

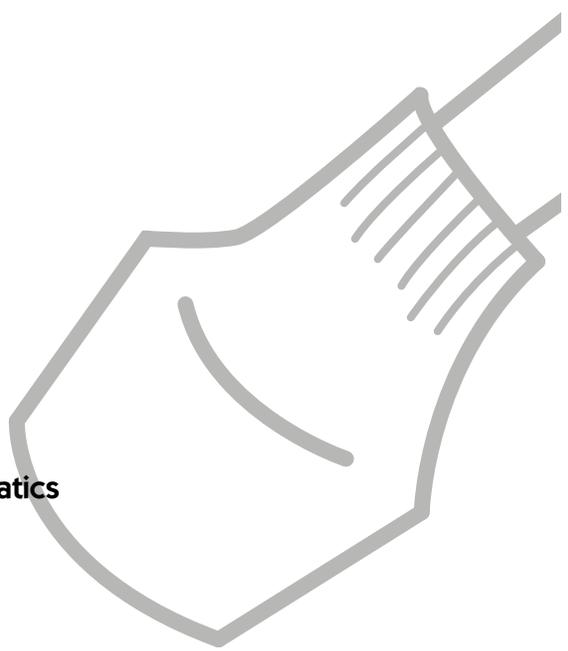


Math with a Sock

Probability and Fractions



Excerpts From Bridges in Mathematics
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Math with a Sock: Probability and Fractions

A Math Learning Center Publication

by Allyn Snider & Donna Burk

illustrated by Tyson Smith

Bridges Breakout Units

Geometry: Shapes, Symmetry, Area and Number

Bugs Across the Curriculum

Sea Creatures Across the Curriculum

Math Buckets: Sorting and Patterning

Crossing the Pond: A Probability Game

Math with a Sock: Probability and Fractions

P0100

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Math With a Sock: Probability and Fractions

Session A Calendar Fractions	1
The Student Book Tile Fractions	4
Session B Shake, Reach & Record	6
Work Place Shake, Reach & Record	10
Session C Pick & Peek: A Probability Experiment	12
Work Place Pick & Peek: Which One Is It?	17
Blackline Masters	
Magnetic Tile Fractions—Graphing Halves	1
Tile Fractions	2
Shake, Reach & Record 6's record sheet	3
Shake, Reach & Record 7's record sheet	4
Shake, Reach & Record 8's record sheet	5
Shake, Reach & Record 9's record sheet	6
Shake, Reach & Record 10's record sheet	7
Pick & Peek	8
Pick & Peek: Which One is It?	9
Overhead Masters	
Shake, Reach & Record 7's record sheet	1
Pick & Peek	2

Bridges Breakouts

Math With a Sock Probability and Fractions

These excerpts from Bridges in Mathematics, Grade 2 are designed to help children in grades 2–4 learn to read and write fractions, create graphs, use experimental data to predict probability, and more. Session A is drawn from the Number Corner; Sessions B, C, and the Work Place come from Volumes Two and Three of the Bridges Teachers Guides.

Each Session can be used whenever it fits into your instruction. The “You’ll need” list outlines supplies you need to gather in order to conduct the lessons. Deluxe Breakout contents are also listed; those who purchased an Economy Breakout will need to collect these items as well.

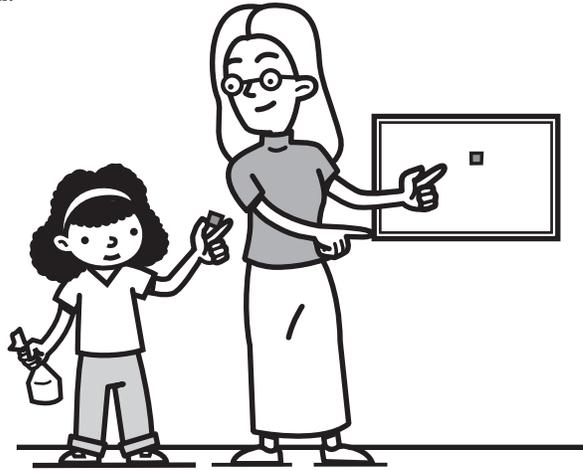
You’ll need

- ★ overhead projector
- ★ set of overhead pens (black, blue, red, and green)
- ★ 3” × 3” sticky notes
- ★ 3” × 5” index cards

Deluxe Breakout includes

- ★ 30 magnetic tile (15 red and 15 green)
- ★ magic wall (metal surface on which to stick magnetic tile)
- ★ paper lunch sacks
- ★ red and blue tile (12 of each color)
- ★ yellow and green tile (300 of each color)
- ★ yellow and green overhead tile (10 of each)
- ★ 6 probability containers (Economy Breakout users can make these containers by slipping plastic pint or quart containers into stretch socks.)

On the day you introduce this new Magnetic Tile activity, explain to your students that you are going to be studying halves during Number Corner. You might even take a minute to find out some of the things your students already know about halves. Then, dump the contents of the probability container and have a couple volunteers count to confirm that there are 15 red and 15 green magnetic tile—equal numbers of both colors. Have a student put the tile back in the container and shake it to mix the contents. Pull out the tile for the day's date and post them on the metal board for all to see. Ask the children whether fewer than half are red, exactly half are red, or more than half are red.



Corey Both of them came out red. That's more than half!

Colby It would have been half red if 1 of the tile was red and the other was green, because half of 2 is 1.

Evelyn Can we try it again and see what happens?

Teacher Sure. What's your prediction?

Dorothy I think they'll both be red again.

Teacher Why?

Dorothy Because red is a stronger color.

