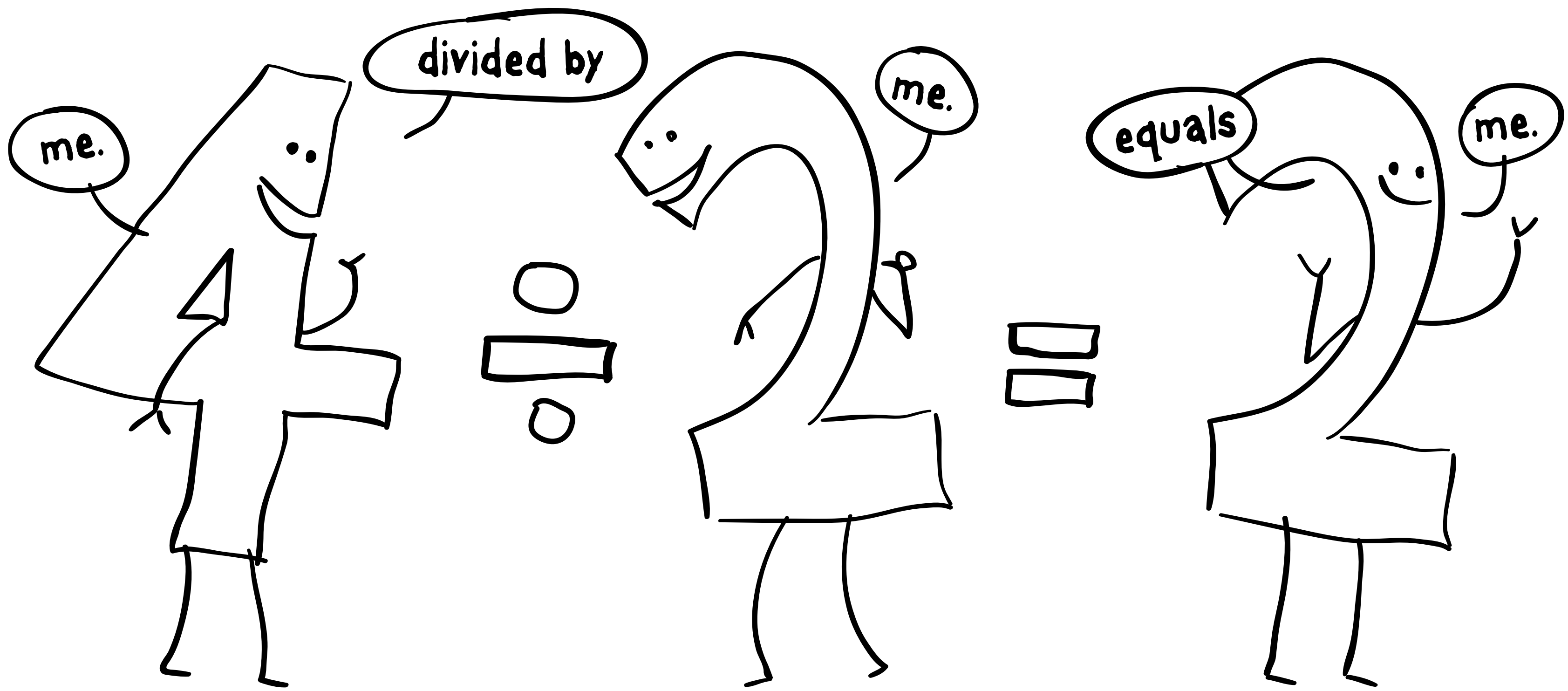


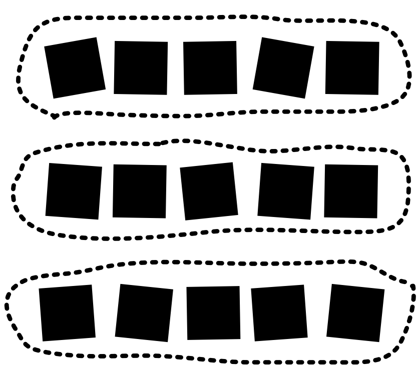
DIVIDING COUNTING NUMBERS

The fifth scene in a series of articles
on elementary mathematics.

