## Learning ProblemSolving Strategies by Using Games

David Moursund

# Learning Problem-solving Strategies by Using Games: A Guide for Educators and Parents 

## David Moursund

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- Assistant Professor and then Associate Professor, Department of Mathematics and Computing Center (School of Engineering), Michigan State University, 1963-1967.
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- Served six years as the first Head of the Computer Science Department at the University of Oregon, 1969-1975.
- Promoted to Full Professor, University of Oregon 1976.
* Retired in 2002, with the last 20 years of his service to the UO being in the College of Education. After retirement, worked $1 / 3$ time in the UO College of Education 2002-2007.
- In 1974, started The Computing Teacher, the publication that eventually became Learning and Leading with Technology, the flagship publication of the International Society for Technology in Education (ISTE).
- Served on the Board of Directors of the Math Learning Center (MLC) since the MLC's inception in 1977.
- In 1979, founded the International Society for Technology in Education (ISTE). Headed this organization for 19 years.
- In 2007, founded the Information Age Education (IAE), an Oregon non-profit organization. IAE works to improve the informal and formal education of people of all ages throughout the world. and website. See http://IAE-Pedia.org and http://I-A-E.org/.
- Author or co-author of about 65 books and several hundred articles in the fields of computers in education and math education. About 40 of the books are available free online at http://iaepedia.org/David_Moursund_Books.
- Presented over 200 workshops and many talks in the fields of computers in education and math education.
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## Preface

In this work, when it shall be found that much is omitted, let it not be forgotten that much likewise is performed. (Samuel Johnson.)

This book is written for people who are interested in helping children learn through games and learn about games. The intended audience includes teachers, parents and grandparents, and all others who want to learn more about how games can be effectively used in education. Special emphasis is given to roles of games in a formal school setting.

As you know, education has many goals, and there is a huge amount of research and practitioner knowledge about teaching and learning. This book is well rooted in this research and practitioner knowledge. Four of the important ideas that are stressed include:

- Learning to learn. Through study and practice, a person can get better at learning.
- Learning about one's strengths and weaknesses as a learner. Even a simple game can provide a good environment in which one can study his or her own learning processes and habits. Metacognition and reflection can be used to identify one's strengths and weaknesses as a learner in this game environment. Specific instruction plus growing cognitive maturity help the learner to transfer learning from the game-learning environment to other learning environments.
- Becoming better at solving challenging problems and accomplishing challenging tasks. Learning some general strategies for problem solving is a unifying theme in this book.
- Intrinsic motivation-students being engaged because they want to be engaged. This idea is illustrated by the following quote from Yasmin Kafai, a world leader in uses of games in education:

If someone were to write the intellectual history of childhood-the ideas, the practices, and the activities that engage the minds of children-it is evident that the chapter on the late 20th century in America would give a prominent place to the phenomenon of the video game. The number of hours spent in front of these screens could surely reach the hundreds of billions. And what is remarkable about this time spent is much more than just quantity. Psychologists, sociologists, and parents are struck by a quality of engagement that stands in stark contrast to the half-bored watching of many television programs and the bored performance exhibited with school homework. Like it or not, the phenomenon of video games is clearly a highly significant component of contemporary American children's culture and a highly significant indicator of something (though we may not fully understand what this is) about its role in the energizing of behavior (Kafai, 2001). [Bold added for emphasis.]

## Educational Goals Copied from Chapter 8

The research on the use of games in education strongly supports the value of having clear learning goals in mind and of specifically teaching to these goals. A few of the ideas in the list given below (copied from chapter 8) may not be completely understandable to you before you have read later parts of the book. They are given here to reassure you that there is some "method to the madness" in the organization and content of this book.

Here is a short list of possible goals for making educational uses of games in a classroom setting.

