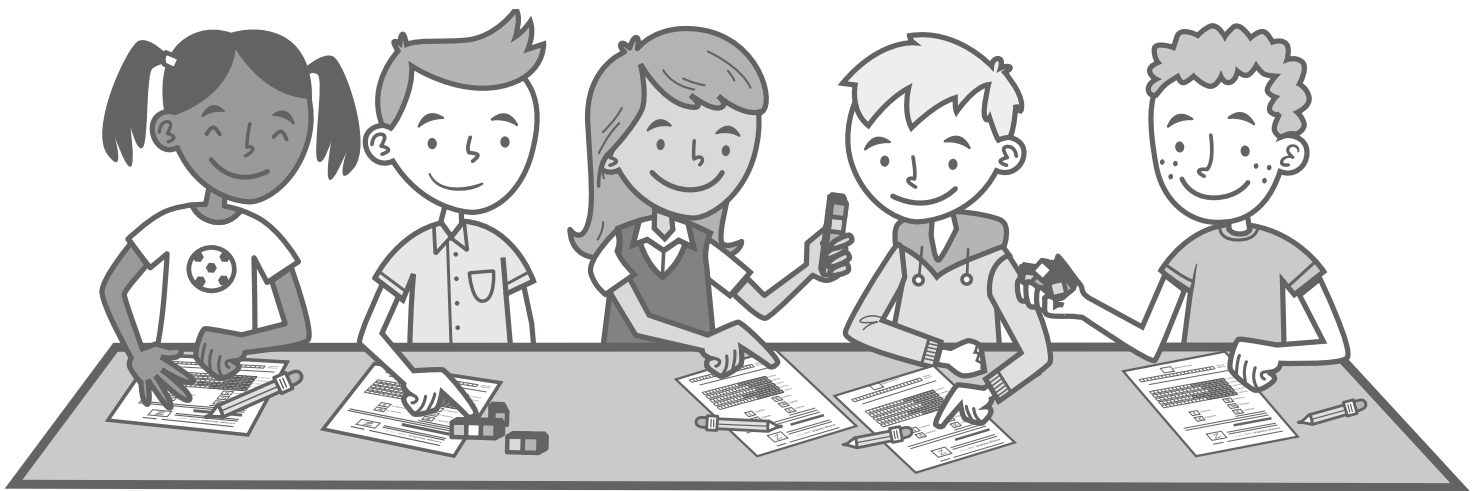


Data Analysis

Exploring Mean, Mode & Range



Adapted from Bridges first edition

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Data Analysis: Exploring Mean, Mode & Range

A Math Learning Center Publication

by Pia Hansen

illustrated by Tyson Smith

Bridges Breakouts include units, games, and activities from Bridges in Mathematics first edition. To find others, visit www.mathlearningcenter.org.

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Data Analysis: Exploring Mean, Mode & Range

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Blacklines

Average of Three Handfuls	DA.1
Average Handfuls, pages 1 and 2	DA.2–3

Data Analysis Exploring Mean, Mode & Range

These activities are adapted from Bridges in Mathematics First Edition and are appropriate for students who have some experience working with fractions. In each session, students explore the concept of *average* or *mean*; in Session 2, they will also explore *mode* and *range*.

Skills & Concepts

- ★ collecting and displaying data
- ★ exploring averaging problems by leveling off columns of cubes or base ten pieces
- ★ determining the mean, mode and range of a set of data
- ★ using a calculator strategically to solve problems

Math Vocabulary

- ★ **mean**: the sum of all numbers in a data set divided by the number of data points; also called the **average**
- ★ **mode**: the value (or values) that appear most often in a set of data; there may be no mode, one mode, or multiple modes in a single set of data
- ★ **range**: the difference between the greatest and least values in a data set
- ★ **median**: the middle value of an ordered set of numerical data; in a set with an even number of data points, the median is the average of the two middle points

Materials

- ★ Average of Three Handfuls (Blackline DA.1, half-class set plus 1 for display)
- ★ Average Handfuls, pages 1 and 2 (Blacklines DA.2–3, class set run back-to-back)
- ★ linking cubes, such as Unifix or Omnifix cubes
(1 bag or bin of about 150 cubes for every 5 students)
- ★ half-class set of measuring tapes or rulers marked in centimeters
- ★ half-class set of base ten pieces **or** devices with MLC's free Number Pieces app
(www.mathlearningcenter.org/resources/apps/number-pieces)
- ★ half-class set of calculators
- ★ spreadsheet software (optional)

Session 1



PROBLEMS & INVESTIGATIONS

Average Handfuls

Overview

Students discuss the concept of averaging and then work as a class and in teams to find the average number of cubes in a handful. Students might use the leveling-off method of averaging, or they might use a grouping method in which all cubes are combined and then divided evenly to determine the mean. The combining method more closely mirrors the process students will use in the next session when they use a calculator to find the average of their foot lengths.