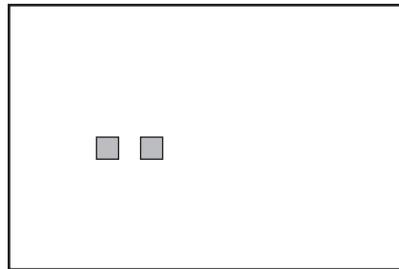
Peter I think it'll be 1 red and 1 green because we put 15 of each in the bag.

Teacher Let's see what does happen. Oh, look—it's 1 red and 1 green this time around.

Children Half are red this time!

You may want to let your students pull several samples out of the container just to see what happens, but in the end, take some time to record what hap

pened the first time around, using standard notation. As you record your results, you'll probably have to explain the symbols you're using, as some children won't be familiar with them.



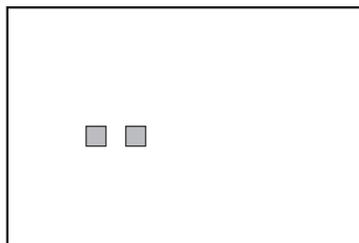
$\frac{2}{2}$ are red
 $\frac{0}{2}$ are green
 More than half are red.

Teacher When I write 2 over 2 the way I have here, it means 2 out of 2. When we pulled 2 tile out of the bag, they were both red—2 out of the 2 were red. How many out of the 2 were green?

Danielle 0?

Teacher That's right. So I've written 0 over 2, or 0 tile out of 2, are green. And what you told me to begin with is true. More than half are red today.

Finally, show the results of the day's first tile sample on the graphing sheet by recording the fraction in the correct column and by shading the box red.



$\frac{2}{2}$ are red
 $\frac{0}{2}$ are green
 More than half are red.

Red		
Less than $\frac{1}{2}$, Exactly $\frac{1}{2}$, or More than $\frac{1}{2}$		
		$\frac{2}{2}$
$< \frac{1}{2}$ less than $\frac{1}{2}$ red	$= \frac{1}{2}$ exactly $\frac{1}{2}$ red	$> \frac{1}{2}$ more than $\frac{1}{2}$ red

On the days that you don't do this lesson with your class, have 2 or 3 student volunteers try it on their own during Work Places or recess. Matters will become more complex and interesting because the sets to be considered each

day will change, and some of the days won't yield exactly half because they're odd. Consider the 9th of December.

Teacher *Eloise, will you and Briana do the tile this morning during recess?*

Eloise *Sure. We have to pull out 9 today, right?*

Teacher *That's right. What do you think will happen?*

Eloise *I think we'll get half red and half green.*

Teacher *How many of each would that be?*

Eloise *5 and 5? No—that makes 10. 4 and 4? That's 8. Hey, wait a minute! 9 is an odd number. We won't be able to get exactly half. It'll either be more or less than half red.*

Teacher *That's true. Don't forget to record your results!*

Red		
Less than $\frac{1}{2}$, Exactly $\frac{1}{2}$, or More than $\frac{1}{2}$		
$\frac{2}{5}$	$\frac{4}{8}$	$\frac{5}{9}$
$\frac{1}{3}$	$\frac{2}{4}$	$\frac{2}{2}$
$< \frac{1}{2}$ less than $\frac{1}{2}$ red	$= \frac{1}{2}$ exactly $\frac{1}{2}$ red	$> \frac{1}{2}$ more than $\frac{1}{2}$ red

It's important to bear in mind, too, that this experience is meant to be an early exploration of fractions, designed to draw on the children's intuitions and to arouse their curiosity and interest. Your students will revisit fractions several times in the Number Corner and study them in much more depth during Unit 7. Understanding fractions and their notation will be a long time in coming.



THE STUDENT BOOK

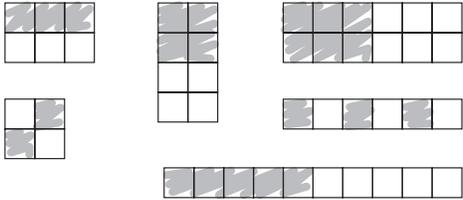
Tile Fractions (Blackline 2)

Take part of a Number corner session later in the month to have children do Blackline 2. It will be fun for them to compare sheets with their classmates as they finish, especially since some of the exercises have multiple solutions.

Blackline 2
 NAME Alex DATE 12/10

 **Tile Fractions**

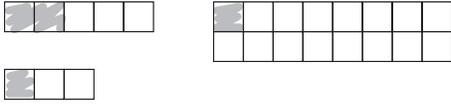
Color in *exactly half* the tile in each set.

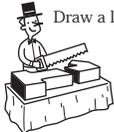


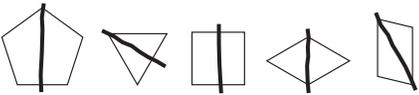
Color in *more than half* of the tile in each set.



Color in *fewer than half* of the tile in each set.



 Draw a line to divide each shape in half.



Briana Why did you color all 6 tiles on the second part?

Alex Well, it said to color in more than half!

Session B



PROBLEMS & INVESTIGATIONS

Shake, Reach & Record

Overview

Students conduct another probability experiment, this time featuring tile pulled out of a bag. The question being investigated is: If you put 10 green tile in a bag and 10 yellow tile, shake them up, and then pull out 7, how many greens are you likely to get? How many yellows? If you repeat this many times, replacing the tile each time and shaking the bag again, are certain combinations of 7 more likely to come up than others? While conducting this experiment, students are seeing and recording all the 2-addend combinations for 7 over and over, as well as creating graphs to show their data. This activity will appear in the next set of Work Places.



This activity is another opportunity to practice addition facts while conducting informal probability investigations. This time, the experiments involve tile sampling. With the record sheet shown on the following page, for example, one would pull 7 tile out of the sack, record how many greens and yellows came out, put the tile back in the sack, shake it, and repeat the process.