1. To help students learn more about themselves in areas such as:
a. Learning to learn and understanding how concentrated, reflective effort over time leads to an increasing level of expertise.
b. Learning about one's cooperative versus independent versus competitive inclinations both in learning and in demonstrating or using one's learning.
c. Learning about oneself as a giver of feedback to others and as a receiver of feedback from others. This includes learning to complete and to make use of both selfassessment and peer assessment.
2. To help students better understand problem-solving strategies and to increase their repertoire of and use of problem-solving strategies. This includes:
a. Learning about low-road (essentially, rote memory) and high-road transfer of learning, especially as they apply to problem solving.
b. Learning how to recognize/identify a problem-solving strategy and explore its possible use across many different problem domains.
c. Learning how to do high-road transfer of learning of problem-solving strategies that cut across many domains.
d. Increasing fluency in making effective use of one's repertoire of domain-independent problem-solving strategies.
3. To help students learn some games and increase their understanding of historical and current roles of games and game playing in our society. This includes:
a. Learning games as an aid to social interaction in small and large groups.
b. Learning games as part of the culture and history of a family or community.
c. Learning games as environments that facilitate communication, collaboration, and peer instruction.
d. Learning how to help other people learn a new game. (Think of the idea that every student plays both learning and teaching roles in life.)

## Brief Overview of Contents

Each chapter ends with a set of activities for the reader of the book, and a set of activities that might be useful with students of varying backgrounds and interests.

The Introduction provides a short discussion of some of the key ideas covered in the book.
Chapter 1 illustrates the idea of thinking outside the box. This idea is important in solving puzzle problems, but it is also essential in solving many real-world problems.

Chapter 2 provides some general educational background needed in the rest of the book.
Chapter 3 uses a puzzle called Sudoku to explore some aspects of puzzles and their roles in education.

Chapter 4 explores some additional puzzles and sources of free puzzles on the Web.
Chapter 5 explores solitaire card games that can be played with ordinary decks of 52 playing cards and/or can be played on a computer.

Chapter 6 explores competitive 2-person games such as checkers, chess, and backgammon. Nowadays, many people play these games using a computer as an opponent.

Chapter 7 explores games that typically involve more than two players, but still only a modest number of players. Examples include poker, bridge, and hearts. These games also can be played using a computer as an opponent.

Chapter 8 discusses the development of game-based lesson plans.
Chapter 9 provides very brief introductions to a miscellaneous collection of ideas related to the topic of games in education. If I were writing a longer book, some of these topics would be developed into individual chapters.

Appendix 1 summarizes the problem-solving strategies and some key terms explored in the book. It also provides additional information about effective ways to use games in education. You may find it helpful to browse this appendix before delving into chapter 1.

Appendix 2 lists the games and puzzles discussed in the book, with a note of the chapter(s) where each can be located.

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Kafai, Y.B. (October 27, 2001). The educational potential of electronic games: From games-toteach to games-to-learn. Cultural Policy Center, University of Chicago. Retrieved 6/24/08 from http://culturalpolicy.uchicago.edu/conf2001/papers/kafai.html. See also:
http://www.gseis.ucla.edu/faculty/kafai/.

## Introduction

All the world's a game,
And all the men and women active players:
They have their exits and their entrances;
And all people in their time play many parts...
(Dave Moursund-Adapted from Shakespeare.)

## A Very Important "Big Idea"

As suggested by the mangled Shakespeare quote given above, I sometimes look at various aspects of life as a game. I make a "move," and people around me make countermoves. My move is designed to improve my "position" in the situation, and their moves are designed to improve their positions or to counter my move.

For an example of a move, suppose I am discussing how my favorite football team did in its game last week, and the conversation involves several people who have other favorite teams. I say something positive that my team did or something negative that some other team did. Perhaps I am trying to "one up" others in the conversation. How will they respond?

Question: Do I think in advance how they will respond? That is a key question in playing many different types of games in which one is "playing" against an opponent. In chess, for example, I can think about how my opponent is apt to respond to a move that I am thinking about making. It is also a key question about all of one's communications with others.

As another example, I am a three-year-old child playing with a friend, also three. We are each individually playing with building blocks, building towers. I deliberately knock down my friend's tower. Do I think in advance what my friend's response will be? Probably not. My "plan ahead" mind has not yet developed to a level that it carefully considers the consequences of a proposed action.

How does a person get better at anticipating the consequences of a proposed action? For most of us, this is a long, slow process-and indeed, many never get very good at it. (Think about a person driving when drunk.)

