theratiotable

# Learning to Think Mathematically with the Ratio Table

## A Resource for Teachers, A Tool for Young Children

Jeff Frykholm, Ph.D.

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## A Resource for Teachers, A Tool for Young Children

Authored by
Jeffrey Frykholm, Ph.D.

Overview: This book prepares teachers with the theoretical basis, practical knowledge, and expertise to use the ratio table as a vigorous model for mathematical learning in grades K–8. A growing body of research in mathematics education points to the ratio table as a fundamentally important model for elementary and middle school learners as we prepare them for success in the beginning algebra course. The ratio table serves as a visual representation—a structural model—that embodies numerous concepts and relationships. This versatile tool promotes proportional reasoning, makes use of equivalent fractions, represents percents, functions as a double number line, and can perform as an elegant computational tool in contexts that require either multiplication or division.

About the Author

Dr. Jeffrey Frykholm has had a long career in mathematics education as a teacher in the public school context, as well as a professor of mathematics education at three universities across the U.S.  Dr. Frykholm has spent the last 22 years of his career teaching young children, working with beginning teachers in preservice teacher preparation courses, providing professional development support for practicing teachers, and working to improve mathematics education policy and practices across the globe (in the U.S., Africa, South America, Central America, and the Caribbean).

Dr. Frykholm has authored over 30 articles in various math and science education journals for both practicing teachers, and educational researchers.  He has been a part of research teams that have won in excess of six million dollars in grant funding to support research in mathematics education.  He also has extensive experience in curriculum development, serving on the NCTM Navigations series writing team, and having authored two highly regarded curriculum programs:  An integrated math and science, K-4 program entitled ***Earth Systems Connections*** (funded by NASA in 2005), and an innovative middle grades program entitled, ***Inside Math*** (Cambium Learning, 2009).  This book, ***Learning to Think Mathematically with the Ratio Table***, is part of his latest series of textbooks for teachers.  Other books in this series include:  ***Learning to Think Mathematically with the Rekenrek;  Learning to Think Mathematically with the Number Line; Learning to Think Mathematically with Multiplication Models;*** and ***Learning to Think Mathematically with the Double Number Line.***

Dr. Frykholm was a recipient of the highly prestigious National Academy of Education Spencer Foundation Fellowship, as well as a Fulbright Fellowship in Santiago, Chile to teach and research in mathematics education.

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# Learning to think Mathematically: An Introduction

## The *Learning to Think Mathematically* Series

One driving goal for K-8 mathematics education is to help children develop a rich understanding of numbers – their meanings, their relationships to one another, and how we operate with them. In recent years, there has been growing interest in ***mathematical models*** as a means to help children develop such number sense. These models (e.g., the number line, the rekenrek, the ratio table) are instrumental in helping children develop structures – or *ways of seeing* – mathematical concepts.

This textbook series has been designed to introduce some of these models to teachers – perhaps for the first time, perhaps as a refresher – and to help teachers develop the expertise to implement these models effectively with children. While the approaches shared in these books are unique, they are also easily connected to more traditional strategies for teaching mathematics and for developing number sense. Toward that end, we hope they will be helpful resources for your teaching. In short, these books are designed with the hope that they will support teachers' content knowledge and pedagogical expertise toward the goal of providing a meaningful and powerful mathematics education for all children.

## How to Use this Book

This is not a typical textbook. While it does contain a number of activities for students, the intent of the book is to provide teachers with a wide variety of ideas and examples that might be used to further their ability and interest in teaching with ratio tables. The book contains ideas about how the ratio table can be used to multiply, to divide, to combine equivalent ratios, to model equivalent fractions, and to structure informal mental math strategies, among other things. Each chapter has a blend of teaching ideas, mathematical ideas, examples, and specific problems for children to engage as they learn to use the ratio table as a mathematical structure.

So in the traditional sense, this is not a "fifth grade" book, for example. Ideas have been divided into several themes – strategies for appropriate use of the ratio table, as well as specific applications of the ratio table toward common mathematical tasks.

We hope that teachers will apply their own expertise and craft knowledge to these explanations and activities to make them relevant, appropriate (and better!) in the context of their own classrooms. In many cases, a lesson may be extended to a higher grade level, or perhaps modified for use with students who may need additional support. Ideas toward those pedagogical adaptations are provided throughout.

## Book Chapters and Content

This book is divided into five sections. The first chapter, *Mental Math and the Ratio Table*, provides the operational building blocks upon which successful work with the ratio table may begin. This chapter introduces students to the key mental strategies they may employ to use ratio tables effectively.

The second chapter, *Multiplication with the Ratio Table*, illustrates how children may complete two and three-digit multiplication problems with the ratio table. Building on common strategies learned in Chapter 1 (e.g., multiplying by 10, doubling, halving, etc.), students develop powerful techniques with the ratio table that they may choose as an alternative to the traditional multiplication algorithm. The benefit of the ratio table as a computational tool is its transparency, as well as its fundamental link to the very nature of multiplication. Multiplication is often described to young learners as repeated addition. Yet, this simple message is often clouded when students learn the traditional multiplication algorithm. A young learner would be hard pressed to recognize the link between the traditional algorithm and "repeated addition" as they split numbers, "put down the zero", "carry", and follow other steps in the standard algorithm that, in truth, hide the very simple multiplicative principle of repeated addition. In contrast, the ratio table builds fundamentally on the idea of "groups of…" and "repeated addition." With time and practice, students develop remarkably efficient and effective problem solving strategies to multiply two and three-digit numbers with both accuracy, ***and*** with conceptual understanding.

The third chapter, *Division with the Ratio Table*, similarly provides an alternative to the traditional, long division algorithm. Building on the intuitive computational strategies developed in Chapter 1, and drawing on students' intuitions about the nature of division as a "fair share," Chapter 3 provides students with a method for dividing that allows them to make meaning of the process in a way that is not often available with the traditional long division algorithm. Ratio table division uses a process very similar to how a child might multiply with the ratio table. Hence, another advantage of the ratio table as a computational tool is its similar application across problem types. While a young student would be hard pressed to see connections between the long division algorithm and the multi-digit multiplication algorithm, multiplication and division with the ratio table are very similar processes. In the case of multiplication, students build groups to find an answer. For example, "I need 20 groups of 12. If 1 group is 12, 10 groups of 12 would be 120. I need 20 groups of 12, which is twice as much as 10 groups of 12, or… 240." In the case of division, students work in the opposite direction, even as they apply similar thinking. For example, the question might be rephrased, "How many groups of 12 are there in 240?" Students use ratio table strategies to build groups of 12 until they arrive at a total of 240.

The fourth chapter of the book, *Fractions and the Ratio Table*, introduces the ratio table as a tool for working with, and understanding, fractions. Of particular interest is the way in which the ratio table can model equivalent fractions.

The final section of the book is an Appendix, which includes numerous problems intended to give students practice applying the ratio table in authentic problem solving contexts. These problem contexts vary in complexity and difficulty to meet the learning goals for a wide range of learners.

# The Ratio Table: An Overview

The mathematics education community has suffered over the years through unproductive debates about the relative merits of teaching for algorithmic proficiency, and teaching for conceptual understanding. The truth is that we need both. The growing popularity of the ratio table may be attributed to the fact that it serves as a conduit between these two schools of thought. On one hand, it is an excellent computational tool that, when understood well by students, can be used quickly, efficiently, and accurately to multiply and divide, calculate percentages, etc. On the other hand, the structure of the model itself promotes conceptual understanding and mathematical connections that are often missing in the standard algorithms that are many times followed by students with little understanding of why they work.

The ratio table is an extremely powerful tool. As you and your students use this book, you'll be able to provide many opportunities for your students to embrace ratio tables not only as a tool for calculations, but also as a *way of thinking* about mathematical relationships.

## WHAT IS A RATIO TABLE? AND, WHY SHOULD WE USE IT?

Consider the following problem:

> "Each week, a farmer sells his fruit at the market. There are 149 apples left in the bottom of the crate. The farmer must put them into boxes of 12 apples each. How many more boxes does he need?"

At first glance, this appears to be a division problem. The most common solution strategy for students trained in traditional mathematics classrooms in the U.S. would be to apply the long division algorithm to divide 149 by 12. This process produces an answer of 12, with a remainder of 5. One of the problems that our students have when encountering these kinds of contexts is that, although they can compute an answer with a remainder, they do not understand what the remainder in the problem *means*. As such, given this problem context, we can expect to see three common incorrect solutions offered by students who either do not have conceptual understanding of division, or possibly do not know how to apply the common division algorithm appropriately. Here are three common misconceptions.

***"Twelve boxes are needed for the apples."***

In this solution, our first student likely "threw away" the remainder in the problem – in this case, five additional apples that need to be put in a box.

***"The farmer needs 12 remainder 5 boxes."***

This student's solution suggests he does not understand the fundamental nature of division, particularly in cases where a remainder is found.

***"The farmer needs 12.42 boxes for the apples."***

Although this answer reflects appropriate application of the long division algorithm to this problem, it is clear that the student is not examining that result in the context of the given situation. One would not likely think in terms of .42 of a box.

Although these solutions may seem nonsensical, they occur all too often because students do not fundamentally understand what division means, or perhaps because they have been drilled in the algorithm without ever being asked to pause and consider whether the answer they obtain is reasonable.

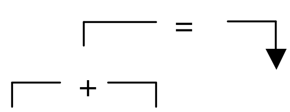
In contrast to these methods, a ratio table can be used to solve the same problem correctly, with a greater likelihood that the student will not fall into one of the common misconceptions described above. One powerful element of the ratio table is that it more clearly illustrates the close relationship between multiplication and division – two methods that are equally suited to solve this problem. Moreover, the very structure of the ratio table, and the thinking it elicits, make it less likely that the student would end up with a solution that contained partial boxes. Let's examine the problem again, modeling the thinking that might be used in conjunction with a ratio table to solve the problem with understanding.

