher Masters T1–T7 according to the instructions at the top of each master.	
	Run a single display copy of Student Book pages 2, 3 & 5.	
Charts	Prior to Calendar Grid Activity 1, prepare the Calendar Grid pocket chart, Calendar Grid Observations Chart, and index card strips according to preparation instructions in the workout.	
	Prior to Calendar Collector Activity 1, create a Calendar Collector Record Sheet according to preparation instructions in the workout.	
Classroom Materials	Prior to Calendar Collector Activity 1, prepare Omnifix cubes according to preparation instructions in the workout.	

September Calendar Grid

Fractions & Decimals

Overview

This month's Calendar Grid focuses on fraction and decimal equivalencies, as well as adding fractions with like denominators and adding decimals (tenths and hundredths). Each day, a student helper reveals the next marker on the Calendar Grid and records information on the Calendar Grid Record Sheet. On days when the Calendar Grid is featured, students share observations about the markers, generate equivalent expressions to match the visuals on the markers, search for and describe emerging patterns in the sequence, and make predictions about future markers based on their observations.

Skills & Concepts

- Recognize equivalent fractions (4.NF.1)
- Demonstrate an understanding that a fraction *a*/*b* is a multiple of the unit fraction 1/*b* (4.NF.4a)
- Express a fraction with denominator 10 as an equivalent fraction with denominator 100 (4.NF.5)
- Write fractions with denominators 10 and 100 in decimal notation (4.NF.6)
- Write decimal numbers with digits to the hundredths place (supports 4.NF)
- Represent decimal numbers with digits to the hundredths place using fraction equivalents (supports 4.NF)
- Add decimals to hundredths, using concrete models or drawings and strategies based on place value and properties of operations (5.NBT.7)
- Reason abstractly and quantitatively (5.MP.2)
- Construct viable arguments and critique the reasoning of others (5.MP.3)

Materials

Activities	Day	Copies	Kit Materials	Classroom Materials
Activity 1 Introducing the Calendar Grid	3	**TM T1** Money & Clock Models	• Calendar Grid pocket chart • Fractions & Decimals Calendar Markers • Month, Day, and Year Cards • Money Value Pieces (1 set)	• 2 sheets of lined chart paper (see Preparation) • erasable marker • 15 index cards (see Preparation) • piece of paper to mask portions of Teacher Master
Activity 2 Equations & Equivalencies	6, 13			
Activity 3 Discussing Predictions & Patterns	18			

TM – Teacher Master, **NCSB** – Number Corner Student Book
Copy instructions are located at the top of each teacher master.

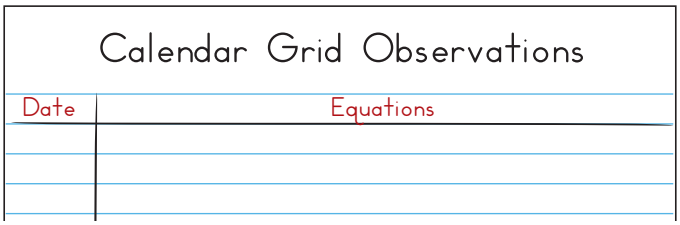
Vocabulary

An asterisk [*] identifies those terms for which Word Resource Cards are available.

denominator*
equivalent expressions*
equivalent fractions*
improper fraction*
mixed number*
numerator*
unit fraction*

Preparation

- Before the first Calendar Grid workout, place the numbered Fractions & Decimals Calendar Markers face-down, in sequence, in the Calendar Grid pocket chart, so that the visuals are hidden from students.
- Make a Calendar Grid Observations Chart from two sheets of lined chart paper. Label the top of one piece "Calendar Grid Observations." Laminate both sheets. Next, use an erasable marker and yardstick to draw two columns on each sheet. Label the columns at the top of the first sheet, as illustrated. Post the chart on your Number Corner display board next to or near the Calendar Grid pocket chart.



- Use the second piece of chart paper to extend the chart midway through the month. Use an erasable marker to record students' observations so that you can re-use the chart each month.
- Cut each of the 15 index cards in half vertically to create a 1 ½" × 5" strip. Store the 30 strips in an envelope or zip-top bag. You will need these each time you conduct a Calendar Grid activity with the class, but students updating the calendar each day will not.
- Develop a system for having students update the Calendar Grid on days when you are not doing a Calendar Grid activity as class. For example, if you have a helper of the day, it can be the helper's job to turn over the Calendar Marker, sometime other than during Number Corner. If you have time, another way to handle updating the Calendar Grid is to take a minute or two to update the grid as a class by having a student turn over the day's Calendar Marker right before or after you do the designated workout. Encourage students to save their observations and ideas about the markers until you do a complete Calendar Grid activity, however.

Mathematical Background

Understanding equivalence is critical to adding and subtracting fractions. We want students to have a variety of meanings come to mind when they see a fraction. For example, when a student encounters ¼, he might think of 1 quarter, 25 cents, \$0.25, half of ½, double ⅛, 25%, dividing something by 4, ¼ of an hour, 15 minutes out of 60 minutes, a distance ¼ of a unit from 0, and so on. Then, when students see ¼ added to another fraction, they can use the meaning that is most helpful, given the denominator of the other fraction. By using money and time as referents, students begin to create connections between those models and fractions, and can then use them to solve problems. As students work with these ordinary, everyday denominators of 100 (money) and 60 (time), they build intuitions about finding common denominators to add or subtract fractions.

For instance, if the problem is $\frac{1}{4} + \frac{1}{10}$, students might think of the fractions in terms of money: \$0.25 + \$0.10 = \$0.35, so $\frac{1}{4} + \frac{1}{10} = \frac{35}{100}$. Also, 35 cents is 7 nickels, so $\frac{35}{100}$ is equivalent to 7 nickels out of 20 nickels. The use of pennies and nickels allows students to see and understand that $\frac{35}{100} = \frac{7}{20}$. If the problem is $\frac{1}{4} + \frac{1}{3}$, students might think of the fractions in terms of time: 15 minutes and 20 minutes is 35 minutes out of 60 minutes, therefore $\frac{1}{4} + \frac{1}{3} = \frac{35}{60}$. Since there are seven 5-minute chunks in 35 minutes and twelve 5-minute chunks in 60 minutes, students can see and understand that $\frac{35}{60} = \frac{7}{12}$.

Money Value Pieces

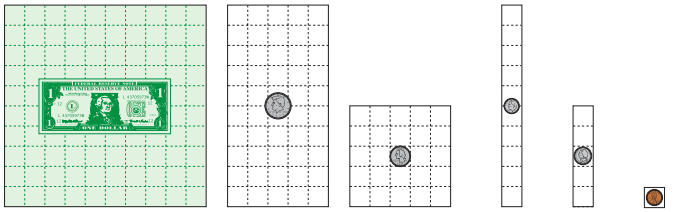
Some of the markers this month feature pictures of Money Value Pieces. These are similar to base ten pieces, with a mat of 100 small squares representing a dollar, a half mat representing 50 cents (literally, a half dollar), a quarter mat representing 25 cents, and so on. The visuals on these markers are designed to support students in understanding the fraction of a dollar represented by each coin in our monetary system. You will find a set of Money Value Pieces in your Number

Key Questions

Learning to search for, describe, and extend patterns facilitates algebraic thinking. Use these questions to help your students investigate this month's pattern.

- What will today's marker look like? What number and model will it show? How do you know?
- What equivalencies can be recorded?
- What patterns can you see so far?
- Does the pattern remain constant, or does it change over time? If so, how does it change?
- How are non-unit fractions related to unit fractions?

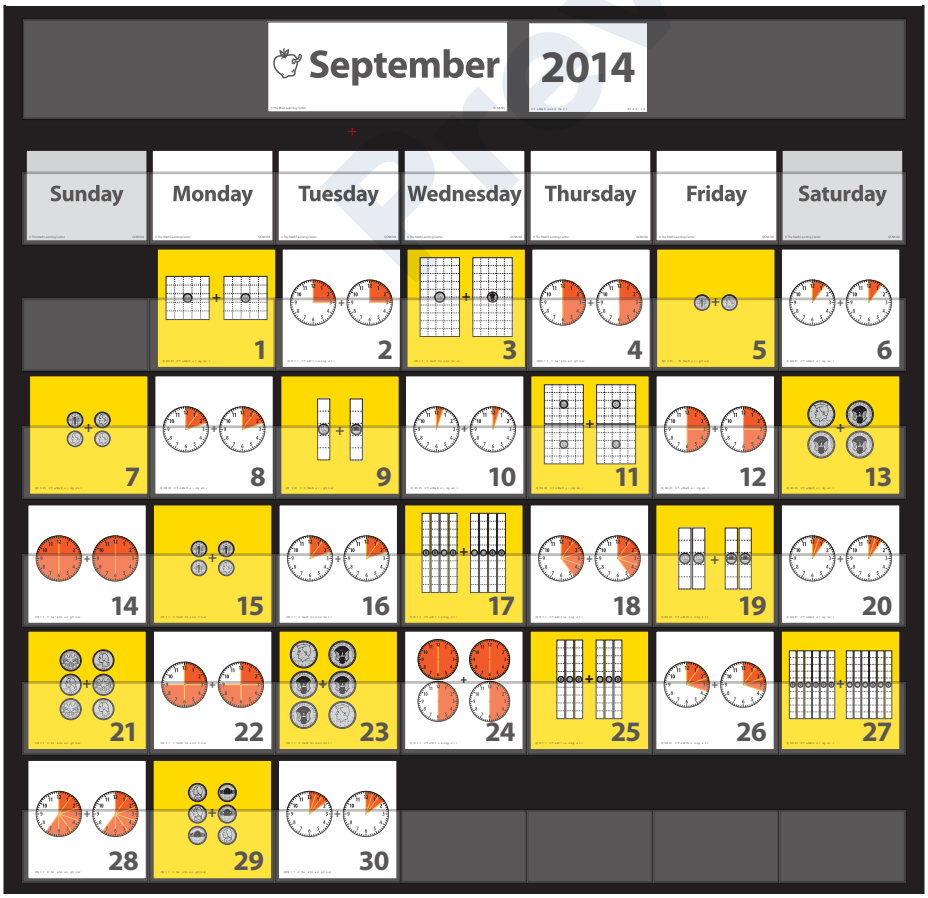
Corner kit to provide further support to students who might not automatically know that a nickel is 1⁄20 of a dollar. Make this set easily accessible to students throughout the month.



About the Pattern

The patterns featured in this month's sequence of markers are described below. Revealing one calendar marker each day allows students to make and test predictions, discovering patterns as new markers are added and their predictions are confirmed or proven false. Don't tell them what the patterns are: instead, allow them to pursue their own ideas and investigations. Don't worry if their ideas seem off base early in the month; as they accumulate information, discuss their observations, and justify their predictions, they will revise and refine their thinking.

- Markers alternate between money and clock models, as well as between goldenrod and white backgrounds.
- Money markers alternate between pictures of money value pieces and pictures of coins in an AABB pattern.
- Pairs of consecutive markers have identical fraction expression labels.
- Markers 1-10 are unit fractions, markers 11-20 have numerators of 2 and represent doubles of markers 1-10, and markers 21-30 have numerators of 3 and represent triples of markers 1-10.
- The denominators feature a repeating pattern of 4, 4, 2, 2, 10, 10, 5, 5, 20, 20.



Fraction Expression Labels

Each time you conduct the Calendar Grid workout with your class, you will work with input from the students to write a fraction expression label to represent the visuals on each marker that has been turned over since the previous activity. For your own reference, here is a list of the expressions you will write through the month.

Markers 1–10	
Marker	**Label**
1	$\frac{1}{4} + \frac{1}{4}$
2	$\frac{1}{4} + \frac{1}{4}$
3	$\frac{1}{2} + \frac{1}{2}$
4	$\frac{1}{2} + \frac{1}{2}$
5	$\frac{1}{10} + \frac{1}{10}$
6	$\frac{1}{10} + \frac{1}{10}$
7	$\frac{1}{5} + \frac{1}{5}$
8	$\frac{1}{5} + \frac{1}{5}$
9	$\frac{1}{20} + \frac{1}{20}$
10	$\frac{1}{20} + \frac{1}{20}$

Markers 11–20	
Marker	**Label**
11	$\frac{2}{4} + \frac{2}{4}$
12	$\frac{2}{4} + \frac{2}{4}$
13	$\frac{2}{2} + \frac{2}{2}$
14	$\frac{2}{2} + \frac{2}{2}$
15	$\frac{2}{10} + \frac{2}{10}$
16	$\frac{2}{10} + \frac{2}{10}$
17	$\frac{2}{5} + \frac{2}{5}$
18	$\frac{2}{5} + \frac{2}{5}$
19	$\frac{2}{20} + \frac{2}{20}$
20	$\frac{2}{20} + \frac{2}{20}$

Markers 21–31	
Marker	**Label**
21	$\frac{3}{4} + \frac{3}{4}$
22	$\frac{3}{4} + \frac{3}{4}$
23	$\frac{3}{2} + \frac{3}{2}$
24	$\frac{3}{2} + \frac{3}{2}$
25	$\frac{3}{10} + \frac{3}{10}$
26	$\frac{3}{10} + \frac{3}{10}$
27	$\frac{3}{5} + \frac{3}{5}$
28	$\frac{3}{5} + \frac{3}{5}$
29	$\frac{3}{20} + \frac{3}{20}$
30	$\frac{3}{20} + \frac{3}{20}$
31	$\frac{4}{4} + \frac{4}{4}$

Update

Have a student helper follow this update procedure every day that the Calendar Grid is not a featured activity.

Procedure

- Turn over one or more calendar markers so that the Calendar Grid is complete up to the current date.
- Update the information on the Calendar Grid Observations Chart.

Note

Let students know that if they are called upon to update the grid and chart on a Monday, they're responsible for revealing the markers for three days rather than one (Saturday, Sunday, and Monday) and recording equations for each of them on the observations chart.

Activity 1

Introducing the Calendar Grid — Day 3

1 Begin by gathering students in front of the Calendar Grid pocket chart and Calendar Grid Observations Chart. Introduce the Calendar Grid by explaining that the class will turn over a new marker for each day of the month, make and record observations and predictions, and look for patterns as the markers are revealed.

Connect to students' previous experience by asking what they remember about calendar patterns from previous years in school (if they have used the Bridges curriculum before).

2 Display the top half of the Money & Clock Models Teacher Master. Give students a few moments to examine the display quietly, and then have them share their observations with a partner.

Note with students that each coin is pictured in regular form, as well as on a money value piece. Money value pieces, which students have encountered in earlier grades, are similar to base ten pieces, and make it possible to easily *see* the fraction and decimal value of each coin with respect to a dollar.

3 Then ask students to consider how each coin can be represented both as a decimal and a fraction. Invite students to share their thinking, and record their suggestions on the teacher master.

Teacher *How can we record the value of each coin as both a decimal and a fraction?*

Akiko *The decimals are easy. It's just like when you write money amounts—.50, .25, .10, .05, and .01.*

Teacher *And how can we record a fraction for each coin?*

Students *The penny would be $\frac{1}{100}$. It's just one cent out of 100. You can see that it takes 100 of the little squares to fit into the dollar, just like with the units and the mats on the base ten pieces.*

Yeah and the quarter is $\frac{25}{100}$.

Or it can be $\frac{1}{4}$, can't it?

Teacher *Which is it? $\frac{25}{100}$ or $\frac{1}{4}$?*

Sam *It's both. If you look at the money value pieces, it's easy to see that the quarter piece is 25 little boxes, which is one-fourth of 100. You can also see that it would take exactly 4 of those quarter squares to fit into the dollar square.*

Teacher *So, $\frac{25}{100}$ and $\frac{1}{4}$ are equivalent, then? I'll write them both. Anything else?*

Whitney *Not for the quarter I don't think. But you can put $\frac{10}{100}$ or $\frac{1}{10}$ under the dime. It's ten cents out of 100, plus you can see that it would take 10 of the dime strips to fit into the dollar square.*

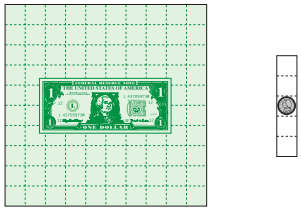
Teacher *What about the nickel?*

Troy *$\frac{1}{5}$!*

Natalie *Wait! It's not $\frac{1}{5}$. That would mean it would only take 5 of them to fit into the dollar mat. Look how little the nickel strips are—they're half the size of the dime strips.*

Troy *Wait a minute—I don't get it. A nickel is 5, right? So it's a fifth of a dollar.*

Teacher *I have a set of Money Value Pieces right here. Let's get them out and see how big the nickel piece is compared to the dollar piece.*



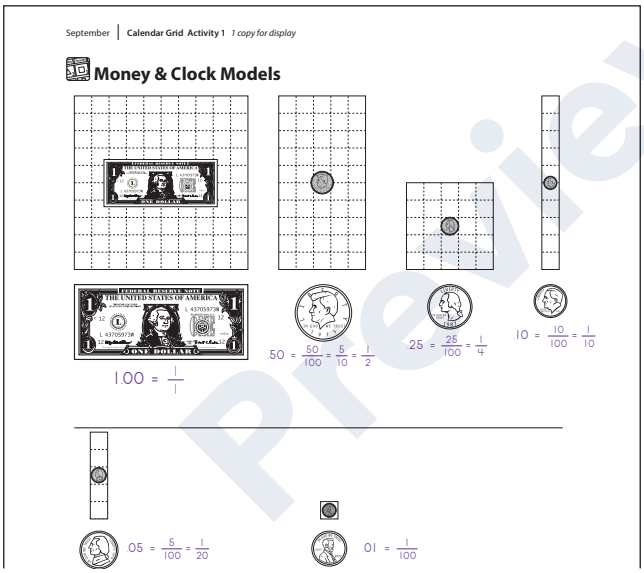
Natalie *See what I mean? The nickel is really tiny. If it was a fifth of a dollar, it would only take 5 of them, but you can see it would take lots more than that to make one of those dollar squares.*

Teacher *Troy, why don't you find out how many of the nickel pieces it takes to make a square the size of the dollar mat?*

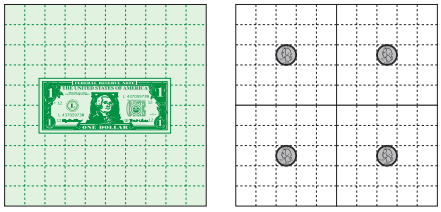
Troy *OK, it's 20. That's a lot more than five!*

Teacher *Does that sound right to everyone?*

Natalie *It does. You need twice as many nickels as dimes because the dime strip is twice as big as the nickel strip. Since you need 10 dimes to make a dollar, it makes sense that you'd need 20 nickels.*



SUPPORT Have the actual Money Value Pieces from your Number Corner Kit out and available during this initial discussion. Support students in their assertions that (for example) a quarter is one-fourth of a dollar by inviting a volunteer to use the quarter pieces to form a square the same size as the dollar mat, or even set the quarter pieces directly on top of the dollar mat.



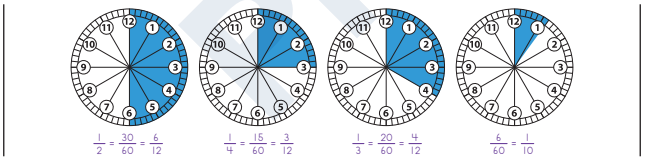
While many students may already know that a quarter is one-fourth of a dollar, and a dime is one-tenth of a dollar, they may need to use the Money Value Pieces to determine that a nickel is one-twentieth of a dollar.

4 Reveal the bottom portion of the Money & Clock Models Teacher Master and draw students' attention to the first clock.

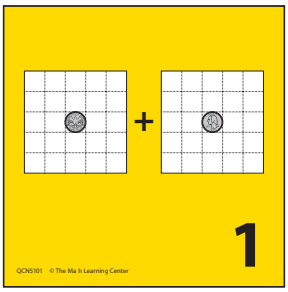
- Have students share ideas, first in pairs and then as a whole group, about the fraction of the clock face that is shaded in.
- Press the students to generate all the equivalent fractions they can justify, given the structure of the clock face.
- Record students' thinking below the clock.