Actions

- 1 The teacher prompts students to discuss the concept of averaging, and students model several ways to average 7 and 11 cubes.
- 2 As a class, students find the average of 3 handfuls of cubes.
- 3 Students work in teams of 5 to take handfuls of cubes and average them.
- 4 The class works together to find the average of all their handfuls.

Skills & Concepts

- ★ collecting and displaying data
- ★ exploring averaging problems by leveling off columns of cubes

You'll need

- ★ Average of Three Handfuls (Blackline DA.1, half-class set plus 1 for display)
- ★ Average Handfuls, pages 1 and 2 (Blacklines DA.2–3, class set run back-to-back)
- ★ linking cubes such as Unifix or Omnifix cubes
(1 bag or bin of about 150 cubes for every 5 students)

Session 1 Average Handfuls (cont.)

The Concept of Average

Begin today's session by building a tower of 7 cubes and another tower of 11 cubes. Ask students what they would do to find the average or mean of the numbers of cubes in these two towers. (*Average* and *mean* are synonymous; you might choose to discuss this with students.) What do they already know about the meaning of average and how to find it?

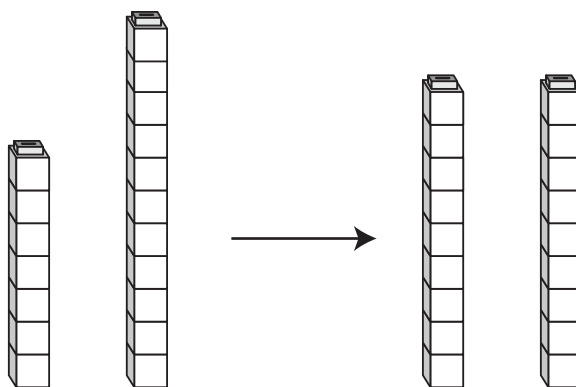
Students *Average is like in the middle.*

It's like what most are. If you're average, you're kind of like everybody else.

I think it's like making them the same. Remember when we did that before?

I've done this before. You make them the same and then the same number is the average.

Invite two volunteers to demonstrate their methods of finding the average. Based on past experiences, students might level off the towers by taking 2 cubes from the taller tower and adding them to the smaller tower so that both are 9 cubes high. They might also combine the cubes and then rebuild two even towers. If no one contributes one of these methods, model it yourself.



Stella *I just counted out 7 and 11. That's 18 cubes. Then I made 2 towers of 9 cubes. They're even, so you know they're equal.*

Andre *I took 2 from the 11 and gave it to the 7 to make 9. When you make them the same, that's the average. The average is 9.*

Ask students to consider reasons that people might want to know the average of something. Are there any averages that they are curious about?

Finding the Average of Three Handfuls as a Class

Let students know that you'll be practicing the concept of averaging with handfuls of cubes. Have them predict about how many cubes you might be able to pick up in one hand. Then ask three volunteers to come up and take a handful of cubes out of a bin or bag full of cubes. Record the number of cubes in each handful on the record sheet.

Session 1 Average Handfuls (cont.)


NAME Ms. Smith NAME _____


Average of Three Handfuls


How many cubes are in a handful?

Record the number of cubes each volunteer took below.

Handful 1 Handful 2 Handful 3

Handful 1 

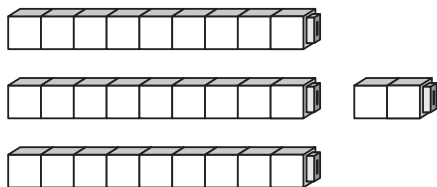
Handful 2 

Handful 3 

Find the average number of cubes in a handful. Record it below.

Ask students to work in pairs to find the average of these three handfuls, using additional cubes. As they work, circulate and listen to their conversations. Students may struggle to decide what to do with “extra” cubes.