> "Each week, a farmer sells his fruit at the market. There are 149 apples left in the bottom of the crate. The farmer must put them into boxes of 12 apples each. How many more boxes does he need?"

## A SOLUTION STRATEGY USING THE RATIO TABLE

"One box holds 12 apples." This beginning point is the foundation for the rest of the informal calculations that a student will make as he/she uses the ratio table as a computational strategy toward the solution of the problem. Given the foundational starting point, the child might easily be persuaded to think the following: "Well, if one box holds 12 apples, then two boxes must hold 24 apples. This thinking strategy, and the steps that follow from it, can be captured in a ratio table as shown below.

**Solution #1**




| | | | | | | |
|---|---|---|---|---|---|---|
| Boxes | 1 | 2 | 10 | 12 | **13** | |
| Apples | 12 | 24 | 120 | 144 | **156** | |

<u>Explanation</u>: In this solution, the student uses mental math to calculate combinations that are based on common numeric relationships. For example, if one box holds 12 apples, then two boxes hold 24. Similarly, 10 boxes would hold 120 apples (10x12). Finding the total number of apples in both sets of boxes (10 boxes of 12 apples, plus 2 more boxes of

12 apples) becomes a straightforward mental calculation: 120 + 24 = 144 apples. So… 12 boxes (10 boxes plus 2 boxes equals 144 apples) gets us close to the 149 apples that started in the crate. There are 5 apples remaining. Therefore, one more box is needed to hold all 149 apples.

The beauty of ratio tables is that students may choose to follow their own paths (based on strengths in their own number sense) to complete the computation. For example, a second method for finding the same solution might have been the following ratio table:

**Solution #2**

| | | | | | | |
|---|---|---|---|---|---|---|
| Boxes | 1 | 2 | 3 | 10 | **13** | |
| Apples | 12 | 24 | 36 | 120 | **156** | |

In this example, the student may have first computed the total number of apples that could be placed into three boxes by adding 12 each time. Then, in new step unrelated to the first three, she found the total number of apples that may be put into 10 boxes (12 x 10 = 120). Adding the results of these last two columns (3 boxes + 10 boxes = 13 boxes; 36 apples + 120 apples = 156 apples) allows the student to determine that 13 boxes would be ample to contain all 144 apples. It is easy to see in this example why students would be much less prone here to make the mistake of calling for 12.42 boxes to transport the apples.

## The Big Ideas: Mathematical Observations

Through this initial example, several essential mathematical ideas may be highlighted.

First, it should be noted that the ratio table ***allows students to take the lead*** in determining a solution strategy that resonates with their own strengths and mathematical intuitions. One student may require 6 columns to solve the problem, whereas a second student might take only 4 columns. Both strategies are entirely appropriate, which emphasizes (perhaps implicitly) that ***students may use more than one strategy*** to solve a problem with ratio tables.

Second, it should also be noted that the ***ratio table promotes mental math strategies in a way that resonates intuitively with students, given the structure of the ratio table itself.*** Some students develop comfort and sophistication with doubling and halving strategies. Some students rely heavily on multiplication by 10's. Some students prefer additive strategies rather than multiplicative ones. The point is that the ratio table fosters mental math strategies, but in a context and through a structure that supports the child's development of mathematical understanding.

Third, notice that each column in a ratio table is in fact a fraction (e.g., 1/12, 2/24), and ***each fraction within a ratio table is an equivalent representation of the same ratio***.

That is, 1/12 = 2/24 = 10/120, and so on. Hence, the ratio table is a wonderful model for promoting fraction equivalency, and those related skills that require ***conceptual understanding*** of fractions. It is crucially important to take advantage of the ratio table to help children understand equivalent fractions, particularly as it relates to fraction addition and subtraction. We have all seen the young child add ½ to ¼, and arrive at an answer of 2/6 (adding the numerators; adding the denominators) – a result that is smaller than one of the addends! By emphasizing fractions as *ratios*, students are much less likely to make that mistake, understanding intuitively, for example, that the “boxes” (the denominator) must be of the same size in order to make any reasonable conclusion about how many “apples” (the numerator) there are in a given number of boxes. In other words, one could determine how many apples are inside 8 full boxes only if he knows how many apples each box contains. If each box holds a different number of apples, we would be left to guess how many apples are contained in all the boxes when put together. Therefore, as students use addition and subtraction operations to complete the tables, they do so with an understanding that this ***addition or subtraction must be done proportionally***.

Through the activities in this book you will lead your students down a path toward deep understanding of the ratio table. Students will become familiar with the table itself – its design, structure, and ability to model multiplication and division problems. Students will also develop great mental math strategies to use the table efficiently – doubling, halving, adding, subtracting, multiplying by ten, etc. By the end of the book, students should be comfortable enough with the ratio table to apply it to widely varying mathematical contexts – each with different needs and objectives – with great success.

# Chapter 1
# Mental Math and the Ratio Table

As noted previously, one of the positive features of the ratio table is the extent to which it fosters students' mental math ability. There are several key arithmetic strategies that are essential to successful work with the ratio table. These strategies are highlighted below, followed by a series of activities that encourage the development of these mental math skills.

As you teach these methods, it is essential that students become comfortable with each of the following strategies. The ratio table can become an instrumental tool in helping children develop confidence with mental math strategies. This will occur, however, only if students are given ample opportunity to experiment with the ratio table, to "play" with different strategies, and to create their own pathways to solutions.

Though guidance is needed at first, eventually the students should take ownership of their chosen strategies. ***Teachers should not force students to use a prescribed set of steps with the ratio table,*** even if those steps are more efficient than the path being taken by the child. With time, students gravitate toward efficiency; they derive great satisfaction in determining their own solution strategy, and will naturally seek to complete the table with as few steps as possible. ***Pushing them toward efficiency too early will stunt the development of their native mathematical intuitions and flexibility with mental math strategies.***

## STRATEGY 1: MULTIPLICATION BY 10

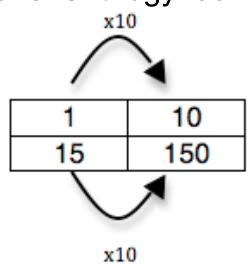
When learning the times tables, most young students quickly gravitate to the ten-facts, noting in particular the pattern that results with multiplication by ten:

4 x 10 = 40 (4 groups of 10 is 40)

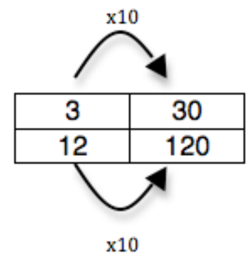
6 x 10 = 60 (6 groups of 10 is 60)

The number relationship inherent in multiplication by ten is comforting to young learners, and it becomes one of the most important and well-used strategies with ratio tables. In ratio table form, the "tens" strategy looks like the following:

Example 1:




Example 2:




Students should be encouraged to adopt the following language to accompany the mental strategy:

Example 1: *"If one group has 15, then 10 groups would have 150."*

Example 2: *"If 3 groups have 12, then 30 groups would have 120."*

## STRATEGY 2: MULTIPLICATION BY ANY NUMBER

Related to the 10's multiplication strategy above, students may also be led to recognize that multiplication by any factor is a viable ratio table strategy. For example:

x 3

| Boxes | 2 | 6 |
|---|---|---|
| Nails | 50 | 150 |

> Student thinking: "I know 2 x 3 = 6. So… I could also multiply 50 by 3: 50 x 3 = 150. That is … if 2 boxes contained 50 nails, then 6 boxes would contain 150 nails."

Quite readily, students realize that multiplication can be a helpful tool in quickly scaling any particular ratio. Moreover, recognize that students may choose various multipliers that they feel comfortable using in this process. Consider the following two examples as illustrations of these important considerations.

Example 1: There are 12 eggs in a carton.
How many eggs are there in 8 cartons?

x4 x 2

| Cartons | 1 | 4 | 8 |
|---|---|---|---|
| Eggs | 12 | 48 | 96 |

*To find the solution to this problem, the student first multiplied by 4, and then in a second step, multiplied by 2.*

Example 2: There are 5 pencils in a box.
How many pencils are in 18 boxes?

x 6 x 3

| Boxes | 1 | 6 | 18 |
|---|---|---|---|
| Pencils | 5 | 30 | 90 |

*To find the solution for this problem, the student first multiplied by 6, followed by multiplication by 3.*

Consider an alternative strategy for the same problem:

Example 3: There are 5 pencils in a box.
How many pencils are in 18 boxes?

| Boxes | 1 | 9 | 18 |
|---|---|---|---|
| Pencils | 5 | 45 | 90 |

*The strategy employed by this student was to first multiply by 9, and then multiply by 2.*

As these examples illustrate, multiplying both rows in a ratio table by the same factor preserves the ratio. Hence, students may multiply by any combination of factors to arrive at an intended solution. The beauty of the ratio table is that one student may choose a different set of factors to multiply than his/her peer. Students will select multipliers based on their comfort with various number facts. These multiple strategies provide opportunities for teachers and students to compare solutions, hence deepening their understanding of the problem, the use of ratio tables, and their own mathematical understanding.

## STRATEGY 3: DOUBLING

A particular subset of the multiplication strategy outlined above is the special case of "doubling" a column in a ratio table. The strategy of ***doubling*** is quite common among children and, once again, is a wonderful mental math strategy that will serve them well throughout their many mathematical explorations in contexts both in and outside of school. Emphasizing doubling as a key step in the use of ratio tables will serve children well. Consider the following example:

Doubling Example 1: I get paid \$8 per hour. How much did I earn after working 20 hours?