> ***Teacher*** *What are you thinking?*
>
> ***David*** *Well, half of the clock is shaded in. So you could write ½.*
>
> ***Teacher*** *I'll write ½ below the clock. You were thinking about the portion of the entire clock. What if I asked about the fraction of minutes that are shaded in on the clock?*
>
> ***Briana*** *Um, it's shaded to 30 minutes. So, 30/60?*
>
> ***Teacher*** *What do you all think of 30/60?*
>
> ***Students*** *Yeah, that would work.*
>
> *If you're thinking about the minute hand going all the way around, it's gone 30 out of 60 minutes. And that's equivalent to ½, so I agree.*
>
> ***Teacher*** *Any other fractions I could record? What about if I was thinking about hours that have passed since noon?*
>
> ***Darius*** *Oh, you could also write 6/12. 6 out of twelve hours on the face of the clock.*
>
> ***Teacher*** *So, again, lots of equivalent fractions represented by this one clock model. The fraction we record varies depending on what we are considering the whole: the clock face itself, the number of hours, or the number of minutes on the clock.*

5 Discuss and record fractions that represent the other three clock models on the teacher master.



6 Invite a volunteer to reveal the first marker by turning it face-up in the pocket chart. Have students study the marker quietly for a few seconds, and then ask them to suggest equations that represent the visual on the card.



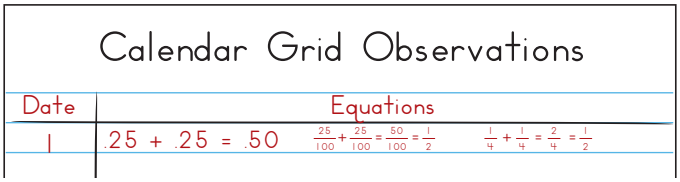
Students *There are 2 quarters.*
You could write 25 cents plus 25 cents equals 50 cents.
Or .25 + .25 = .50.

Teacher *Anything else? Could I record a fraction equation that represents this picture?*

Students *You could write ¼ + ¼ = ²⁄₄, and that equals ½.*

You could also write ²⁵⁄₁₀₀ + ²⁵⁄₁₀₀ = ⁵⁰⁄₁₀₀ because each quarter is 25 out of 100 cents that make a dollar.

7 Record students' equation suggestions on the Calendar Grid Observations Chart, suggesting and supplying any that are not mentioned.

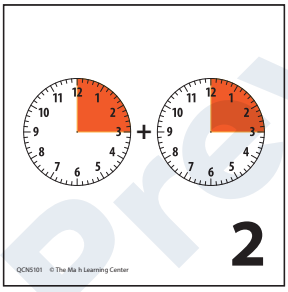


8 Then, let students know that one expression to represent each marker will be recorded on half an index card and slid into the pocket chart in front of the picture.

Record the expression ¼ + ¼ on half an index card, placing it in front of marker 1.

9 Invite a volunteer to reveal the second marker and then ask students to study the marker quietly for a few seconds.

Discuss the clock model represented, then ask students to share what equations could be recorded on the Calendar Grid Observations Chart.



Teacher *For this marker, a clock model is used. What equations can we record that represent the marker?*

Carlos *Each clock is showing 15 minutes shaded, so you could write ¹⁵⁄₆₀ + ¹⁵⁄₆₀.*

Teacher *And if I was going to write that as an equation?*

Serafina *¹⁵⁄₆₀ + ¹⁵⁄₆₀ = ³⁰⁄₆₀.*

Teacher *OK, anything else?*

Darryl *If you think about hours, you could write ³⁄₁₂ + ³⁄₁₂ = ⁶⁄₁₂.*

Teacher *What about the portion of the clock that is shaded?*

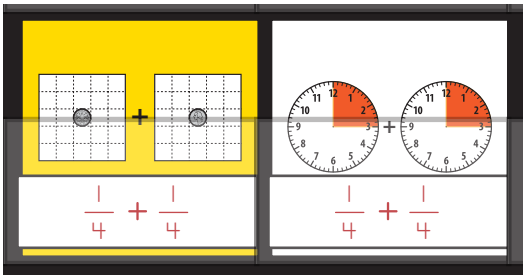
Mei *¼ of each clock is shaded, so ¼ + ¼ = ²⁄₄.*

Teacher *Interesting. So, we have several equations that can be represented by this clock? How can that be?*

Cody *All of the answers are the same as a half. ²⁄₄ is half, and so is ³⁰⁄₆₀ and ⁶⁄₁₂.*

Teacher *Let's record all those equations you found.*

10 Record the expression ¼ + ¼ on half an index card, placing it in front of marker 2. Then invite volunteers to describe what they see, including any relationships they notice between the markers.

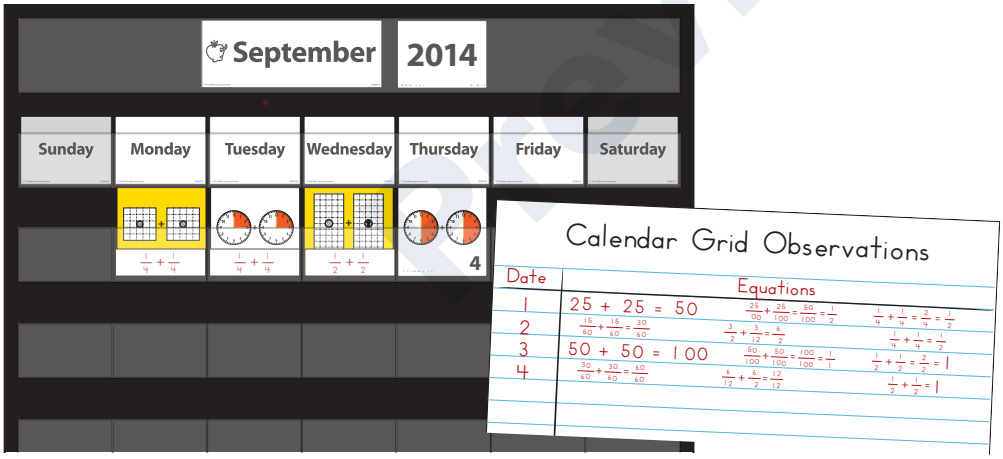


Teacher *What do you notice about these first two markers?*

Students *They both add up to a half.*
Some of the equations are the same, but not all of them.
Both markers have a lot of equivalencies.
Both markers show ¼ + ¼.

11 Then, invite students to turn over calendar markers one by one until the calendar is showing a marker for each day of September that has passed so far. Each time a marker is turned over, pause to allow students to make observations and generate equations that can be recorded on the Calendar Grid Record Sheet.

For marker 3, add an index card label that shows ½ + ½. Do not add index cards to additional markers, as they will be the focus of discussion in the second activity.



12 Wrap up Calendar Grid today by explaining how students will update the Calendar Grid when it is not a focus of discussion during Number Corner.

- Each day, one student will turn over a calendar marker and record on the Calendar Grid Observations Chart any decimal and fraction equations that represent the visual on the marker. These students will not be responsible for creating an index card to add to the Calendar Grid pocket chart.
- Explain the system you have set up to identify which student is responsible for updating the Calendar Grid each day (except for the days when it a focus of instruction).

Note

Post the Money & Clock Models Teacher Master you filled in with the class today on or near the Number Corner display for students' reference during the month.

Activity 2

Equations & Equivalencies — Days 6, 13

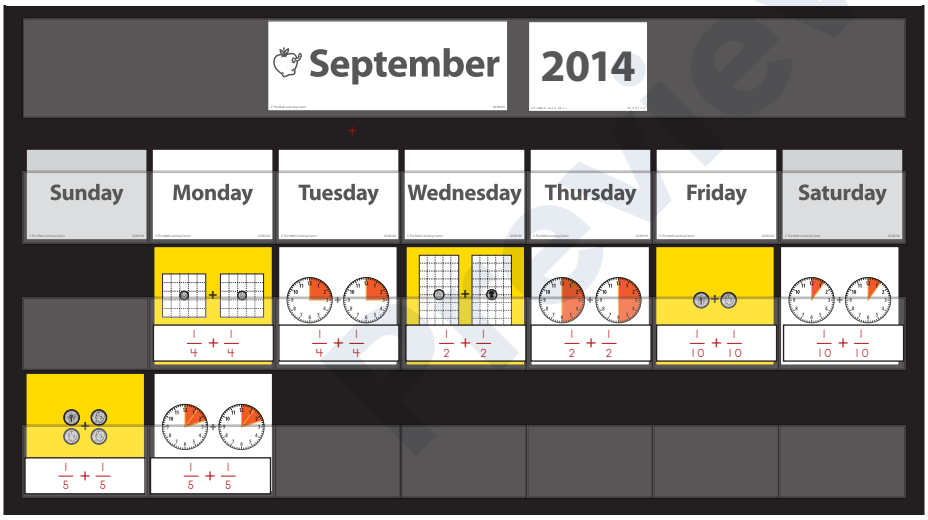
1 Gather students in front of the Calendar Grid pocket chart and Calendar Grid Observations Chart and set the goal for today's activity.

Let students know that today's activity will be focused on:

- Making predictions about today's marker based any patterns they have observed so far.
- Looking at the equations recorded on the Calendar Grid Observations Chart for the days of the month that have passed so far.
- Discussing equivalencies among the markers and equations.
- Looking for patterns across the Calendar Grid and adding an index card label to each of the markers that has been posted since the last time the class discussed the grid during Number Corner.

2 Begin by asking students to share predictions about today's marker, first in pairs, and then as a whole group.

3 After students have had a few moments to share predictions, ask several students to share their predictions with the class.



Press students to explain or justify their predictions.

> ***Sergio*** *So far, it's a pattern with money and clocks. It goes back and forth. Yesterday was clocks, so today it has to be money.*
>
> ***Kiara*** *I think it's going to be turquoise with those money pieces on it instead of regular coins. We had two like that, and then two that had regular money.*
>
> ***Xavier*** *Maybe it'll be pieces with nickels or pennies because we haven't had any of those yet.*
>
> ***Teacher*** *Thumbs up if you agree that today's marker will have nickels or pennies on it. I see some doubtful looks here and there. Do we have enough information yet to know for sure what kind of coins we'll see on the next marker?*
>
> ***Sara*** *I don't think so, unless maybe the pattern starts over. It could be quarters, half-dollars, dimes, dimes, and then start over today. Maybe it'll be something like 4 quarters instead of 2, and it'll be those money pieces.*

Lin *I'm trying to figure out what fractions we'll have, but I don't think you can tell yet. We had fourths, then halves. Then we had tenths, and then fifths. So far you have a fraction, and then you have a fraction that's twice as big.*

Kelsey *Oh, yeah—that's true! It was fourths and then halves on the first 4 markers, and a half is the same as 2 fourths. Then on the next 4 markers, it was tenths, and then fifths, and a fifth is the same as 2 tenths.*

Lin *Right, but then I can't tell what the fraction will be today. It was fourths on Marker 1 and tenths on Marker 5, so maybe it'll be something smaller than a tenth today.*

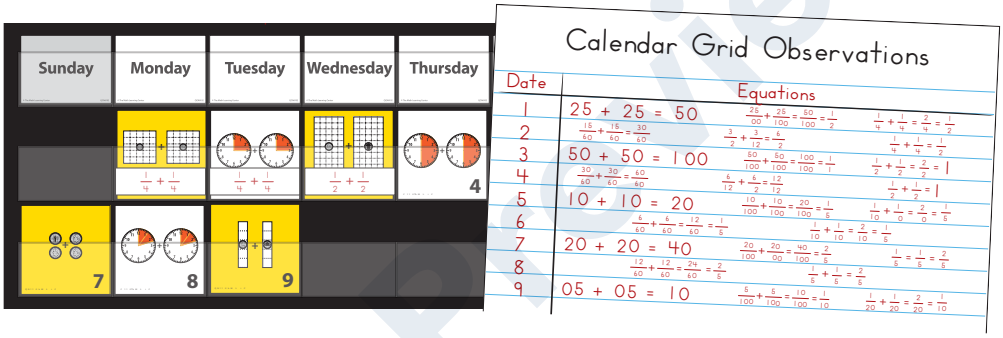
Raven *So maybe Xavier is right! Maybe it will be nickels or pennies today, because those fractions are 1/20 and 1/100, definitely smaller than 1/10.*

4 When a few students have shared their predictions, have your helper reveal today's marker.

Ask students to study the marker quietly for a few seconds. Then ask them to share equations that could be recorded on the Calendar Grid Observations Chart to represent the visual.

5 After recording students' suggestions, ask them to spend a minute taking a closer look at the Calendar Grid markers and the equations recorded on the Calendar Grid Observations Chart.

Then ask them to comment on any incorrect or additional equations that could be added to the observations chart for previous markers.



6 Let students know that index cards need to be placed in the pocket chart for each day, and ask them to look carefully for any patterns that might help determine which expressions would best fit.

Discuss an appropriate label for Marker 4.

Teacher *We have a long list of equations to represent each of the markers, but we are only going to record one expression on this index card. The first marker was labeled ¼ + ¼, and the second marker was also labeled ¼ + ¼. Then, the third marker was labeled ½ + ½. What do you think we should record on the index card for the fourth marker?*

Max *Probably ½ + ½ again.*

Teacher *Why is that?*

Max *The first two markers had the same label, so I think the next two markers might have the same label.*

Elisa *Yeah, there was a money one, then a clock one, then a money one, and this one is a clock one. But the fraction sentence is the same for the first two and then the same for the next two.*

> ***Teacher*** *Let's write ½ + ½ and add it to the pocket chart. Does that expression match the visual?*

7 Work with input from the class to create fraction expression labels for Markers 5 and 6, then invite discussion about the label for Marker 7 by wondering aloud about existing patterns.

If none of the students comment that all the fractions labeled so far are unit fractions, suggest it yourself in the discussion.

> ***Teacher*** *So far, we have these labels on our index cards: ¼ + ¼, ¼ + ¼, ½ + ½, ½ + ½, 1/10 + 1/10, and 1/10 + 1/10. Pairs of the same expression! And I also notice that they all seem to be unit fractions so far, fractions where the numerator is one. I'm looking at the next picture and I am wondering about the index card label we should add. I see two dimes plus two dimes. What would that be?*
>
> ***Brandon*** *2/10 + 2/10.*
>
> ***Teacher*** *Does that expression match the picture? Yes? And what about the pattern of fraction expressions on our little labels so far… does the expression 2/10 + 2/10 fit? Is there an equivalent expression we could record that fits the pattern of expressions and the visual on Marker 7 both?*
>
> ***Ivan*** *Two dimes is 20 cents and that's 1/5 of a dollar, so we could write 1/5 + 1/5.*
>
> ***Teacher*** *What do you think, everyone?*

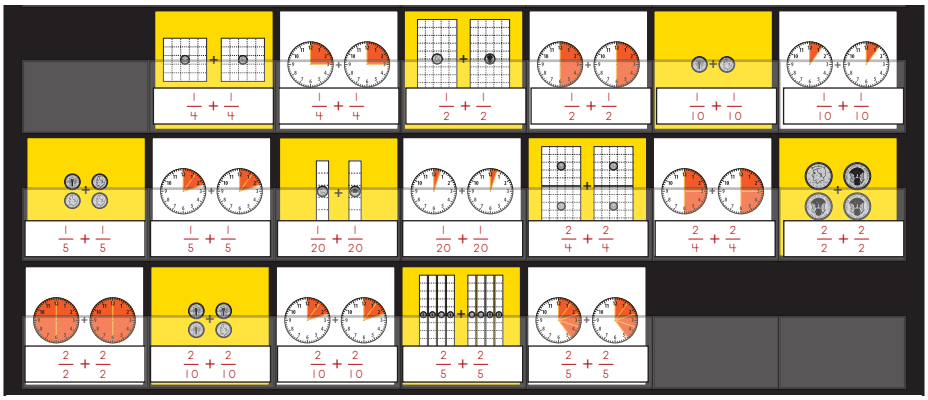
8 Continue creating index cards labels for the rest of the markers up to today's date, discussing equivalencies and patterns used to help determine the best label.

When discussing marker 9, be sure conversation occurs that solidifies why a nickel is called 1/20 rather than 1/5.

9 Wrap up today's activity by reminding students that someone will update the Calendar Grid each day. Ask students to take a final moment to make predictions about the next few markers and patterns that the labels will follow.

10 When you conduct this activity again later in the month, repeat the actions outlined above.

At Marker 11, students should observe that unit fractions change to fractions with a numerator of two.



Here is how the calendar markers and fraction expression labels would look on the 13th instructional day of the month if you started teaching on September 2.

Activity 3

Discussing Predictions and Patterns — Day 18

1 Gather students in front of the Calendar Grid pocket chart and Calendar Grid Observations Chart and review a few of the equations recorded on the chart in the previous days.

- Focus discussion on equivalencies listed for each marker.
- Take this opportunity to discuss any inaccuracies recorded or crucial equations missing for some markers.

If students are impatient to see today's marker, let them know that they'll make predictions and a classmate will reveal the marker for today a little later in the activity.

2 Then work with input from the class to label each marker that has been posted since your last Calendar Grid activity with a fraction expression that fits into the pattern established earlier in the month.

If you conducted Activity 2 for a second time on your 13th instructional day, you will need to create fraction expression labels for about 7 markers.

3 Next, challenge the students to use any patterns they have found, and any that come to light in the next few minutes, to predict what some, or even all the rest of the markers this month will look like.

- Ask students to share any patterns they have noticed as the markers have been revealed over the past few weeks.
- List students' suggestions on the board, and add some of your own if necessary. Students should notice that:
  - » Markers alternate between money and clock models, as well as between goldenrod and white backgrounds.
  - » Coins appear on Money Value Pieces twice, and then in isolation twice, in a repeating AABB pattern.
  - » Pairs of markers have identical fraction expression labels.
  - » The visuals on Markers 1–10 can all be represented with unit fractions. The visuals on Markers 11–20 can all be represented with fractions that have numerators of 2. The visuals on Markers 21–30 can be represented with fractions that have numerators of 3.
  - » The denominators feature a repeating pattern of 4, 4, 2, 2, 10, 10, 5, 5, 20, 20.

4 Ask students to turn and talk to a neighbor about the patterns listed, and use them to make as detailed a prediction about today's marker as possible.

Here are some questions you might use to spark students' thinking:

- What model should we see, clock or money? If it's money, will it appear on Money Value Pieces or as regular coins?
- What might the model show? (e.g., How many coins might there be and what denomination? What fractional part(s) of the clocks might be shaded in?)
- What equivalent fractions or decimals might we be able to list on the Calendar Grid Record Sheet?
- What will the fraction expression label likely be today?

5 Allow students to share their predictions and justify their thinking. Then reveal today's marker, and ask students to compare the actual marker with their predictions.

6 Have students continue to make predictions about upcoming markers through the end of the month, or as far as time allows.

After a number of predictions have been shared for a marker, turn it over to confirm students' thinking.

7 Wrap up by asking students to summarize some of the things they learned about fractions this month.

Let students know a new pattern and set of markers will be introduced next month.

Note

Even though students will have seen some or perhaps all of the markers for the rest of the month, turn them back over so the markers show up through today's date. Have one student each day update the Calendar Grid and the observations chart as usual through the end of the month.

September Calendar Collector

Layer a Day

Overview

This month, students collect data about rectangular prisms as they are built one layer at a time. Students explore the relationships between dimensions, area, and volume, and have opportunities to apply the associative and commutative properties of multiplication in the process.

Skills & Concepts

- Write and evaluate numerical expressions with parentheses (5.OA.1)
- Interpret numerical expressions without evaluating them (5.OA.2)
- Demonstrate an understanding that a cube with edge length of 1 unit is called a "unit cube," has 1 cubic unit of volume, and can be used to measure the volumes of other solid figures (5.MD.3a)
- Demonstrate an understanding that a solid figure that can be packed without gaps or overlaps by *n* unit cubes has a volume of *n* cubic units (5.MD.3b)
- Measure the volume of a solid figure by counting the number of cubic units that fill it, with no gaps or overlaps (5.MD.4)
- Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes; show that the volume can be found by multiplying the edge lengths or by multiplying the area of the base by the height; and represent the product of three whole numbers as the volume of a right rectangular prism whose edge lengths are equal to those three whole numbers (5.MD.5a)
- Reason abstractly and quantitatively (5.MP.2)
- Look for and make use of structure (5.MP.7)

Materials

Activities	Day	Copies	Kit Materials	Classroom Materials
Activity 1 Introducing the Calendar Collector	4		• Omnifix cubes (see Preparation)	• erasable marker • 1 piece of lined chart paper (see Preparation) • ruler or straight edge
Activity 2 Revisiting the Collection	9			• erasable markers • Calendar Collector Record Sheet
Activity 3 What's Missing?	14	**NCSB 1** What's Missing?		
Activity 4 Analyzing Layers	19			

TM – Teacher Master, **NCSB** – Number Corner Student Book
Copy instructions are located at the top of each teacher master.