The kinds of games discussed in this book create environments in which a player can anticipate the possible consequences of a move and can practice planning ahead. With proper guidance, instruction, and help from a parent or teacher, the player can not only get better at planning ahead in the game, but can also make a transfer of learning of this skill to using it in "real life" settings. The game, with the help of a human teacher, becomes a very powerful aid to helping the player to become a more self-responsible person. In brief summary, this type of learning from games is the purpose of this book.

## Problem Solving is the Key Focus and Purpose of the Book

I think of the study of problem solving as a unifying theme in education and in the study of each specific discipline. Good education exposes students to the problems and knowledge of a very wide range of disciplines. It focuses on helping students learn to learn and also learn to take responsibility for their own education.

The title of this book is Learning Problem-solving Strategies by Using Games: A Guide for Educators and Parents. The book's focus is on learning problem-solving strategies through the use of games. The goal is to gain useful knowledge and skills in problem solving, and games are the vehicle.

Games tend to be intrinsically motivating and fun to play. In the book, I use games as a vehicle to explore joy and fun, two very important aspect of teaching and learning problem solving. For more information about joy and fun in education, see the IAE Newsletter series that began with its December, 2015, issue. The titles of the first two newsletters in this series are The Joy of Learning: An Introduction (Sylwester, December, 2015) and Joy in Learning and Playing Games (Moursund, December, 2015).

There is much about problem solving and much about uses of games in education not covered in the book. Later in this section are brief introductions to other topics that could have been included-but weren't.

## Games

The word game means different things to different people. In this book, I explore a variety of board games, card games, dice games, word games, and puzzles that many children and adults play. Many of these games come in both non-electronic and electronic formats. This book places special emphasis on electronic games and the electronic versions of games originally developed in non-electronic formats.

This book does not explore all types of games. For example, I do not explore sports games, such as baseball, basketball, football, and soccer, or any of the sports in the summer and winter Olympic Games.

Since my early childhood, I have enjoyed playing a wide variety of games. Indeed, at times I have had a reasonable level of addiction to various games. In retrospect, I feel I learned a great deal from the board games, card games, puzzles, and other types of games I played as a child.

In recent years, a number of educators and educational researchers have come to realize that games can be an important component of both informal and formal education. This has become a legitimate area of study and research.

Oodles of games are now available in electronic format. While many are distributed commercially, many others are available for free play on the Web, and many can be downloaded at no cost. In this book, I am especially interested in games available at little or no cost, and that also have significant educational value.

Some electronic games are merely computerized versions of games that existed long before computers. Others only exist in a computer format. Computer networks have made possible games that allow many thousands of players worldwide to be participating simultaneously. For example, over its long lifetime The World of Warcraft had a peak number of subscribers of about 12 million in 2010, and in 2015 it still had about 5.6 million worldwide-people who pay to play the game online. The computerized animation and interaction in these games bring a previously unknown dimension to games.

## Puzzles

A puzzle is a type of game. To better understand the purpose of this book, think about some popular puzzles such as crossword puzzles, jigsaw puzzles, and logic puzzles (often called brain
teasers). In every case, the puzzle-solver's goal is to solve a particular mentally challenging problem or accomplish a particular mentally challenging task.

This definition of a puzzle (a particular mentally challenging problem) means that there are puzzles in every discipline of study and in every human mental activity. Some of these problems have no solution, some have one solution, and some have more than one solution.

Consider, for example, the following two math problems:

1. Find two odd integers whose sum is an odd integer. That is a perfectly well-defined math problem/puzzle. It does not have a solution.
2. Find two integers between 4 and 9 inclusive that add to an even integer. This math problem/puzzle has several solutions.
The message is that not every puzzle or problem is solvable, and a puzzle or problem might have more than one solution. I like to add a twist to this. A puzzle may be solvable, but not within the framework of the knowledge and skills that I have. When I am doing a crossword puzzle, there are apt to be many clues and their corresponding puzzle entries that are completely outside of my knowledge and experience. I have no way of knowing whether the puzzle is solvable by others (the puzzle maker may have deliberately or accidently made a mistake), but I can convince myself that I cannot solve it.

This is a Big and Profound Idea. Contrast this reality with the usual school curriculum designed to help students learn to solve a wide variety of problems and accomplish a wide variety of tasks. The curriculum tends to be designed so that students are able to gain the knowledge and skills needed to solve the problems and accomplish the tasks that are being presented to them. Thus they tend to believe that every problem is solvable, if only they can master the content they are being taught.

