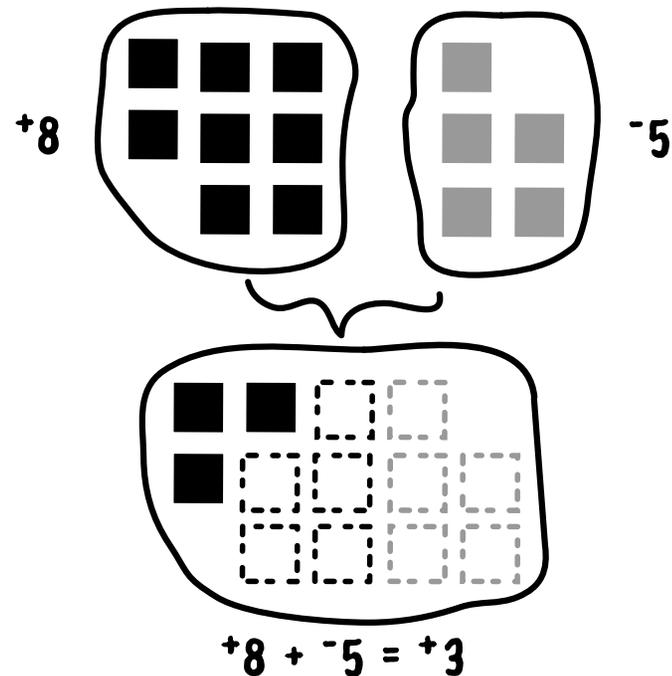
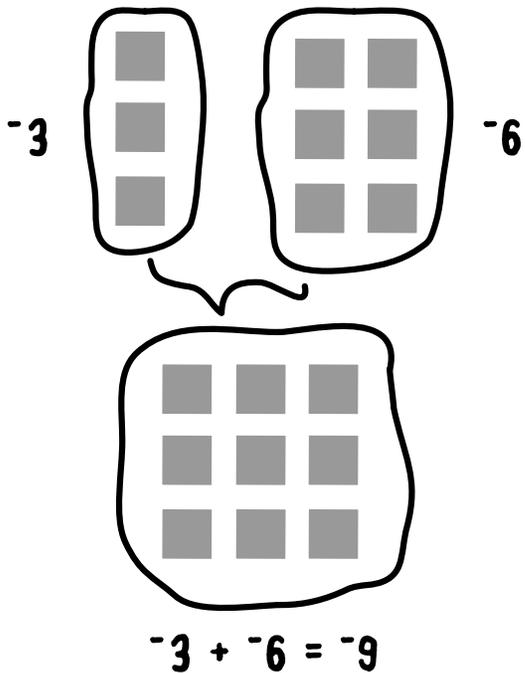
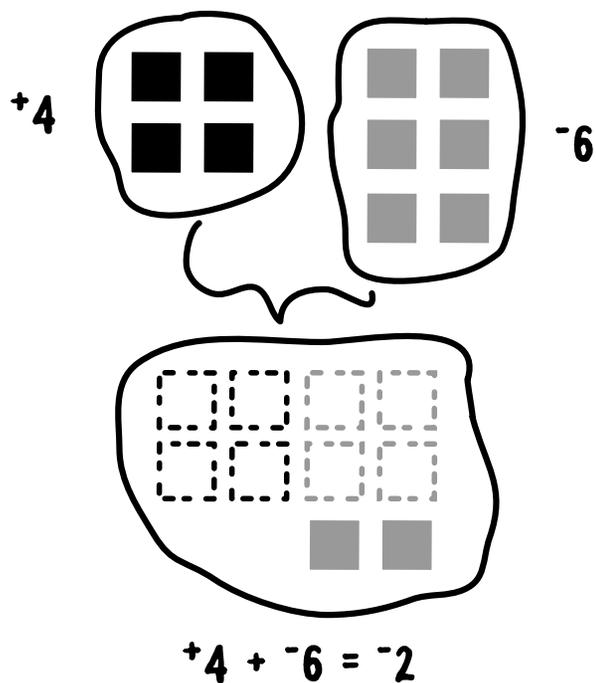