You'll need

- ★ Shake, Reach & Record 7's record sheet (Overhead 1)
- ★ 10 yellow overhead tile and 10 green overhead tile in a paper lunch sack
- ★ 3" sticky notes

Each child will need

- ★ Shake, Reach & Record 7's record sheet (Blackline 3, 1 copy)
- ★ 10 yellow tile and 10 green tile in a paper lunch sack (Use the tile from your base ten kits.)

Skills

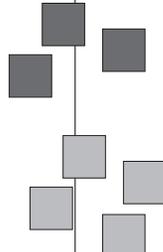
- ★ seeing and recording all the 2-addend combinations for 7 using standard notation
- ★ recording data on a graph
- ★ using experimental data to predict probability

NAME Leslie DATE Nov. 7

Shake, Reach & Record 7's record sheet

			3 + 4				
		2 + 5	3 + 4	4 + 3			
0 + 7	1 + 6	2 + 5	3 + 4	4 + 3	5 + 2	6 + 1	7 + 0

Bridges 5-MP1-2-8



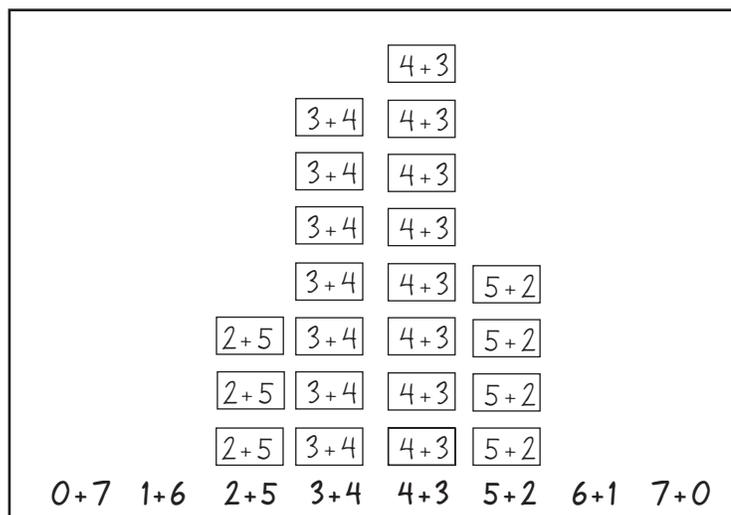
Leslie I pulled out 3 greens and 4 yellows this time, so I'll have to write down $3 + 4$ on my record sheet. I've gotten $3 + 4$ two times now. I wonder what I'll get next time.

To start the lesson, show your overhead transparency of the record sheet. As always, ask children to talk with one another about what they notice. Then take a minute to have a few volunteers share with the group. Your students may notice that each combination along the bottom of the sheet adds up to 7. They're almost sure to notice some patterns in the numbers and will be curious to know what you're planning to do with the sheet.

Explain that this is another game to help them practice addition facts. Show them your sack and, while they watch, count 10 green and 10 yellow overhead tile into the sack. Shake the sack well, reach in, and pull out 7 tile. Set them on the overhead platform for all to see, and explain that you want to record the results of your sample by writing down how many greens you got first, and then how many yellows. Once the numbers have been recorded, place the 7 tile back in the bag, shake it again, and draw out 7 more. Record your results, put the tile back, and repeat several more times. Be sure to emphasize the fact that you're returning the tile to the bag each time, shaking it well before each draw, and counting the greens *before* the yellows as you record.

Once you've gone through the steps 4 or 5 times, ask the children what they think might happen as you continue to work. Do they think that you're more likely to pull any particular combinations out of the bag? Do they see any combinations they think might be harder to get? Why? The fact that you have equal numbers of green and yellow may make this experiment simpler to think about than others, but if your students are anything like ours, they'll find it easier to observe the results of probability than to explain them. Nevertheless, we think that questions involving probability are worth pursuing.

When you think most of your students understand what to do, send them out to work on their own 7's sheets, with the understanding that they're to shake, reach, and record 7's until two of their columns have reached the top. Again, have them keep track of first and second place winners on their sheets. As children finish, have them record their first place winners on sticky notes and graph them on the board, as shown below. Discuss the class graph at the end of the math period. What do students notice about the graph? Why did so many children pull combinations of 3 and 4 or 4 and 3 out of the bag? Why weren't there more combinations of 0 and 7 or 1 and 6 pulled out? If you repeated this experiment tomorrow, would the results be similar or different? Why?



NAME Evan DATE Nov. 13

Shake, Reach & Record 7's record sheet

			1 st 3 + 4	2 nd 4 + 3			
			3 + 4	4 + 3			
			3 + 4	4 + 3	5 + 2		
		2 + 5	3 + 4	4 + 3	5 + 2		
		2 + 5	3 + 4	4 + 3	5 + 2	6 + 1	
0 + 7	1 + 6	2 + 5	3 + 4	4 + 3	5 + 2	6 + 1	7 + 0

Instructional Considerations for Shake, Reach & Record

In this Work Place, children can choose among sheets for 6's, 7's, 8's, 9's, and 10's. You, of course, can also assign sheet levels to particular students, but we find that given the choice children make pretty wise decisions for themselves. Youngsters who aren't very solid with facts for 6's and 7's tend to choose those sheets. Children who are more confident with addition facts will usually go for the 8's, 9's, and 10's.

You might want to have children begin each sheet by placing a star at the top of the column they believe will fill first. Even though some are likely to erase their stars and switch them to the winning column midway through, the mere act of making a prediction about the column that's most likely to fill to the top first leads to some nice intuitive thinking about probability.

