Epilogue: Completeness

The conclusion to the eighteen part series of articles on elementary mathematics.

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We have come to the end of our play on numbers.

We began with the ***counting numbers***. With only counting numbers at our disposal, however, we had no solution to equations such as $6 + x = 2$. To overcome this deficiency, we introduced zero and the negative integers, which, with the counting numbers, constitute the set of ***integers***.

However, within the integers, there is no solution to equations such as $3x = 2$, which led to the introduction of fractions representing the quotient of integers. These along with the integers constitute the set of ***rational numbers***, which, we saw. are equivalent to the collection of terminating and repeating decimals.

However, within the set of rationals there is no solution to equations such as $x^2 = 2$, hence the need for irrational numbers which, collectively, are equivalent to the set of decimals which neither terminate nor repeat. The irrationals along with the rationals form the set of ***real numbers.***

Within the real numbers, there are no solutions to equations such as $x^2 = -4$. This led to the introduction of the ***complex numbers***, that is, numbers of the form $a + bi$ where and b are real numbers and $i^2 = -1$.

Here , quite remarkably, our story ends. No new numbers are needed to obtain solutions to simple equations since it can be shown that the set of complex numbers is ***algebraically complete***. To put it another way, if $\boldsymbol{n}$ is a positive integer and $\boldsymbol{a}_0$, $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, ..., $\boldsymbol{a}_n$ is any set of $\boldsymbol{n} + 1$ complex numbers, then there is always a complex number $\boldsymbol{x}$ such that

$$\boldsymbol{a}_0 + \boldsymbol{a}_1\boldsymbol{x} + \boldsymbol{a}_2\boldsymbol{x}^2 + \ldots + \boldsymbol{a}_n\boldsymbol{x}^n = 0.$$

Thus, we don't need to create any more numbers and so our play ends.

"It can be shown that the set of complex numbers is algebraically complete."

END of A PLAY ON NUMBERS

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