# ADDING & SUBTRACTING INTEGERS

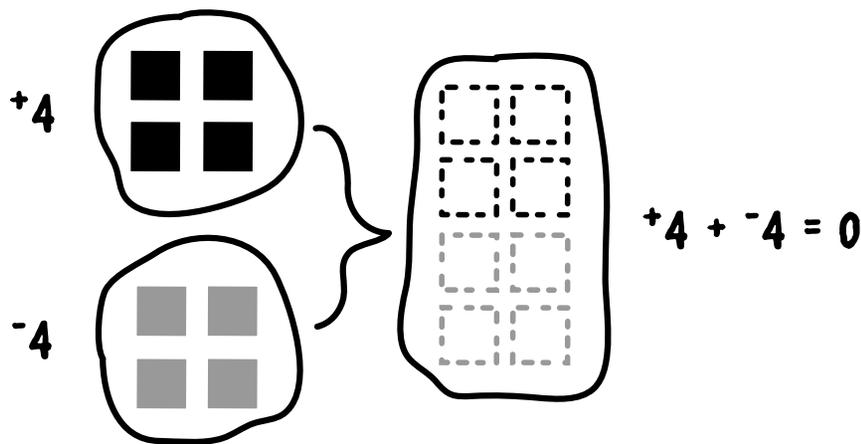
The seventh scene in a series of articles  
on elementary mathematics.

written by Eugene Maier  
designed and illustrated by Tyson Smith

To add two integers, combine collections that have the values of these integers and find the value of the combined collection. Here are some examples:



Note that the sum of an integer and its opposite is 0, e.g.  $+4 + -4 = 0$ .



There are several ways to subtract integers. One way uses the “take-away” method of subtraction. To find  $+4 - +6$  using this method, one takes away a collection whose value is  $+6$  from a collection whose value is  $+4$ . Since the minimal collection for  $+4$  doesn't contain enough black tile to take away 6 of them, one augments the number of black in the collection, without changing its value, by adding an equal number of black and red tile. Adding two of each produces a collection of 6 black and 2 red (Figure 1). Taking 6 black tile from this collection leaves 2 red (Figure 2). Hence,  $+4 - +6 = -2$ .

Figure 1

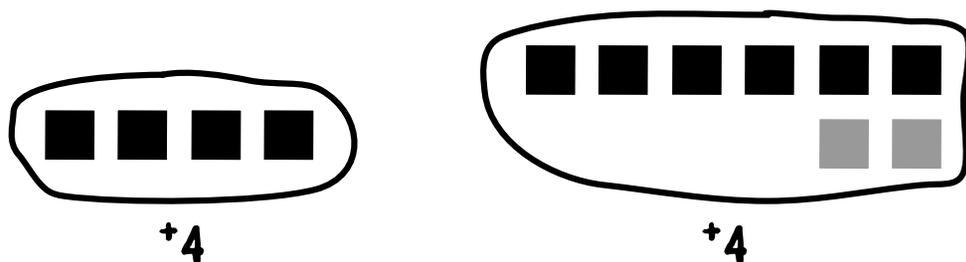
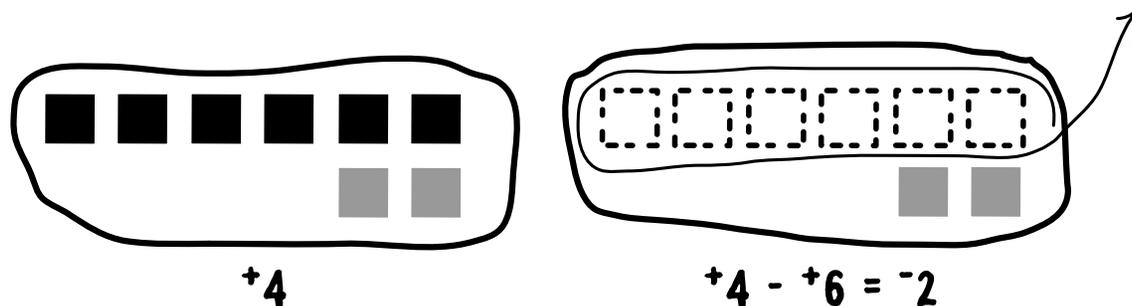
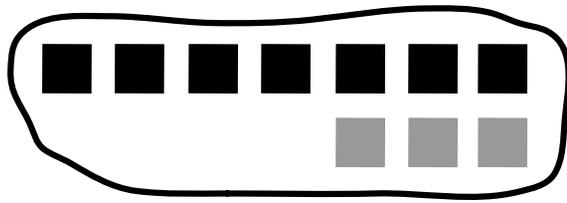