Doubling Strategy

| | | | | | | |
|---|---|---|---|---|---|---|
| Hours | 1 | 2 | 4 | 5 | 10 | 20 |
| Wages | 8 | 16 | 32 | 40 | 80 | 160 |

*A "doubling" strategy is used repeatedly toward the solution for this problem. Each of the arrows above the table indicates that a doubling calculation between adjacent columns has taken place. On the top row, then, the student has completed the following doubles facts: 1 x 2 = 2; 2 x 2 = 4; 5 x 2 = 10; 10 x 2 = 20. Across the bottom row, we see the following numbers were likewise doubled: 8 to 16; 16 to 32; 40 to 80; and 80 to 160. Doubling is a powerful strategy with wide reaching application and, again, easily motivated and developed through the use of ratio tables.*

## STRATEGY 4: HALVING

The inverse operation to doubling the entries in one column to another is to "***halve***" a column. Again, this is a powerful mental math strategy that children use intuitively in many contexts. Given that ***halving*** makes a quantity smaller, it should be noted therefore that this halving strategy is often used to reduce quantities in a ratio table. Hence, as discussed later in the book, halving strategies are instrumental in using the ratio table for division. Consider the following example:

Halving Example 1: 120 new baseballs must be split evenly between 8 teams. How many new baseballs does each team get?

Halving Strategy

| | | | | |
|---|---|---|---|---|
| Teams | 8 | 4 | 2 | 1 |
| Baseballs | 120 | 60 | 30 | 15 |

A "halving" strategy is used repeatedly toward the solution for this problem. Each of the arrows above the table indicates that a **halving** calculation between adjacent columns has been performed. On the top row, then, the student has completed the following halving calculations: Half of 8 is 4; half of 4 is 2; half of 2 is 1. Likewise, across the bottom row we see the following calculations: half of 120 is 60; half of 60 is 30; half of 30 is 15. Therefore, each team receives 15 baseballs. **Halving** is a powerful strategy with wide-reaching application and, again, is easily motivated and developed through the use of ratio tables.

Be aware that halving strategies are not exclusive to division problems. Sometimes a student will use a halving strategy as part of a string of calculations toward a desired end. Consider the following example in which the student is trying to determine how many eggs are in 6 cartons:

| | | | | |
|---|---|---|---|---|
| Cartons | 1 | 10 | 5 | 6 |
| Eggs | 12 | 120 | 60 | 72 |

In the context of the solution strategy for this problem, the student has "halved" 10 to get 5, and subsequently taken half of 120 to arrive at 60. The student is trying to arrive at 6 groups of 12 (for a total of 72).

The thinking might go as follows: "I know that one group is 12. I am trying to find the total for 6 groups. Well, 10 groups would be 120. Half of that is 5 groups, or, 60. So, if 5 groups is 60, then one more group, six in all, would be 60 + 12 = 72 eggs." In this example we see how a halving strategy might very well be appropriately applied in a problem that is, by nature, multiplicative.

## STRATEGY 5: COMBINING COLUMNS (ADDING AND SUBTRACTING)

The final strategy consists of combining columns – either by addition, or by subtraction. The basic idea that we want children to understand is that we are, in a sense, "combining buckets." For example, imagine the following ratio table that indicates the number of cherries that may be found in various combinations of baskets (15 cherries per basket).

| | | | | | |
|---|---|---|---|---|---|
| Baskets | 1 | 10 | 2 | 12 | 8 |
| Cherries | 15 | 150 | 30 | 180 | 120 |

We begin with our initial ratio: 1 basket holds 15 cherries. Using our multiplication by ten strategy, our next column indicates that 10 baskets would therefore hold 150 cherries. The third column simply doubles the first column: if one basket holds 15 cherries, 2 baskets would hold 30.

Now, at this point, we are ready to combine columns. Observe the $4^{th}$ column. We see 12 in the top row, and 180 in the bottom row. How did we arrive at those figures? We did so by combining the previous columns. If 10 baskets contain 150 cherries, and 2 baskets hold an addition 30 cherries, then we could pool the two baskets together, arriving at 12 baskets (10 + 2), with 180 cherries (150 + 30).

10 + 2 = 12

\+ =

| | | | | | |
|---|---|---|---|---|---|
| Baskets | 1 | 10 | 2 | 12 | 8 |
| Cherries | 15 | 150 | 30 | 180 | 120 |

= +

150 + 30 = 180

Now, how do we arrive at the final column: 8 baskets contain 120 cherries? We use similar thinking. Instead of combining baskets of cherries, however, we subtract them. That is, returning to the column with 10 baskets and 150 cherries, if we took away two baskets, that would mean we would need to take away 30 cherries. So… we end up with 8 baskets (10 – 2 = 8), and 120 cherries (150 – 30 = 120). See the table below:

10 - 2 = 8

| | | | | | |
|---|---|---|---|---|---|
| Baskets | 1 | 10 | 2 | 12 | 8 |
| Cherries | 15 | 150 | 30 | 180 | 120 |

150 - 30 = 120

***We must be very cautious when introducing this strategy***. It is easy for students to develop a misconception regarding addition and subtraction of fractions. Consider the following ratio table.

| | | | | |
|---|---|---|---|---|
| students | 1 | 2 | | |
| # of feet | 2 | 4 | | |

In the third column, we might very well put in 3 on the top, and 6 on the bottom. In other words, if 1 student has 2 feet, and 2 students have 4 feet, we can add those respective amounts together, and arrive at the ratio 3:6 (3 students will have 6 feet). And yet, if we pulled these fractions outside of a ratio table, we do not want students to believe that 1/2 + 2/4 = 3/6. That is a nonsensical answer. If we had ½ of a candy bar, and we added another ½ (or the equivalent 2/4ths) of the candy bar, how could we end up with where we started: 3/6 (or one-half) of the candy bar? We would not have 3/6 (one-half) of a candy bar; rather, we would have the equivalent of a whole candy bar since we added ½ of the bar to another ½ of the bar.

The important concept to emphasize is that a ratio table, in its entirety for a given problem, is representing ***ratios***, and using proportional reasoning. Since each column in a ratio table is equivalent to every other column – that is, each column in the ratio table is the same fraction as every other column in the ratio table (e.g., 1/2 … 2/4 … 10/20 … 15/30), when we are combining columns, we are maintaining the existing ratio. ***We are not changing the value of the ratio***… we change the form of the ratio. Any new numbers we add in a column of a ratio table must preserve the given ratio. Hence, we manipulate the ratio table until we arrive at a column that expresses our ratio in terms that we desire.

## STRATEGY SUMMARY

In the previous pages, we have explored a number of unique strategies:

- Multiplying by 10
- Multiplying by any number
- Doubling
- Halving
- Adding
- Subtracting

While students will naturally gravitate to some strategies over others, it is important that they understand how the various strategies work, and how they may be used together to solve a given problem. Teachers should make every attempt to compare and share the various solution strategies used by students to solve the same problem. This is a powerful teaching methodology not only to emphasize how the ratio table works, but to reinforce the notion that there are always multiple ways to solve a problem, and that there will likely always be some student who is thinking about the problem in a unique way, employing a unique strategy. ***Encourage this diversity of strategy use***. **Be sure to regularly model multiple approaches to a given problem**. This will pay dividends down the road. As students begin to trust their own intuitions and native strategies, they develop an expanded vision of mathematical thinking, of their own efficacy as young mathematicians, of the ratio table itself as a mathematical tool, and powerful mental math strategies.

The following Student Activity Sheets provide numerous practice problems for students.

**Activity Sheet 1**: Using Multiplication by 10 to Solve a Ratio Table
**Activity Sheet 2**: Using Multiplication by Any Factor to Solve a Ratio Table
**Activity Sheet 3**: Using Doubling Strategies to Solve a Ratio Table
**Activity Sheet 4**: Using Halving Strategies to Solve a Ratio Table
**Activity Sheet 5**: Using Addition to Solve a Ratio Table
**Activity Sheet 6**: Using Subtraction to Solve a Ratio Table
**Activity Sheet 7**: Solving Problems with Ratio Tables
**Activity Sheet 8**: Solving Problems with Ratio Tables
**Activity Sheet 9**: Solving Problems with Ratio Tables
**Activity Sheet 10**: Ratio Table Strategy Review

Name: ___________________________

**Activity Sheet 1**: Using Multiplication by 10

**Directions**: Solve each problem with a ratio table. Use the strategy of multiplying by 10. You may not need to use every column in the ratio table.

**Example**: There are 5 pieces of gum in a pack. How many pieces of gum are in 10 packs?

x 10

| | | | | |
|---|---|---|---|---|
| Pack | 1 | 10 | | |
| Pieces | 5 | 50 | | |

x 10

1) There are 6 desks per row. How many desks are there in 10 rows?

| | | | | |
|---|---|---|---|---|
| Rows | 1 | 10 | | |
| Desks | 6 | | | |

2) There are 3 birds per nest. How many birds are there in 10 nests?

| | | | | |
|---|---|---|---|---|
| Nests | 1 | | | |
| Birds | 3 | | | |

3) There are 4 students per table. How many students are there at 10 tables?

| | | | | |
|---|---|---|---|---|
| Tables | | | | |
| Students | | | | |

4) Two trays contained 12 ice cubes. How many ice cubes are there in 20 trays?

| | | | | |
|---|---|---|---|---|
| Trays | 2 | | | |
| Ice Cubes | 12 | | | |

5) Jon gets paid $4 for every 3 hours he works. How much will he get paid if he works 30 hours?

| | | | | | |
|---|---|---|---|---|---|
| Hours worked | | | | | |
| Dollars earned | | | | | |

Name: ___________________________

**Activity Sheet 2**: Using Multiplication by Any Number

**Directions**: Solve each problem with a ratio table. Use multiplication to help you arrive at the answer. You may not need to use every column in the ratio table.