Vocabulary

An asterisk [*] identifies those terms for which Word Resource Cards are available.

area*
associative property of multiplication*
base*
commutative property of multiplication*
cubic unit*
dimension*
face*
parentheses*
prism*
volume*

Preparation

Calendar Collector Record Sheet

To make a record sheet for use throughout the year, record the title "Calendar Collector Record Sheet" at the top of a sheet of lined chart paper. Laminate the chart paper. Next, use an erasable marker and straight edge to draw six columns on the sheet. Label the columns at the top of the sheet, as illustrated here.

Layer a Day Record Sheet

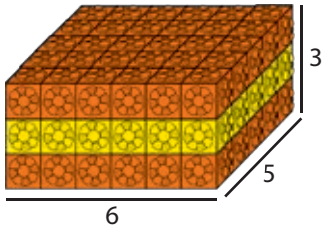
Day	Dimensions of the Base	Area of the Base	# of Layers	Volume	Dimensions of the Prism

Preparing the Omnifix Cubes

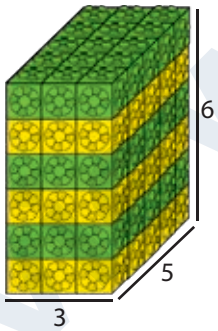
This month, students will use Omnifix cubes to build rectangular prisms. If you haven't yet used these cubes, you'll find that they come packaged as flat nets that have to be snapped together to form the cubes. Consider having the students or parent volunteers do this for you prior to conducting your first Calendar Collector activity. You will need a total of 340 cubes for the month, and it would probably be helpful to have a few extra on hand.

Mathematical Background

By building rectangular prisms one layer at a time and noting the relationships between the dimensions, the area of the base and the number of layers, students develop understandings about volume as an attribute of three-dimensional space. On each of the first three days in class, the group collects a 6×5 layer of cubes. Each layer is added to the previous layer, so by the third day the class will have built a rectangular prism with three 6×5 layers: $(6 \times 5) \times 3 = 90$ cubic units. Over the following six days in class, students will build a second rectangular prism, collecting a 3×5 layer each day to create a prism that is $(3 \times 5) \times 6$. Comparing the two prisms they will have collected by the ninth day in class, students discover that the volumes of the two are equal, because although they are oriented differently, the two prisms are congruent; they have the same dimensions.

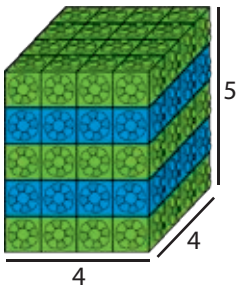


Prism 1: $(6 \times 5) \times 3 = 90$ cubic units

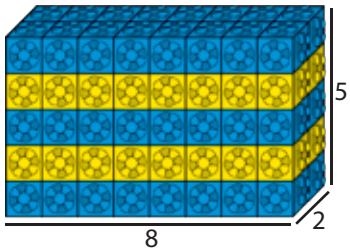


Prism 2: $(3 \times 5) \times 6 = 90$ cubic units

Over the next 5 days in class, students collect layers that are 4 cubes by 4 cubes, each layer added to the previous to form a third rectangular prism that is $(4 \times 4) \times 5$. For the remaining 5 days, students collect layers that are 8 cubes by 2 cubes, to form a fourth prism that is $(8 \times 2) \times 5$. When they compare prisms 3 and 4, they will notice that the length of the fourth prism is double that of the third, the width is half, and the heights of the two are the same. Thus, even though these two prisms are not congruent, they both have a volume of 80 cubic units.



Prism 3: $(4 \times 4) \times 5 = 80$ cubic units



Prism 4: $(8 \times 2) \times 5 = 80$ cubic units

Key Questions

- How is the volume of the prism related to the volume of one layer?
- How is the volume of the prism related to the number of layers?
- What happens when you double one dimension of a prism, halve another, and keep the third dimension the same?
- How can we use the dimensions of a prism to determine its volume in ways that are efficient and effective?

Toward the Formula for Volume

In writing expressions to represent the dimensions and number of each layer in each arrangement, students are developing a conceptual understanding of the formula for finding the volume of a rectangular prism ($V = b \times h$, where b is the area of the base and h is height). For the purpose of notation during this exploration, we write the measurements of the base in parentheses, expanding the expression for the formula to $V = (l \times w) \times h$.

Update

Starting after Activity 1, have student helpers complete this update procedure every school day that the Calendar Collector is not a featured activity.

Procedure

- The student helper adds a new layer to the rectangular solid already formed, then fills in the information on the Calendar Collector Record Sheet.
- Each prism should stay intact and be displayed next to the record sheet for other students to view. Prisms collected earlier in the month should also stay intact so by the end of the month, there are four different prisms on display.

Note

Layers are collected only on school days, so helpers making the updates on Mondays are only responsible for adding one layer to the prism in process and recording the information. Also, the activities that feature Calendar Collector are timed so that the class will either start building each of the new prisms together, or the teacher will give the student helper for the following day instructions about how to start the next prism in the series.

Activity 1

Introducing the Calendar Collector — Day 4

Notes About This Activity

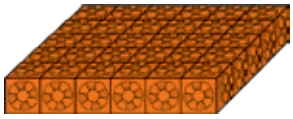
Construct a $(6 \times 5) \times 1$ prism out of Omnifix cubes in a single color prior to conducting this activity with the class.

1 To introduce the Calendar Collector, explain that this year, just as they may have done in earlier grades, students will make a new collection each month. This month, they will collect a layer of cubes each day they're in class. The layers will be added to one another to build a set of 4 rectangular prisms over the course of the month.

2 Ask students to join you in the Number Corner discussion area and seat themselves so they can see the Calendar Collector Record Sheet.

3 Hold up one Omnifix cube for students to see. Discuss the fact that the dimensions of the cube—the length, width, and height—are each one unit, and that the amount of space taken up by the cube, or its *volume*, is one *cubic unit*.

4 Display a $(6 \times 5) \times 1$ rectangular prism, constructed from Omnifix cubes of one color. Have students share observations about the prism, first in pairs and then as a whole class.



5 Draw students' attention to the Calendar Collector Record Sheet posted on the wall, and give them a few moments to take note of the column headings.

6 Work with input from the students to collect and record information about the prism, emphasizing and defining terms such as *area, base, dimension,* and *volume* as they come up during your discussion.

ELL/SUPPORT Post Word Resource Cards for these terms near the Number Corner discussion area to support students with language needs.

Teacher *Since this is our first Calendar Collector acivity, let's work together to fill in the information about this rectangular solid on our record sheet. The first column we need to fill in asks for the dimensions of the base of this prism.*

Kelsey *The base is the part it sits on, right? That's a 6 × 5 or a 5 × 6, depending on how you look at it.*

Teacher *Next, we need to record the area of the base. Can someone remind us what area is and how to find it?*

Brandon *Area is how many square units it takes to cover something. You get it by multiplying the length times the width—6 × 5 is 30, so the area of the base is 30 square units.*

Teacher *This next column is interesting to me. It is asking for the number of layers. What do you think about that?*

Akiko *There's just 1 layer. We should write 1 for that question.*

Teacher *The next column asks for volume. Can anyone help us with the meaning of that word?*

Ivan *That's how many cubes you need to build the prism. You said that one cube has a volume of one… what did you call it?*

Teacher *One cubic unit.*

Ivan *Yeah, one cubic unit. So if you use 30 of them to make the prism, it has a volume of 30 cubic units.*

Teacher *OK, the last column says that we need to fill in the dimensions of the rectangular prism.*

Sara *I think you should write 6 × 5 × 1, since it has one layer.*

Layer a Day Record Sheet

Day	Dimensions of the Base	Area of the Base	# of Layers	Volume	Dimensions of the Prism
1	6 × 5	30 sq. units	1	30 cubic units	6 × 5 × 1

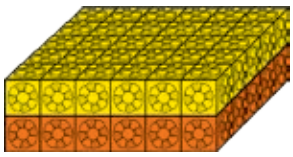
7 Let students know that they will work together right now to bring the collection up to date, given that this is the fourth instructional day of the month.

8 Invite a student to the front of the group to add a second, differently colored, layer of Omnifix cubes to the rectangular prism. As the student is building, ask the rest of the class to view the chart and make predictions about the information that will be filled in for Day 2.

Before examining the dimensions and related information for the updated rectangular prism, allow students time to share with a neighbor or with the whole class about their thinking.

ELL Listen for students who struggle to use key vocabulary and direct them to Word Resource Cards, as needed.

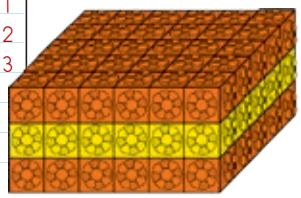
9 Display the rectangular prism and work with input from the class to enter the information for Day 2 on the Calendar Collector Record Sheet.



10 Invite another student volunteer to quickly add a third layer to the rectangular prism while the rest of the class makes predictions about the missing information. Then, fill in the Day 3 data together.

Layer a Day Record Sheet

Day	Dimensions of the Base	Area of the Base	# of Layers	Volume	Dimensions of the Prism
1	6 × 5	30 sq units	1	30 cubic units	6 × 5 × 1
2	6 × 5	30 sq units	2	60 cubic units	6 × 5 × 2
3	6 × 5	30 sq units	3	90 cubic units	6 × 5 × 3



11 Before collecting and entering the information for Day 4, ask students to share observations about any patterns they notice so far on the Calendar Collector Record Sheet.

Here are some questions you might use to spark students' thinking about the information on the record sheet:

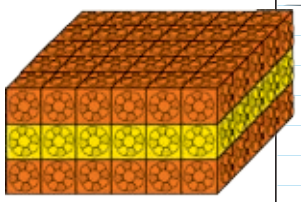
- What happens to the dimensions of the base as layers are added? (They remain the same.)
- What happens to the area of the base as layers are added? (The area of the base remains the same.)
- How are the dimensions of the prism related to the dimensions of the base and the number of layers?
- By how much does the volume of this particular prism increase as each layer is added? Why?

12 Let students know that on, or directly after, the days when Calendar Collector is the featured workout, they will start a rectangular prism with new dimensions.

Explain that this is one of those days, so right now, you will leave the (6 × 5) × 3 prism intact for future reference, and start a new prism.

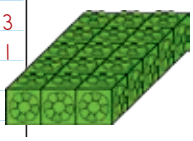
13 Invite a volunteer to create a one-layer prism with the base dimensions 3 × 5.

Then, have the class provide the information for Day 4 as you enter it on the Record Sheet.



Layer a Day Record Sheet

Day	Dimensions of the Base	Area of the Base	# of Layers	Volume	Dimensions of the Prism
1	6 × 5	30 sq units	1	30 cubic units	6 × 5 × 1
2	6 × 5	30 sq units	2	60 cubic units	6 × 5 × 2
3	6 × 5	30 sq units	3	90 cubic units	6 × 5 × 3
4	3 × 5	15 sq units	1	15 cubic units	3 × 5 × 1



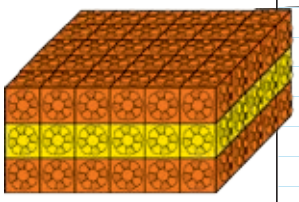
14 Close the activity by letting students know that a student helper will add another layer to the new rectangular prism, and enter the information on the Record Sheet each day. The class will reconvene to discuss Calendar Collector again in a few days, at which time, you will start a third rectangular prism together.

Activity 2

Revisiting the Collection — Day 9

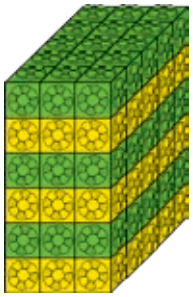
1 Invite students to join you in the Number Corner discussion area. Give them a few moments to quietly examine the Calendar Collector Record Sheet and the two rectangular prisms that have been constructed so far.

At this point, the Calendar Collector Record Sheet should have been updated daily through Day 8 by a student helper. If the data through Day 8 has not been collected and entered prior to meeting in the discussion area, quickly build the prisms a layer at a time and fill in the chart with the students.



Layer a Day Record Sheet

Day	Dimensions of the Base	Area of the Base	# of Layers	Volume	Dimensions of the Prism
1	6 × 5	30 sq units	1	30 cubic units	6 × 5 × 1
2	6 × 5	30 sq units	2	60 cubic units	6 × 5 × 2
3	6 × 5	30 sq units	3	90 cubic units	6 × 5 × 3
4	3 × 5	15 sq units	1	15 cubic units	3 × 5 × 1
5	3 × 5	15 sq units	2	30 cubic units	3 × 5 × 2
6	3 × 5	15 sq units	3	45 cubic units	3 × 5 × 3
7	3 × 5	15 sq units	4	60 cubic units	3 × 5 × 4
8	3 × 5	15 sq units	5	75 cubic units	3 × 5 × 5



2 Ask students to turn and talk with a neighbor about any patterns they notice on the class record sheet. Then invite volunteers to share their observations with the class.

During discussion, draw out conversation about the relationship between the area of the base and the volume of the prism, and any connection students can make between prisms with the same volume but different dimensions.

If students struggle to share substantial observations, the following questions may spur additional conversation:

- How can you find the volume of each prism?
- What connections can be made between the numbers on each row of the chart?
- What patterns, if any, do you notice in the information entered for Days 4–8? Days 1–8?
- How might you use the given information to make a prediction about today's prism?

> ***Mei*** *I noticed that for the first three days, the volume went 30, 60, 90. It went up by 30 each time.*
>
> ***Teacher*** *Do you have an idea about why that might be? Anyone?*
>
> ***Cody*** *The first layer had a volume of 30 cubes, so if you are just adding a new layer on top each time, it makes sense that you'll keep getting 30 bigger with each layer.*
>
> ***Imani*** *Then, for the next prism, it went 15, 30, 45, 60, 75. The volume went up by 15 every time it got another layer.*
>
> ***Max*** *I also noticed something else. The prism on Day 1 and the prism on Day 5 have the same volume.*

Teacher *What do you make of that?*

Max *Well, you need the same number of cubes to build both of them.*

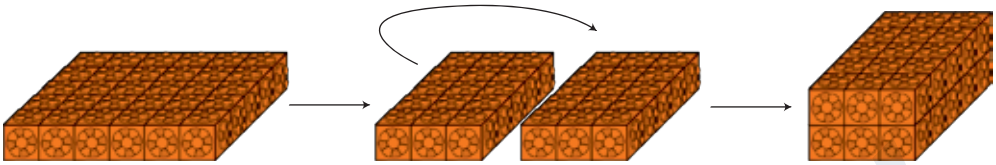
Teacher *Interesting. But aren't they different prisms?*

Max *Yes, but they must be the same size, sort of.*

Teacher *Turn and talk to a neighbor about what Max just pointed out. Can anyone provide a suggestion about how these two prism are the same size, sort of?*

Kiara *Well, the first prism has a 6 × 5 base, and just one layer, so that's 30 cubes on the bottom. The second prism only has 3 × 5 on the bottom and that's 15 cubes. But it also has two layers, so that's 30 cubes.*

Carlos *Since the bottom layer isn't as big, you need two layers to make them match. See, if you break the 6 × 5 layer in half and stack the halves, it will look like the 3 × 5 × 2.*



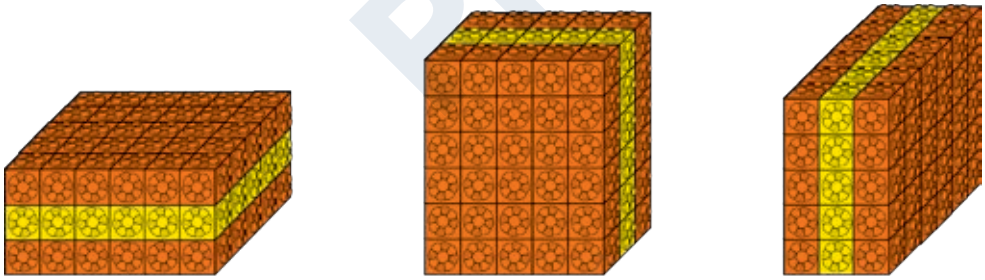
Teacher *Are there any other prisms that we might be able to match like this?*

Elisa *Well, the 6 × 5 × 2 and the 3 × 5 × 4 both use 60 cubes.*

Teacher *OK, and are you thinking that there is a connection between those two prisms?*

Elisa *Yep! The first one has a 6 × 5 base with two layers, and that's 30 two times. But since the 3 × 5 base is only 15 cubes, you need twice as many layers, 4 of them, to get the 60 cubes.*

3 Hold up the (6 × 5) × 3 prism. Rotate the prism while students observe, so that they can view it as a (5 × 3) × 6 prism and a (3 × 6) × 5 prism.



(6 × 5) × 3 (5 × 3) × 6 (3 × 6) × 5

4 Ask students to turn and talk to a neighbor about whether or not any of the information on the Calendar Collector Record Sheet has to change if a prism is rotated or flipped, as you have just done.

Give student pairs a minute to discuss the question, and then invite volunteers to share their thinking with the class.

Briana *We think that everything on the chart would have to stay the same, because all you did was turn the prism around—it's not like you took any of the cubes away or anything.*

Teacher *That's true—I did rotate the prism. Yes, Craig, what are you thinking?*

Craig *I know that some of the information will stay the same, like the volume, but some of the other numbers would have to change because every time you put the prism a different way, the base was different, like the first way was 6 × 5, and the next way was 5 × 3.*

Teacher *Ah. So you are saying that even though the total number of cubes used to build the prism, the volume, does not change, the face that the prism is resting on does change. So we have the same prism, but we can rest it on different faces. That means we can think about the dimensions differently, depending on how I'm holding it. Is that correct?*

Craig *Yep.*

5 Introduce the use of parentheses as a way to represent the base and the height of a rectangular prism.

- Hold up the 6 × 5 × 3 prism positioned so that the base is 6 × 5, while the height is 3. Record the expression (6 × 5) × 3 on the board, and explain that you've placed parentheses around the numbers that show the dimensions of the base.
- Now rotate the prism so that, from where the students are sitting, the base is 5 × 3, and the height is 6, and work with input from the students to write the corresponding expression: (5 × 3) × 6.
- Finally, rotate the prism so that the base is 3 × 6 and the height is 5. Again, work with input from the students to write the corresponding expression: (3 × 6) × 5.

As you work with the students to write an expression for each position, note with them that the numbers in the parentheses can be switched without altering the dimensions of the base or the height. In other words, the expression (6 × 5) × 3 represents a prism with the same base and height as a prism represented with the expression (5 × 6) × 3. On the other hand, the expression (5 × 3) × 6 represents a prism with a different base and height than one represented with the expression (3 × 6) × 5, even though the two are congruent.

$\underbrace{(6 \times 5)}_{\text{base}} \times \underset{\text{height}}{3}$ $\underbrace{(5 \times 3)}_{\text{base}} \times \underset{\text{height}}{6}$ $\underbrace{(3 \times 6)}_{\text{base}} \times \underset{\text{height}}{5}$

6 Write parentheses on the Record Sheet to indicate the dimensions of the base for each of the prisms constructed by the class so far. Then invite a student to add one more layer to the (3 × 5) × 5 prism from Day 8, and have that student record the information for Day 9, with input from her classmates.

Invite conversation about which columns of information will not change as an additional layer is added. Be sure the student volunteer includes parentheses as she records the dimensions of the prism in the last column for Day 9.

Layer a Day Record Sheet

Day	Dimensions of the Base	Area of the Base	# of Layers	Volume	Dimensions of the Prism
1	6 × 5	30 sq units	1	30 cubic units	(6 × 5) × 1
2	6 × 5	30 sq units	2	60 cubic units	(6 × 5) × 2
3	6 × 5	30 sq units	3	90 cubic units	(6 × 5) × 3
4	3 × 5	15 sq units	1	15 cubic units	(3 × 5) × 1
5	3 × 5	15 sq units	2	30 cubic units	(3 × 5) × 2
6	3 × 5	15 sq units	3	45 cubic units	(3 × 5) × 3
7	3 × 5	15 sq units	4	60 cubic units	(3 × 5) × 4
8	3 × 5	15 sq units	5	75 cubic units	(3 × 5) × 5
9	3 × 5	15 sq units	6	90 cubic units	(3 × 5) × 6

7 Close the activity by asking students to consider the following questions:

- What information do you need to determine the volume of a prism?
- If you were given the volume of the prism and the number of layers, would you be able to determine the dimensions of the base?
- If you were given the volume of the prism and the number of layers, would you be able to determine the area of the base?
- If you were given the dimensions of the base and the total volume, would you be able to determine the number of layers?