I am reminded of the quotations:
If at first you don't succeed [in solving a particular problem], try, try again. Then quit. There's no use being a damn fool about it. (W.C. Fields)

Here is my version of this quotation:
If at first you don't succeed in solving a particular problem, try, try again. Then if you still have not succeeded and the problem is important to you, seek help or put the problem away and save it for a time when you have gained more knowledge and skills relevant to this problem situation. (David Moursund, adapted from W.C. Fields)

Puzzles and other types of games can be used as vehicles to help students learn this very important idea.

Many people are hooked on certain types of puzzles. For example, some people cannot start the day without spending time on the crossword puzzle in their morning newspaper. In some sense, they have a type of addiction to crossword puzzles. The fun is in meeting the challenge of the puzzle - making some or a lot of progress in completing the puzzle.

Crossword puzzles draw upon one's general knowledge, recall of words defined or suggested by short definitions or pieces of information, and spelling skill. Through study and practice, a person learns some useful strategies and can make considerable gains in crossword puzzlesolving expertise. Doing a crossword puzzle is like doing a certain type of brain exercise. In
recent years, research has provided evidence that such brain exercises help stave of the dementia and Alzheimer's disease that are so common in the elderly.

From an educational point of view, it is clear that solving crossword puzzles helps to maintain and improve one's vocabulary, spelling skills, and knowledge of many miscellaneous tidbits of information. Solving crossword puzzles tends to contribute to one's self esteem. For many people, their expertise in solving crossword puzzles plays a role in their social interaction with other people.

## Knowledge-Building Communities

For many years now, a number of people have been doing research in the field of games in education. Scardamalia and Bereiter (1994) provide a good foundation for some of this research. Their article includes a focus on three important aspects of education that are also important aspects of using games in education. As you consider uses of games in your teaching, consider three important underlying educational ideas. Quoting from the article:

## 1. Intentional learning.

Although a great deal of learning is unintentional, important kinds of school learning appear not to take place unless the student is actively trying to achieve a cognitive objective-as distinct from simply trying to do well on school tasks or activities (Bereiter \& Scardamalia, 1989; Chan, Burtis, Scardamalia, \& Bereiter, 1992; Ng \& Bereiter, 1991).
[Comment from Moursund: As pointed out elsewhere in this book, effective use of games in education requires that they be used in an intentional learning environment.]

## 2. Expertise is a process.

Although expertise is usually gauged by performance, there is a process aspect to expertise, which we hypothesize to consist of reinvestment of mental resources that become available as a result of pattern learning and automaticity, and more particularly their reinvestment in progressive problem solving-addressing the problems of one's domain at increasing levels of complexity (Bereiter \& Scardamalia, 1993; Scardamalia \& Bereiter, 1991b). Progressive problem solving characterizes not only people on their way to becoming experts, but it also characterizes experts when they are working at the edges of their competence. Among students, the process of expertise manifests itself as intentional learning.
[Comment from Moursund: We want students to develop their levels of expertise in many different areas. Research indicates that students should understand this educational goal, understand the meaning of expertise, and be actively engaged in developing their own expertise.]

## 3. Schools as knowledge-building communities.

The process of expertise is effortful and typically requires social support. By implication, the same is true of intentional learning. Most social environments do not provide such support. They are what we call first-order environments. Adaptation to the environment involves learning, but the learning is asymptotic. One [a teacher] becomes an old timer, comfortably integrated into a relatively stable system of routines (Lave \& Wenger, 1991).

Competitive sports and businesses are examples of second-order environments, in which the accomplishments of participants keep raising the standard that the others strive for. More relevant examples in education are the sciences and other learned disciplines in which adaptation involves making contributions to collective knowledge. Because this very activity increases the collective knowledge, continued adaptation requires contributions beyond what is already known, thus producing non- asymptotic learning. The idea of schools as knowledge-building communities is the idea of making them into second-order environments on this model.
[Comment from Moursund: One of the key ideas here is that of a steadily rising bar. The totality of human knowledge is steadily growing. Many people talk about the idea of an information overload-that there is too much information that we need to deal with. Our schools should be helping students learn to deal with this information overload. For more information see my article, Information Underload and Overload (Moursund, 2015).]