**Example**: There are 5 apples in each bag. How many apples are in 16 bags?

x 2 x 4 x 2

| | | | | |
|---|---|---|---|---|
| Bags | 1 | 2 | 8 | 16 |
| Apples | 5 | 10 | 40 | 80 |

x 2 x 4 x 2

1) There are 4 chairs per table. How many chairs are there for 20 tables?

| | | | | | | |
|---|---|---|---|---|---|---|
| Tables | 1 | | | | | |
| Chairs | 4 | | | | | |

2) There are 12 eggs in every carton. How many eggs are there in 8 cartons?

| | | | | | | |
|---|---|---|---|---|---|---|
| Cartons | 1 | | | | | |
| Eggs | 4 | | | | | |

3) Three people can fit in each rowboat. How many people can fit into 12 rowboats?

| | | | | | | |
|---|---|---|---|---|---|---|
| Rowboats | | | | | | |
| People | | | | | | |

4) You can buy 8 balloons for $1. How many balloons can you buy for $8?

Here is Melia's solution strategy:

| | | | | | | |
|---|---|---|---|---|---|---|
| Dollars | 1 | 2 | 4 | 8 | | |
| Balloons | 8 | 16 | 32 | 64 | | |

Explain Melia's Strategy

Melia's strategy:

Now… solve this problem in a ***different*** way.

| | | | | | | |
|---|---|---|---|---|---|---|
| Dollars | | | | | | |
| Balloons | | | | | | |

Name: ________________________

**Activity Sheet 3**: Using the Doubling Strategy

**Directions**: Solve each problem with a ratio table. Use “doubles” to help you arrive at the answer. You may not need to use every column in the ratio table.

**Example**: Gasoline costs $4 per gallon. How much does it cost to buy 8 gallons?

x 2 x 2 x 2

| | | | | |
|---|---|---|---|---|
| Gallons | 1 | 2 | 4 | 8 |
| Dollars | 4 | 8 | 16 | 32 |

x 2 x 2 x 2

1) Each student has 2 shoes. How many shoes are there for 8 students?

| | | | | | | |
|---|---|---|---|---|---|---|
| Students | 1 | | | | | |
| Shoes | 2 | | | | | |

2) There are 12 eggs in every carton. How many eggs are there in 4 cartons?

| | | | | | | |
|---|---|---|---|---|---|---|
| Cartons | 1 | | | | | |
| Eggs | 12 | | | | | |

3) There are 8 M&M’s in every mini-bag. How many M&M’s are there in 16 mini-bags?

| | | | | | | |
|---|---|---|---|---|---|---|
| mini-bags | 1 | | | | | |
| M&M’s | 8 | | | | | |

4) There are 6 chairs for every 3 tables. How many chairs are there for 12 tables?

| | | | | | | |
|---|---|---|---|---|---|---|
| Tables | 3 | | | | | |
| Chairs | 6 | | | | | |

5) 5 students need 20 crayons. How many crayons do 40 students need?

| | | | | | | |
|---|---|---|---|---|---|---|
| Students | | | | | | |
| Crayons | | | | | | |

Name: ___________________________

**Activity Sheet 4**: Using the Halving Strategy

**Directions**: Solve each problem with a ratio table. Use "halves" to help you arrive at the answer. You may not need to use every column in the ratio table.

**Example**: 10 bottles of juice cost $20. How much does it cost for 5 bottles of juice?

÷2

| | | | | |
|---|---|---|---|---|
| Bottles | 10 | 5 | | |
| Cost | $20 | $10 | | |

÷ 2

1) It takes 10 hours to ride a bike 40 miles. How far can you ride in 5 hours?

| | | | | | | |
|---|---|---|---|---|---|---|
| Hours | 10 | | | | | |
| Miles | 40 | | | | | |

2) 60 eggs fit into 4 baskets. How many eggs fit into one basket?

| | | | | | | |
|---|---|---|---|---|---|---|
| Baskets | 4 | | | | | |
| Eggs | 60 | | | | | |

3) You can buy 12 apples for $4. How much does it cost to buy 6 apples?

| | | | | | | |
|---|---|---|---|---|---|---|
| Apples | | | | | | |
| Cost | | | | | | |

4) You can buy 20 oranges for $6. How much would it cost to buy 5 oranges?

| | | | | | | |
|---|---|---|---|---|---|---|
| Oranges | 20 | | | | | |
| Cost | | | | | | |

5) 32 students need to sell 80 raffle tickets. If the students split it up evenly, how many tickets does each student need to sell on his or her own?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Students | 32 | | | | | | |
| Tickets to sell | 80 | | | | | | |

Name: ___________________________

**Activity Sheet 5**: Using Addition

**Directions**: Solve each problem with a ratio table. Use addition across columns to help you arrive at the answer. You may need to use some other strategies as well (like doubling, multiplication, etc.). You may not need to use every column in the ratio table.

**Example**: 1 bottle of juice costs $4. How much does it cost for 3 bottles of juice?

| | | | | |
|---|---|---|---|---|
| Bottles | 1 | 2 | 3 | |
| Cost | $4 | $8 | $12 | |

1 + 2 = 3

$4 + $8 = $12

1) It takes 1 minute to travel 2 miles on the high speed train. How many miles can you travel in 12 minutes?

| | | | | | | |
|---|---|---|---|---|---|---|
| Minute | 1 | 2 | 10 | 12 | | |
| Miles | 2 | 4 | | | | |

2) It takes 2 minutes to run 1 lap around the track. How long would it take to run 5 laps?

| | | | | | | |
|---|---|---|---|---|---|---|
| Laps | 1 | 2 | 3 | 5 | | |
| Minutes | 2 | | | | | |

6) One t-shirt costs $6. 3 shirts cost $18. How much does it cost to buy 4 shirts?

| | | | | | | |
|---|---|---|---|---|---|---|
| Shirts | 1 | 3 | 4 | | | |
| Cost | $6 | $18 | | | | |

7) You can buy 10 cherry tomatoes for $6. How much would it cost to buy 15 tomatoes?

| | | | | | | |
|---|---|---|---|---|---|---|
| Tomatoes | 10 | 5 | | | | |
| Cost | $6 | | | | | |

8) Each player on the basketball team needs has to shoot 5 free-throw shots at practice. There are 12 players on the team. How many shots in all will be taken by the team?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Players | | | | | | | |
| Free throws | | | | | | | |

Name: __________________________

**Activity Sheet 6**: Using Subtraction

**Directions**: Solve each problem with a ratio table. Use subtraction across two columns to help you arrive at the answer. You may need to use some other strategies as well (like doubling, multiplication, etc.). You may not need to use every column in the ratio table.

**Example**: 1 bottle of medicine costs $4. How much does it cost for 9 bottles?

| | | | | |
|---|---|---|---|---|
| Bottles | 1 | 10 | 9 | |
| Cost | $4 | $40 | $36 | |

10 - 1 = 9

$40 - $4 = $36

1) It takes 1 minute to pump 2 gallons of gas. How many minutes does it take to pump 19 gallons of gas?

| | | | | | | |
|---|---|---|---|---|---|---|
| Minutes | 1 | 10 | 20 | 19 | | |
| Gallons | 2 | 4 | | | | |

2) Each pizza contains 6 slices. How many slices in 8 pizzas?

| | | | | | | |
|---|---|---|---|---|---|---|
| Pizzas | 1 | 2 | 10 | 8 | | |
| Slices | 6 | 12 | | | | |

3) In a big storm, rain fell at the rate of 2 centimeters every hour for an entire day. How much rain had fallen after 18 hours?

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Hours | 1 | 10 | 20 | 2 | 18 | | | |
| Centimeters | 2 | | | | | | | |

4) One baseball cap costs $5. How much would 14 caps cost?

| | | | | | | |
|---|---|---|---|---|---|---|
| Caps | 1 | 10 | 5 | 15 | 14 | |
| Cost | $5 | $50 | | | | |

5) You can buy 10 bananas for $6. Therefore, 50 bananas cost $30. How much would it cost to buy 45 bananas?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Bananas | | | | | | | |
| Cost | | | | | | | |

Name: ________________________

**Activity Sheet 7**: Solving Problems with Ratio Tables

*Sammy is helping his parents plant a garden. He goes to the store and finds that corn seeds come in small packs. Each pack of seeds contains 12 seeds. How many seeds will he get if he buys 6 packs? He solves the problem 4 different ways.*

Solution Strategy #1

| Packs | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Seeds | 12 | 24 | 36 | 48 | 60 | 72 |

Explain Sammy's strategy? How did he use the ratio table to solve the problem?

Solution Strategy #2

| Packs | 1 | 2 | 3 | 6 |
|---|---|---|---|---|
| Seeds | 12 | 24 | 36 | 72 |

Explain Sammy's strategy? How did he use the ratio table to solve the problem?

Solution Strategy #3

| Packs | 1 | 10 | 5 | 6 |
|---|---|---|---|---|
| Seeds | 12 | 120 | 60 | 72 |

Explain Sammy's strategy? How did he use the ratio table to solve the problem?

Solution Strategy #4

| Packs | 1 | 2 | 4 | 8 | 6 |
|---|---|---|---|---|---|
| Seeds | 12 | 24 | 48 | 96 | 72 |

Explain Sammy's strategy? How did he use the ratio table to solve the problem?

Now YOU solve the problem, using any strategy you prefer.

*Sammy is helping his parents plant a garden. He goes to the store and finds that corn seeds come in small packs. Each pack of seeds contains 12 seeds. How many seeds will he get if he buys 6 packs?*

| Packs | 1 | | | | | |
|---|---|---|---|---|---|---|
| Seeds | 12 | | | | | |

Explain your strategy.

Why did you choose the strategy that you did?