8 Let students know that in future Calendar Collector workouts, they will continue to explore the relationships on the record sheet.

Tell student helper who is in charge of updating the Calendar Collector tomorrow that he will start the next new prism, with a base of 4 × 4, and a height of 1. Then, on Days 11–13, additional layers will be added to the new prism.

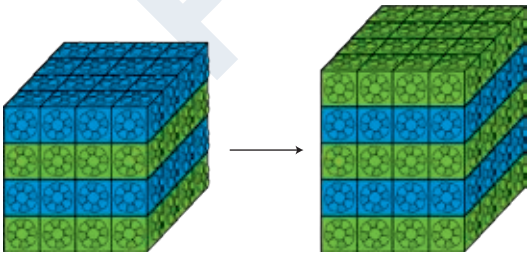
Activity 3

What's Missing? Day 14

1 Open by explaining to students that they are going to quickly update the Calendar Collector for Day 14, and then do a page in their Student Number Corner Book today, rather than meeting in the discussion area for the entire workout time.

At this point, the class chart should have been updated daily through Day 13 by a student helper. If the data through Day 13 has not been collected prior to Number Corner time, consider filling it in yourself to allow for enough time for students to complete today's activity.

2 Let students know that another layer will be added to the current prism for Day 14. Ask students to predict what the dimensions and volume of the prism will be, as you add another layer to the existing 4 × 4 × 4 prism.



3 When you are finished adding the next layer, invite a volunteer to describe the dimensions of the prism. Then ask students to consider how they would determine the total volume of the prism.

During discussion, elicit efficient strategies including some or all of the following:

- Add 16 (the volume of the new layer) to the existing volume for the prism constructed on Day 13.
- Reassociate the numbers to make use of well-known combinations, e.g., (4 × 4) × 5 = 4 x (4 × 5) = 4 × 20 = 80.
- Multiply 4 × 4, and then split 16 into 10 and 6, and multiply each of these numbers by five: (4 × 4) × 5 = 16 × 5 = (10 + 6) × 5 = (10 × 5) + (6 × 5) = 50 + 30 = 80.

4 Work with the students to enter the information for Day 14 on the Calendar Collector Record Sheet.

Layer a Day Record Sheet

Day	Dimensions of the Base	Area of the Base	# of Layers	Volume	Dimensions of the Prism
1	6 × 5	30 sq units	1	30 cubic units	(6 × 5) × 1
2	6 × 5	30 sq units	2	60 cubic units	(6 × 5) × 2
3	6 × 5	30 sq units	3	90 cubic units	(6 × 5) × 3
4	3 × 5	15 sq units	1	15 cubic units	(3 × 5) × 1
5	3 × 5	15 sq units	2	30 cubic units	(3 × 5) × 2
6	3 × 5	15 sq units	3	45 cubic units	(3 × 5) × 3
7	3 × 5	15 sq units	4	60 cubic units	(3 × 5) × 4
8	3 × 5	15 sq units	5	75 cubic units	(3 × 5) × 5
9	3 × 5	15 sq units	6	90 cubic units	(3 × 5) × 6
10	4 × 4	16 sq units	1	16 cubic units	(4 × 4) × 1
11	4 × 4	16 sq units	2	32 cubic units	(4 × 4) × 2
12	4 × 4	16 sq units	3	48 cubic units	(4 × 4) × 3
13	4 × 4	16 sq units	4	64 cubic units	(4 × 4) × 4
14	4 × 4	16 sq units	5	80 cubic units	(4 × 4) × 5

5 Then have the students locate their What's Missing? Student Book page, as you place a copy on display.

Read the directions at the top of the page, and then ask students to quietly examine the table of missing information.

6 Review the rest of the problems briefly with the class, and clarify as needed. When the students understand what to do, have them go to work.

SUPPORT Encourage students who want or need to use Omnifix cubes to help solve some of the problems on the Student Book page to do so.

CHALLENGE Ask students to consider the minimum amount of information needed for each item.

7 As students complete the What's Missing? Student Book page, have them meet with a classmate to compare solutions and strategies.

You will probably not have time to discuss this sheet with the class as a whole, and some students may need to take it home, or work during a seatwork period the following day, to finish it. If you can find the time to debrief the assignment with the class at some point, however, here are some questions you might pose.

- What information is necessary to determine the volume of a prism?
- If you were given the volume of the prism and the number of layers, would you be able to determine the dimensions of the base? How?
- If you were given the volume of the prism and the number of layers, would you be able to determine the area of the base? How?
- If you were given the dimensions of the base and the total volume, would you be able to determine the number of layers? How?

8 Let students know that a student helper will continue to update the Calendar Collector each day.

Tell the helper who is in charge of updating the Calendar Collector tomorrow that he will start the next new prism, with a base of 8 × 2, and a height of 1. Then, on Days 16–18, additional layers will be added to the new prism.

Activity 4

Analyzing Layers — Day 19

1 Invite students to join you in the Number Corner discussion area and give them a few moments to examine the Calendar Collector Record Sheet quietly.

At this point, the class chart should have been updated daily through Day 18 by a student helper. If the data through Day 18 has not been collected prior to meeting in the discussion area, quickly fill in the chart together.

Layer a Day Record Sheet

Day	Dimensions of the Base	Area of the Base	# of Layers	Volume	Dimensions of the Prism
1	6 × 5	30 sq units	1	30 cubic units	(6 × 5) × 1
2	6 × 5	30 sq units	2	60 cubic units	(6 × 5) × 2
3	6 × 5	30 sq units	3	90 cubic units	(6 × 5) × 3
4	3 × 5	15 sq units	1	15 cubic units	(3 × 5) × 1
5	3 × 5	15 sq units	2	30 cubic units	(3 × 5) × 2
6	3 × 5	15 sq units	3	45 cubic units	(3 × 5) × 3
7	3 × 5	15 sq units	4	60 cubic units	(3 × 5) × 4
8	3 × 5	15 sq units	5	75 cubic units	(3 × 5) × 5
9	3 × 5	15 sq units	6	90 cubic units	(3 × 5) × 6
10	4 × 4	16 sq units	1	16 cubic units	(4 × 4) × 1
11	4 × 4	16 sq units	2	32 cubic units	(4 × 4) × 2
12	4 × 4	16 sq units	3	48 cubic units	(4 × 4) × 3
13	4 × 4	16 sq units	4	64 cubic units	(4 × 4) × 4
14	4 × 4	16 sq units	5	80 cubic units	(4 × 4) × 5
15	8 × 2	16 sq units	1	16 cubic units	(8 × 2) × 1
16	8 × 2	16 sq units	2	32 cubic units	(8 × 2) × 2
17	8 × 2	16 sq units	3	48 cubic units	(8 × 2) × 3
18	8 × 2	16 sq units	4	64 cubic units	(8 × 2) × 4

2 Ask students to turn and talk with a neighbor about any patterns they notice on the class record sheet, and then invite volunteers to share their observations with the class.

If students struggle to share substantial observations, the following questions may spur additional conversation:

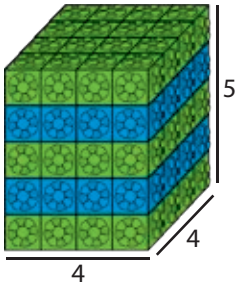
- How can the volume of each prism be quickly determined?
- How is the volume of the prism related to the volume of one layer?
- Do you notice any patterns or connections between the information recorded for Days 10–13 and the information recorded for Days 15–18?
- How can you use the given information to make a prediction about the prism to be recorded on Day 19?

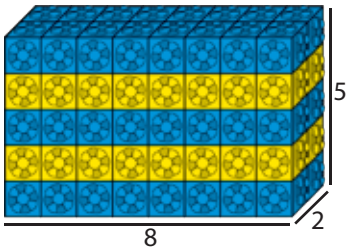
3 Invite a student to add an additional layer to the (8 × 2) × 4 prism from Day 18 and have that student record the information for Day 19, with input from her classmates.

Invite conversation about which columns of information will not change as an additional layer is added. Be sure the student volunteer includes parentheses in their recording in the final column of the chart.

4 Ask students to share the strategies they used for finding the volume of the prism just built and recorded.

If it doesn't come from the students, draw out the connection between the third and fourth prisms, and note with the class that their height is the same, while the one dimension of the 4 × 4 base has been doubled, and the other halved.





Teacher *We just helped Jade fill in the information for Day 19. I'm curious to know how you determined the volume of this prism.*

Lin *I saw the numbers that were used and thought about 8 × 5, then times 2.*

Serafina *I knew that it was just one more layer, so I added 16 more cubes to the day before.*

Darryl *I knew that it was that is was going to be 80 without even doing any adding or multiplying.*

Teacher *Can you tell us how you know?*

Darryl *Sure. I saw that pattern. The prism for today had 5 layers just like the other one did. And since the bases have the same number of cubes—16 for both of them—they are going to have the same number of cubes when they both have five layers.*

5 Close the activity by asking students to summarize some of the things they have learned during this month's Calendar Collector workout.

September Computational Fluency

Multiple Game

Overview

To open the first Computational Fluency activity this month, the teacher introduces the Multiple Game. Students play with the teacher in Activity 1, then play with a partner in activity 2. In Activity 3, students complete a Student Book page about the game, multiples, products, primes, and composite numbers.

Skills & Concepts

- Find all factor pairs for a whole number between 1 and 100 (4.OA.4)
- Demonstrate an understanding that a whole number is a multiple of each of its factors (4.OA.4)
- Determine whether a whole number between 1 and 100 is a multiple of a given 1-digit number (4.OA.4)
- Determine whether a whole number between 1 and 100 is prime or composite (4.OA.4)
- Use appropriate tools strategically (5.MP.5)
- Look for and express regularity in repeated reasoning (5.MP.8)

Materials

Activities	Day	Copies	Kit Materials	Classroom Materials
Activity 1 Introducing the Multiple Game	2	**NCSB 2*** Multiple Game Board		• crayons or colored pencils in red and blue (1 of each color)
Activity 2 Partner Multiple Game	12	**NCSB 2** Multiple Game Board		• 2 different colored crayons or colored pencils for each student pair
Activity 3 Maddy's Multiples	15	**NCSB 3** Maddy's Multiples		

TM – Teacher Master, **NCSB** – Number Corner Student Book
Copy instructions are located at the top of each teacher master. ** Run 1 copy of this page for display.*

Vocabulary

An asterisk [*] identifies those terms for which Word Resource Cards are available.

composite number*
factor*
multiple*
prime number*

Preparation

Familiarize yourself with the Multiple Game by playing one round or more before introducing it to the class.

Mathematical Background

This workout provides a game environment that allows students to review factor pairs and practice basic multiplication facts in an engaging context. There is ample opportunity for developing winning strategies, helping to ensure that even the most capable students will be challenged. The review in this month's Computational Fluency activities sets the stage for upcoming work with division, as well as addition and subtraction of fractions with unlike denominators, where students will need to identify common multiples and factors to do the computation and simplify the results.

Activity 1

Introducing the Multiple Game — Day 2

1 Introduce the workout by name—Computational Fluency.

- Explain that someone with computational fluency is able to work very quickly and easily with numbers.
- Let students know that they will work with a variety of Computational Fluency activities throughout the year, all designed to encourage efficient and effective strategies for solving number problems.

2 Review key vocabulary (multiple, factor, prime, composite) by asking students to turn and talk to a neighbor about their own definitions of the words. Elicit examples of each.

SUPPORT/ELL Display the Word Resource Cards for each of these terms, and leave posted in or near the Number Corner through the month.

3 Introduce the Multiple Game to the class, summarizing the directions and object of the game.

Player 1 chooses a target number on the The Multiple Game Board and circles it with her colored pencil. Player 2 then identifies all the numbers on the game board for which the target number is a multiple and circles those numbers with his colored pencil. Players alternate until the game board is filled and then find the sum of their circled squares to determine a winner.

4 Display your copy of the Multiple Game Student Book page. Give students a moment to look it over, and then explain that you're going to play the game with the class right now.

- Let them know that they'll play as one team, and you'll play as the other today.
- Explain that each team will work in a different color, either red or blue. Decide with the class which color each team will use.

5 Use your copy of the Multiple Game page to play the game with the class.

- Take the first turn to choose a target number on the game board and circle it with your color.
- Invite one or more students to come up and use their team's color to circle all the numbers on the board for which the target number you selected is a multiple, not including the target number itself.
- Next, have one of the students circle a target number for the class. Then use your color to circle all the numbers for which the students' target number is a multiple.
- Take turns with the class to choose target numbers and circle factors.

> ***Teacher*** *Today we are going to play a new game that will help us review multiples, factors, and multiplication facts. I'm going to be Player 1, and you will all work together as Player 2. The instructions say that I should choose a number to be the target number and circle it. I choose 16, and I'm going to circle it with my color—blue. Now it's your job to find all the numbers on the sheet that are factors of 16. In other words, you need to find all the numbers that divide evenly into 16. Talk with your neighbor about that please.*
>
> ***Teacher*** (after allowing a few moments for discussion) *Who can come up and circle a factor of 16 in red, the color your team is using today?*

Key Questions

Use the following questions to guide students' discussion this month:

- What is a multiple?
- What is a factor?
- How are factors and multiples related?
- What do you notice about the multiples of ___?
- What do you notice about the factors of ___?
- What are prime and composite numbers?
- What strategies can be employed when playing the Multiple Game to ensure a winning score?

September | Computational F uency Activity 1 & 2

NAME | DATE

Multiple Game Board

Player 1 Mr. G Player 2 The Class

2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22

DJ *But what about 16? Isn't 16 a multiple of 16?*

Teacher *Good thinking! Yes, it is. But since I circled the 16 already, and each number is only used one time, you aren't able to circle it. Is 16 a prime or composite number? How do you know?*

Jade *It has to be composite. We found more than just two factors. It it was prime, it would only have 1 and itself for factors.*

Teacher *Now it's your turn to circle a target number, and my turn to circle any factors of that number I can find on the sheet. Would anyone like to propose a target number for the class?*

Craig *Let's pick 27 as our target number. Can I circle it?*

Teacher *Sure! So that means I need to find the numbers for which 27 is a multiple; numbers that are factors of 27. I know that 1 × 27 is 27, but there's no 1 on the sheet, and 27 is already circled. I also know that 3 × 9 is 27. I'll circle those. You picked a composite number also!*

September | Computational F uency Activity 1 & 2

NAME | DATE

Multiple Game Board

Player 1 Mr. G Player 2 The Class

2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

Xavier *So who is winning so far?*

Teacher *The score for your team is the sum of the numbers that are circled in your color. So you have 2, 4, 8, and 27. That's how much?*

Students *41!*

Teacher *And I have 16, 3, and 9. How much is that?*

Students *28. We're ahead!*

6 Elicit student participation as the class plays against you, and generate conversation about factors, multiples, prime and composite numbers while playing.

Pose questions like the following to promote discussion of factors and multiples while you play:

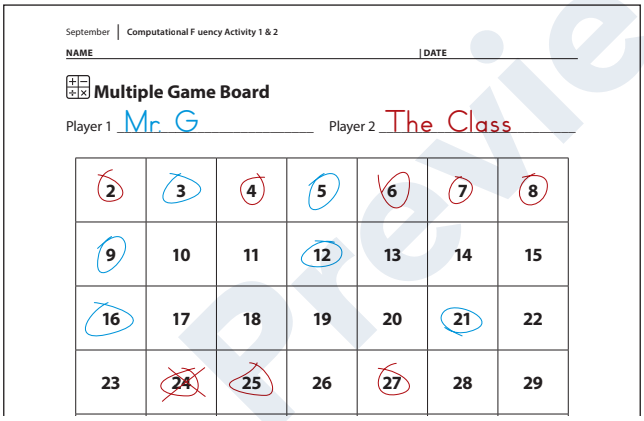
- If you are not sure what numbers to circle, what can you do?
- How can writing multiplication facts help determine the factors?
- How can you be sure you circled all the factors for your target number?

- What can you do to get a higher score than your opponent?

7 Continue to take turns with the class choosing target numbers and circling factors until no further plays can be made.

If a team chooses a target number for which there are no factors that can still be circled, that number must be crossed out and the team does not get points for that turn.

> ***Teacher*** *OK, it's your turn to circle the next target number. Andrea, can you come up and circle a target number for the class?*
>
> ***Cody*** *I'm going to circle 24 for our target number.*
>
> ***Teacher*** *So now I need to clarify one of the rules of the game. If you circle a target number for which there are no factors left on the board, you have to cross out the number, and you don't get any points for it. Are there any factors of 24 left to be circled on the board?*
>
> ***Students*** *Well, 24 is 4 × 6, but both of those numbers are circled already.*
> *Eight and 3 are gone too.*
> *There's still 12 times 2.*
> *Oh no! Two and 12 are both circled*
> *That's not fair—we didn't understand about that rule!*
>
> ***Teacher*** *Don't worry. You do now, and I'm sure I'll get stuck too. Just wait! Andrea, please cross out the 24. Now it's my turn to circle a target number. I'd better choose one that still has some factors left to circle!*


September | Computational F uency Activity 1 & 2

NAME | DATE

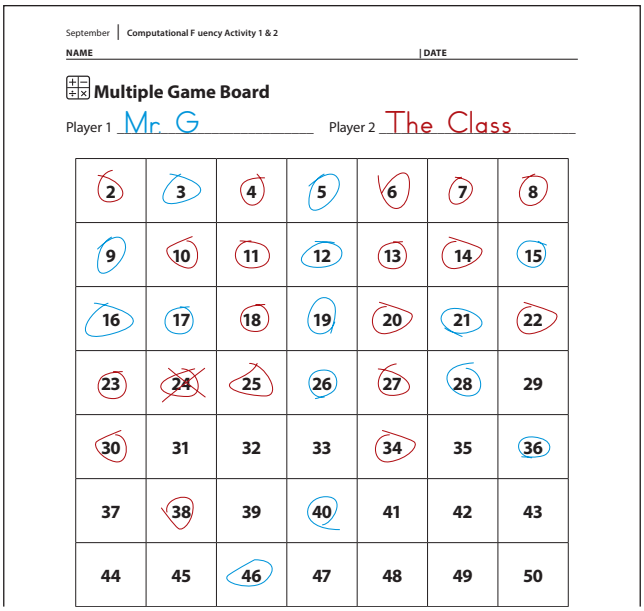
Multiple Game Board

Player 1 Mr. G Player 2 The Class

2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

8 When the numbers remaining on the sheet are not multiples of any uncircled numbers (i.e., when no further plays can be made) the game is over.

> ***Teacher*** *It's your turn to choose a target number now. Talk to the person sitting next to you about which number you think the class ought to circle.*



September | Computational F uency Activity 1 & 2

NAME | DATE

Multiple Game Board

Player 1 Mr. G Player 2 The Class

2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	~~24~~	25	26	27	28	29
30	31	32	33	34	35	36
37	38	39	40	41	42	43
44	45	46	47	48	49	50

Students *I think we're stuck.*

I think you're right. I don't see any number we can choose that has any factors left. Like 29, 31, 37, 41, 43… none of them work because they're prime. If we circle any of them, we won't get points.

What about 32?

I think all the factors for 32 are already circled. Let's see… 2 and 16, 4 and 8… yep, they're all taken.

Teacher *What about 39? Could you circle that for your target number?*

Students *No, because you'd need a 3 or a 13, and they're both taken.*

Oooohh, but what about 42 or 48? Those both have lots of factors, and maybe some of them are still left to circle.

OK, for 42 it would be 2 and 21, 3 and 14, or 6 and 7… all gone. For 48, you'd need 2 and 24, 3 and 16, 4 and 12, 6 and 8, and all of those are gone too. I think we're stuck.

Teacher *I agree. I don't see anything that will work for you. That means it's my turn, but there's nothing I can circle for a target number either. The game is over when both teams are stuck.*

9 At the game's conclusion, have the students find the sum of the numbers circled in the class color, while you find the sum of the numbers circled in your color.