written by Eugene Maier
designed and illustrated by Tyson Smith

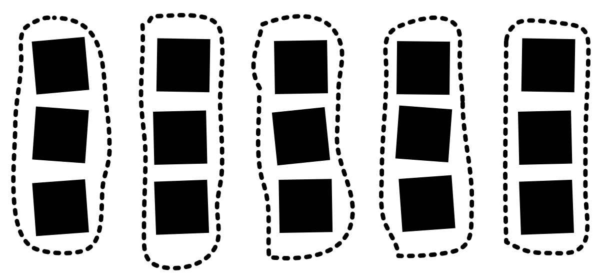
To find 15 ÷ 3, I can split up 15 tile into 3 equal shares and determine how many tile are in each share (the ***sharing*** method), or I can form 15 tile into groups of 3 and determine how many groups are obtained (the ***grouping*** method), or I can arrange 15 tile in an array in which one dimension is 3 and determine the other dimension (the ***array*** or ***area*** method).

15 divided into
3 equal shares



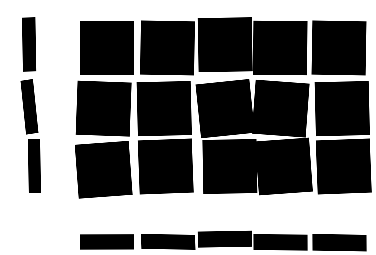
There are 5 in each
share, so 15 ÷ 3 = 5

15 split into
groups of 3



There are 5 groups,
so 15 ÷ 3 = 5

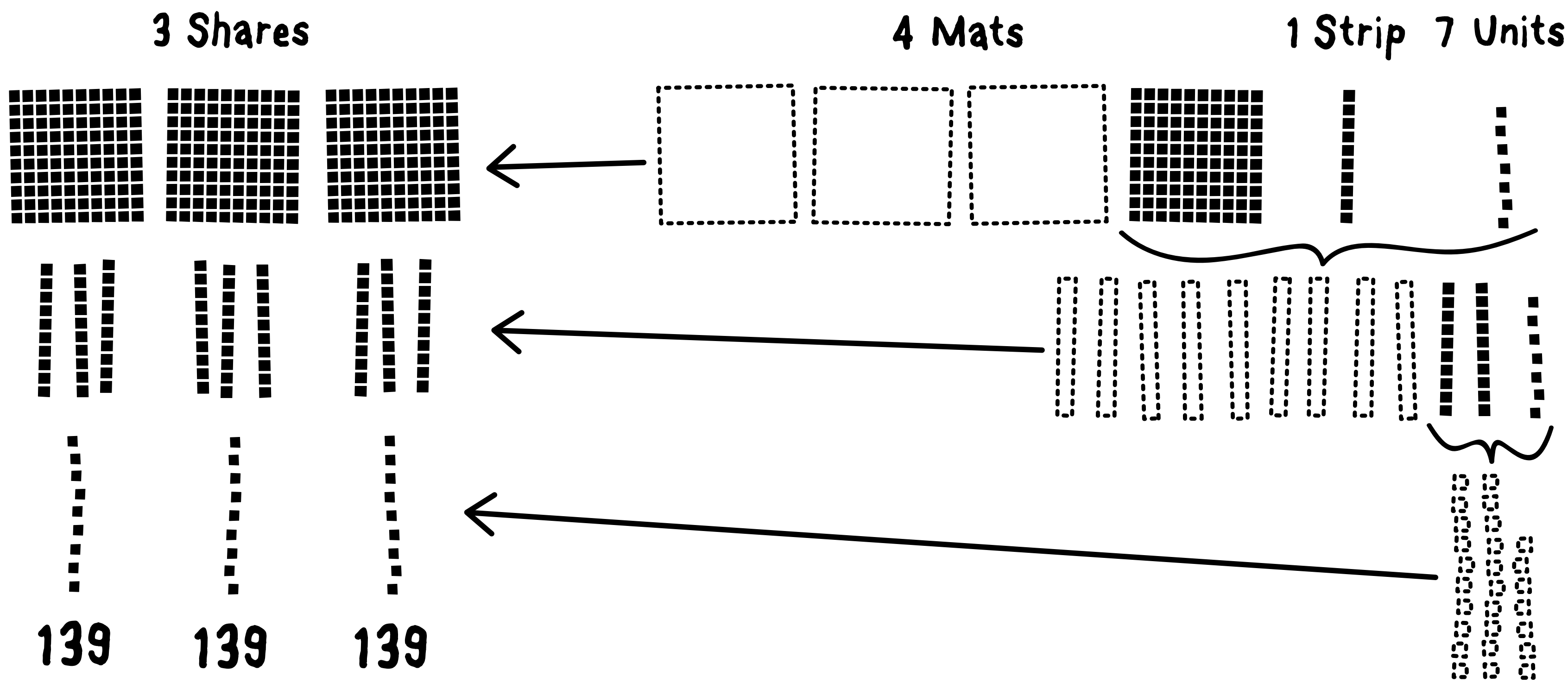
15 arranged in
an array with
one dimension 3