Maria Okay, if we take one from the 10 and give it to the 8, they're both 9s. Then we can take the extra cubes off the 11 and have all three be 9s, but we have 2 left over. I don't know what to do with them.



Sara We could just leave them there and say the average is 9 with a little bit left over. Hey Zofia, what did you and Ginny do with these extra cubes?

Zofia Well, we thought that the average would be 9 and then not quite 1 more for each. If there were 3 left over, the average would be 10 because you could give another cube to each one. But it's going to be a little less than that because you just have 2.

Ginny Oh hey, if we broke each cube into 3 pieces, like fractions, we could give part to each one. So we could give each one a third and then another third. That's 9 and two-thirds for each.

After students have had time to work through the problem, invite volunteers to share their thinking and record the average for all to see. In this case, you could record the average as “between 10 and 11” or “10 and two-thirds,” depending on your students’ comfort level with and discoveries about fractional parts.

Session 1 Average Handfuls (cont.)

Finding the Average of Five Handfuls in Teams

Now divide students into teams of 5 and give each team a bag or bin of cubes. Give each student a record sheet. Ask teams to take turns drawing a handful of cubes and counting them. Then have them work together to find the average of their handfuls, recording their work on their record sheets.

As they work, circulate throughout the room, listening to their conversations and offering help where needed. Invite students to talk about their strategies, and press them to decide how to handle fractional parts when the average is not a whole number. Have students edit each other's record sheets to make sure they are complete and legible.



Use a Spreadsheet

You might model using spreadsheet software to record the handfuls and calculate the class average.

Explain your formula or calculations as you go. Plan to have students work in pairs to calculate averages using spreadsheet software during the next session.

Finding the Average of All the Handfuls as a Class

When teams have finished, ask each student to share the number of cubes in their handful. Record these numbers on the second record sheet and have students record them along with you on their copies of the sheet. Then invite the class to average the numbers using cubes.

Have several volunteers share their strategies with the class. Students' conceptual grasp of averaging is strengthened when they must find the average of different numbers of samples.



CHALLENGE

Finding the Median

Invite students to find the median of the class data.

The median is the middle of the data. To find it, arrange all of the values in order. The value in the middle, with an equal number of values above and below it, is the median. If you have an even number of values, find the mean (average) of the two numbers closest to center.

Session 2



PROBLEMS & INVESTIGATIONS

Finding the Average Length of Our Feet

Overview

Students work in pairs to measure their own feet. They find their average foot length in pairs, in groups of four, and then as a class. Students begin by using base ten pieces to average their foot lengths. As they find the average of larger numbers of feet, they use a calculator. Students also determine the mode and range of foot lengths in the class.

Actions

- 1 The teacher introduces foot measurements by sharing the average length of a newborn's foot.
- 2 Students work in pairs to measure their right feet and find the average.
- 3 Students work in groups of 4 to find the average foot length in their group.
- 4 Students use calculators to find the average foot length for the whole class.
- 5 Students examine the class data to determine the minimum, maximum, range, and mode.

Skills & Concepts

- ★ collecting and displaying data
- ★ exploring averaging problems by leveling off columns of base ten pieces
- ★ determining the mean, mode and range of a set of data
- ★ using a calculator strategically to solve problems

You'll need

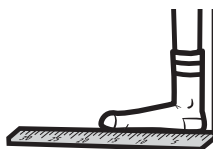
- ★ half-class set of centimeter measuring tapes or rulers
- ★ half-class set of base ten pieces **or** devices with MLC's free Number Pieces app
- ★ half-class set of calculators

Session 2 Finding the Average Length of Our Feet (cont.)

Introducing Foot Measurements

Begin today's lesson by asking how many students have seen a brand new baby. Chances are many of your students have experience with infants, either their own siblings, other family members, or friends. Ask them to imagine the baby's foot and think about its length. About how many centimeters long is a new baby's foot? Have students make estimates and share them with a partner, and then let them know that the foot of an average newborn is about 8 centimeters long. Let them know that today they will measure their own feet and find the average foot length in their class to see about how much their feet have grown, on average, since they were born.

Ask a volunteer to help you show students a method for accurately measuring their feet. First, have the volunteer remove her right shoe. Then place the zero end of the measuring tape or ruler against the wall. Ask the volunteer to place her heel against the wall and put the inside edge of her foot beside the measuring tape or ruler to take a measurement from her heel to the tip of her big toe. Most likely, the volunteer's foot will not be a whole number of centimeters. Let students know that in this case they'll need to round the measurement to the nearest half centimeter.