The side with the greater total is the winner.

10 After playing the Multiple Game whole class, wrap up today's activity by letting students know they will play the Multiple Game with a partner during the next Computational Fluency activity.

Activity 2

Partner Multiple Game — Day 12

1 Open today's activity by letting students know they will play the Multiple Game against a partner today.

2 Quickly review how to play the Multiple Game.

As you review the game instructions with the class, remind students of the following:

- Once a number on the game board has been circled, it may no longer be used.
- If a player chooses a target number that has no factors left that have not been circled, that player must cross out a number. The player loses a turn and does not get the points for the number he or she selected as a target.
- When the numbers remaining on the sheet are not multiples of any uncircled numbers (i.e., when no further plays can be made) the game is over.

3 Ask students to turn to a neighbor to review the directions, and take a minute to answer any remaining questions they might have about the game.

4 When students understand what to do, have them play the game in pairs.

- Give them a few moments to locate the Multiple Game Board Student Book page in their books and get a crayon or colored pencil.
- Assign or have students choose partners.
- Explain that each pair only needs one copy of the Multiple Game Board Student Book page to share. Have them decide whose copy they will use, and save the other in case they have time to play the game again today or at another time.

5 As students play, circulate around the room, making observations and offering differentiated instruction as needed.

ELL Help ELL students understand the directions for the game by playing a round with them, modeling each step and emphasizing what to do on their record sheets. Pair ELL students with supportive partners or allow them to observe a game being played before playing it on their own.

SUPPORT If students are having a difficult time determining the factors of their chosen target number, help them to create an organized list of factor pairs or a "factor rainbow". Encourage students to work together, rather than competitively. Alternatively, players may work on a board with numbers 1-30.

CHALLENGE Encourage students to talk about how they choose target numbers to keep their partner from scoring high amounts. Players may also challenge themselves with a Multiple Game board with numbers up to 100.

6 At the end of Number Corner, have students clean up and put away materials. Conclude the activity by asking students to share any observations, insights, or tips they have for playing the Multiple Game.

Some students may note that it's a good idea to choose a prime number or a composite number with very few factors (such as 16 or 39) early in the game. Others may observe that it's wise to choose composite numbers later in the game, when many of the factors have already been crossed out. Your students may have other observations or tips to share as well.

Activity 3

Maddy's Multiples! Day 15

1 Open by explaining to students that they are going to do a page in their Student Number Corner Book today, based on their experience playing the Multiple Game whole class and with a partner.

2 Have students locate their Maddy's Multiples Student Book page, as you place a copy on display.

3 Review the questions briefly with the class, and clarify as needed. When students understand what to do, have them go to work.

CHALLENGE After students complete question 3, ask them to consider which numbers on the Multiple Game board are the best and worst first moves.

4 As students complete the Maddy's Multiples Student Book page, have them meet with a classmate to discuss their responses and explain their thinking.

5 When the majority of students have completed the Maddy's Multiples Student Book page, or when there are a few minutes left in Number Corner time, revisit the following questions:

- How are factors and multiples related?
- How do prime and composite numbers affect the Multiple Game?
- What strategies can be employed when playing the Multiple Game to ensure a winning score?

SUPPORT Provide additional time for students who were not able to complete the assignment during Number Corner to finish their work within the next day or two.

September Solving Problems

Solving Problems Using Multiples & Factors

Workout Overview

The Problem Solving Workout this month features two sets of problems. Students are given time to solve the problems and then discuss their solutions and problem solving strategies as a class. The mathematical content focuses mainly on using multiples and factors, particularly least common multiples and greatest common factors. In addition, the problems themselves will almost certainly help students identify diagrams and organized lists as useful problem-solving tools.

Skills & Concepts

- Find all factor pairs for a whole number between 1 and 100 (4.OA.4)
- Demonstrate an understanding that a whole number is a multiple of each of its factors (4.OA.4)
- Determine whether a whole number between 1 and 100 is a multiple of a given 1-digit number (4.OA.4)
- Determine whether a whole number between 1 and 100 is prime or composite (4.OA.4)
- Generate two numerical patterns given two different rules, and identify relationships between their corresponding terms (5.OA.3)
- Make sense of problems and persevere in solving them (5.MP.1)
- Model with mathematics (5.MP.4)
- Look for and make use of structure (5.MP.7)

Materials

Activities	Day	Copies	Kit Materials	Classroom Materials
Activity 1 Introducing Solving Problems	5	**NCSB 3*–4** Rock Hopping		• a piece of copy paper to mask portions of the Student Book page
Activity 2 Discussing Rock Hopping	10	**NCSB 3–4** Rock Hopping		
Activity 3 Solving Another Problem	16	**NCSB 5*–6** Field Trip Snacks		• a piece of copy paper to mask portions of the Student Book page
Activity 4 Discussing Field Trip Snacks	20	**NCSB 5–6** Field Trip Snacks		

TM – Teacher Master, **NCSB** – Number Corner Student Book
Copy instructions are located at the top of each teacher master. ** Run 1 copy of this page for display.*

Vocabulary

An asterisk [*] identifies those terms for which Word Resource Cards are available.

factor*
factor pair(s)*
multiple*
composite number*
prime number*

Preparation

Between Activities 1 and 2 and between Activities 3 and 4, you will look at student work and decide which students should share their work. See Activities 2 and 4 below for additional guidance in selecting student work to feature during discussions.

Mathematical Background

This month's problems focus on factors and multiples. The first set of problems use a scenario of frogs jumping onto successive rocks, where each frog lands on a multiple of a number, with DJ landing on every second rock and Freddy landing every third rock. Students are asked to investigate which rocks both frogs would land on. Since the numbers 2 and 3 share no common factors other than 1, the frogs will both land on each number that has both 2 and 3 as factors. Because 6 is the least common multiple of 2 and 3, all of these rocks are multiples of 6. This kind of thinking about the common multiples of two numbers and how they relate to the least common multiple helps students create a structure of number relationships that is very helpful with respect to multiplication in general, and also in finding equivalent fractions and common denominators for fractions.

The second set of problems uses the context of setting up snack bags for a class where there are different numbers of ingredients: 36 fruit cups and 24 bananas. Students investigate the different numbers of snack bags that can be made if each bag includes the same quantities of fruit cups and bananas as all the other bags. Since neither fruit cups nor bananas can be cut in pieces and stored, students will find that you can make 12 snack bags with 3 fruit cups and 2 bananas in each bag, 6 snack bags with 6 fruit cups and 4 bananas, 4 snack bags with 9 fruit cups and 6 bananas, 3 snack bags with 12 fruit cups and 8 bananas, 2 snack bags with 18 fruit cups and 12 bananas, or the silly case of 1 snack bag with 36 fruit cups and 24 bananas. Since 12 is the greatest common factor of 36 and 24, the maximum number of snack bags that can be created is 12. As students investigate the different possibilities, they build intuitions about common factors, including greatest common factors, of numbers.

The Student Book pages related to this workout are designed with a couple of main problems and a few related problems. This helps ensure that all students will be able to finish the main problems, but also keeps all students engaged for the entire activity. It is likely that not all students will finish every problem, and you will probably not discuss every problem.

Key Questions

- What is the problem asking?
- What information in the problem will help you figure it out?
- What strategy can you use to figure it out?
- What model can you use to represent your thinking?
- Is your answer reasonable?
- How can you check your work after you have solved the problem?
- Can you write a story problem that uses the same math skills and concepts?

Activity 1

Introducing Solving Problems — Day 5

1 Open this activity by explaining the Solving Problems workout.

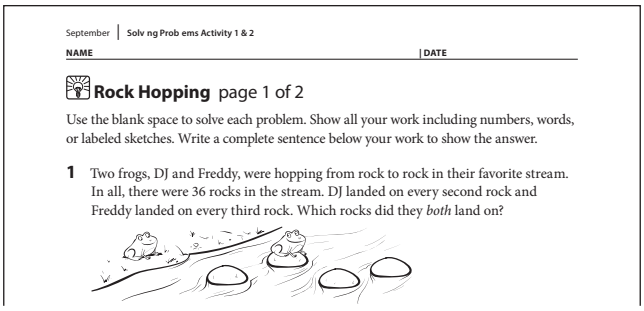
Let students know that when they do Solving Problems activities, they will work on skills such as interpreting problems, identifying important information, and determining which strategy is best for the problem. They will also work on practices for checking their work to make sure their answers are reasonable.

2 Ask students to think about what problem solving means to them. After a minute of quiet thinking time, invite several students to share their ideas.

> ***Students*** *I think it is like when we solve story problems.*
>
> *I think it means to find an answer to something.*
>
> *We use problem solving when we play games.*
>
> *And when we have to figure something out, even when we are not in math.*

3 Next, display your copy of the Frog Hopping Student Book page, revealing the first problem only.

Read the general instructions at the top of the page to the class, and then invite one of the students to read the problem aloud.



September | Solv ng Prob ems Activity 1 & 2

NAME | DATE

Rock Hopping page 1 of 2

Use the blank space to solve each problem. Show all your work including numbers, words, or labeled sketches. Write a complete sentence below your work to show the answer.

1 Two frogs, DJ and Freddy, were hopping from rock to rock in their favorite stream. In all, there were 36 rocks in the stream. DJ landed on every second rock and Freddy landed on every third rock. Which rocks did they *both* land on?

4 Ask students to turn to a partner and talk about what the problem is asking them to figure out, and identify the important information that will help them to solve the problem. Then, invite a few pairs to share their ideas.

If students jump right to figuring out the problem, re-focus their attention to make sure they can verbalize what the problem is asking.

> ***Students*** *The question asks which rocks they both landed on.*
>
> *And there are two frogs, but they jump on different rocks.*
>
> *We also know that there are only 36 rocks in the stream.*

5 Once you are sure all students understand what the first problem is asking, have them find the Rock Hopping pages in their Number Corner Student Books and read the rest of the problems on both pages of the assignment.

6 Have students turn to a partner and begin thinking and talking about how they might go about solving these problems.

Encourage them to think about what strategies they would use and what they might put on their paper to show their thinking.

7 Tell students that they will have the rest of today's workout time to solve the problems with their partner. Remind them to show their work. Ask them if they have any questions and then have them get started.

8 As students work, circulate around the room, making observations, answering questions, and offering differentiated instruction.

ELL/SUPPORT Check with students to make sure they understand what the questions are asking. Help ELL students with vocabulary such as "every third rock," as needed.

CHALLENGE In the allotted time, most students should be able to finish problems 1 and 2. If students determine the answers immediately, showing adequate work and writing a sentence, ask them to solve the questions related to problem 3. Not all students will have adequate time to solve problem 3 in its entirety. Have students complete what they can in the time left.

9 As students finish the problems, have them check their work. Encourage them to think about whether or not their answers are reasonable.

10 Wrap up this activity by letting students know they will discuss their work they next time they do a Solving Problems activity in a few days.

SUPPORT If some of your students weren't able to complete the first two problems on the sheet, give them additional time to finish before the next Solving Problems activity on Day 10.

CHALLENGE Press capable students to complete the entire problem set, and give them extra time to do so before the next Solving Problems activity.

Activity 2

Discussing Rock Hopping — Day 10

Look over students' work from Day 5. Look for a student pair who listed the multiples of 2, 3 and 6 separately and found common multiples. Look for any students who used a sketch or diagram to model the problem. If no students did this, introduce the strategy yourself, in order to compare for efficiency. Finally, look for student work that represents a more abstract understanding of the relationship between the multiples of 2, 3, and 6.

1 Open today's activity by letting students know that they will share and discuss their work on the Rock Hopping problems.

2 Display your copy of Rock Hopping, page 1, and ask students to find their work in their own Student Books.

Take a few moments to review the problems students solved during the first Solving Problems activity.

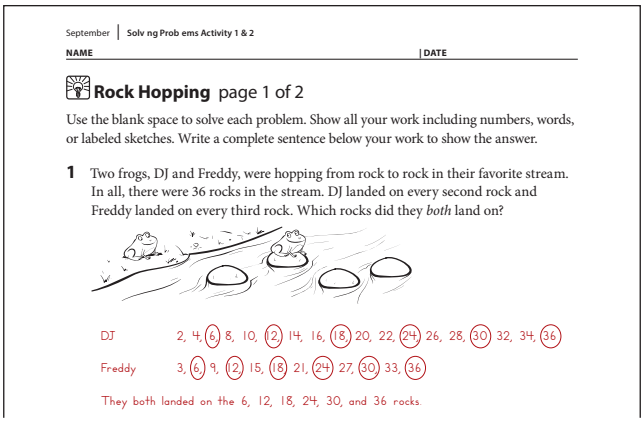
3 Then, let students know that you looked at their work and are anxious to have them share their thinking with the class. First set a few guidelines for sharing:

- Some students, but not all, will share how they solved the problem.
- Each pair of students who shares will have a chance to explain and show their work.
- The rest of the class will have a chance to ask questions about how they solved the problem.
- Then, the teacher may ask other students to summarize what the pair of students did.
- Then, another pair of students will present.
- As more students share, the dialogue will grow and the class will collect and compare the strategies shared.
- All students are responsible for being a respectful as they listen to others as well as being responsible for trying to understand their classmates' work. If they do not understand what other students did or why, they should ask a question.

4 Ask students if they have any questions about the guidelines. Then invite the first pair of students, who listed the multiples separately, to present. Ask them to describe their strategy for solving the first problem.

Have students put their work under a document camera or show how they solved the problem on your whiteboard or chalkboard.

> ***Darius*** *We decided to make a list of all the rocks that each of the frogs landed on.*
>
> ***Troy*** *So, first we made a list of DJ's rocks. We wrote 2, 4, 6, 8… and kept going until we got to 36.*
>
> ***Darius*** *Then we made a list of Freddy's rocks, counting by 3s. So, 3, 6, 9, 12…*
>
> ***Troy*** *And once we made our lists, we just circled the numbers on both lists.*
>
> ***Darius*** *We got 6, 12, 18, 24, 30, and 36.*
>
> ***Teacher*** *And what do those numbers mean?*
>
> ***Troy*** *Those are the rocks that both of the frogs landed on.*



September | Solv ng Prob ems Activity 1 & 2

NAME | DATE

Rock Hopping page 1 of 2

Use the blank space to solve each problem. Show all your work including numbers, words, or labeled sketches. Write a complete sentence below your work to show the answer.

1 Two frogs, DJ and Freddy, were hopping from rock to rock in their favorite stream. In all, there were 36 rocks in the stream. DJ landed on every second rock and Freddy landed on every third rock. Which rocks did they *both* land on?

DJ 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36

Freddy 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36

They both landed on the 6, 12, 18, 24, 30, and 36 rocks.

Teacher *Does anyone have a question for Troy and Darius?*

Imani *I do. Why did you count by 2s for one frog and 3s for the other frog?*

Darius *Because the problem said that DJ landed on every second rock. That means he landed on rock number 2, then rock number 4, then…*

Imani *Oh, OK. And Freddy's list has skip-counting by 3 because he landed on every third rock.*

Troy *Right.*

Kendra *So, then, when you solved problem 2, what did you do?*

Student B *We made another list for Sue's rocks, counting by 6s. Then we just had to compare all three lists to see the rocks that everyone landed on.*

Teacher *Any other questions? No?*

5 Invite a second pair of students who used a model for both lists of multiples, to share, then compare the two strategies for efficiency.

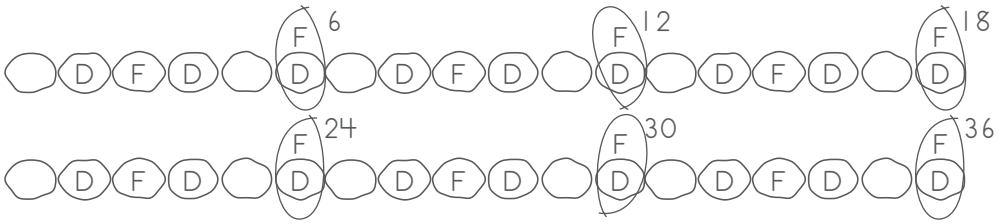
Teacher *OK, Elisa and Sara, please share with us what you did to solve this problem.*

Elisa *When we read this problem, the first thing we thought of was that we needed to be able to see the rocks that they frogs were hopping on. We decided to draw all 36 rocks.*

Sara *Right. And then we labeled the rocks that each frog landed on by writing DJ or F, for Freddy.*

Elisa Then, we just had to list the ones that both frogs landed on.

Sara *We got the same answer as the first group.*



They both landed on 6, 12, 18, 24, 30, and 36.

Teacher *How does this strategy compare with the one Troy and Darius shared? How are the strategies alike, and how are they different?*

Carlos *The first group made a new list for each frog, but the second group used the same picture of rocks for both of the frogs.*

Kiara *They both counted by 2s, 3s and 6s.*

Sergio *They got the same answers.*

Darius *I think Elisa and Sara might have done less work than us, since they used the same picture for both frogs.*

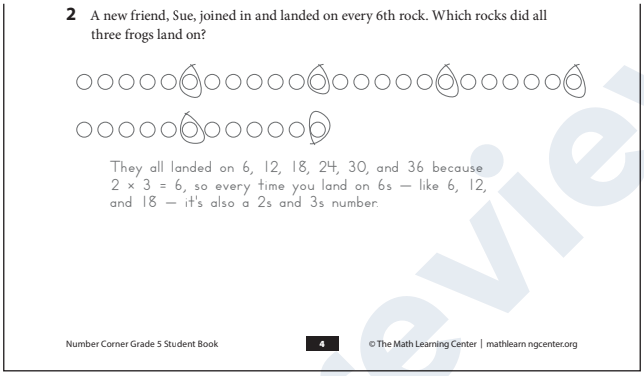
Teacher *That's an interesting thought, Darius. I'd like to have one more group share.*

6 Invite one more pair of students to share, ideally students whose work indicated some kind of understanding about the connection between the multiples of 2 and 3, and the multiples of 6.

Teacher *Let's have one more pair share Darryl and Natalie, will you please talk to us about what you discovered?*

Natalie *We solved the first problem by drawing the rocks and making labels, kind of like Elisa and Sara. Then when we worked on the second problem, we noticed that Sue landed on the same rocks that DJ and Freddy both landed on.*

Darryl *Yeah, if DJ and Freddy both landed on a rock, then so did Sue.*



2 A new friend, Sue, joined in and landed on every 6th rock. Which rocks did all three frogs land on?

They all landed on 6, 12, 18, 24, 30, and 36 because 2 × 3 = 6, so every time you land on 6s — like 6, 12, and 18 — it's also a 2s and 3s number.

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Whitney *So, now that you say that, I can see it, but how did you know?*

Natalie *When we noticed it, we thought about 2 times 3 is 6, and so every time you would land on a 6, you would also be landing on a 2 or 3.*

Teacher *Natalie, are you saying that every time you land on a multiple of 6, you also land on a multiple of 2 and a multiple of 3?*

Natalie *Yes, so if you land on a rock that DJ landed on, that's a multiple of 2. And if you land on a rock that Freddy landed on, that's a multiple of 3.*

Darryl *So, then that rock is a multiple of both 2 and 3 so it has to be a multiple of 6 too!*

Teacher *Does anyone need more help clarifying their strategy? Does anyone have a question or comment about it?*

7 Help students understand the connection between the first two problems and the least common multiple.

The least common multiple of two numbers is the smallest number that is a multiple of both numbers. Since 6 is the smallest multiple of 2 and 3, it is the least common multiple of 2 and 3.

You can extend student thinking with the following questions:

- What if you had been told about Sue first? How could you use this idea of a least common multiple to help you find what rocks DJ and Freddy both landed on?

- What if you were told about a new friend, Joanne, who lands on every 20th rock, and then you were asked to find which rocks both Ann and Bette landed on if Ann jumps on every 4th rock and Bette lands on every 5th rock?

8 If you have time, invite students to use the strategies discussed to solve or reexamine problem 3, then share their thinking.

9 At the end of your Number Corner time today, summarize the first two Solving Problems activities, and explain that students will repeat this sequence of working on a problem set on one day and then sharing their work a few days later.