Figure 2

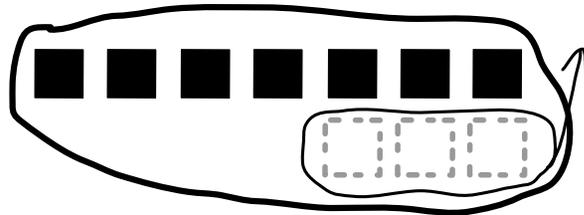


Here are three other examples of "take-away":  $+4 - -3 = +7$  (Figure 3);  
 $-5 - +2 = -7$  (Figure 4);  $-3 - -5 = +2$  (Figure 5).

Figure 3

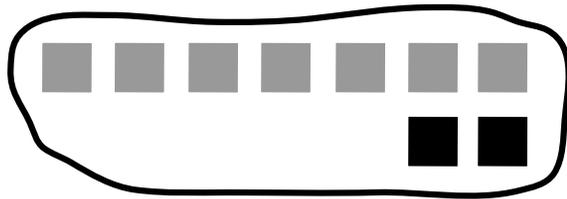


$$+4$$

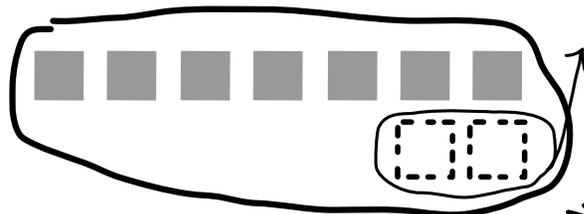


$$+4 - -3 = +7$$

Figure 4

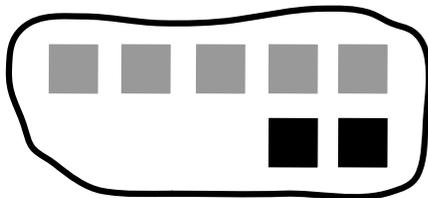


$$-5$$

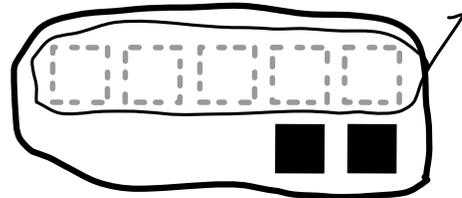


$$-5 - +2 = -7$$

Figure 5



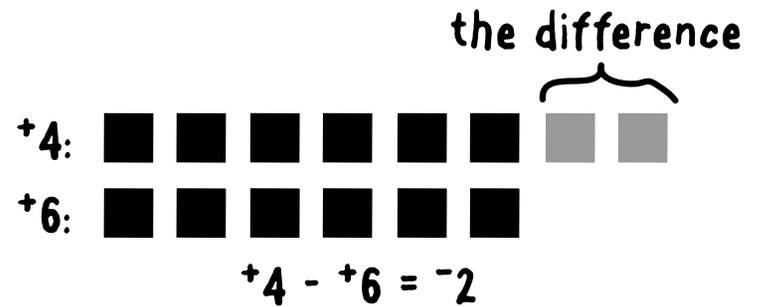
$$-3$$



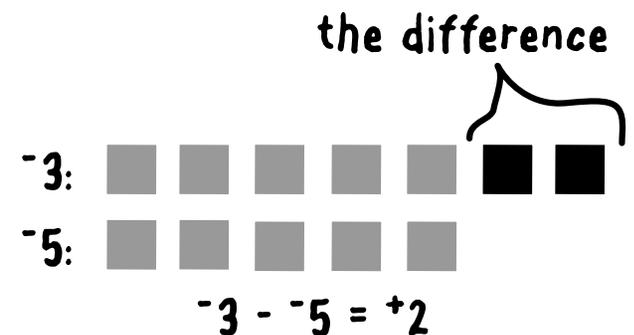
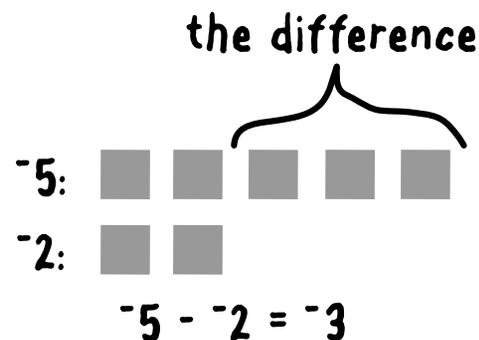
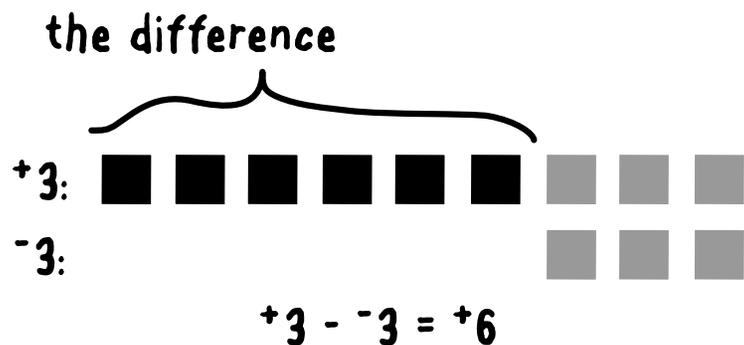
$$-3 - -5 = +2$$