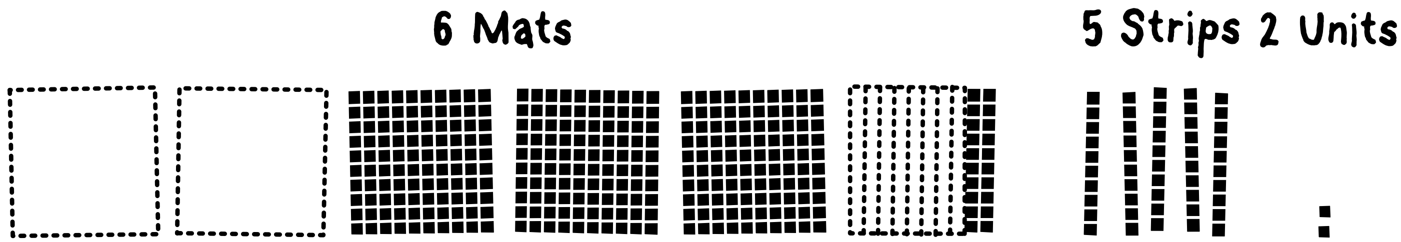
The other dimension
is 5, so 15 ÷ 3 = 5

In a particular instance, one of the methods of dividing may be more efficient than the others. For example, to model 417 ÷ 3 with base 10 pieces, rather than dividing a collection for 417 into groups of 3 or arranging it into an array for which one dimension is 3, it is easier to divide the collection into 3 equal shares, as described here:

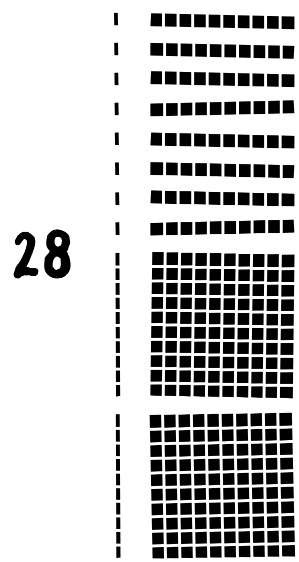
A collection for 417 contains 4 mats, 1 strips and 7 units as shown on the right. We begin by placing 1 mat in each share. Converting the remaining mat into 10 strips gives us a total of 11 strips, We place 3 of these in each share and convert the two remaining strips into units, giving us a total of 27 units. These 27 units are then distributed, 9 to each share. Thus each share contains 1 mat, 3 strips and 9 units. Hence 417 ÷ 3 = 139.



In other instances, forming an array is helpful, as in the following example where 652 ÷ 28 is computed by arranging the equivalent of 6 mats, 5 strips and 2 units into a rectangular array in which one dimension is 28. We proceed as follows:



We first lay out an edge whose value is 28. We then place two mats along side the edge and, to fill in the remaining 8 rows above the mats, we trade a mat for 10 strips and place 8 of them above the mats. We are left with 3 mats, 7 strips and 2 units.