> ***Teacher*** *We've done two Solving Problems activities this year. In the first one, we read and discussed our first problem set and you worked with a partner to solve them. In the second one, some of you shared your work, presenting how you solved the problem. The rest of the class asked questions and made comments. We will continue this process of working through problems and then sharing work during the Solving Problems workout all year long.*

Activity 3

Solving Another Problem — Day 16

1 Follow the procedure you established in Activity 1 to introduce this week's problem set.

- Display your copy of the Field Trip Snacks Student Book page, revealing the first problem only. Read the general instructions at the top of the page to the class, and then invite a student to read the problem aloud.
- Ask students to turn to a partner and talk about what the problem is asking them to figure out, and identify the important information that will help them to solve the problem. Then, invite a few pairs to share their ideas.
- Once you are sure all students understand what the first problem is asking, have students find the Field Trip Snacks pages in their Student Books and read the rest of the problems.
- Have students turn to a partner and begin thinking and talking about how they might solve the problems in this set.
- Solicit any questions students may have before they get started.

2 When students understand what to do, give them the time remaining to solve the problems with their partner.

Remind them to show their work.

3 As students work, circulate around the room, making observations, answering questions, and offering differentiated instruction.

ELL/SUPPORT Check with students to make sure they understand what the questions are asking. Help ELL students with vocabulary, as needed. Make manipulatives available to students so they can create a model of identical bags.

CHALLENGE In the allotted time, most students should be able to finish problem 1. As time allows, challenge students to complete problems 2 and 3, and to model an efficient way to create a list of the number of bags the teacher could make. Encourage these students to read question 4 and summarize their work.

4 As students finish the problems, have them check their work. Encourage them to think about whether or not their answers are reasonable.

5 Wrap up this activity by letting students know they will discuss their work they next time they do a Solving Problems activity in a few days.

SUPPORT If students did not finish the Student Book pages (particularly problem 1), give them additional time to work on the assignment before the next Solving Problems Activity on Day 20.

Activity 4

Discussing Field Trip Snacks — Day 20

Look over students' work from Day 16. Look for a pair of students who used a guess-and-check method for determining the number of bags that could be made. Look for another pair of students who used an organized system or list to keep track of the possibilities.

1 Open today's activity by letting students know that they will share and discuss their work on the Field Trip Snacks problems.

2 Display your copy of Field Trip Snacks, page 1, and ask students to find their work in their own Student Books.

Take a few moments to review the problems students solved during the previous Solving Problems activity.

3 Then, let students know that you looked at their work and are excited to have them share their thinking with the class. Review the guidelines and expectations you have set for sharing.

4 Invite the pairs of students whose work you selected ahead of time to share with the class.

- Have each pair their work under a document camera or show how they solved the problem on your whiteboard or chalkboard.
- Encourage the other students to
  - » Ask clarifying questions
  - » Summarize each strategy after it's presented
  - » Compare the strategies, noting differences and likenesses, as well as potential for being efficient and effective

Student Work/Strategy	Comments

Guess & Check

September | Solving Problems Activity 3 & 4

NAME | DATE

Field Trip Snacks page 1 of 2

Use the blank space to solve each problem. Show all your work including numbers, words, or labeled sketches. Write a complete sentence below your work to show the answer.

1 A parent donated 36 fruit cups and 24 bananas to the fifth grade. The teacher wanted to make field trip snack bags with the donated food and wondered about the ways that the snacks could be packed. To be fair, the teacher wants to makes sure that all the bags are exactly the same.

a What is the greatest number of snack bags that the teacher can make, if each bag is identical? How do you know?

2 bags 36 ÷ 2 = 18 fruit cups, and 24 ÷ 2 = 12 bananas

12 bags 36 ÷ 12 = 3 fruit cups, and 24 ÷ 12 = 2 bananas

~~8 bags doesn't work because you can't divide 36 by 8 evenly~~

~~9 bags doesn't work because you can't divide 24 by 9 evenly~~

We think 12 bags is the most, because no number more than 12 can go into 24 and 36 at the same time.

b What other numbers of snack bags could she make? How do you know?

3 bags... 36 ÷ 3 = 12 and 24 ÷ 3 = 8, so 12 FC and 8 bananas

5 bags... NO

7 bags... NO

Ivan *So, we know she can make 2 bags or 12 bags or 3 bags. 12 is the most.*

Teacher *Class, who can explain Ivan and Xavier's strategy for determining the number of bags that the teacher could make?*

Jade *They picked a number of bags and then tried to see if they could split the fruit cups and bananas evenly. And then they tried another number of bags.*

Teacher *Ivan and Xavier, my question to you is this: How do you know that you found all of the bags that she could make?*

Xavier *Um, I'm not sure. I think we tried everything.*

Ivan *Yeah, those were the only bags that worked out.*

Big Idea

Using a guess-and-check strategy is one way to approach a problem, but it can be challenging to know if and when all the possible solutions have been found.

Organized List

September | Solving Problems Activity 3 & 4

NAME | DATE

Field Trip Snacks page 1 of 2

Use the blank space to solve each problem. Show all your work including numbers, words, or labeled sketches. Write a complete sentence below your work to show the answer.

1 A parent donated 36 fruit cups and 24 bananas to the fifth grade. The teacher wanted to make field trip snack bags with the donated food and wondered about the ways that the snacks could be packed. To be fair, the teacher wants to makes sure that all the bags are exactly the same.

a What is the greatest number of snack bags that the teacher can make, if each bag is identical? How do you know?

Bags	Fruit Cups	Bananas
1	36 ÷ 1 = 36	24 ÷ 1 = 24
2	36 ÷ 2 = 18	24 ÷ 2 = 12
3	36 ÷ 3 = 12	24 ÷ 3 = 8
4	36 ÷ 4 = 9	24 ÷ 4 = 6
5 No		
6	36 ÷ 6 = 6	24 ÷ 6 = 4
7 No		
8 No		
9 No		

b What other numbers of snack bags could she make? How do you know?

10 No		
11 No		
12	36 ÷ 12 = 3	24 ÷ 12 = 2
13 No		
14 No		

The most bags she can make is 12. She can also make 1, 2, 3, 4, and 6 bags.

Number Corner Grade 5 Student Book 6 © The Math Learn ng Center | mathlearningcenter.org

Teacher *Akiko, you said you tried all the bags until you were done. Can you say more about that? How did you know when you were done?*

Akiko *Well, at first we thought we had to go all the way to 24 because that's how many bananas there were and 24 bags would mean one banana in each. But after we thought about 13 and 14 bags we realized we could stop.*

Raven *When there are 12 bags, each bag has 2 bananas, and if there were 24 bags, they would each have one banana, but anything in between that wouldn't work.*

Teacher *Can someone summarize the strategy that these two students used to find the number of bags possible?*

Sam *They made a list and went in order, trying each bag until they got to a place they could stop.*

Teacher *Ah, so making a list and going in order. And how was that helpful?*

Xavier *They were able to make sure they got them all. We missed some because we skipped around.*

Big Idea

Using an organized list makes it easier to determine when and if you've found all the possible solutions to a problem.

5 After a second pair of students have shared, ask students to turn their focus the fourth problem in the set.

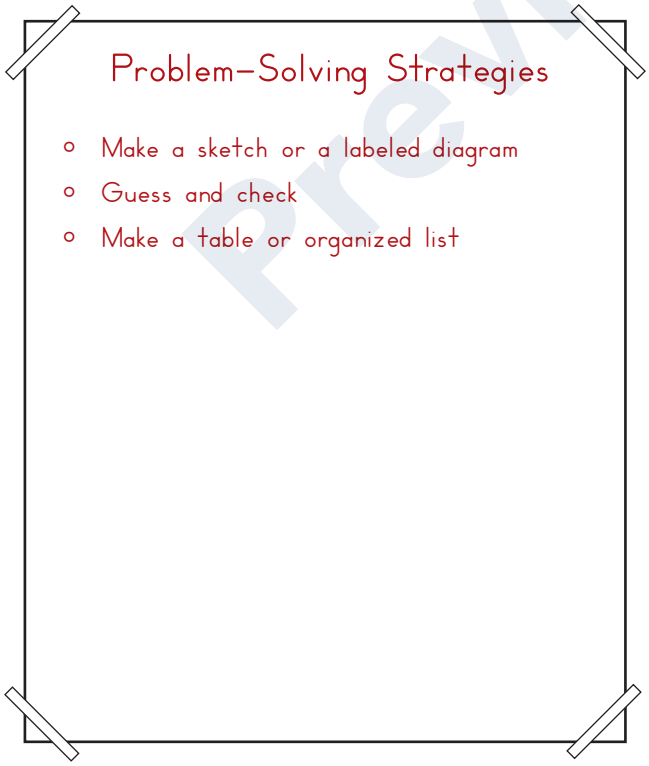
- Invite one students to read the question aloud. Then facilitate a conversation about fact that the numbers of snack bags that work are the common factors of 24 and 36.
- Here are some questions and prompts you might use to drive the discussion:
  - » So, this question is asking what the numbers of snack bags that the teacher can make have to do with the number of fruit cups and bananas. What do you think?
  - » Let's focus on the bananas for a moment. What are the factors of 24, the numbers by which 24 can be divided evenly? Let's write those on the board. (1, 2, 3, 4, 6, 8, 12, 24)
  - » Now let's think about the 36 fruit cups, and list the factors of 36 on the board. (1, 2, 3, 4, 6, 9, 12, 18, 36)
  - » So, again, what do the numbers of snack bags that can be made, which we said were 1, 2, 3, 4, 6, and 12, have to do with the number of fruit cups and bananas? (Those numbers are factors of both 24 and 36.)

6 If you have time, invite students to use the strategies discussed to solve or reexamine problems 2 and 3, then share their thinking.

7 Close the activity by listing the types of problem-solving strategies students have shared this month.

Remind students that developing a repertoire of problem-solving skills will allow them to have choice and become more flexible thinkers.

You might leave this list posted somewhere in the classroom, where you can add to it through the school year.



September Problem Strings

Addition & Subtraction Strategies

Overview

This month, students solve and discuss problem strings designed to elicit efficient strategies for adding and subtracting whole numbers and decimal numbers.

Skills & Concepts

- Add and subtract decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and the relationship between addition and subtraction (5.NBT.7)
- Use written numbers and symbols to represent strategies for computing with decimals to hundredths (5.NBT.7)
- Explain the reasoning behind strategies for computing with decimals to hundredths (5.NBT.7)
- Construct viable arguments and critique the reasoning of others (5.MP.3)
- Model with mathematics (5.MP.4)
- Look for and express regularity in repeated reasoning (5.MP.8)

Materials

Activities	Day	Copies	Kit Materials	Classroom Materials
Activity 1 Problem String 1	1	**NCSB Appendix** Problem String Work Space **TM T2** Problem String Work Space		• chart paper or space to write on the whiteboard
Activity 2 Problem Strings 2 & 3	11, 17			

TM – Teacher Master, **NCSB** – Number Corner Student Book
Copy instructions are located at the top of each teacher master.

Vocabulary

An asterisk [*] identifies those terms for which Word Resource Cards are available.

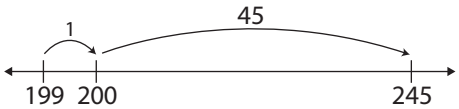
decimal number
decimal point
difference*
hundredth*
tenth*
whole number*

Mathematical Background

The first problem string this month invites students to consider the idea of taking some amount from one of the addends in a multi-digit addition combination and giving it to the other addend in order to make the problem easier to solve. The second string deals with the fact that subtraction can be thought of in terms of removing or taking away one number from another, but also as a process of finding the difference between two numbers. The third string builds on the differencing model for subtraction. For all three strings, you will represent students' thinking using equations, as well as the open number line, which is a particularly effective tool for modeling and solving multi-digit addition and subtraction problems.

Give and Take

Certain addition combinations lend themselves to taking a little from one of the addends and giving it to the other to create an easier combination. For example, in solving 199 + 46, one might remove 1 from 46 and "give" it to the 199, so that the combination is 200 + 45. Here are two different ways to represent the strategy:



$$\begin{array}{rcl} 199 & + & 46 \\ +\ 1 & & -\ 1 \\ \hline 200 & + & 45 = 245 \end{array}$$

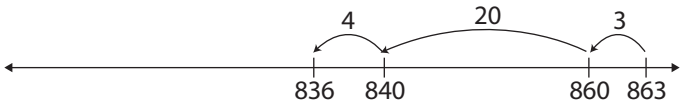
The give and take strategy works equally well with certain decimal combinations as well. For example, in solving 4.8 + 2.9, one might remove 1 tenth from 4.8 and "give" it to 2.9, resulting in an easier combination: 4.7 + 3. This line of thinking can be modeled with equations as shown here.

$$\begin{array}{rcl} 4.8 & + & 2.9 \\ -\ 0.1 & & +\ 0.1 \\ \hline 4.7 & + & 3.0 = 7.7 \end{array}$$

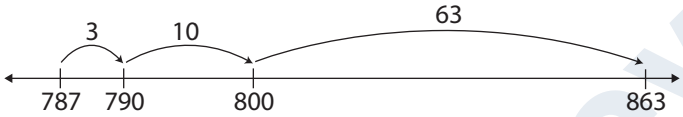
Removal vs. Differencing

The problems in the second string alternate between combinations that are best solved using a removal strategy and combinations that are solved more efficiently by finding the difference between two numbers. Students will discover that removal is more efficient and effective when the numbers are far away from each other, while finding the difference is more efficient when the numbers are close together.

Both strategies are easily modeled on the open number line. The removal strategy involves hopping backward the amount specified by the subtrahend, preferably to a friendly number first, and then by chunks that are as efficient as possible. To solve 863 – 27, for example, one might take 3 away to get to 860, then remove 20 more, and finally remove the last 4 to get an answer of 836.



The removal strategy becomes inefficient when the minuend and the subtrahend are close, however. Consider 863 – 787, for example. Starting at 863 and removing 787, no matter how large the increments, is tedious at best. On the other hand, a differencing strategy, which involves adding on to find the difference—literally the distance—between the subtrahend and the minuend is much easier.

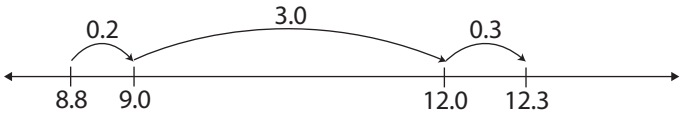


3 + 10 + 63 = 76, so 863 – 787 = 76

Constant Differences

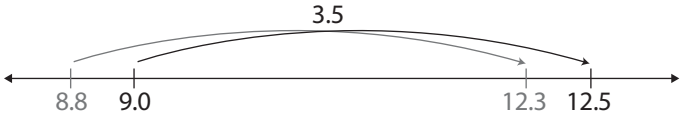
The third string features the constant difference strategy, which involves creating an equivalent problem that is easier to solve than the original problem by shifting each of the numbers by the same amount. Students will discover that this strategy is most effective when the subtrahend can be shifted to a "nice number" (i.e., a whole number or a multiple of 10, 100, 1000, and so on).

The open number line again serves as a particularly effective way to model and solve problems using the constant differences strategy. Consider the problem 12.3 – 8.8, which can be solved by finding the difference or distance between the subtrahend and the minuend, as shown here.



0.2 + 3.0 + 0.3 = 3.5, so 12.3 – 8.8 = 3.5

The combination becomes even easier to solve if we shift the 8.8 up the line by 2 tenths to become 9.0. We have to make the same shift for 12.3 so the difference between the subtrahend and the minuend will remain constant. The resulting combination, 12.5 – 9.0 is very easy to compute mentally; much more so than 12.3 – 8.8.



$$\begin{array}{r} 12.3 + 0.2 = 12.5 \\ -\ 8.8 + 0.2 = -\ 9.0 \\ \hline 3.5 \end{array}$$

Key Questions

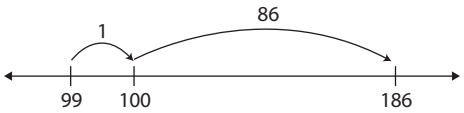
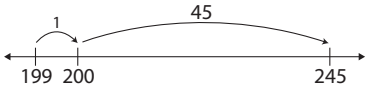
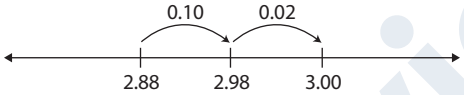
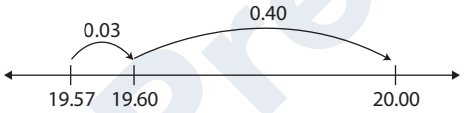
Use these questions to help your students investigate this month's strings.

- What do you know that could help you solve this problem?
- What strategy could you use?
- How can you show your thinking?
- What model could you use to show your thinking?
- How can solving one problem in a string help you solve another problem later in the string?
- What is the big idea of this string?
- How can your work with this string help you with other problems?

Activity 1

Problem String 1 — Day 1

Today you will deliver a problem string consisting of the combinations listed here. Sample strategies are shown and explained in detail in the dialog below. Problems and answers are provided in the left column of this table for your convenience. When you present the problems to students, do not include the sums.

Problems	Sample Strategies & Recording	Connections
99 + 87 = 186		These combinations are selected to elicit give-and-take strategies from students.
199 + 46 = 245	 199 + 46 + 1 − 1 200 + 45 = 245	
4.8 + 2.9 = 7.7	4.8 + 2.9 − 0.1 + 0.1 4.7 + 3 = 7.7 4.8 − 0.1 = 4.7 2.9 + 0.1 = 3 7.7	
2.88 + 0.12 = 3	Students may naturally think of problems like this one in terms of currency. Others will see combinations to 100 in the hundredths, adding .88 and .12 to make one whole. 	
2.88 + 1.56 = 4.44	2.88 + 1.56 + 0.12 − 0.12 3.00 + 1.44 = 4.44 2.88 + 0.12 = 3.00 + 1.56 − 0.12 = 1.44 4.44	
19.57 + 0.43 = 20.00		
19.57 + 8.74 = 28.31	19.57 + 8.74 + 0.43 − 0.43 20.00 + 8.31 = 28.31 19.57 + 0.43 = 20.00 + 8.74 − 0.43 = 8.31 28.31	
1,992 + 2,996 = 4,988	1,992 + 8 = 2,000 + 2,996 − 8 = 2,988 4,988 or 1,992 − 4 = 1,988 + 2,996 + 4 = 3,000 4,988	
2.99 + 3.65 = 6.64	2.99 + 3.65 + 0.01 − 0.01 3.00 + 3.64 = 6.64 2.99 + 0.01 = 3.00 + 3.65 − 0.01 = 3.64 6.64	

1 Open by gathering students in your discussion area with their Number Corner Student Books and a pencil. Tell students that they will have a Problems Strings workout as part of Number Corner. Briefly review problem strings and how they work:

- A problem string is a series of related problems that students will solve and discuss one at a time.
- Strings usually start out with an easier problem and then the problems get more challenging as the string continues.
- The problems at the beginning of the string can often be used to help solve the problems toward the end of the string.

- Solving the problems in a string involves thinking like a mathematician because students want to find smart and efficient ways to solve the problem. Efficient strategies are quick, and can be explained clearly and easily.
- There is a process the class will use to solve each problem, share strategies and answers, and discuss each other's thinking.
- Students will do their work on special pages at the back of their Student Books.
- When students explain their thinking, the teacher will usually represent their work for everyone to see.

2 Display your copy of a Problem String Work Space page as students find the first page of work space in the back of their Number Corner Student Books.

3 Explain that each time the class does a problem string during Number Corner, the students will use these pages to show their work. When starting a new string, students should always find the next unused Problem String Work Space page and write the date.

4 Introduce the string by posing and writing the first problem, 99 + 87, on the board or a piece of chart paper.

- Ask students to write the problem in their journals and to put their thumb up in front of their chest when they have an answer.
- Tell students that if they just know they answer, they don't need to show any work, but remind them that you will ask them to show their thinking in other problems.

5 When you see several thumbs up, invite a few students to share the answer.

Record all answers without comment or indication that any of them are correct or incorrect.

6 Then, ask several volunteers to explain how they figured it out.

Record students' thinking on the board (or a piece of chart paper) for everyone to see, using equations and number line diagrams.

Teacher *Who would like to explain how you got the answer?*

Brandon *I lined up the numbers and added them together. I got 186.*

Teacher *OK. Whitney, can you tell us what you did?*

Whitney *I thought about adding one more to 99 to make it 100, then just adding what I had left, 86, to get 186.*

Teacher *Let me model what you just said. I'm going to represent your thinking on an open number line.*

7 Pose the next problem in the string, 199 + 46. Allow students time to solve the problem. Then solicit answers. When you have recorded all the answers, invite several volunteers to explain how they figured it out.