Name: ________________________

**Activity Sheet 8**: Solving Problems with Ratio Tables

Problem: Silver City Middle School needs a new gym floor. The gym floor tiles come in boxes of 45 tiles per box. Mr. Sheffield, the gym teacher, ordered 16 boxes of tiles. Three students used a ratio table to find out how many tiles were in all 16 boxes.

Becky solved the problem this way:

| Boxes | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 16 |
|---|---|---|---|---|---|---|---|---|---|
| Tiles | 45 | 90 | 135 | 180 | 225 | 270 | 315 | 360 | 720 |

Explain Becky's thinking. How did Becky use the ratio table?

Ann solved the problem this way:

| Boxes | 1 | 2 | 4 | 8 | 16 |
|---|---|---|---|---|---|
| Tiles | 45 | 90 | 180 | 360 | 720 |

Explain Ann's thinking. How did Ann use the ratio table?

Brian solved the problem this way:

| Boxes | 1 | 10 | 2 | 6 | 16 |
|---|---|---|---|---|---|
| Tiles | 45 | 450 | 90 | 270 | 720 |

Explain Brian's thinking. How did Brian use the ratio table?

**Question**: Which of these strategies do you like the best? Why?

Name: ___________________________

**Activity Sheet 9**: Solving Problems with Ratio Tables

Directions: Use a ratio table to solve the following problems. Use any combination of strategies you'd like to use. You may not need all the columns in the tables provided. Be prepared to share and explain your strategy with a partner.

1. A tube contains 3 tennis balls. How many tennis balls are there in 32 tubes?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Tube | | | | | | | |
| Tennis Balls | | | | | | | |

2. Sunglasses cost $8. How much will it cost to buy 12 sunglasses?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Sunglasses | | | | | | | |
| Cost | | | | | | | |

3. At the market, 2 tomatoes sell for $3. How much would it cost to buy 12 tomatoes?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| | | | | | | | |

4. At the zoo, the lions eat 25 pounds of meat a day. How much would they eat in one week (7 days)?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| | | | | | | | |

5. At the same zoo, 50 visitors are allowed into the monkey exhibit each hour. How many visitors can see the monkeys in a day? The zoo is open for 8 hours each day.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| | | | | | | | |

6. It costs $12 per student to get into the zoo. How much would it cost for a group of 21 students to visit the zoo?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| | | | | | | | |

7. Every 4 hours, the doctor sees 8 patients. How many patients does the doctor see after 11 hours?

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Hours | | | | | | | | | |
| Patients Seen | | | | | | | | | |

Name: ___________________________

**Activity Sheet 10**: Ratio Table Strategy Review

You have solved many problems in this book with ratio tables. You probably used familiar strategies like these:

**Multiply by 10**

| | |
|---|---|
| 1 | 10 |
| 15 | 150 |

**Multiplying**

| | |
|---|---|
| 2 | 6 |
| 50 | 150 |

**Doubling**

| | |
|---|---|
| 4 | 8 |
| 15 | 30 |

**Halving**

| | |
|---|---|
| 20 | 10 |
| 30 | 15 |

**Adding**

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 25 | 50 | 75 |

**Subtracting**

| | | |
|---|---|---|
| 1 | 10 | 9 |
| 12 | 120 | 108 |

Find the missing numbers in the shaded boxes in the ratio tables below. Then write which of the above strategies you used.

1.

| | | |
|---|---|---|
| 1 | 10 | 9 |
| 18 | 180 | |

Strategy:______________________

2.

| | | |
|---|---|---|
| 1 | 10 | 5 |
| 18 | 180 | |

Strategy:______________________

3.

| | | |
|---|---|---|
| 1 | 10 | 20 |
| 12 | 120 | |

Strategy:______________________

4.

| | |
|---|---|
| 2 | 12 |
| 8 | |

Strategy:______________________

5.

| | |
|---|---|
| 4 | 40 |
| 18 | |

Strategy:______________________

6.

| | | |
|---|---|---|
| 2 | 10 | 12 |
| 8 | 40 | |

Strategy:______________________

7.

| | |
|---|---|
| 4 | 8 |
| 18 | |

Strategy:______________________

8.

| | |
|---|---|
| 12 | |
| 8 | 4 |

Strategy:______________________

# Chapter 2
# Multi-Digit Multiplication with the Ratio Table

In the previous chapter, single-digit multiplication was introduced as a strategy for solving problems that require proportional reasoning. The very same thinking that allows a child to use single-digit multiplication to solve these sorts of problems can be used equally effectively with larger numbers.

For example, recall this problem from a previous section:

**Example**: There are 5 apples in each bag. How many apples are in 16 bags?

x 2 x 3 x 4

| Bags | 1 | 2 | 6 | 24 |
|---|---|---|---|---|
| Apples | 5 | 10 | 30 | 120 |

x 2 x 3 x 4

In this problem, students may follow reasoning such as the following:

**Student**: "Well, let's see. I know that there are 5 apples in every bag. If I have 2 bags, then I must have (2 x 5) = 10 apples. To get to 6 bags, I would multiply by 3. So, 6 bags would be (10 x 3) = 30 apples."

This very reasoning is the key to using ratio tables for multi-digit multiplication. Consider the following problem, and the mathematical thinking that it elicits as a student uses the ratio table to determine a solution.

Problem: 22 x 35 = ? → Solution Strategy

| Groups | 1 | 2 | 20 | 22 |
|---|---|---|---|---|
| Total | 35 | 70 | 700 | 770 |

The accompanying student reasoning for the solution strategy shown in this ratio table (of course, there would be other perfectly appropriate paths of reasoning as well), would be the following:

Step 1 (column 1): I need 22 groups of 35. I will **start with 1 group** of 35.
Step 2 (column 2): Now, I double. If one group is 35, **two groups would be** 70.
Step 3 (column 3): Now, I multiply by 10. If two groups is 70, **then 20 groups would be** (70 x 10) = 700.
Step 4 (column 4): Now I can add what I already have. I know that 2 groups is 70. I know that 20 groups is 700. Therefore, 22 groups would be 770. So, 22 times 35 is 770.

**Consider a second example: 13 x 41 = ?**

Ratio Table Solution: I need to find **13 groups of 41**.

| Groups | 1 | 2 | 3 | 10 | 13 |
|---|---|---|---|---|---|
| Total | 41 | 82 | 123 | 410 | 533 |

Step 1: One group is 41.
Step 2: Two groups is 82.
Step 3: Three groups is (82+41) = 123.
Step 4: Ten groups of 41 would be (10 x 41) = 410.
Step 5: To get 13 groups, I add what I got for 10 groups (410) to what I got for 3 groups (123). So… 13 x 41 = 533.

With moderate practice, using this strategy can be every bit as automatic and preferred by students as the traditional method. Moreover, there are reasons to believe that this method might actually promote a deeper level of mathematical understanding than the traditional model typically taught in our schools.

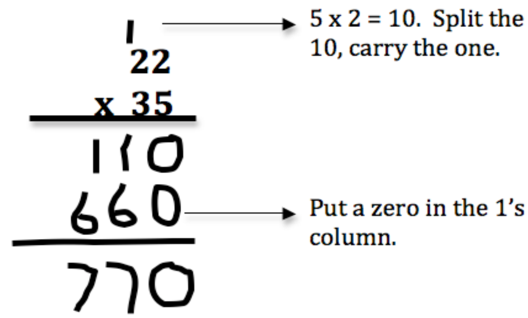
***Consider the differences*** between the two strategies on the first problem highlighted above:

Problem: 22 x 35 = ?

Ratio Table Strategy

| Groups | 1 | 2 | 20 | 22 |
|---|---|---|---|---|
| Total | 35 | 70 | 700 | 770 |

Traditional Method




One can argue that both methods can be used quickly and efficiently by students. The question to linger upon, however, is the degree to which one of these methods might promote a higher level of mathematical understanding about the nature of mathematics, and multiplication in particular. Research has suggested that the traditional multiplication algorithm, though a process that students can replicate effectively, can be difficult for young learners to ***comprehend***. "Carrying the one"… "put down a zero" … separating the ones from the tens… Understanding these steps can be a challenge for students. Make no mistake – students can ***learn*** these steps, and perform multi-digit multiplication effectively with the traditional model. The question remains, however, do they fundamentally ***understand*** what they have done?

***In contrast***, one might argue that the thinking that goes into the use of the ratio table as a method for multi-digit multiplication is much more transparent. For example: "I need 22 groups of 35. Let me start with on group of 35. Two groups would be 70. 20 groups would be 700. 22 groups would be 770." That is, the thinking reflected in the appropriate use of a ratio table is *multiplicative in nature.* If multiplication is to be understood as "repeated addition" as it is often described to young learners, then the ratio table is a model that directly reflects the repeated addition necessary to solve any given multi-digit multiplication problem.

There is an additional, compelling reason to promote the use of ratio tables with young children. Whereas the traditional multiplication algorithm requires a very specific and sequenced set of steps, the ratio table can be used flexibly to solve a problem in multiple ways. There is widespread agreement among mathematics educators that children who can solve problems in more than one way are likely to have a more nuanced, if not deeper (and lasting), understanding of the problem at hand when compared to the student who can solve a problem with only one strategy. Consider, for example, the three very different solution strategies a teacher might see from students on the following multi-digit multiplication problem: 39 x 44 = ?