You can emphasize or downplay the probability angle, depending on the needs and interests of your class. Children who are still grappling with standard notation and facts to 10 may need to concentrate on the basic activity. Children who are quite proficient with addition facts may enjoy collecting data from their own records and those of their classmates to ferret out trends and patterns. They can be challenged to figure out whether some combinations are more likely to be pulled out of containers loaded equally with green and yellow tile. Changing the tile proportions may further student thinking too. A container of tile with 10 green and 10 yellow seems to yield lots of $2 + 4$'s, $3 + 3$'s, and $4 + 2$'s if you're pulling out 6 at a time. What would happen if all the tile in the container were green? What if the container had two or three times as many green as yellow tile (14 greens and 7 yellows, for instance)? These are explorations that may transform an otherwise humdrum activity into a meaningful investigation for some of your more able students.

Session C



PROBLEMS & INVESTIGATIONS

Pick & Peek A Probability Experiment

Overview

What will happen if you put 2 blue tiles and 6 red tiles in a sack, give it a shake, and pull 1 out without looking? Theoretical probability says that you're 3 times as likely to draw a red as a blue. If you repeat the experiment 10 times, you could pull red out of the bag 10 times, or even the reverse—a blue every time (although the chances of that happening are about one in a million!). More than likely, you'll pull red more often than blue because the bag's been loaded that way. Pick & Peek introduces this experiment, engages children in prediction and speculation, and sets the stage for a more complex investigation.

You'll need

- ★ Pick & Peek (Overhead 2)
- ★ a probability container
- ★ a red, a blue, and a black overhead pen
- ★ Pick & Peek (Blackline 8, run half a class set)
- ★ a paper lunch sack for every 2 children in your classroom
- ★ red and blue 1" tile
- ★ pencils, red and blue crayons
- ★ 3" x 5" index cards for graphing labels

Skills

- ★ exploring probability
- ★ finding fractions
- ★ creating and interpreting graphs

Open this session by gathering children into a discussion circle. As they watch, place 6 red tiles and 2 blue tiles in the probability container and give the container a good shake. Explain that you're going to have a volunteer reach into the container and pull out one of the tile, but before you do, you'd like children to predict which color will be drawn. Students' responses will probably vary, some based on the actual contents of the container and others on somewhat magical thinking.

Children *It'll be red. Red is stronger and heavier. It'll have to be red, 'cause there are way more red tiles in the container. It could be blue. Yeah, but it would be hard to get blue. There are only 2 blues in there. It could be blue, but it will probably be red. I think my hand can feel the difference—I think I could get a blue.*

After some speculation, have a volunteer pull one tile out of the container and show it to the group.

Children *It is red—I knew it!
That was my idea—red.
I still think it could have been blue.
I bet it'll be blue next time.*

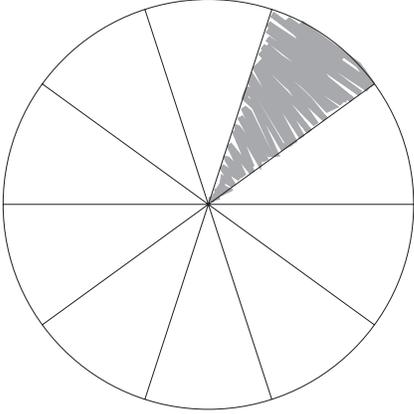
Next, show the Pick & Peek transparency at the overhead and record the results of the first trial. Then read the instructions at the top of the sheet together and record some of the group's thoughts.

Overhead 2

Pick & Peek

Put 6 red tile and 2 blue tile in a probability container. Shake well. Pull out a tile and record its color by filling in 1 of the sections on the pie graph below. Return the tile to the container, shake it again, and pull out another tile. Do this 10 times. Be sure to shake the container each time. What do you predict will happen? Why?

Red will come out way more times because there are
6 reds in the sack and only 2 blues.



Red came out $\frac{\quad}{10}$'s of the time Blue came out $\frac{\quad}{10}$'s of the time

Have student volunteers help you repeat the tile sampling sequence nine more times, returning the tile to the container and shaking well between each trial. Record the results on the pie graph and then discuss.

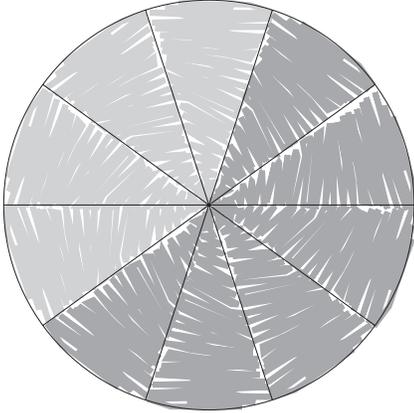


Overhead 2

Pick & Peek

Put 6 red tile and 2 blue tile in a probability container. Shake well. Pull out a tile and record its color by filling in 1 of the sections on the pie graph below. Return the tile to the container, shake it again, and pull out another tile. Do this 10 times. Be sure to shake the container each time. What do you predict will happen? Why?

Red will come out way more times because there are 6 reds in the sack and only 2 blues.



Red came out $\frac{6}{10}$'s of the time Blue came out $\frac{4}{10}$'s of the time

Children Red did come out the most!

I knew it!

It had to be because there are 6 reds in the container and only 2 blues. It's funny, though. We got red 6 times and blue 4 times—they weren't that much different.

Teacher Do you think it'll always work this way? Do you think you'll always get 6 reds and 4 blues?