## Computational Thinking

Your mind/brain learns by developing and storing patterns. As you work to solve a problem or accomplish a task, (as you think) you draw upon these stored patterns of data, information, knowledge, and wisdom.

Beginning more than 5,000 years ago, reading and writing have become more and more important as a mind/brain aid. In the past few decades, computers have contributed substantially to mind/brain processes by providing improved access to information, improved communication, and aids to automating certain types of "thinking" processes.

Notice how the thinking of mind/brain and the thinking (information processing) of computers are melded together in the following brief discussion of computational thinking (Wing, March, 2006):

Computational thinking builds on the power and limits of computing processes, whether they are executed by a human or by a machine. Computational methods and models give us the courage to solve problems and design systems that no one of us would be capable of tackling alone. Computational thinking confronts the riddle of machine intelligence: What can humans do better than computers? What can computers do better than humans? Most fundamentally it addresses the question: What is computable? Today, we know only parts of the answers to such questions.
Computational thinking is a fundamental skill for everybody, not just for computer scientists. To reading, writing, and arithmetic, we should add computational thinking to every child's analytical ability.

Games provide an excellent environment to explore ideas of computational thinking. The fact that many games are available both in a non-computerized form and in a computerized form helps to create this excellent learning environment. A modern education prepares students to be productive and responsible adult citizens in a world in which mind/brain and computer working together is a common approach to solving problems and accomplishing tasks.

## Computer Programming as a Game

Chapter 9 of this book briefly discusses computer programming as a game. Children who are introduced to programming languages suited to their level of cognitive development have a great deal of fun "playing at" computer programming. In that sense, learning and using a programming language is somewhat similar to learning and playing a game.

Quite early on in the development of computer programming languages, the languages BASIC and Logo were taught to a great many elementary school and older students. However, in the United States such instruction did not gain a solid foothold in most school districts, and did not become part of state and national assessment. In recent year, the teaching of computer programming to precollege students has had a major resurgence. Languages that even young children can use to create and combine sound and graphics in animations have also proven useful in introductory college computer science courses.

## Growing Up With Computer Games

The parents of today's K-12 students did not grow up with Smartphones. Steve Jobs introduced Apple's iPhone in January of 2007. Since then several billion Smartphones have been manufactured and sold, and the available features have been increased and improved.

Many quite young children gain access to a Smartphone. Wow! Taking pictures and sharing them with others is fun. To a child, a Smartphone is like an electronic game. There are lots of fun games that run on a Smartphone. Talking to friends is fun. Learning some reading and writing, and an increasing level of maturity, help prepare a child to engage in Smartphone and/or computer-based social networking.

In addition, many of today's very young children play with computerized toys and games. Their early insights into capabilities and limitations of computers are based on their enjoyable play activities. They have fun both learning to play the games and playing the games.

As they grow a little older, they learn to distinguish between the physical hardware and programs (apps, games) that run on different types of hardware. Most children in such environments find that their playtime is restricted by adults and that they do not get to have all of the machines and games that they see advertised and/or that their friends have. They experience situations of delayed gratification or perhaps complete inability to get what they want.

To today's children, learning to play the Smartphone and other electronic games is just as "natural" as the processes their parents and grandparents experienced in learning the games of their time. However, there is a huge difference. The games that their parents and grandparents learned to play tended to be games that their parents and grandparents also played.

Many of today's children grow up gaining game-playing knowledge and skills that most of their parents and grandparents may not have. They learn to do things with computer-based technologies that many of their parents know little about. Indeed, parents and grandparents often seek help from the children. One of the important educational aspects of games is that learners can learn from each other and can help others to learn.

## Final Remarks

The games discussed in this book are used as vehicles to learn problem-solving ideas that cut across the school curriculum. You do not need to be an expert in any of the games to use them as
an aid to improve your problem-solving knowledge and skills. The same statement holds true for your students and/or children.

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## Chapter 1

## Thinking Outside the Box

We can't solve problems by using the same kind of thinking we used when we created them. (Albert Einstein.)
The vertical thinker says: "I know what I am looking for." The lateral thinker says: "I am looking but I won't know what I am looking for until I have found it." (Edward de Bono.)