Strategy #1

| | | | | | |
|---|---|---|---|---|---|
| Groups | 1 | 10 | 20 | 40 | 39 |
| Total | 44 | 440 | 880 | 1760 | 1716 |

x 10 **Double** Double **Subtract 1 group of 44**

Strategy #2

| | | | | | |
|---|---|---|---|---|---|
| Groups | 1 | 2 | 4 | 40 | 39 |
| Total | 44 | 88 | 176 | 1760 | 1716 |

**x 2** x 2 **x 10** **Subtract 1 group of 44**

Strategy #3

**3 x 10**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Groups | 1 | 2 | 3 | 4 | 5 | 30 | 35 | 39 |
| Total | 44 | 88 | 132 | 176 | 220 | 1320 | 1540 | 1716 |

**Four times... add 1 group of 44** **132 x 10** **Add 5 groups** **Add 4 groups**

As one might observe, in each case, students arrived at the correct answer, though each employed a different solution strategy. One must be careful when using ratio tables to allow students as many column calculations as necessary to solve the problem. It might be easy to assume that since Strategy #3 required more steps than the previous two strategies, it is less sophisticated. We must avoid this faulty assumption. The student that completed strategy #3 has evidenced not only conceptual understanding of the problem, but also has used a rich set of mental strategies to arrive at the correct answer. One strength of the ratio table, then, is its ability to elicit multiple solution paths.

The following activity sheets contain a number of practice problems that will help students develop confidence with the ratio table as a computational model for multi-digit multiplication.

Name: ___________________________

**Activity Sheet 11**: Solving Multi-Digit Multiplication Problems with the Ratio Table

**Directions**: Use a ratio table to solve the following problems. Use any combination of strategies that make sense to you. ***Some*** possible steps have been started for you. (You may use other steps too.) Be prepared to share and explain your strategy with a partner.

1. What is the answer to the following problem: 12 x 12?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Groups | 1 | 2 | 10 | 12 | | | |
| Total | 12 | | | | | | |

2. What is the answer to the following problem: 13 x 14?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Groups | 1 | 10 | 5 | 15 | 14 | 13 | |
| Total | 14 | | | | | | |

3. What is the answer to the following problem: 20 x 32?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Groups | 1 | 10 | 20 | | | | |
| Total | | | | | | | |

4. What is the answer to the following problem: 15 x 19?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Groups | | 10 | 5 | 15 | | | |
| Total | 19 | | | | | | |

5. What is the answer to the following problem: 23 x 41?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Groups | 1 | 10 | 20 | 2 | 3 | 23 | |
| Total | | 410 | | 82 | | | |

6. What is the answer to the following problem: 15 x 16?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Groups | | 10 | | 15 | | | |
| Total | 16 | | 80 | | | | |

7. What is the answer to the following problem: 21 x 33? (or … 33 x 21)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Groups | 1 | 10 | 20 | 21 | | | |
| Total | 33 | | | | | | |

8. What is the answer to the following problem: 41 x 31?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Groups | 1 | 10 | 20 | 30 | 31 | | |
| Total | 41 | | | | | | |

Name: ___________________________

**Activity Sheet 12**: Solving Multi-Digit Multiplication Problems with the Ratio Table

**Directions**: Use a ratio table to solve the following problems. Use any combination of strategies you'd like to use. You may not need all the columns in the tables provided. Be prepared to share and explain your strategy with a partner.

1. What is the answer to the following problem: 12 x 23?

| Groups | 1 | 2 | 10 | 12 | | | |
|---|---|---|---|---|---|---|---|
| Total | 23 | | | | | | |

2. What is the answer to the following problem: 14 x 14?

| Groups | | | | | | | |
|---|---|---|---|---|---|---|---|
| Total | | | | | | | |

3. What is the answer to the following problem: 22 x 24?

| Groups | | | | | | | |
|---|---|---|---|---|---|---|---|
| Total | | | | | | | |

4. What is the answer to the following problem: 16 x 16?

| Groups | | | | | | | |
|---|---|---|---|---|---|---|---|
| Total | | | | | | | |

5. What is the answer to the following problem: 31 x 9?

| Groups | | | | | | | |
|---|---|---|---|---|---|---|---|
| Total | | | | | | | |

6. What is the answer to the following problem: 13 x 41?

| Groups | | | | | | | |
|---|---|---|---|---|---|---|---|
| Total | | | | | | | |

7. What is the answer to the following problem: 21 x 61?

| Groups | | | | | | | |
|---|---|---|---|---|---|---|---|
| Total | | | | | | | |

8. What is the answer to the following problem: 9 x 104?

| Groups | | | | | | | |
|---|---|---|---|---|---|---|---|
| Total | | | | | | | |

Name: ___________________________

**Activity Sheet 13**: Solving Multi-Digit Multiplication Problems

**Directions**:  Use a any method you prefer to solve the following multiplication problems. Be prepared to share and explain your strategy with a partner.

1. 12 x 23 =

2. 13 x 31 =

3. 21 x 21 =

4. 19 x 20 =

5. 18 x 18 =

6. 32 x 51 =

# Chapter 3
# Division with the Ratio Table

In the previous chapter, the ratio table was introduced not only as a ***viable*** strategy for multiplication, but perhaps the most mathematically ***intuitive*** strategy for young learners to use when understanding multiplication as "*repeated addition*." A similar case may be made for the operation of division.

There is no question among mathematics educators that the traditional long division method is the most challenging algorithm for young students to comprehend and use. There are multiple steps in the algorithm that, while efficient, are nonetheless very difficult for children to understand. Hence, because they often do not understand the algorithm itself, it is often the case that at the end of the problem, children do not know whether the answer they obtained is correct, or even reasonable. The traditional long division algorithm, while efficient, masks the very nature of division.

Similar to the case that was made for multi-digit multiplication, the ratio table can be used to solve division problems, ***with imbedded meaning***. Please consider several examples.

**<u>The Traditional Method: Long Division</u>**

Consider the methodology applied when using the long division algorithm to solve the following problem: 256 ÷ 16

| | |
|---|---|
| **16**<br>**16 ⟌ 256**<br>**-<u>16</u>**<br>**96**<br>**<u>- 96</u>**<br>**0** | Student thinking, step by step…<br><br>Step 1: How many times does 16 go into 25? Ans: 1<br>Step 2: Place the 1 on top of the 5.<br>Step 3. Multiply 1 x 16, and put the answer under the 25.<br>Step 4: Subtract: 25 – 16 = 9<br>Step 5: Put the 9 under the 6.<br>Step 6: "Bring down" the 6.<br>Step 7: Think: How many times does 16 go into 96? Ans: 6<br>Step 8: Write the 6 next to the 1 on the top line.<br>Step 9: Multiply 6 x 16.<br>Step 10: Write the 96 under the 96.<br>Step 11: Subtract 96 – 96.<br>Step 12: Write 0 under the 96.<br><br>Problem Completed. Answer: 256 ÷ 16 = 16. |

As adults, and after using the traditional algorithm many times, we have become accustomed to its various steps, and can likely reproduce them quickly and with confidence. Imagine, however, how a young child would come to ***understand*** each of the steps outlined in the table above – what mathematical significance they have, and why we might want to complete them. Indeed, the traditional method, though efficient and trustworthy (upon many repetitions), is very challenging for most young children to fully comprehend.

In contrast, imagine if we were to teach children the real meaning of division. Consider the previous example. The statement "256 ÷ 16" ***means*** … "How many groups of 16 are there in 256?"

Once children develop this understanding of division as a parallel operation to multiplication, the very same thinking that has been applied in the earlier use of ratio tables will again serve students well.

To solve this problem with a ratio table, only one small shift in thinking needs to occur (when compared to multiplication and ratio tables). Specifically, the difference is that, instead of seeking the answer (i.e., the "total" which may be found in the bottom row of the last column), we are seeing the **final number of groups necessary to obtain** the given answer. Consider the following ratio table for the problem: 256 ÷ 16.

| **Number of Groups** | 1 | | | | **??** |
|---|---|---|---|---|---|
| **Total** | 16 | | | | 256 |

We start in the same fashion… 1 group is 16.

Now… division asks a different question than multiplication. We already know our final answer. What we need to find out is <u>how many groups of 16 ***are required*** to obtain 256?</u>

Once this subtle shift in understanding is obtained, then students may use the very same strategies they have become comfortable with in previous work with the ratio table. One strategy that a student might use to solve the problem would be the following:

| **Number of Groups** | 1 | 10 | 5 | 15 | **16** |
|---|---|---|---|---|---|
| **Total** | 16 | 160 | 80 | 240 | 256 |

By the time students are ready to use the ratio table for division, they should be very comfortable with each of the steps used above: multiplying by 10, dividing by 2, and adding groups of 16.

Consider another example: 576 ÷ 18 = ?

A ratio table can be used to solve this division problem through the following steps.

Solution Strategy: 576 ÷ 18

1) 576 ÷ 18 … **This means**: "How many groups of 18 are there in 576?"
2) I can start with 1 group of 18. Now I need to find out how many more groups of 18 are necessary to arrive at a total of 576.
3) I begin by recording that 1 group equals 18. My **target** is 576.

| **Number of Groups** | 1 | | | | | ? |
|---|---|---|---|---|---|---|
| **Total** | 18 | | | | | 576 |

4) Using common ratio table strategies, I work my way toward a total of 576.

| **Number of Groups** | 1 | 2 | 10 | 20 | 30 | 31 | **32** |
|---|---|---|---|---|---|---|---|
| **Total** | 18 | 36 | 180 | 360 | 540 | 558 | 576 |

5) The final answer: There are 32 groups of 18 in 576.

As a final example, consider three different solution strategies to solve the following division problem: 323 ÷ 17 = ?