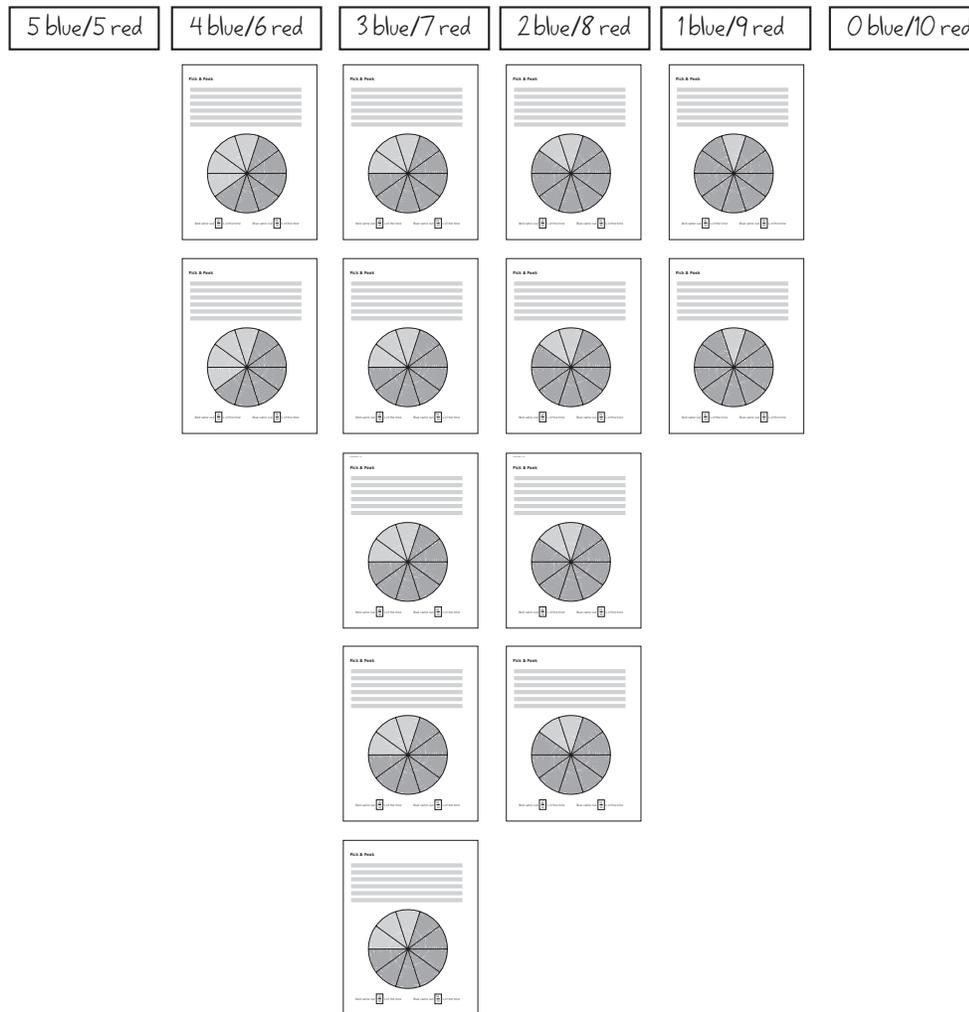
Children Yep! Red will always be the most.

I think it could be blue that wins sometimes, but it would be hard.

I think you could get even more reds sometimes, like maybe 8 times, and blue only 2 times because there are only 2 blues in the container.

After some discussion about the results of this experiment, send children out in partners to try this experiment on their own. Each pair of students will need 6 red tile, 2 blue tile, 1 paper sack, 1 record sheet, and some crayons.

As the children finish, have them lay their sheets on the floor in the appropriate column to create a class graph of the results. Discuss.



The results of this experiment will vary a bit from class to class, but will tend to the right-hand side of the graph. More often than not, children will have drawn red 6 or more times and blue 4 or fewer. (The theoretical probability of getting red 6 times and blue 4 times from a bag loaded with 6 reds and 2 blues is .15. The theoretical probabilities of getting red 7 or 8 times and blue 3 or 2 times are .25 and .20 respectively. The chances of getting red 9 times and blue only once are surprisingly high at .19, while the chances of getting red all 10 times are slim, at .06.) It's certainly not necessary to share the theoretical probabilities with your students, but many will have some sort of intuitive understanding that a container heavily loaded with red tiles will produce these results. You can draw their understandings out a bit in your discussion by gently nudging students towards some generalizations.

Teacher Which column came out with the most on our class graph?

Children 3 blues and 7 reds. But 2 blues and 8 reds got almost as many.

Teacher What do you think would happen if we all went back and tried the experiment again? If we collected twice the evidence?

Children 3 and 7 would keep being the winner.

It could be 2 and 8.

Maybe 4 and 6 would pull ahead.

Teacher I notice that none of you are talking about getting things like 7 blues and 3 reds or 8 blues and 2 reds. Why not?

Children Because there are so many reds in the container. How could you pull blue out so many times when there are only 2 blue tiles in the container?

It's just going to be kind of the same—you're always going to pull red out more times because there are so many more reds in the container.

Teacher Well, okay. What would happen if we changed the numbers of tile in the container to 2 red and 6 blue?

Children 6 blues and only 2 reds?

You'd pick blue way more often—you'd have to.

It would be the opposite of what we had before.

Teacher What would happen to the shape of the graph?

Children It would go the other way!

The hump would be over on the other side of the graph, where there are lots of blues!

It would be the opposite of what we had before.

With many experiences like this, students' intuitive and explicit understandings of probability will grow and they'll gradually relinquish some of their magical thinking about tile sampling and other such activities.