Do you believe education can be joyful and fun for all students? For an interesting discussion, see Robert Sylwester's article, The Joy of Learning: An Introduction (Sylwester, December, 2015).

Now, consider these two statements:

- Education has many goals. Few people would list "to have fun" as one of the main goals of education. Instead, people tend to say things like "it is good for you" and "no pain, no gain.".
- Many games are used as a form of play. Games are for fun.

Now, think back to your childhood. I'll bet you can think of games that you played that were fun and made significant contributions to your learning. Monopoly comes immediately to my mind. I spent hundreds of hours playing this and other games that involved play money.

As a child I enjoyed playing many different card games, board games that involved dice or spinners, and board games such as checkers, chess, and go that do not depend on randomness. As a young adult I learned to play hearts, poker, and bridge, and in more recent years have learned to play a wide variety of computer games. To read about my exploits in learning to play the computer game DragonVale, see http://i-a-e.org/iae-blog/entry/what-i-learned-from-learning-to-play-a-new-computer-game.html.

Games have contributed significantly to my informal and formal learning. Game playing was such an important part of my childhood that I made sure it was a part of my children's childhoods. Throughout my life, playing games that involved two or more people has been an important component of my social development and social life.

In recent years, computers have made possible many new types of games. In addition, computers have made many older games more accessible. For example, many two-player games can now be played by one person playing against the computer.

As you read this book, please allow yourself to suspend some of your suspicions and beliefs about educational and other values of games. Open your mind to new possibilities. For example, there now are computer-based games in which tens of thousands of people simultaneously play in a combination of cooperative and competitive manners. This is made possible by the Internet and by the development of games designed to accommodate huge numbers of simultaneous players. Whether it is just a few people or a few thousand people playing a computer-based game, they are learning to communicate and interact in a computer-supported environment. Learning to play and work in this new environment is now an important educational goal.

How are computer-based games relevant to education? Think outside the box! Our children are growing up in a world in which it is common for teams of people, with members located throughout the world, to work together on complex problems and tasks. You have undoubtedly heard the African proverb, "It takes an entire village to raise a child." Combine this idea with that of global village and you can see that, nowadays, the whole world is involved in raising and educating our children. Our children need an education that prepares them to be effective participants in this global village.

## Puzzle Problems

This book will expose you to a variety of games. One type of game is called a puzzle. A puzzle is a problem or enigma mainly designed for entertainment. Often one can solve a puzzle without having to draw upon deep knowledge of any discipline. A jigsaw puzzle and a Rubik Cube provide good examples of this.

A child doing a jigsaw puzzle is engaged in tasks that involve looking for patterns, using spatial visualization skills (colors, depictions, outlines). This puzzle playing may be done individually or in a small group. In the latter case, there is a strong social education aspect of putting together a jigsaw puzzle. (Remember the person who would hide a piece in order to be the one putting in the last piece?)

Other types of puzzles require a broad and deep background. Contrast a jigsaw puzzle or a Rubik Cube with a crossword puzzle from The New York Times, especially the Friday and Saturday puzzles. The crossword puzzle draws upon reading, spelling, word definitions, and word-suggestion clues.

In some cases, a particular type of puzzle has many variations. There are lots of different interlocking jigsaw puzzles, and there are lots of different crossword puzzles.

In other cases, a puzzle will be one-of-a kind. Once you have figured out how to solve the puzzle, it is no longer a challenge. Here is an example of a brain-teaser puzzle that you may have seen before.

Problem: You are at a river that you want to cross with all of your goods. Your goods consist of a chicken, a bag of grain, and your large dog named Wolf. You have to cross the river in your canoe but can only take one passenger (chicken, dog, bag of grain) with you at a time. You can't leave the chicken alone with the grain, as the chicken will eat the grain. You can't leave your dog Wolf alone with the chicken, as Wolf will eat the chicken. However, you know that Wolf does not eat grain. How do you get everything across the river intact?

Words of Advice: Spend some time trying to solve this puzzle before looking at a solution. The educational value of such puzzle problems is not in knowing a solution. It is in the mental challenge of figuring out how to solve the puzzle. This gets at the heart of discovery-based learning. I believe a good education is an appropriate balance between learning to learn and improving one's learning and problem-solving skills, and learning content that you and others deem worthwhile.