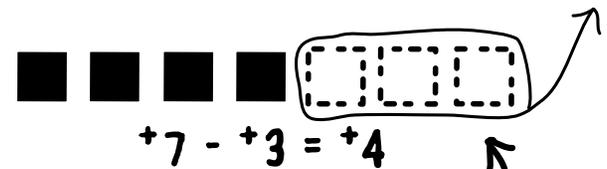
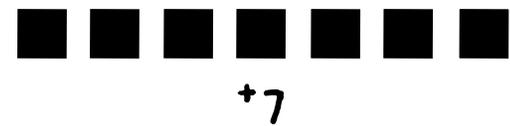
One can also subtract integers by “comparing” or, as it is sometime called, “finding the difference.” To find  $+4 - +6$  by this method, one compares collections whose values are  $+4$  and  $+6$  and determines the difference. Using a collection representing  $+4$  that contains 6 black tile facilitates the comparison.



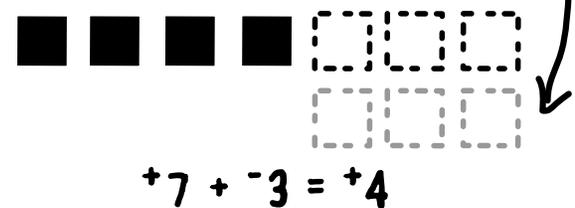
Here are some other subtractions done by the comparison method. Notice that collections other than minimal collections are often used to make the differences more apparent.