Solution Strategy: Student #1, Ratio Table

| **Number of Groups** | 1 | 2 | 3 | 10 | 5 | 15 | 18 | **19** |
|---|---|---|---|---|---|---|---|---|
| **Total** | 17 | 34 | 51 | 170 | 85 | 255 | 306 | 323 |

Solution Strategy: Student #2, Ratio Table

| **Number of Groups** | 1 | 10 | 20 | **19** | | | | |
|---|---|---|---|---|---|---|---|---|
| **Total** | 17 | 170 | 340 | 323 | | | | |

Solution Strategy: Student #3, Long Division

$$\begin{array}{r|l} & \;\;19 \\ \hline 17 & 323 \\ & \underline{17} \\ & 153 \\ & \underline{153} \\ & \;\;\;0 \end{array}$$

Each of these strategies is correct – all lead to the right answer (i.e., there are 19 groups of 17 in 323). Yet, each exhibits unique thinking and strategy use. Student #1 employs a ratio table. This student is evidently comfortable adding groups of 17, inching closer to the target of 323. While there is nothing wrong with this approach (and, indeed, it illustrates the child's understanding of the nature of division), it is clear that Student #2 employs different thinking that ultimately solves the problem, with a ratio table, in a few short steps. In the case of the second student, the strategy includes actually overshooting the target, and then subtracting one group of 17 to ultimately arrive at the solution. The power of the ratio table is that it is a model that allows for multiple solution strategies – each of which can fruitfully lead to correct answers.

Imagine, in contrast, the solution strategy of Student #3: the traditional long division algorithm. While there is nothing inherently wrong with the this method, the question must be asked again: "What about conceptual understanding of the both the process, and the nature, of division?" In particular, consider the third step of this problem, highlighted in red below.

Guess and Check with the Traditional Method

```
      1
17 ⟌ 323
     17
     153   <——   Step 3: Now… How many times
                 does 17 go into 153? Hmm…
```

This step can be very frustrating for students, particularly those with limited mathematical sense. Many times students need to apply trial-and-error techniques until they narrow in on the fact that 17 can go into 153 nine times. So, while the traditional method may appear to be efficient in its representation, without the very number sense and mental math skills (which can be easily developed through the use of the ratio table), students will often struggle to complete the algorithm effectively as they employ "guess and check" methods.

**In summary**, one can argue that both the traditional long division algorithm, and the ratio table, can be used quickly and efficiently by students to solve division problems. The question to linger upon, however, is the degree to which either of these methods might promote a higher level of mathematical understanding about the nature of mathematics, and division in particular. The traditional long-division algorithm, though a process that students can replicate effectively with practice, can be difficult for young learners to ***comprehend***. Much like the multi-digit multiplication algorithm, various steps in the process do not inherently embody mathematical understanding. Students can ***learn*** these steps, and perform long division effectively. The question remains, however, do they fundamentally ***understand*** what they have done? A second question that remains is how well students might learn – and effectively apply – the ratio table as a tool for multi-digit division if given the same amount of practice and the same number of repetitions as the typical math textbook provides for the learning of the long division algorithm.

With minimal practice, students can use the ratio table extremely effectively as an operational tool. Along the way, they also acquire mental math strategies, mathematical insights, and a sharpness about number relationships that is difficult to cultivate through the use of traditional algorithms. Bear in mind that most traditional algorithms were originally designed out of a perceived need for "mathematical efficiency." Mathematical understanding that might have been transparent or inherently conveyed in the algorithms was considered to be a secondary objective. With the ratio table, no such compromise is necessary. With proper instruction and adequate opportunity to practice, students will compute efficiently, accurately, ***and*** with meaning.

Name: ___________________________

**Activity Sheet 14**: Solving Multi-Digit Division Problems with the Ratio Table

**Directions**: Use a ratio table to solve the following problems. Use any combination of strategies that make sense to you. ***Some*** possible steps have been started for you. (You may use other steps too.) Be prepared to share and explain your strategy with a partner.

1. What is the answer to the following problem: 144 ÷ 12?

| Groups | 1 | 2 | 10 | | | | |
|---|---|---|---|---|---|---|---|
| Total | 12 | | | 144 | | | |

2. What is the answer to the following problem: 182 ÷ 14?

| Groups | 1 | 10 | 5 | | | | |
|---|---|---|---|---|---|---|---|
| Total | 14 | | | | 182 | | |

3. What is the answer to the following problem: 640 ÷ 32?

| Groups | 1 | 10 | | | | | |
|---|---|---|---|---|---|---|---|
| Total | | | | | 640 | | |

4. What is the answer to the following problem: 285 ÷ 19?

| Groups | | 10 | 5 | | | | |
|---|---|---|---|---|---|---|---|
| Total | 19 | | | | | 285 | |

5. What is the answer to the following problem: 943 ÷ 41?

| Groups | 1 | 10 | | | | | |
|---|---|---|---|---|---|---|---|
| Total | | 410 | | | | | |

6. What is the answer to the following problem: 240 ÷ 16?

| Groups | | | | | | | |
|---|---|---|---|---|---|---|---|
| Total | 16 | | | | | | |

7. What is the answer to the following problem: 693 ÷ 33?

| Groups | 1 | | | | | | |
|---|---|---|---|---|---|---|---|
| Total | 33 | | | | | | |

8. What is the answer to the following problem: 1271 ÷ 31?

| Groups | | | | | | | |
|---|---|---|---|---|---|---|---|
| Total | | | | | | | |

Name: ___________________________

**Activity Sheet 15**: Solving Multi-Digit Division Problems with the Ratio Table

**Directions**: Use a ratio table to solve the following problems. Use any combination of strategies you'd like to use. You may not need all the columns in the tables provided. Be prepared to share and explain your strategy with a partner.

1. What is the answer to the following problem: 276 ÷ 23?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Groups | 1 | | | | | | |
| Total | 23 | | | | 276 | | |

2. What is the answer to the following problem: 196 ÷ 14?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Groups | | | | | | | |
| Total | | | | | | | |

3. What is the answer to the following problem: 528 ÷ 24?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Groups | | | | | | | |
| Total | | | | | | | |

4. What is the answer to the following problem: 289 ÷ 17?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Groups | | | | | | | |
| Total | | | | | | | |

5. What is the answer to the following problem: 279 ÷ 31?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Groups | | | | | | | |
| Total | | | | | | | |

6. What is the answer to the following problem: 533 ÷ 41?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Groups | | | | | | | |
| Total | | | | | | | |

7. What is the answer to the following problem: 1281 ÷ 61?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Groups | | | | | | | |
| Total | | | | | | | |

8. What is the answer to the following problem: 936 ÷ 9?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Groups | | | | | | | |
| Total | | | | | | | |

Name: ___________________________

**Activity Sheet 16**: Solving Multi-Digit Division Problems

**Directions**: Use any method you prefer to solve the following multiplication problems. Be prepared to share and explain your strategy with a partner.

1. 276 ÷ 23 =

2. 403 ÷ 13 =

3. 441 ÷ 21 =

4. 650 ÷ 25 =

5. 870 ÷ 30 =

6. 1632 ÷ 51 =

# Chapter 4
# Fractions and the Ratio Table

The ratio table can also be used both to understand and to work with fractions.

When you compare columns in a ratio table, you will find something interesting about the way the numbers are related. For example, look at this ratio table, in which a doubling strategy was used to move from 1 group of 2, to 8 groups of 2.

| | | | |
|---|---|---|---|
| 1 | 2 | 4 | 8 |
| 2 | 4 | 8 | 16 |

Now, let's look at each column of the table, written as a fraction.

$$\frac{1}{2},\ \frac{2}{4},\ \frac{4}{8},\ \frac{8}{16}$$

What is evident is that each column of the ratio table is in fact an equivalent fraction. This holds true for any ratio table, as illustrated in the following examples:

| | | | | | |
|---|---|---|---|---|---|
| Packs | 1 | 2 | 4 | 8 | 6 |
| Seeds | 12 | 24 | 48 | 96 | 72 |

Each column is a fraction equivalent to 1/12.

| | | | | |
|---|---|---|---|---|
| Gallons | 1 | 2 | 4 | 8 |
| Dollars | 4 | 8 | 16 | 32 |

Each column is a fraction equivalent to 1/4.

After working with ratio tables in various ways as illustrated in the previous chapters of this book, the step students must take to begin to use ratio tables as a model for equivalent fractions is not a large one. Indeed… as the name of the model implies, the intent of these tables is to use a series of ***equivalent ratios*** to solve problems involving multiplication and division.

Ratio tables can be used (as above) to find equivalent fractions. Ratio tables may also be used to **simplify** a fraction and ***reduce it to its simplest form***. For example, as shown below, the following fraction may be reduced to simplest terms using a ratio table: 16/64.

| | | | | |
|---|---|---|---|---|
| **16** | 8 | 4 | 2 | 1 |
| **64** | 32 | 16 | 8 | 4 |

In each of these columns, a "halving" strategy was used to ***reduce*** the original fraction, while retaining its value.

## And now… A word of caution!

It is imperative that students understand the difference between using the ratio table ***to combine ratios***, and using the ratio table ***to add fractions*** (which they cannot do!).

If there is a potential pitfall to the ratio table, this issue would be the one. Recall that one of the strategies that is quite useful in working with ratio tables is to add columns. For example, if 1 carton holds 12 eggs, then 2 cartons would hold 24 eggs. Now, we can add these quantities together:

> *"If one carton has 12 eggs, then 2 cartons have 24 eggs. If we put the cartons together, there would be 3 cartons, together containing 36 eggs."*

**Please note**: There is nothing wrong with this combination; in this case, two equivalent ratios are being used to create a third, equivalent ratio. ***But, this is very different from fraction addition***! For example, we would be remiss if children walked away from this context believing that:

$$\frac{1}{12} + \frac{2}{24} = \frac{3}{36}$$

This result is nonsensical: $\frac{1}{12}$ has the <u>*same value*</u> as $\frac{3}{36}$. It could not be the case that adding any positive value to a fraction would result in the very same value that we began with.

Rather, If we were adding fractions, we might use some other model. For example, $\frac{1}{12}$ of an apple pie, plus an another slice equal to $\frac{1}{12}$ of the pie, would result in $\frac{1}{6}$ of the apple pie ($\frac{1}{12} + \frac{1}{12} = \frac{2}{12}$, or… $\frac{1}{6}$ of the pie).