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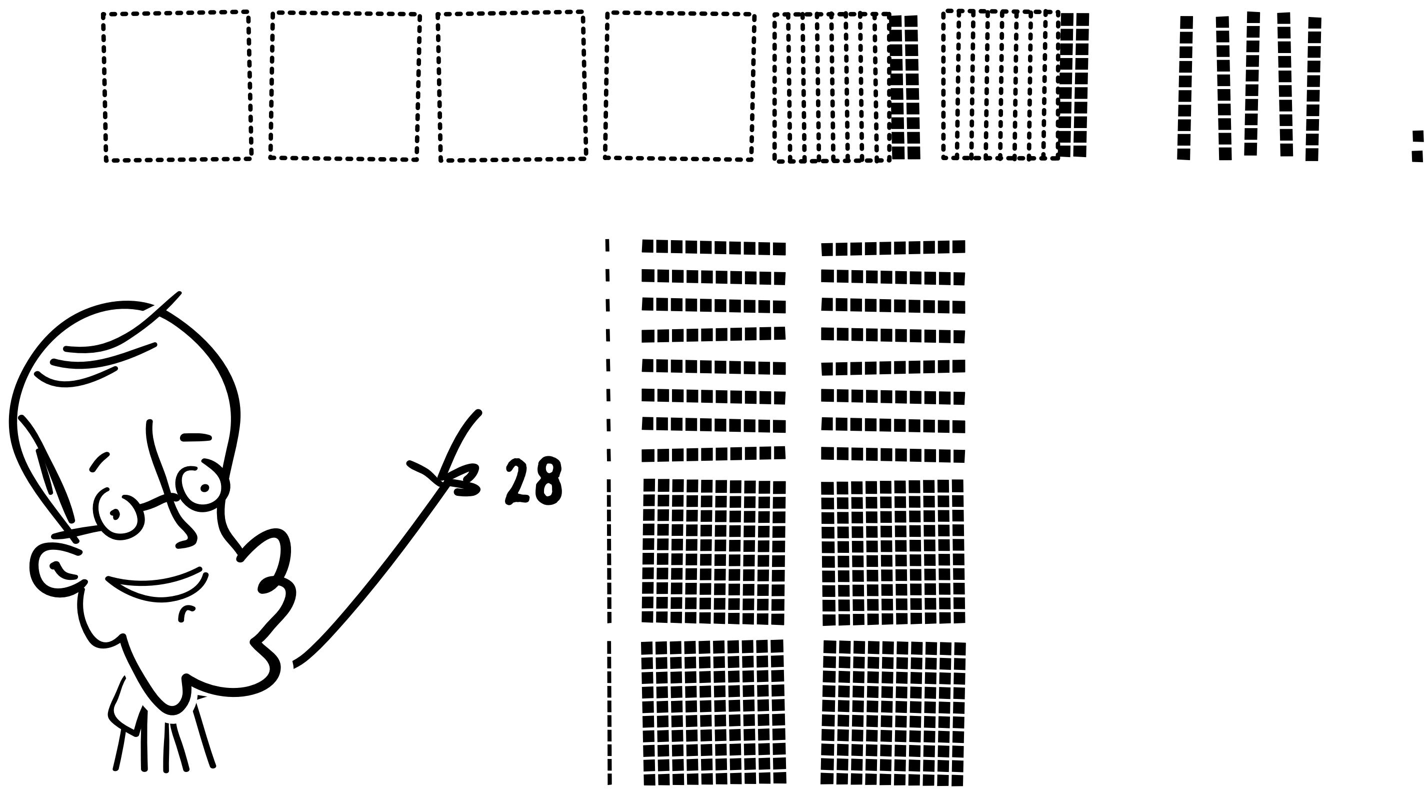
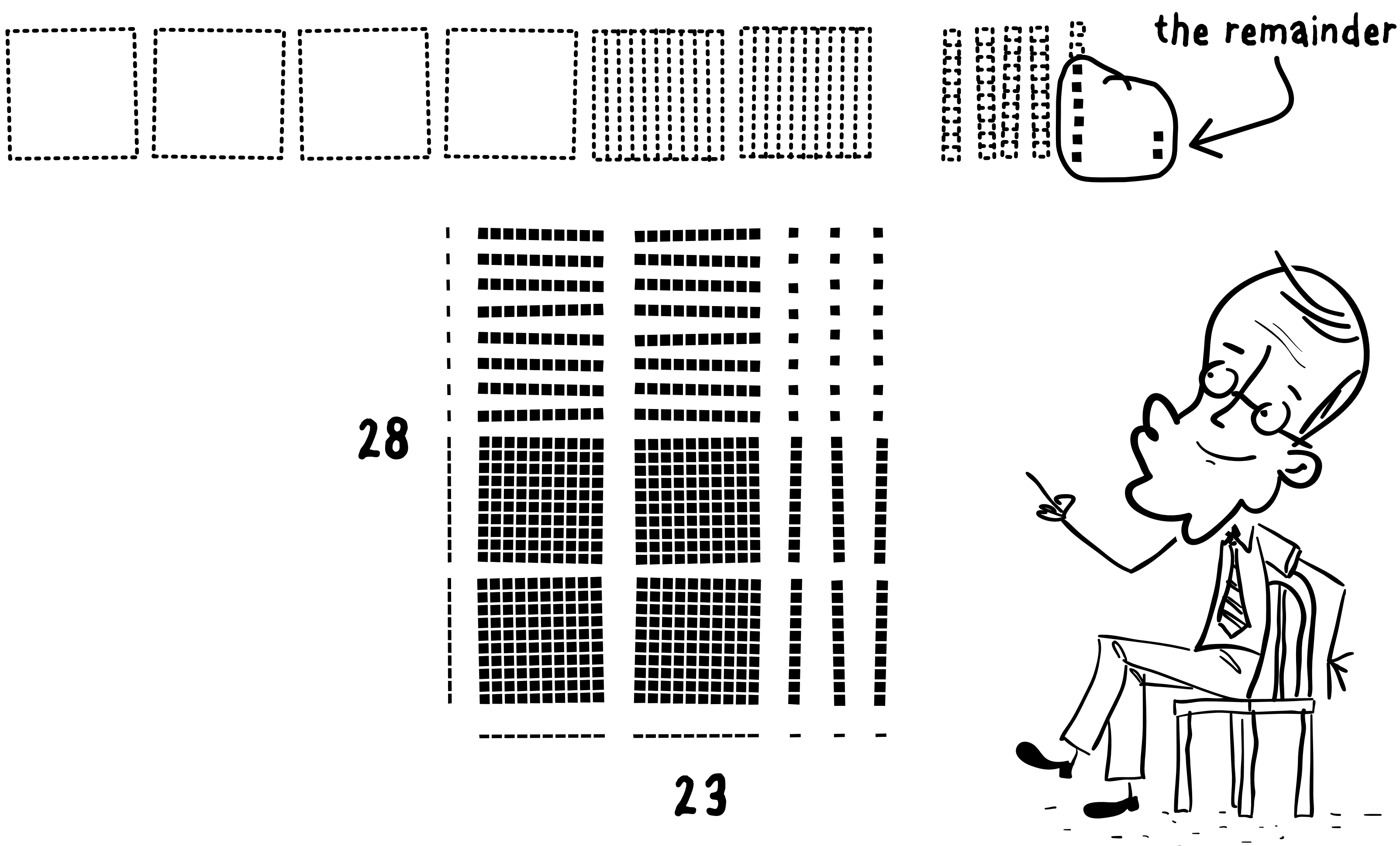