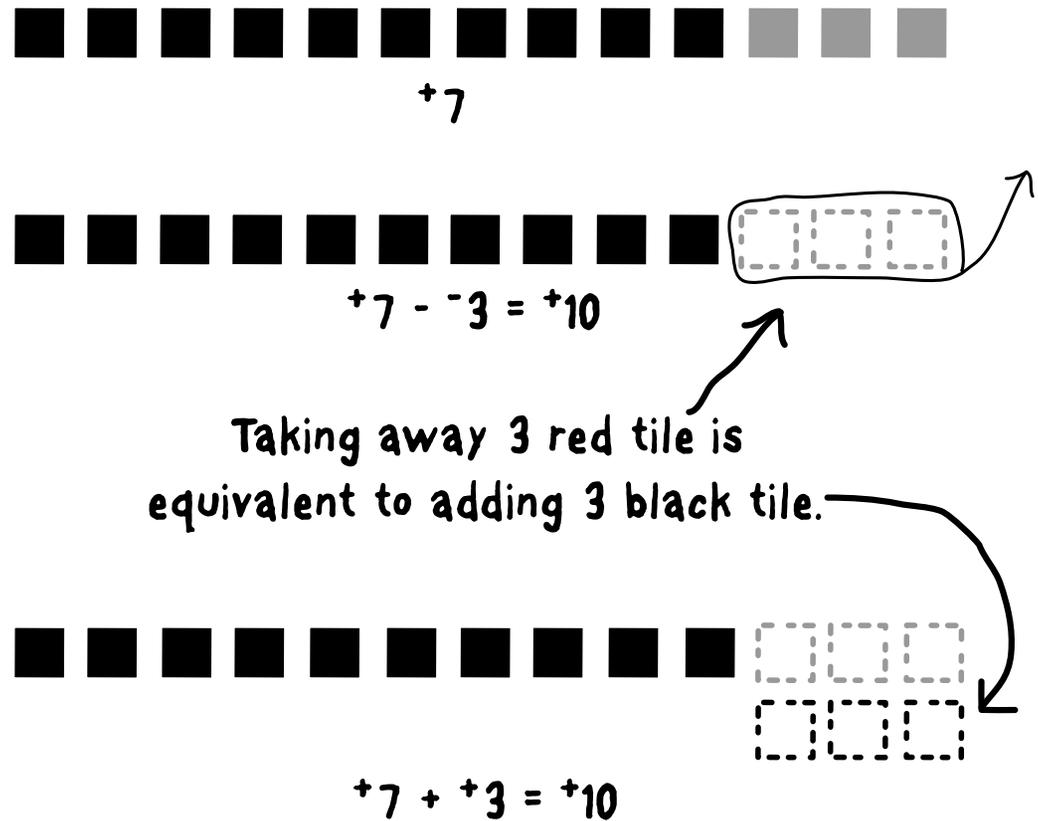
A third way to perform a subtraction is a variation of the take-away method. To carry out the subtraction  $+7 - +3$  by the take-away method, one would start with a collection whose value is  $+7$  and remove from it a collection whose value is  $+3$ , and then determine the value of the remaining collection. However, instead of removing a collection whose value is  $+3$  and thereby eliminating its value, the value of this collection could be eliminated by adding to it a collection whose value is  $-3$ . Thus, instead of removing 3 black tile from a collection of 7 black tile to evaluate  $+7 - +3$  one could add the opposite of 3 black tile, namely, 3 red tile, to the collection of 7 black. Thus,  $+7 - +3 = +7 + -3 = +4$ .



Taking away 3 black tile is equivalent to adding 3 red tile.



Similarly to evaluate  $+7 - -3$ , instead of changing the value of a collection for  $+7$  by removing 3 red tile, one could accomplish the same change in value by adding the opposite of 3 red tile, namely 3 black tile, to the collection. Thus,  $+7 - -3 = +7 + +3 = +10$ .



As the above examples illustrate, instead of performing a subtraction by removing from a collection a second collection of a given value, one can arrive at the same result by adding to the first collection a collection whose value is the opposite of the second collection. Thus, in the integers, every subtraction can be converted to an addition. Some examples:  $-5 - -3 = -5 + +3 = -2$ ;  $-3 - -5 = -3 + +5 = +2$ ;  $+3 - -5 = +3 + +5 = +8$ ;  $+3 - +5 = +3 + -5 = -2$ . In general, if  $m$  and  $n$  are counting numbers  $m - n = m + o(n)$ .



**END of SCENE 7: ADDING & SUBTRACTING INTEGERS**

For comments and questions please email  
Gene Maier at [genem@mathlearningcenter.org](mailto:genem@mathlearningcenter.org)

coming up next...

**SCENE 8: MULTIPLYING & DIVIDING INTEGERS**