So, while the ratio table can be extremely valuable in helping children understand (and learn how to find) equivalent ratios, one must take great care to prevent this very common misconception.

The following activity sheets will help student practice with, and understand, the ratio table as a tool for fraction operations.

Name: ___________________________

**Activity Sheet 17**: Fraction Equivalence and the Ratio Table

Directions: Use your ratio table strategies (e.g., doubling, multiplying, halving, etc.) to solve the following problems.

1. Find 5 equivalent fractions for 1/3.

| | | | | | |
|---|---|---|---|---|---|
| 1 | | | | | |
| 3 | | | | | |

2. Find 5 equivalent fractions for 1/4.

| | | | | | |
|---|---|---|---|---|---|
| 1 | | | | | |
| 4 | | | | | |

3. Find 5 equivalent fractions for 2/3.

| | | | | | |
|---|---|---|---|---|---|
| 2 | | | | | |
| 3 | | | | | |

4. Find 5 equivalent fractions for 2/5.

| | | | | | |
|---|---|---|---|---|---|
| 2 | | | | | |
| 5 | | | | | |

5. Find 5 equivalent fractions for 3/5.

| | | | | | |
|---|---|---|---|---|---|
| 3 | | | | | |
| 5 | | | | | |

6. Find 5 equivalent fractions for 3/8.

| | | | | | |
|---|---|---|---|---|---|
| 3 | | | | | |
| 8 | | | | | |

7. Find 5 equivalent fractions for 4/5.

| | | | | | |
|---|---|---|---|---|---|
| 4 | | | | | |
| 5 | | | | | |

8. Find 5 equivalent fractions for 3/4.

| | | | | | |
|---|---|---|---|---|---|
| 3 | | | | | |
| 4 | | | | | |

9. Find 5 equivalent fractions for 1/2.

| | | | | | |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |

10. Find 5 equivalent fractions for 3/2.

| | | | | | |
|---|---|---|---|---|---|
| 3 | | | | | |
| 2 | | | | | |

Name: ___________________________

**Activity Sheet 18**: Reducing Fractions with the Ratio Table

Directions: Use your ratio table strategies (e.g., doubling, multiplying, halving, etc.) to reduce each fraction to its simplest terms. Each problem may require a different number of columns to complete.

1. Reduce.

| | | | | | |
|---|---|---|---|---|---|
| 24 | | | | | |
| 80 | | | | | |

2. Reduce.

| | | | | | |
|---|---|---|---|---|---|
| 60 | | | | | |
| 90 | | | | | |

3. Reduce.

| | | | | | |
|---|---|---|---|---|---|
| 18 | | | | | |
| 24 | | | | | |

4. Reduce.

| | | | | | |
|---|---|---|---|---|---|
| 48 | | | | | |
| 60 | | | | | |

5. Reduce.

| | | | | | |
|---|---|---|---|---|---|
| 8 | | | | | |
| 16 | | | | | |

6. Reduce.

| | | | | | |
|---|---|---|---|---|---|
| 24 | | | | | |
| 96 | | | | | |

7. Reduce.

| | | | | | |
|---|---|---|---|---|---|
| 40 | | | | | |
| 50 | | | | | |

8. Reduce.

| | | | | | |
|---|---|---|---|---|---|
| 60 | | | | | |
| 150 | | | | | |

9. Reduce.

| | | | | | |
|---|---|---|---|---|---|
| 100 | | | | | |
| 2000 | | | | | |

10. Reduce.

| | | | | | |
|---|---|---|---|---|---|
| 60 | | | | | |
| 40 | | | | | |

# Appendix
# Application Problems with the Ratio Table

Throughout the previous sections of this book, many applications of the ratio table have been presented.  The following activities ask students to apply their understanding of the ratio table toward solutions for a wide variety of problems, many of which are nested in imaginable problem contexts.

Name: ___________________________

**Activity Sheet 19**: Tickets

**Directions**: Use ratio table strategies (e.g., doubling, multiplying, halving, etc.) to solve each of the following problems.

1. One ticket to get into Wet World Water Park costs $3.50. How much does it cost to buy 10 tickets? How much for 18 tickets?

| People | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Cost | | | | | | | | |

10 tickets? ________________ 18 tickets? __________________

2. Tickets for the school play are $1.75 each. Together, Jenny and Sarah sold 28 tickets. Both Jenny and Sarah started ratio tables to learn how much money they earned together, but they did not finish their work. Help Jenny and Sarah complete the tables by filling in the shaded boxes.

Jenny's method

| Tickets | 1 | 2 | | 28 | | | |
|---|---|---|---|---|---|---|---|
| Cost | $1.75 | | $7.00 | | | | |

Sarah's method

| Tickets | 1 | 10 | 20 | 5 | 2 | 30 | 28 |
|---|---|---|---|---|---|---|---|
| Cost | $1.75 | $17.50 | | | $3.50 | | |

What was Jenny's method? What strategies did she use in her ratio table?

_____________________________________________________________________________

_____________________________________________________________________________

_____________________________________________________________________________

What was Sarah's method? What strategies did she use in her ratio table?

_____________________________________________________________________________

_____________________________________________________________________________

_____________________________________________________________________________

Name: ___________________________

**Activity Sheet 20**: Money at the Market

**Directions**: Use ratio table strategies (e.g., doubling, multiplying, halving, etc.) to solve each of the following problems.

1. Joel went to an orchard to pick fruit. Three pounds of peaches sold for $2. How much did Joel have to pay for 12 pounds? **Answer**: $__________

| Pounds of Peaches | 3 | | | | |
|---|---|---|---|---|---|
| Cost | $2.00 | | | | |

2. How much would Joel have to pay for 33 pounds of peaches? **Answer**: $ _________

| Pounds of Peaches | 3 | | | | |
|---|---|---|---|---|---|
| Cost | $2.00 | | | | |

3. You can buy two watermelons for $3. How many watermelons can you buy if you have only $7.50? **Answer**: ______ watermelons

| Watermelon | 2 | | | | |
|---|---|---|---|---|---|
| Cost | $3.00 | | | | |

4. Four pounds of apples cost $2.50. You have $10 to spend on apples. How many pounds can you buy? **Answer**: _______ pounds

| Pounds of Apples | 4 | | | | |
|---|---|---|---|---|---|
| Cost | $2.50 | | | | |

5. A vendor sells hot dogs at the ballgame, 2 for $1.50. How many hot dogs can you buy with $20? **Answer**: ___________ hot dogs

| hot dogs | | | | | | | |
|---|---|---|---|---|---|---|---|
| cost | | | | | | | |

Explain your strategy:

Name: ___________________________

**Activity Sheet 21**: Ratio Table Puzzles

**Directions:** The ratio tables below are puzzles. Try to solve the puzzles by correctly filling in all the empty cells in each table. You **must use exactly the number of columns provided**! You can use any strategy you have learned about ratio tables – adding, multiplying by ten, halving, doubling, etc. See if you can figure them out!

1.

| | | | |
|---|---|---|---|
| 1 | | | 8 |
| 6 | | | |

2.

| | | | | | |
|---|---|---|---|---|---|
| 3 | | | | | 36 |
| 7 | | | | | |

3.

| | | | |
|---|---|---|---|
| 2 | | | 60 |
| 5 | | | |

4.

| | | |
|---|---|---|
| 3 | | 15 |
| 2 | | |

5.

| | | | |
|---|---|---|---|
| 1 | | | 17 |
| 3 | | | |

6.

| | | | |
|---|---|---|---|
| 1 | | | 11 |
| 5 | | | |

7.

| | | | |
|---|---|---|---|
| 1 | | | 42 |
| 2 | | | |

8.

| | | | | |
|---|---|---|---|---|
| 2 | | | | 22 |
| 6 | | | | |

The mathematics education community has spent years debating the relative merits of teaching for algorithmic proficiency, versus teaching for conceptual understanding. The truth is that we need both. The growing popularity of the ratio table may be attributed to the fact that it serves as a conduit between these two schools of thought. On one hand, it is an excellent computational tool that, when understood well by students, can be used quickly, efficiently, and accurately to multiply, divide, calculate percentages, reduce fractions, etc. On the other hand, the structure of the model itself promotes conceptual understanding and mathematical connections that are often missing in the standard algorithms that are many times rehearsed repeatedly by students, with little understanding of why they work.

The ratio table is a powerful tool. This book highlights many opportunities for students to embrace ratio tables not only as a tool for calculations, but also as a way of thinking about mathematical relationships. Students will...

- Develop proportional reasoning
- Cultivate mental math strategies (e.g., doubling, halving, etc.)
- Multiply with the Ratio Table
- Divide with the Ratio Table
- Use the Ratio Table to work with fractions
- Apply the Ratio Table to find solutions to real world problems

**Example Problem**

120 new baseballs must be split evenly between 8 teams.
How many new baseballs does each team get?
The ***Halving*** Strategy

| | | | | |
|---|---|---|---|---|
| **Teams** | **8** | **4** | **2** | **1** |
| **Baseballs** | **120** | **60** | **30** | **15** |

Jeffrey Frykholm, Ph.D.

An award winning author, Dr. Jeffrey Frykholm is a former classroom teacher who now focuses on helping teachers develop pedagogical expertise and content knowledge to enhance mathematics teaching and learning. In his ***Learning to Think Mathematically*** series of textbooks for teachers, he shares his unique approach to mathematics teaching and learning by highlighting ways in which teachers can use mathematical models (e.g., the rekenrek, the ratio table, the number line, etc.) as fundamental tools in the teaching and learning of mathematics. These books are designed with the hope that they will support teachers' content knowledge and pedagogical expertise, toward the goal of providing a menaingful and powerful mathematics education for all children