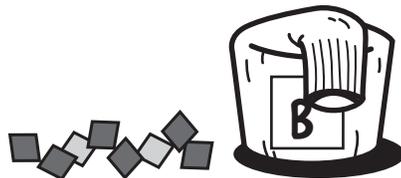
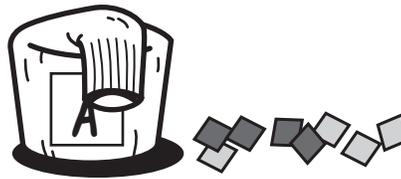

WORK PLACE

Pick & Peek Which One Is It?

This Work Place basket will need

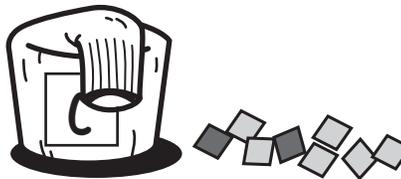
- ★ 3 probability containers prepared in the following way:

Cut three 3" square tagboard labels and label them "A," "B," and "C," respectively. Use safety pins to fasten these labels securely to the socks that cover the 3 containers. In container A, put 4 red and 4 blue tile. In container B, put 6 red and 2 blue tile. In container C, put 2 red and 6 blue tile.



- ★ Pick & Peek: Which One Is It? (Black-line 9, run 30 copies and place in a folder)

- ★ red and blue crayons



Skills

- ★ exploring probability
- ★ finding fractions
- ★ creating and interpreting graphs

To Work

1. With a partner, choose one of the containers and find that letter on your record sheets. There is space on the Pick & Peek sheet to record the results of sampling tile from each container. Whether you choose to start with the A, B, or C container, make sure you're recording in the right spot. Each of you need to keep your own sheet.

2. Go through the tile sampling procedure ten times using the container you've chosen. That is, give the container a good shake, pull out a single tile without looking, and record the color you got by coloring in one of the sections on the pie graph for that container. Then return the tile to the container, shake the container again, pull out a second tile, and record the color you got. Repeat this sequence eight more times, being sure to put the tile back in the container and give it a good shake each time. Finally, record the results of your experiment using the fraction boxes below the pie graph (see next page).

NAME Kevin DATE May 15

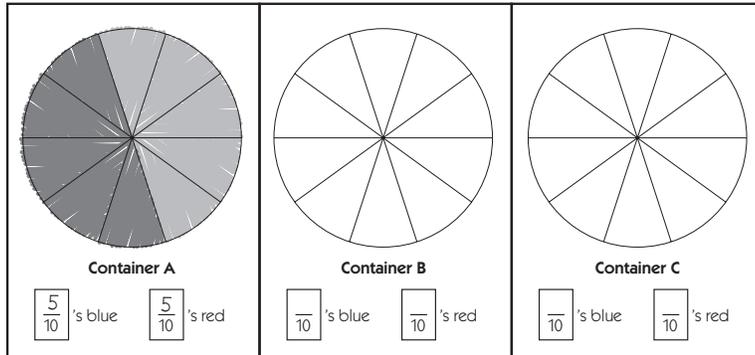
Pick & Peek Which One Is It?



Your mission is to identify the container that best fits the mystery profile:

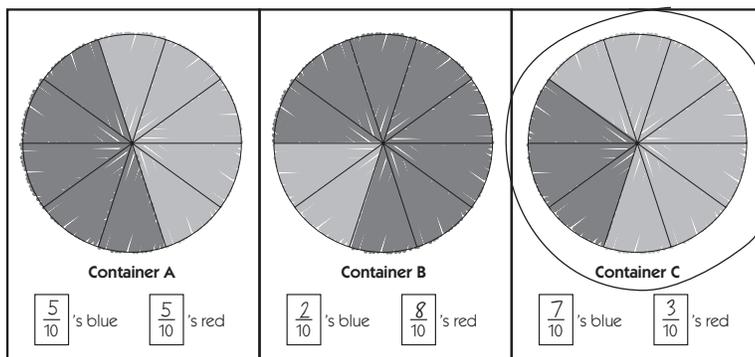
6 blue and 2 red

Sample the contents of each container 10 times and record your findings below. Based on your evidence, circle the graph you think best fits the mystery profile.



“Hmm...I got red 5 times and blue 5 times. This probably isn't the container with 6 blues and 2 reds or I would have gotten blue more often. I bet this container really has half and half.”

3. Repeat the whole tile sampling routine with the other two containers, recording as you go. What you're trying to figure out is which container actually has 6 blue and 2 red tile in it, but none of your results will match exactly because you're taking ten samples and there are only eight tile in the bag. Circle the graph of the container you think *probably* has 6 blues and 2 reds.



“Boy, this is hard. Which container *probably* has 6 blues and 2 reds? It can't be Bag A—I only got half reds and half blues on that one. And the middle one came out 8 reds and 2 blues—that couldn't be it. It must be that last one 'cause I pulled blues out so many times.”

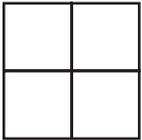
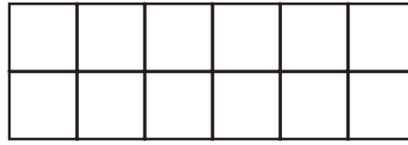
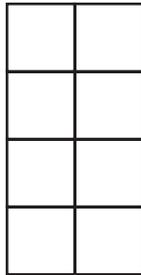
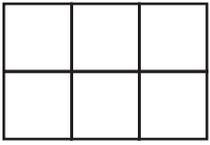
Instructional Considerations for Pick and Peek: Which One Is It?

It might be a bit of a stretch for some of your students to predict which container really has 6 blue tiles and 2 reds based on the results of their work. Since none of their sampling outcomes will match the “mystery profile,” children will just have to pick the one that’s closest. You might want to collect the sheets from this Work Place as students finish them and post them on a wall so children can see the growing body of evidence. As the data pile up, it may become evident to many students that C is, in fact, the mystery container.