Look for a student who made use of the proximity of 199 to 200.

Teacher *Sam, I noticed that you got the answer very quickly. Can you explain what you did?*

Sam *I got 245.*

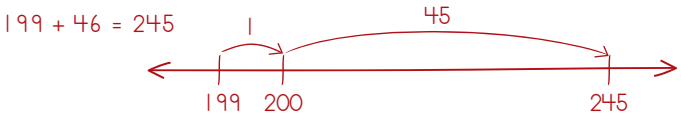
Teacher *And how did you get your answer?*

Sam *I saw that 199 was really close to 200, so I added 1 more to 199 to get 200. Then, since I already added 1, I just made a jump of 45.*

Teacher *Where did the 45 come from?*

Sam *I used 1 from the 46 to get from 199 to 200, so I only needed to add 45 more.*

Teacher *Oh, OK. Thank you. Let me model what you said for everyone to see.*



Teacher *Mei, what Sam did was similar to your thinking, wasn't it? But not quite identical. Tell us about your thinking please.*

Mei *I also thought about how 199 was close to 200, but I changed both numbers at the same time. I guess it's kind of like making a new problem. I knew that I needed to find 199 + 46, but if you take one from 46 and give it to 199, you have made a new problem 200 + 45. And that's easy to do in your head.*

Teacher *And you can do that? Make a new problem?*

Mei *Sure. You are adding it all together anyway, so you would get the same thing.*

Teacher *Let me record your thinking. I'll write this equation to show how you changed both numbers simultaneously, or at the same time.*

$$\begin{array}{rcrcl} 199 & + & 46 & & \\ + \ 1 & & - \ 1 & & \\ \hline 200 & + & 45 & = & 245 \end{array}$$

Teacher *That's pretty efficient, isn't it? Just take some from one of the numbers and give it to the other number so that you are left with numbers that can be added easily in your head. I wonder if anyone else can think about an addition problem in that way.*

8 Repeat for 4.8 + 2.9 by posing the problem, giving students time to work, soliciting and recording answers, and modeling student strategies.

- If necessary, encourage students to share strategies for working with decimals, such as thinking in terms of dollars, dimes, and pennies, or ones, tenths, and hundredths.
- Look for a student who used the give and take strategy to adjust the numbers.

Teacher *I saw some of you frown a little when you saw this problem. Why is that?*

Briana *There are decimals!*

Teacher *How do you think about problems with decimals?*

Students *I think of money sometimes.*
I take the decimal point out and add the numbers. Then I put the decimal point back in.
I don't really do anything different. It's still addition.

Teacher *Max, tell us about how you solved this problem, please.*

Max *I thought about what Mei did on the last problem. So, I took one-tenth from the 4.8 and gave it to the 2.9. Then the problem I had was 4.7 and 3. That's way easier. It's just 7.7!*

Teacher *Ah, that is nice, isn't it? I want to record that with an equation in two ways.*

$$\begin{array}{r} 4.8 + 2.9 \\ -\ 0.1 \quad +\ 0.1 \\ \hline 4.7 + 3.0 = 7.7 \end{array} \qquad \begin{array}{r} 4.8 - 0.1 = 4.7 \\ 2.9 + 0.1 = 3.0 \\ \hline 7.7 \end{array}$$

Teacher *Maria Jose, you also got 7.7, but I saw that you wrote something different down. Will you tell us about your thinking?*

Maria Jose *When Max said he did the same thing as Mei, I thought it would be the same as mine, but it wasn't. I also used give and take to make the numbers easier to do in my head, but I made 4.8 into 5 by taking 2 tenths from the 2.9. So the new problem I made was 5 + 2.7.*

$$\begin{array}{r} 4.8 + 2.9 \\ +0.2 \quad -\ 0.2 \\ \hline 5.0 + 2.7 = 7.7 \end{array}$$

Teacher *Class, what do you think about what Maria Jose said? Did she do the same thing as Max?*

Students *She made a different problem.*
But she did the same kind of thinking.
Yeah, she did give and take to make the numbers easier. She just made the 4.8 a nice number, and Max made the 2.9 a nice number.

9 Pose the next problem in the string, 2.88 + 0.12. Allow students time to solve the problem, then solicit strategies for solving.

Cody *I thought about this one like it was money. I added 10 cents to 2.88 to get 2.98 and then 2 more cents makes 3 dollars.*

2.88 + 0.12 = 3.00

Kelsey *I just knew that 88 and 12 makes 100, so .88 and .12 makes one whole. I think I am pretty good at finding combinations of 100 and that helps with a lot of problems.*

10 Pose the next problem in the string, 2.88 + 1.56. Allow students time to solve the problem. Then solicit answers. When you have recorded all the answers, invite a volunteer who made use of the previous combination to share his strategy.

Darius *At first, I couldn't think of anything really easy to do for this one, but then I looked back and saw we had just solved 2.88 + 0.12. So I used that. I took 12 hundredths from the 1.56 and gave it to the 2.88, so that I had one really easy number. Then I just had to solve 3 + 1.44.*

$$\begin{array}{r} 2.88 + 1.56 \\ +\ 0.12 \quad -\ 0.12 \\ \hline 3.00 + 1.44 = 4.44 \end{array} \qquad \begin{array}{r} 2.88 + 0.12 = 3.00 \\ +\ 1.56 - 0.12 = 1.44 \\ \hline 4.44 \end{array}$$

11 Continue the string with the next two problems, 19.57 + 0.43 and 19.57 + 8.74.

As you work with the class, reinforce the idea of using the first of the two combinations to help solve the second.

12 Wrap up the string with the two problems 1,992 + 2,996 and 2.99 + 3.65, which should elicit and reinforce the give and take strategy.

Some students may think of 1,992 + 2,996 as the equivalent 2,000 + 2,988, while others may think of the problem as 1,988 + 3,000. Similarly, some students will likely use 3.00 + 3.64 or 2.64 + 4.00 to solve 2.99 + 3.65. Note with students that the give and take strategy allows for flexible thinking.

13 Ask students to summarize a big idea that came out of doing this string.

When adding, students should recognize that the give and take strategy can be used to create equivalent problems that are easier to solve mentally.

Activity 2

Problem Strings 2 & 3 — Days 11 & 17

1 Follow the procedure you established in Activity 1 to deliver the second and third problem strings for the month.

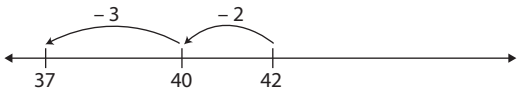
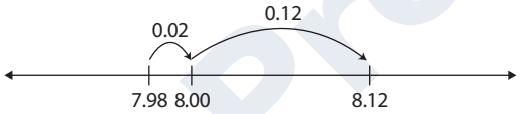
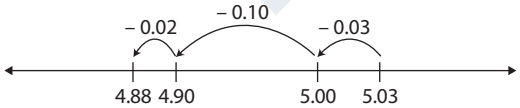
- Ask students turn to the next blank page of the Problem String Workspace in their Number Corner Student Books and write today's date.
- Deliver each problem in the string one at a time.
  - » Have students record and solve the problem in their Student Books.
  - » Have them put up their thumbs when they are ready to share their solutions and strategies.
  - » When you see several thumbs up, invite a few students to share the answer. Record all answers without comment or indication that any of them are correct or incorrect.
  - » Then invite two or three volunteers to explain how they got their answers.
  - » Record students' thinking with equations or number lines on the board (or a piece of chart paper) for everyone to see.

2 Conclude the activity by asking students to reflect briefly on the string.

- Ask students to share, first in pairs and then as a group, a one-sentence statement about something they learned in today's string.
- Invite students to write a simple statement beneath their work in the Student Book describing what they learned while doing this string.

> ***Students*** [Referring to Problem String 2] *When you only have to take off a little bit, it's better to do take away.*
>
> *When the first number is big and the number you are taking off is small, like 463 – 27, it's way easier hop backward on a number line, just taking 27 off, than trying to find the difference between the two.*
>
> *If the numbers are both big or both small, it's better to find the difference.*
>
> *Yeah, like with 746 – 687, it's way easier to start at 687 and go forward to find out how far it is from there to 746.*
>
> ***Students*** [Referring to Problem String 3] *With constant differences, you can move numbers to better places on the number line to make a problem easier as long as you move them both the same distance.*
>
> *You want to move them so the first number is nice.*
>
> *No, the second number; the one you're taking away.*
>
> *Yeah, 7.0 – 3.5 is pretty easy, but 7.5 – 4.0 is even easier.*

Problem String 2 — Day 11

Problems	Sample Strategies & Recording	Connections
42 – 5 = 37		By posing problems that are radically different in terms of how the subtrahend compares to the minuend, you immediately invite the strategies of removal and differencing. **Big Idea** When the subtrahend is much smaller than the minuend, as in 42 – 5, it's quite efficient to use a removal strategy, which can be modeled using backward hops on an open number line. On the other hand, when the subtrahend is quite close to the minuend, it is far easier and more efficient to find the difference or distance between the two by adding on or moving forward along the number line. You can help students make generalizations about the two strategies by asking if anyone used a removal or take away strategy to solve 7.2 – 6.8, or found the difference for 9.3 – 0.4. Discuss why or why not, and ask students to begin thinking about when they would probably prefer one of the strategies over the other.
51 – 49 = 2		
9.3 – 0.4 = 8.9		
7.2 – 6.8 = 0.4		
8.12 – 7.98 = 0.14		Before students solve each of the problems at left, ask them to think about whether it would be quicker to find the distance or remove, and explain why. **Big Idea** See above.
5.03 – 0.15 = 4.88		

Notes About This Activity

Students will share a variety of strategies for these problems. Use the open number line and equations to represent their thinking. Work with the class to think about combinations for which removal or take away—which can be represented using backward hops on the open number line—makes more sense than finding the difference, and vice versa.

Sample Dialog

Diego *I just started at 42 and counted back 2 to get to 40, then back 3 more to get to 37.*

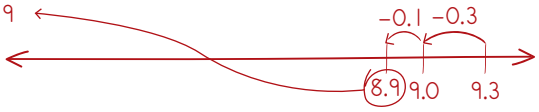
42 – 5 = 37

Tavis *For that one, I just counted up. I started at 49 and added 1 to get to 50, then one more to get to 51.*

51 – 49 = 2

Milla *I thought about removing 0.3 from 9.3, and that gave me 9. But then I needed to get rid of one more tenth, so I was at 8.9.*

9.3 – 0.4 = 8.9



Lin *I said that I needed .2 more to get to 7. Then I jumped .2 more to get to 7.2. My answer is 0.4.*

7.2 – 6.8 = 0.4

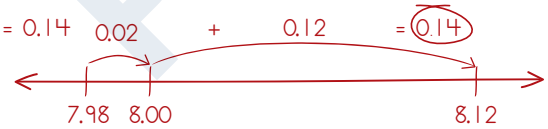
Imani *Wait, you added? But I got the same answer you did, and I didn't add the numbers.*

Lin *I added, but not like you think. I didn't add the two numbers from the problem. I added to get from 6.8 up to 7.2.*

Teacher *Lin actually found the distance between those two numbers, didn't he? Interesting. That seemed pretty fast.*

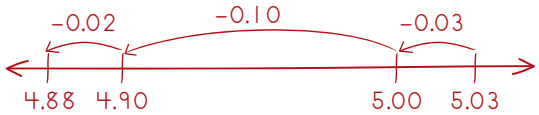
Cody *I started at 7.98 and added up, so I found the distance between the numbers, if you're thinking about them on a line.*

8.12 – 7.98 = 0.14



Sergio *I removed on this problem. You're taking such a little amount away, it's easy to just hop backward on the number line. Also, I thought about money to make it easier to do the decimals. The problem is like 5 dollars and 3 cents take away 15 cents. First I took off 3 cents. That got me to 5 dollars. Then I took off a dime, and I was at $4.90, and then I took off the last 2 cents. I got 4.88.*

5.03 – 0.15 = 4.88

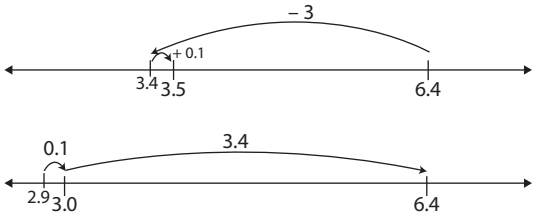
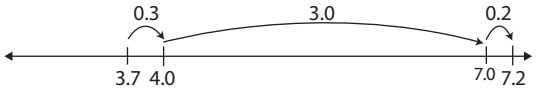
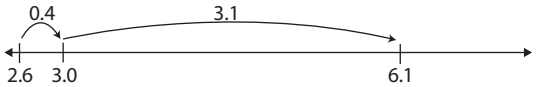
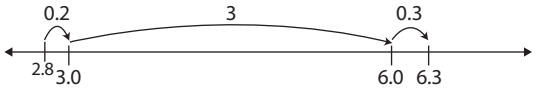


Teacher *Did anyone find the difference, or the distance, between these two numbers?*

Kiara *I think it's much easier to take off 0.15 instead of finding the distance. It's a long way from 0.15 up to 5.03.*

Problem String 3

Day 17

Problems	Sample Strategies & Recording	Connections
6.4 – 2.9 = 3.5		The first five problems in this string all result in the same answer, 3.5. This is a deliberate move, designed to help the teacher introduce the idea of constant differences later in the string. After the fifth problem, draw students' attention to the number lines you've recorded and ask for observations. After students discuss the fact that the answer is the same in all five problems, introduce the idea of "constant difference" which involves shifting the subtrahend and the minuend by the same amount to create an easier combination.
7.2 – 3.7 = 3.5		
6.1 – 2.6 = 3.5		
6.3 – 2.8 = 3.5		
7.1 – 3.6 = 3.5		
7.0 – 3.5 = 3.5	Students will likely be able to solve this problem without use of a number line or give-and-take strategy, either because of the constant difference relationships determined in the problems above or because it can be solved by halving.	Ask students to discuss which of the first six problems they would prefer to solve and why. Discuss the idea that shifting the position of both numbers by the same amount can be used to make an easier subtraction problem.
		This problem string progression continues after the sample dialog below.

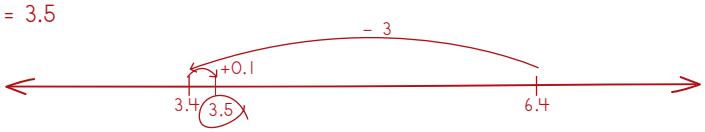
Notes About This Activity

This string is designed to introduce constant differences, and draws heavily on the differencing strategy from String 2. In presenting and discussing the first problem, call on students who have used either removal or differencing. For the problems that follow, call on students who have used a differencing strategy. Model students' thinking on open number lines throughout the string.

Sample Dialog

Craig *I did takeaway on that one, but I found something really cool. 2.9 is almost like 3, so I took 3 away from 6.4, which was 3.4, and then added back in a tenth, because we're really only supposed to subtract 2.9. I got 3.5 for my answer.*

6.4 – 2.9 = 3.5



Natalie *I got the same answer as Craig, but I found the difference between the two numbers. I started at 2.9 and added a tenth to get to 3. It was easy to get from 3 to 6.4 by adding 3.4. So the total difference was one-tenth plus 3.4, which is 3.5.*

6.4 – 2.9 = 3.5

Lin *I started at 3.7 and jumped up 3 tenths to get to 4. Then I knew I had to get up to 7.2, so I added 3 more to get to 7 and 2 tenths after that to get to 7.2. Then I added up the hops, so the distance between 3.7 and 7.2 is 3.5.*

7.2 – 3.7 = 3.5

Diego *I got 3.5 for this one. I started at 2.6, jumped up 4 tenths to get to 3, and then I added another 3.1 to get up to 6.1.*

6.1 – 2.6 = 3.5

Connie *This one is also 3.5. I added onto 2.8 to see how far to get up to 6.3. First I added 2 tenths to get up to 3. Then I added 3, which got me up to 6, and then I added 3 more tenths. If you put that together, it's 3.5.*

6.3 – 2.8 = 3.5

Shane *OK, this one is 3.5 also, and I think I know why. It's just like the one before last with 6.1 subtract 2.6, just each of the numbers is 1 bigger. So the difference is still 3.5, see?*

7.1 – 3.6 = 3.5

Teacher *I'm thinking about what you said about the distance between the numbers being 3.5 for each of the problems we have done so far. You are correct. It's almost like there is a bubble, or distance, of 3.5 that is sliding up and down the number line. The size of the distance stays the same, 3.5, but it's moving up and down the number line so it has a different starting and ending point. Isn't that strange? I wonder why or when that might ever be useful? What can you say about all of these problems? Are they equivalent?*

Students *Yes, because the answers are the same.*
Yes, they are all the same distance apart.

Teacher *So, the problems are equivalent. I will write that:*
6.4 – 2.9 = 7.2 – 3.7 = 6.1 – 2.6 = 6.3 – 2.8 = 7.1 – 3.6.
Let me give you another problem: 7.0 – 3.5.

Darryl *It has to be 3.5. I don't even have to work it out, because it's the same as the one we just did. You just took a tenth off both numbers.*

Jade *Also, I already know that 3.5 + 3.5 is 7, so it's easy to do the subtraction.*

Teacher *Wow, the same distance as the other problems we have solved. I'm looking back at the problems and wondering if there is one that you would prefer to solve over the others. David?*

David *The one we just did, for sure.*

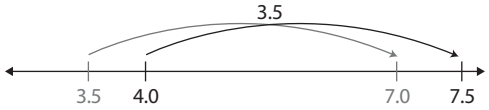
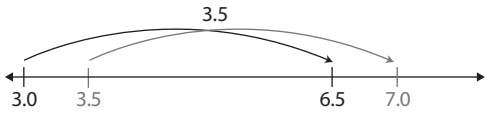
Teacher *You like 7.0 – 3.5 the most? Why is that?*

David *Because I know that 3.5 and 3.5 equals 7.*

Teacher *So if I had given you the first problem and this problem at the same time, you think you would have liked to solve this last one first? Could we use this idea to help us solve the less friendly problems?*

Pose the next two problems, 7.5 – 4 and 6.5 – 3. Ask students to compare these two problems to the last one in the set above (7.0 – 3.5) and share which they think are easier to solve. Ask students to make a generalization about whether it's easier to make the first or the second number in the combination "nice" (i.e., a whole number).

Problem String 3, Part 2

Problems	Sample Strategies & Recording	Connections
7.5 – 4 = 3.5		**Big Idea** A combination may be easiest to solve mentally if you can shift the numbers so that the subtrahend is a whole number.
6.5 – 3 = 3.5		

Ambrose *That one was really easy. 7 – 4 is 3, and then you can add back in the 5 tenths, so the answer is 3.5.*

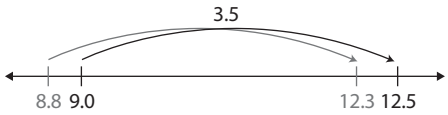
Maria Jose *It's 3.5, just like all the others.*

Teacher *Which of the last 3 problems was easiest to solve?*

DJ *We all agreed that 7.0 – 3.5 was easier than the ones you had already put up. But then you put those last two up and we all knew the answer right away. I think some people still had to think about the problem when the first number was nice, like 7.0. But when the second number was nice, we all got it immediately.*

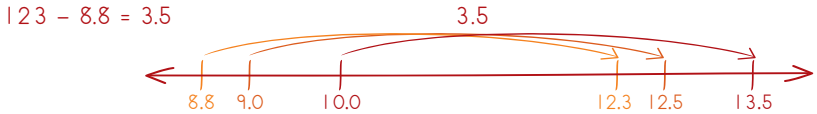
Finish the string work by asking students to think about the problem 12.3 – 8.8 and the distance between the two numbers. Challenge the group to figure out how to shift the position of both numbers by the same amount to create an easier combination.

Problem String 3, Part 3

Problems	Sample Strategies & Recording	Connections
12.3 – 8.8 = 3.5		**Big Idea** A combination may be easiest to solve mentally if you can shift the numbers so that the subtrahend is a whole number.

Carlos *If you add 2 tenths to 8.8, it'll be 9, but you have to add 2 tenths to the other number too. So then it would be 12.5 – 9, which is just 3.5.*

Kendra *You could even make the number you're taking away be 10 if you add 1.2. That would be 13.5 – 10, and that's 3.5.*



September Assessment

Baseline Assessment

Overview

During the second week of school, the teacher administers a written assessment to the entire class, half in place of Number Corner workouts one day, and the other half in place of workouts the following day. This Baseline Assessment is designed to help teachers ascertain students' current skills in areas that were targeted for mastery by the end of fourth grade.