Measuring Feet and Finding Averages in Pairs

Now ask students to partner up. Give each pair a centimeter measuring tape or ruler and a set of base ten pieces. Ask them to title a new journal page *Average Foot Length*. They'll record foot measurements for themselves and their partners on this page. Because often a person's left and right feet are different lengths, ask everyone to measure only their right foot. Once students have taken both measurements with the centimeter side of the ruler, ask them to find and record the average of their own and their partner's foot lengths. Invite them to use whatever tools they are comfortable with, including the base ten pieces, to find the average of these two lengths.

Give students some time to explore the process and record their results in their math journals. Invite them to talk with one another about their various methods, and ask them to consider which methods are most efficient. You may want to remind them that the base ten pieces represent the number of centimeters in their foot length, but not the actual measurement itself.

Roberto *Hey, how come you guys are using base ten pieces?*

Midori *Well, my foot was 17 centimeters and Leela's was 21. So we didn't want to have to count all those pieces out. We just went 10 and*

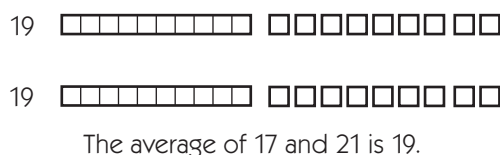
Session 2 Finding the Average Length of Our Feet (cont.)

added 7 for me. And then we used 2 strips and a unit for Leela. See? It was easier that way.



Omar *But your feet aren't that big!*

Leela *You're right, but we just wanted to use 10s and 1s because it was faster. The numbers are still the same. Then when we averaged, we traded 1 of my strips for 10 units. And then we evened them out. It was 19. So there were lots of units. But still, it was easier, because we both still had a strip.*



.....

Best Practice Tip

Generating and testing hypotheses increases student achievement. When students make predictions and draw conclusions, they analyze their prior knowledge and make new connections.

Computing Averages in Groups of Four

Now group your students in teams of four. Ask them to predict whether the average for the group will remain the same as the average for their pairs. They can take a look at one another's feet or measurements first to inform their predictions. Invite them to think privately and then share with their team members. Then select a few students to share their ideas with the class. Be sure to have them justify their predictions.

Students *We think the average will be higher for the group, because Andre's foot looks a lot bigger than either of our feet.*

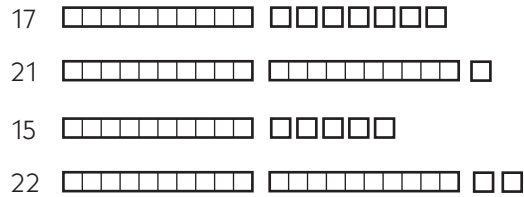
My foot is about the same as everyone in the group, so maybe the average will be my number.

The average will be somewhere in the middle. Not the longest and not the shortest, but somewhere in between.

Ask each group member to use the base ten pieces to show the number that represents the length of their own foot. Then ask the teams to work together to find the average by redistributing their units. Remind students that they must keep a row of base ten pieces for each student in their group. By now many students will begin with the largest number and try to give some units to the smallest number. Others might want to level all the rows and divide the remaining units among the students fairly. Pose any of the following questions to help students get started: Who has the most? Who has the fewest? How could you divide up the units to make all the rows the same?

Session 2 Finding the Average Length of Our Feet (cont.)

In many cases, the team average will not be a whole number. Allow students to decide if they want to round up or down, or deal with fractional parts or decimals.



Students *Our smallest is 15 and the longest is 22.*

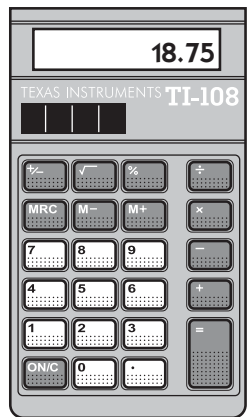
I think for both of them in the 20s we should break the tens into units.

Everybody has at least 15, so let's kind of show 15. Then we can decide how to use the rest.

Okay, so let's just put them together and give one to each person and keep doing it like that till they're even.



Give each group two calculators. Ask pairs to use the calculators to find the average for their group. How could they use the calculator to carry out the same process of averaging more quickly? When they've had some time to work, ask volunteers to explain what they did with the calculator and why. Some may employ the traditional method of adding the measurements together and dividing by the number of measurements. Others may invent their own approach. Be sure to connect their sequence of computations on the calculator to the models they constructed with base ten pieces earlier.