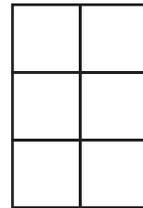


Tile Fractions

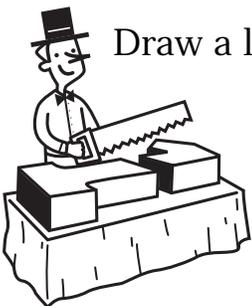
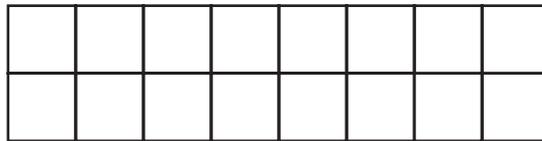
Color in *exactly half* the tile in each set.



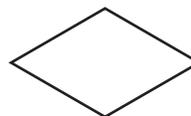
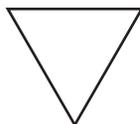
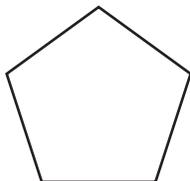
Color in *more than half* of the tile in each set.



Color in *fewer than half* of the tile in each set.



Draw a line to divide each shape in half.



NAME _____ DATE _____

Shake, Reach & Record 7's record sheet

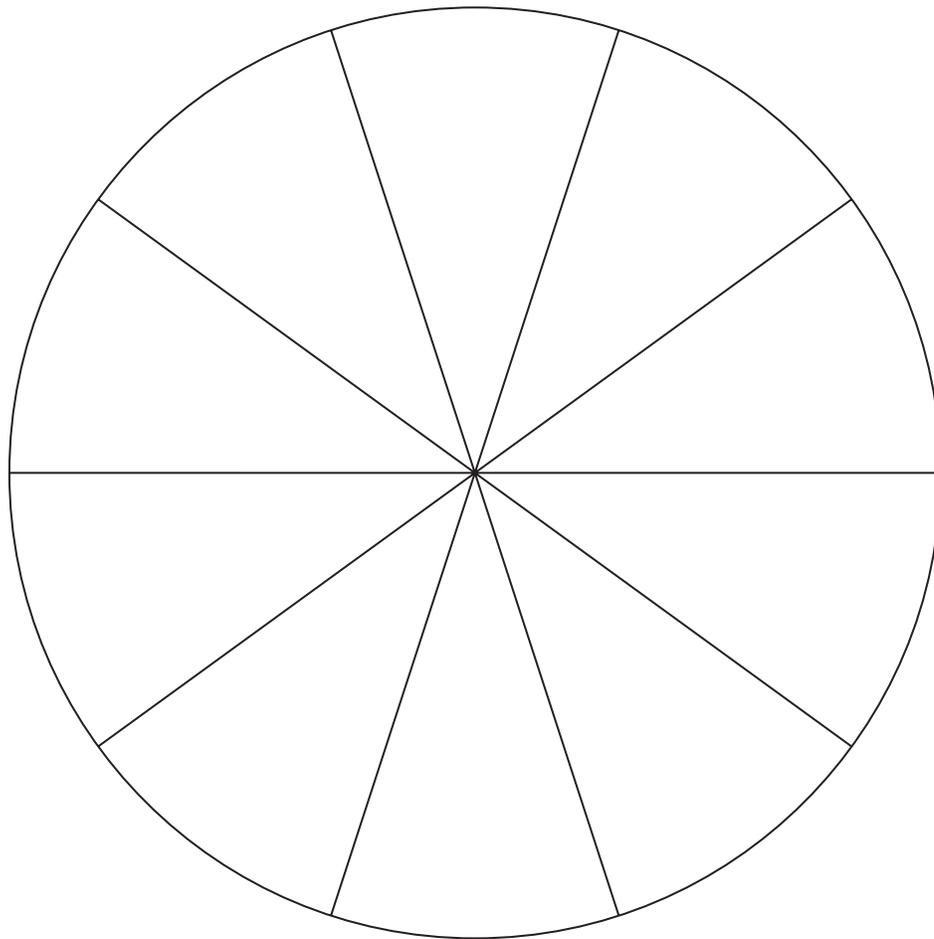
								0 + 7
								1 + 6
								2 + 5
								3 + 4
								4 + 3
								5 + 2
								6 + 1
								7 + 0

NAME _____

DATE _____

Pick & Peek

Put 6 red tile and 2 blue tile in a probability container. Shake well. Pull out a tile and record its color by filling in 1 of the sections on the pie graph below. Return the tile to the container, shake it again, and pull out another tile. Do this 10 times. *Be sure to shake the container each time.* What do you predict will happen? Why?

Red came out $\frac{\boxed{}}{10}$'s of the timeBlue came out $\frac{\boxed{}}{10}$'s of the time

NAME _____ DATE _____

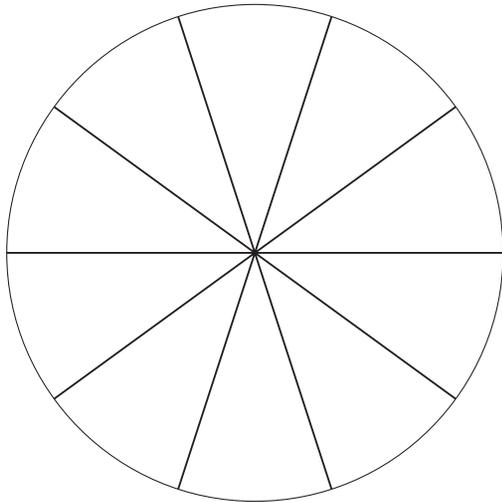
Pick & Peek Which One Is It?