Skills & Concepts

- Recall from memory all products of two 1-digit numbers (3.OA.7)
- Fluently divide with dividends to 100 using strategies (3.OA.7)
- Solve story problems involving a multiplicative comparison using multiplication or division (4.OA.2)
- Solve multi-step story problems involving only whole numbers, using addition, subtraction, multiplication, and division with remainders; represent problems with equations; assess reasonableness of answers (4.OA.3)
- Demonstrate understanding of factors, multiples, prime, and composite numbers (4.OA.4)
- Use the standard algorithms with fluency to add and subtract multi-digit whole numbers (4.NBT.4)
- Multiply a 3-digit whole number by a 1-digit whole number, and two 2-digit numbers using strategies based on place value and the properties of operations (4.NBT.5)
- Divide 2- and 3-digit numbers by 1-digit numbers, using strategies based on place value, the properties of operations, or the relationship between multiplication and division (4.NBT.6)
- Use a visual model to explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ (4.NF.1)
- Compare two fractions with different numerators and different denominators; use the symbols >, =, and < to record comparisons, and explain why one fraction must be greater than another (4.NF.2)
- Add and subtract mixed numbers with like denominators (4.NF.3c)
- Solve story problems involving subtraction of fractions referring to the same whole and with like denominators (4.NF.3d)
- Multiply a fraction by a whole number (4.NF.4b)
- Solve story problems that involve multiplying a fraction by a whole number (4.NF.4c)
- Express a fraction with denominator 10 as an equivalent fraction with denominator 100 (4.NF.5)
- Add a fraction with denominator 10 to a fraction with denominator 100 by rewriting the first fraction as an equivalent fraction with denominator 100 (4.NF.5)
- Write fractions with denominator 10 or 100 in decimal notation (4.NF.6)
- Compare two decimal numbers with digits to the hundredths place; use the symbols >, =, and < to record comparisons (4.NF.7)
- Solve story problems involving liquid volume, distance, and length using addition, subtraction, and multiplication of whole numbers and fractions (4.MD.2)

Materials

Assessments	Day	Copies	Classroom Materials
Baseline Assessment, Part 1 Completing Pages 1–3	7	**TM T3–T5** Baseline Assessment, Pages 1–3	• scratch paper
Baseline Assessment, Part 2 Completing Pages 4–6	8	**TM T6–T8** Baseline Assessment, Pages 4–6	• scratch paper

TM – Teacher Master, **NCSB** – Number Corner Student Book
Copy instructions are located at the top of each teacher master.

Vocabulary

An asterisk [*] identifies those terms for which Word Resource Cards are available.

addition
composite number*
divide*
division
equation*
equivalent fractions*
estimate*
factor*
milliliter (ml)*
multiple*
multiplication
multiply*
prime number*
remainder*
standard algorithm
subtraction
times as many

Mathematical Background

The Baseline Assessment gives you an opportunity to gauge incoming students' proficiency with essential numeracy and computation skills that were targeted for mastery in fourth grade. The Baseline Assessment is intended to guide your instruction by providing information about which students can (and cannot) demonstrate fluency with basic multiplication and division facts, solve story problems involving all four operations, compare and order fractions, add and subtract fractions and mixed numbers with like denominators, multiply fractions by whole numbers, and write and compare decimals to hundredths. It is 6 pages long, designed to be administered in two 20-minute periods.

After conducting this assessment, you will be in a better position to plan daily instruction and make the minute-to-minute instructional decisions so crucial to good teaching. On the basis of students' strengths and weaknesses, you might decide to emphasize certain aspects of Number Corner instruction while minimizing others, and you will have at least some of the information needed to pitch questions and prompts at levels appropriate to different students. The Baseline Assessment may also be considered an early warning system. While it is risky to make hard-and-fast judgments about incoming fifth-graders, you will want to keep a close eye on students who are unable to perform more than a few of the assessment tasks, as some of these children may emerge as candidates for special services this year if they haven't been identified previously.

Baseline Assessment, Part 1

Completing Pages 1–3 — Day 7

1 Open the session by explaining what a baseline assessment is and describing how you'd like students to work on the baseline assessment they will start today during Number Corner and complete tomorrow.

Explain that a baseline assessment is a way of finding out where everyone is in math at the very start of the school year. The problems on this assessment involve skills the children studied last year in fourth grade. The assessment will help you and the students see what they still remember and what they may need to review or study again this year. It will help you do a better job of teaching because you'll see more clearly what each student already knows, and what they still need to work on.

Explain that you would like students to do the following things as they work on the baseline assessment:

- Listen carefully to the instructions for the checkup.
- Work independently.
- Raise your hand if you have a question.
- Try to answer all of the problems, even those you don't fully understand.
- Explain how you solved a problem when the directions ask you to. You can use pictures, numbers, and words in your explanations.

2 Display your copy of the first page of the Baseline Assessment, and give each student a copy of the first three pages of the assessment.

- Give students a few moments to quietly examine the three pages as you place a small stack of scratch paper at each table or near each cluster of desks for their use.
- Using your copy of the first page, show students where to write their name and date at the top of the page.

3 Introduce and have students complete the first item on page 1.

Assessment Guide

See the Grade 5 Assessment Guide for scoring and intervention suggestions.

- Have them look at the top portion of the first page, and ask them not to begin working until you ask them to.
- Explain that they will have one minute to solve these 20 multiplication problems.
- Tell students not to worry if they cannot solve them all in one minute. Reassure them that this is just one way for them to demonstrate what they know about multiplication.
- Encourage them to solve the problems out of order if they wish; they can skip around and find the combinations they know first.
- When everyone is ready, have them begin. Give them one minute to complete the 20 multiplication facts in item 1.

4 After one minute has passed, have students put down their pencils and relax for a moment.

5 Then, explain that they will have the rest of the Number Corner time to work on the rest of page 1, as well as pages 2 and 3.

- Display pages 2 and 3, one at a time.
- Give students time to read the items.
- Clarify the instructions and items as needed.
- Tell students not to worry if they don't finish all three pages today; they will have more time on another day.

6 Ask students if they have any questions about the assessment. Then, remind them to work quietly and independently and have them get started.

- While students work independently, circulate to observe how they work, and answer questions that may arise.
- Ask students who finish before their classmates to double-check their work, and then read or draw quietly until everyone else is done.

ELL If necessary, read questions aloud for ELL students and help them interpret what the questions are asking.

SUPPORT Apart from the first item, this assessment does not need to be timed. If you have students who are unable to complete the first three pages of the assessment in the given amount of time, you can give them additional time to complete it later, perhaps during choice time or a seatwork period.

Baseline Assessment, Part 2

Completing Pages 4–6 — Day 8

1 Let students know that they are going to do the second half of the Baseline Assessment today.

- Have them get out their pencils in preparation for taking this part of the assessment
- Ask helpers to give each student a sheet of scratch paper.

2 Display your copy of the Baseline Assessment, Pages 4–6, and give each student a copy.

- Give students a few moments to examine the Pages quietly.
- Provide clarification as needed, and answer students' questions.

3 When students understand what to do, give them time to complete the second half of the assessment.

- While students work independently, circulate to observe how they work and answer questions as needed.
- Ask students who finish before their classmates to double-check their work, and then read or draw quietly until everyone else is done.

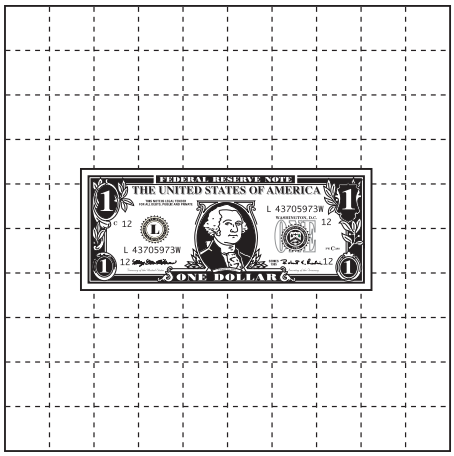
SUPPORT None of the items on this part of the assessment need to be timed. If there are students who are unable to complete the second part of the assessment in the given amount of time, have them do so later, perhaps during math stations or a seatwork period.

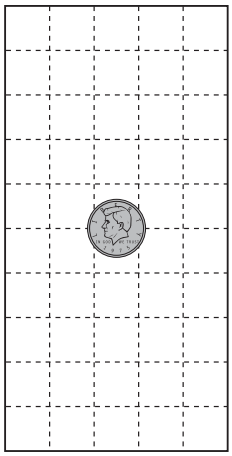
Teacher Masters

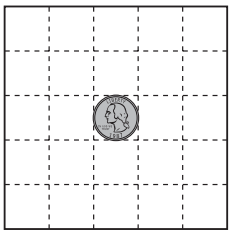
GRADE 5 – SEPTEMBER

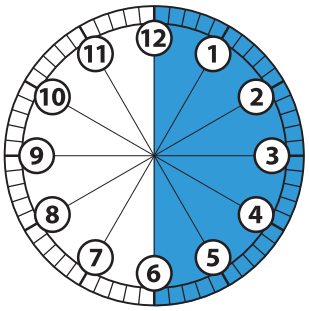
NUMBER® CORNER

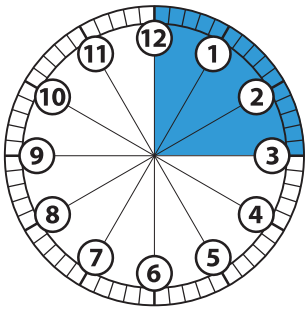
Money & Clock Models

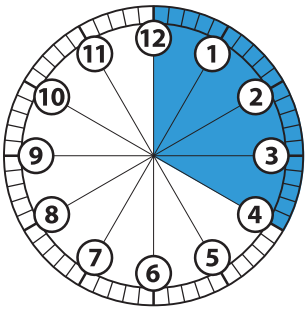


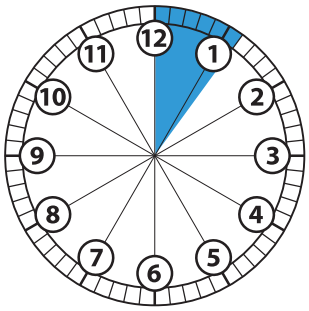












Problem String Work Space

NAME | **DATE**

Baseline Assessment page 1 of 6

1 Solve as many of these multiplication problems as you can in one minute.

8 × 6	7 × 9	4 × 6	6 × 6	9 × 5	4 × 7	8 × 3
10 × 6	9 × 6	9 × 9	6 × 7	7 × 7	8 × 9	5 × 8
9 × 4	9 × 3	5 × 7	6 × 5	8 × 4	7 × 8	

2 Divide.

45 ÷ 5 = ___	28 ÷ 7 = ___	27 ÷ 3 = ___	36 ÷ 6 = ___
28 ÷ 4 = ___	48 ÷ 6 = ___	49 ÷ 7 = ___	32 ÷ 8 = ___
30 ÷ 6 = ___	36 ÷ 9 = ___	56 ÷ 8 = ___	63 ÷ 7 = ___

3 True or False?

a 6 and 9 are both factors of 36 and 54 ____

b 42 is a multiple of 4 ____

c 15 is a prime number ____

d 21 is a composite number ____

e 1, 2, 3, 4, 6, 8, 12, and 24 are all of the factors of 24 ____

(continued on next page)

NAME | **DATE**

Baseline Assessment page 2 of 6

4 Use the standard addition and subtraction algorithms to solve these problems. Show your work.

1,379	46,736	514	406,582
+ 2,945	+ 57,409	– 368	– 84,966

5 Use an efficient strategy to solve each multiplication problem. Show your work.

458	34
× 9	× 28

6 Use an efficient strategy to solve this division problem. Show your work.

232 ÷ 8

(continued on next page)

NAME | **DATE**

Baseline Assessment page 3 of 6

7 Cody has 126 milliliters of red paint. Brandon has 3 times as many milliliters of blue paint.

a If Cody and Brandon pour their paint into the same container, how many milliliters will they have in all? Show your work. Label your answer with the correct units.

b Write an equation, including the answer, to represent this problem.

8 Some of the parents are taking the kids in the After-School Club to the art museum tomorrow.

a If each car can hold 4 people (not including the driver), how many cars do they need to take all 27 kids in the After-School Club to the museum? Show your work using numbers, sketches, or words.

b If there was a remainder, how did you handle it? Why?

9 Jensen says that $\frac{2}{3}$ and $\frac{4}{6}$ are equivalent fractions. Do you agree with Jensen? Why or why not? Include a labeled sketch in your explanation.

(continued on next page)

NAME | **DATE**

Baseline Assessment page 4 of 6

10 Use the symbols <, >, and = to compare each pair of fractions.

$\frac{3}{4}$ ____ $\frac{7}{8}$ $\frac{4}{8}$ ____ $\frac{5}{10}$ $\frac{2}{4}$ ____ $\frac{5}{12}$ $\frac{9}{6}$ ____ $1\frac{2}{6}$

11 Franklin says that $\frac{5}{8}$ is less than $\frac{6}{12}$ because both of the numbers in $\frac{5}{8}$ are less than the numbers in $\frac{6}{12}$. Do you agree with Franklin? Why or why not? Include a labeled sketch in your explanation.

12 Solve the following problems.

a $5\frac{3}{5} + \frac{2}{5} =$ _____

b $6 \times \frac{5}{8} =$ _____

c $5\frac{2}{8} - 2\frac{6}{8} =$ _____

d $3\frac{5}{6} + 5\frac{4}{6} =$ _____

13 True or false?

$\frac{7}{3} = 7 \times \frac{1}{3}$ ____ $6 \times \frac{2}{3} = 18 \times \frac{1}{2}$ ____ $\frac{4}{5} \times 3 = \frac{1}{5} \times 12$ ____

14 What is $\frac{3}{10} + \frac{28}{100}$? Show your work.

(continued on next page)

NAME | **DATE**

Baseline Assessment page 5 of 6

15 Heather is 45 $\frac{1}{3}$ inches tall. Neena is 42 $\frac{2}{3}$ inches tall. How much taller is Heather than Neena? Show your work. Label your answer with the correct units.

16 Julia bought 6 pieces of ribbon for an art project. Each piece of ribbon is $\frac{3}{4}$ of a yard.

a How many yards of ribbon did Julia buy in all? Show your work. Label your answer with the correct units.

b Write a multiplication equation, including the answer, to represent this problem.

17 Fill in the blanks.

$\frac{6}{10} = \frac{\square}{100}$ $\frac{70}{100} = \frac{\square}{10}$ $\frac{3}{10} = 0.\_\_\_$ $\frac{35}{100} = 0.\_\_\_$

18 Use the symbols <, >, and = to compare each pair of decimals.

0.6 ___ 0.78 0.40 ____ 0.04 2.15 ____ 1.99 0.5 ____ 0.50

(continued on next page)

NAME | **DATE**

Baseline Assessment page 6 of 6

19 Dan is training for a bike race. He wants to bike 300 miles over the next couple of weeks. Dan biked 29 miles a day for 3 days. Then, he biked 32 miles a day for 4 days. How many more miles does he still have to bike to get to his goal of 300 miles?

a Sara estimated that Dan would need to bike about 80 more miles to get to his goal of 300 miles. Is this a reasonable estimate? Why or why not?

b Write an equation for this problem with a letter standing for the unknown quantity.

c Solve the problem. Show your work.

Student Book

GRADE 5 – SEPTEMBER

NUMBER® CORNER

NAME | DATE

What's Missing?

1 Raj's class is collecting information about prisms they are building, but he is missing some of the data. Fill in the missing information in the table below.

Building Prisms				
Dimensions of the Base	**Area of the Base**	**Number of Layers**	**Volume**	**Dimensions of the Prism**
4×2		2		
		2		$(3 \times 3) \times 2$
2×10			40 cubic units	
$4 \times$ ___	36 sq. units	3		
5×2			100 cubic units	
	35 sq. units			$(5 \times 7) \times 4$
6×5		8		
___ $\times 5$	25 sq. units	4		
		3		$(2 \times 9) \times 3$
				$(2 \times 7) \times 4$

2 Help Raj and his partner fill in the blanks in the equations below.

a $(4 \times 3) \times 6 = (2 \times 3) \times$ _____

b $9 \times (6 \times 6) = 9 \times ($ _____ $\times 3)$

c $(3 \times 4) \times 6 = 24 \times$ _____

3 Raj and his partner disagree about the following equations. Tell whether each is true or false.

a $(5 \times 2) \times 4 = 2 \times (4 \times 5)$ _____

b $8 \times (10 \times 2) = 16 \times (5 \times 1)$ _____

c $2 \times (9 \times 3) = 3 \times 18$ _____

NAME | DATE

Multiple Game Board

Player 1 _____ Player 2 _____

2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31	32	33	34	35	36
37	38	39	40	41	42	43
44	45	46	47	48	49	50

Player 1 Total _____ Player 2 Total _____

NAME | DATE

Maddy's Multiples

1 Maddy is playing the Multiple Game. She goes first. She chooses the number 36.

a How many points does Maddy get? _____

b How many points does Maddy's partner get? _____

c Was 36 a good choice for Maddy's first turn? Explain.

Multiple Game Board						
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31	32	33	34	35	36
37	38	39	40	41	42	43
44	45	46	47	48	49	50

2 List the factors for each number below. Write P next to numbers that are prime and C next to numbers that are composite.

	29	
	24	

	25	
	23	

3 Which of the four numbers in problem 2 would you choose if you were going first in the Multiple Game? Why?

4 List the factors each pair of numbers below has in common.

ex What factors do 24 and 36 have in common? 2, 4, 6, 12

a What factors do 20 and 28 have in common? _____

b What factors do 18 and 32 have in common? _____

5 List two multiples that each pair of numbers below has in common.

ex What are 2 multiples that 6 and 12 have in common? 12, 24

a What are 2 multiples that 3 and 5 have in common? _____

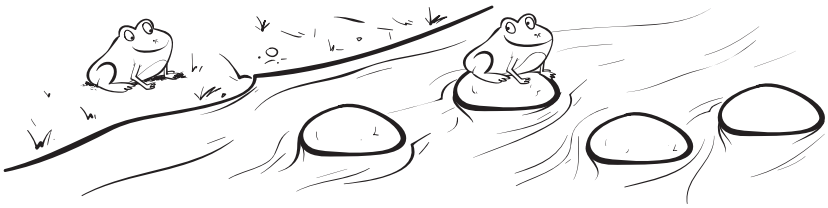
b What are 2 multiples that 4 and 7 have in common? _____

NAME | DATE

Rock Hopping page 1 of 2

Use the blank space to solve each problem. Show all your work including numbers, words, or labeled sketches. Write a complete sentence below your work to show the answer.

1 Two frogs, DJ and Freddy, were hopping from rock to rock in their favorite stream. In all, there were 36 rocks in the stream. DJ landed on every second rock and Freddy landed on every third rock. Which rocks did they *both* land on?



2 A new friend, Sue, joined in and landed on every 6th rock. Which rocks did all three frogs land on?

NAME | DATE

Rock Hopping page 2 of 2

3 If the three friends keep jumping:

a Will DJ land on the 51st rock? How do you know?

b Will Freddy land on the 51st rock? How do you know?

c What is the first rock that all three frogs will land on after the 51st rock? How do you know?

NAME | DATE

Field Trip Snacks page 1 of 2

Use the blank space to solve each problem. Show all your work including numbers, words, or labeled sketches. Write a complete sentence below your work to show the answer.

1 A parent donated 36 fruit cups and 24 bananas to the fifth grade. The teacher wanted to make field trip snack bags with the donated food and wondered about the ways that the snacks could be packed. To be fair, the teacher wants to makes sure that all the bags are exactly the same.

a What is the greatest number of snack bags that the teacher can make, if each bag is identical? How do you know?

b What other numbers of snack bags could she make? How do you know?

NAME | DATE

Field Trip Snacks page 2 of 2

2 Another parent also donated 24 bananas, so there are 48 bananas total. Now what is the greatest number of snack bags that can be made?

3 The teacher realized that she miscounted and had only 30 fruit cups. How many snack bags can she make with 48 bananas and 30 fruit cups?

4 What do the different numbers of snack bags that can be made have to do with the number of fruit cups and number of bananas?