We next place 2 of the remaining mats alongside those already in the array and trade the remaining mat for 10 strips, 8 of which we place above the mats. We now have filled in 20 columns of the array and are left with 9 strips and 2 units.

(continued)

Finally, we arrange the remaining pieces into as many additional columns as possible, trading strips for units as necessary. After 3 columns are formed, only 8 units remain and no further columns are possible. Our array has 23 columns with 8 units left over. Thus, 652 ÷ 28 = 23 with a remainder of 8.

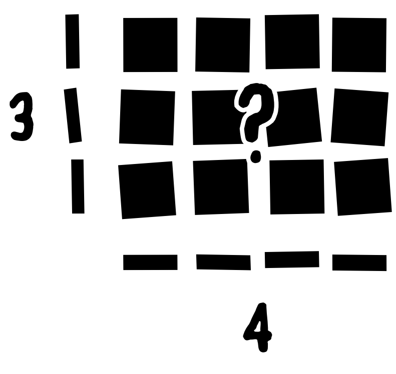


Note that we began constructing the array ten columns at a time. Since each group of 10 columns will contain 280 tile, we can determine how many 10-wide columns we can put in our array by determining how many 280's we can get out of 652 tile. Since 2 × 280 is 560, we can form two of these 10-wide columns and have 92 tile yet to place. Since a single column is composed of 28 tile and 3 × 28 is 84, with these 92 tile we can form 3 single columns and have 8 tile remaining. Recording this as follows leads to one of the standard paper-and-pencil algorithms for division:

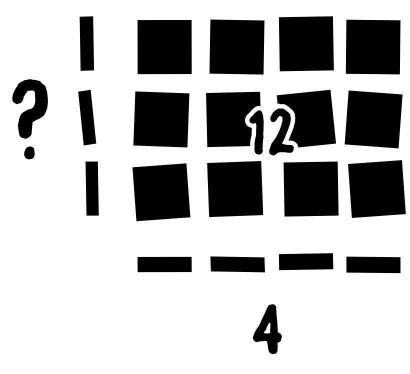
	the number of 10-wide columns
23	← the number of single columns
28) 652	
560	← number of tile in 10-wide columns
92	← number of tile remaining
84	← number of tile in single columns
	number of tile remaining.
8	←

This method can be extended to larger numbers, for example, if the divisor is a 3-digit number we begin by asking how many 100-wide columns can we place. However, such divisions are most efficiently done on a calculator. The calculator will give the remainder as a decimal fraction, a topic we will address later.

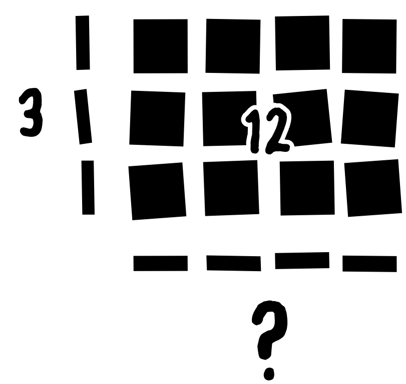
Gene says: In the array model of multiplication, one is given the dimensions of the array and asked for the number of tile in the array. In the array model of division, one is given the number of tile in the array along wih one of its dimensions and asked to find the other dimension. Notice that every array is associated with one multiplication statement and two division statements.



$3 \times 4 = 12$



$12 \div 3 = 4$



$12 \div 4 = 3$

FYI: Sometimes one may wish to give instructions to carry out a succession of computations. In such cases, parentheses, or other ***grouping symbols***, are used to avoid ambiguity. For example, if one wants to give the instruction to add 5 and 7 and multiply that result by 3 one would write, "Compute $(5 + 7) \times 3$," to indicate that 5 and 7 are to be added before multiplying by 3. If one wrote, "Compute $5 + (7 \times 3)$", one is giving the instruction to add 5 to the product of 3 and 7. Note that $(5 + 7) \times 3 = 12 \times 3 = 36$ while $5 + (7 \times 3) = 5 + 21 = 26$.

In an instruction involving parentheses, the convention is to carry out computations within parentheses first. Sometimes a set of parentheses is contained in another set. This indicates the computation within the innermost set is to be done first: $5 + (3 \times (4 + 7)) = 5 + (3 \times 11) = 5 + 33 = 38$. There is also the so-called ***order of operations*** convention: if no parentheses occur in an expression, the indicated multiplications and divisions are to be done before the additions and subtractions. Thus $2 \times 3 + 8 \div 2$ means $(2 \times 3) + (8 \div 2)$; hence $2 \times 3 + 8 \div 2 = 6 + 4$.

Incidentally, division is neither commutative (e.g., $10 \div 5 \neq 5 \div 10$) nor associative (e.g., $(24 \div 6) \div 2 = 4 \div 2 = 2$ whereas $24 \div (6 \div 2) = 24 \div 3 = 8$.)

END of SCENE 5: DIVIDING COUNTING NUMBERS

For comments and questions please email
Gene Maier at genem@mathlearningcenter.org

coming up next...

SCENE 6: THE INTEGERS