Your mission is to identify the container that best fits the mystery profile:

6 blue and 2 red

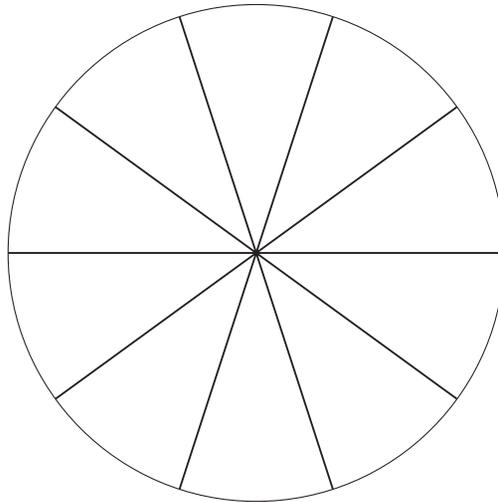
Sample the contents of each container 10 times and record your findings below.
Based on your evidence, circle the graph you think best fits the mystery profile.



Container A

$\frac{\quad}{10}$'s blue

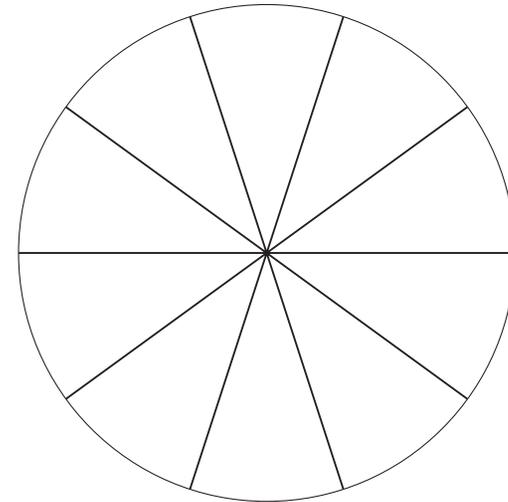
$\frac{\quad}{10}$'s red



Container B

$\frac{\quad}{10}$'s blue

$\frac{\quad}{10}$'s red



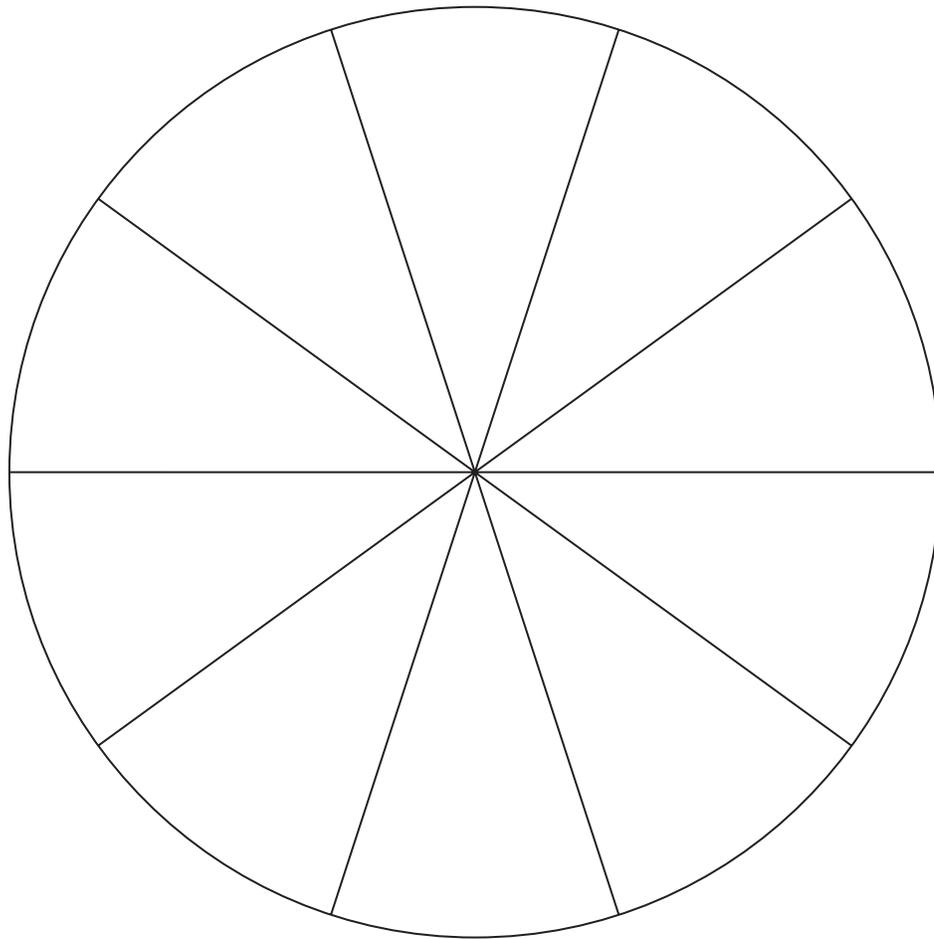
Container C

$\frac{\quad}{10}$'s blue

$\frac{\quad}{10}$'s red

Pick & Peek

Put 6 red tile and 2 blue tile in a probability container. Shake well. Pull out a tile and record its color by filling in 1 of the sections on the pie graph below. Return the tile to the container, shake it again, and pull out another tile. Do this 10 times. *Be sure to shake the container each time.* What do you predict will happen? Why?



Red came out $\frac{\boxed{}}{10}$'s of the time

Blue came out $\frac{\boxed{}}{10}$'s of the time