Session 2 Finding the Average Length of Our Feet (cont.)

$$\begin{array}{r}
 17 \\
 21 \\
 15 \\
 + 22 \\
 \hline
 75
 \end{array}$$

$$75 \div 4 =$$

Computing the Class Average

Now ask students to consider what the class average might be. Will their own foot length be below, above, or about the same as the class average? Ask them to make some predictions about these questions and share their thinking with their partners.

Ask students to report their own foot lengths one at a time. Choose a capable student to record each measurement in the order they are reported on the whiteboard. As students report their measurements, ask each pair of students to enter the numbers into the calculator. Model how this is done with a computer's calculator app. Have students check their totals as you go. Students who are off track can clear their calculators and begin with the class's running total. When everyone's data has been entered, divide the total by the number of students in your class. The result is your class average.

Ask students to consider your class average. What is the average length of a classmate's foot? How many students had foot lengths the same as the class average? How many foot lengths were shorter or longer than the average?

Ask students to consider where and how this information might be valuable. Who would want to know the average foot length of (for example) a 10-year-old? Why?

**CHALLENGE****Comparing the Group Average to the Class Average**

Calculate the average of teams' averages. Compare this new average to the class average. Is it the same or different? Why?

Use a Spreadsheet

You might have pairs work together using spreadsheet software to record the measurements and calculate the class average.

Session 2 Finding the Average Length of Our Feet (cont.)**Finding the Mode, Maximum, Minimum, and Range**

The mode is the measurement that appears most frequently in the class data. As your students were sharing their information with the class, what measurement came up most frequently? Ask students to look at the data on the board and share their ideas with a neighbor. The mode may not be obvious since the measurements were recorded in the order in which they were reported instead of numerical order. Ask students to suggest how you might reorganize the data to make it easier to read.

Students *You should put the numbers together, so you don't have them all mixed up.*

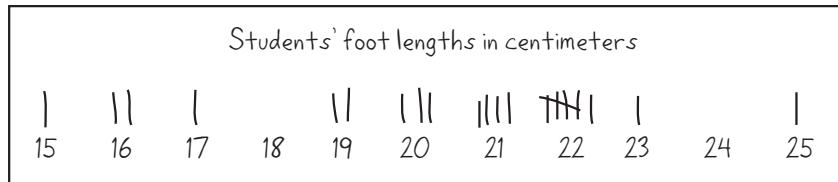
Make a line plot!

But start with 15. That was the smallest number.

Put a tally by each number when someone says it.

Teacher *It sounds like a tally chart might be a nice way of organizing our information. Should I make a vertical or horizontal list?*

List all the numbers on the board, starting with the smallest measurement and running through the largest measurement in your class. If there were no students with a given measurement, don't record a tally for that measurement. When you have finished, ask the students to determine the mode. Which foot length was most common in your class? In the example below, the mode is 22, because 6 students had feet measuring 22 centimeters.



Then have students consider how the mode from their data compares to the class average. When mathematicians analyze data, they may choose to report the mode (the most frequent value), the mean (average), or some other measure of center or combination of measures of center. In our sample, the mode was only 1 centimeter away from the average (21), but your data will probably be different than ours.

The range is the difference between the minimum and maximum values in the data set. Ask students to determine the minimum and maximum foot lengths of students in your class and use these two measurements to determine the range. The range for the example on the previous page is 10 centimeters ($25 - 15 = 10$).

If you have time at the end of the session, ask students to think about how much they have grown if their feet were about 7 or 8 centimeters when they were born. Have their feet doubled in length? More than doubled in length?

NAME _____

NAME _____

Average of Three Handfuls

How many cubes are in a handful?

Record the number of cubes each volunteer took below.

Handful 1

Handful 2

Handful 3

Handful 1

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

Handful 2

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

Handful 3

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

Find the average number of cubes in a handful. Record it below.

Average
Handful

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

Average Handful

Average Handfuls page 2 of 2

How many cubes are in a handful?

Share the number of cubes you took in your handful with the rest of the class.
Record the number of cubes each classmate took in their handful below.

_____	_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____

Now find the average number of cubes in a handful for the class. Show your work.

Average Handful



CHALLENGE

Do you think all grade five classes would have a similar average? Why or why not?