Solution: Take the chicken across the river first and leave it on the other side. Return to where you have left Wolf and the grain.

Next, take Wolf across the river, and leave him there, but bring the chicken back with you.

Next, leave the chicken where you started. Take the bag of grain across the river and leave it with Wolf.

Finally, go back and get the chicken, and take it with you across the river.
This brainteaser requires you to think outside the box. Many people do not think about the idea that in solving this puzzle you might bring something back on a return trip. They never consider this possibility, and they are unable to solve the puzzle problem. Note also that the learning gained from solving this particular puzzle probably will help you solve similar puzzles.

Here is another brainteaser puzzle that requires thinking outside the box:
Problem: Using pencil and paper, arrange nine distinct dots into a $3 \times 3$ pattern as illustrated in Figure 1.1. The task is to draw four straight line segments with the beginning of the second starting at the end of the first, the beginning of the third starting at the end of the second, and the beginning of the fourth starting at the end of the third, arranged so that the total sequence of line segments passes through each dot.

Figure 1.1. Nine dots in a $3 \times 3$ square pattern.
See if you can solve this puzzle before reading farther.
To begin, you may think about how easy it is to complete the task using five line segments. A solution is given in Figure 1.2. After studying this solution, you can easily find other five-line segment solutions.


Figure 1.2. A 5-line segment solution for the 9-dots puzzle.
How can one possibly complete the task with only four line segments? As with the rivercrossing puzzle, it is necessary to think outside the box. In this case, the layout of the puzzle tends to create a visual box. Many people do not think about drawing line segments that go outside this visual box. A solution using four line segments is shown in Figure 1.3.


Figure 1.3. A 4-line segment solution for the 9-dots puzzle.
I suspect that most parents, teachers, and other adults really don't care whether students learn how to solve this 9 -dots, 4 -line segment puzzle problem. I don't ever recall encountering a similar real-world problem during my lifetime.

However, many people care about helping students learn to think outside the box. Thus, they want students to have an informal and formal educational system that will help students learn to think outside the box.

In this book, we will explore real-world problems and game-world problems. Of course, the games are part of our real world, so the distinction is somewhat silly. However, the goal in this book is to learn to make better use of the game world to learn about solving problems in the real world.

The think outside the box strategy is illustrated by the two puzzles described above. However, these two examples are useful in education mainly if the learner makes a connection between the examples and real-world problems. Young students seldom make such connections on their own. Merely having students work to solve these two puzzles and then showing them solutions will not help the typical young student to make such connections.

This is where a teacher enters the picture. A good teacher can help students to discover personal examples of thinking outside the box. The teacher might be a parent, a schoolteacher, a sibling, or a peer. The point is that the teacher does a valuable service for the student. With proper instruction, most students can gain increased skill in making such connections by themselves. Clearly, this is an important goal in education!

Here is another 9-dot challenge. See if you can use just three connected line segments to draw through all of the dots. As before, think about this before going on. Think outside the box!

The chances are that you are like many other people, in that you have studied math for many years, starting in preschool or elementary school. Thus, you can probably tell me the difference between a dot and a mathematical point. A dot has size, while a point does not. This puzzle was stated in terms of using nine distinct dots (not nine points). A 3-line segment solution is illustrated in Figure 1.4. To make the illustration easier to understand, I have enlarged the dots in the puzzle.


Figure 1.4. A 3-line segment solution for the 9-dots puzzle.

This solution not only illustrates thinking outside the box, it also illustrates the importance of precise vocabulary and that the problem solver must understand the meaning of the precise vocabulary. This is a tricky puzzle problem, because many people tend to think of a dot as a mathematical point.

Here is another challenge. Can the problem be solved using only two line segments? Prove your assertion!

The request for a proof is, of course, a standard component in mathematics courses. However, proof is an important concept in many other disciplines. A lawyer works to prove a client is not guilty, and a researcher in science works to prove a scientific theory is correct. One way to prove that a problem can be solved is to actually solve it. Demonstrate to other people how to solve the problem, and do so in a manner that makes it possible for them to solve the problem. That is what I did in the 3 -line segment solution to the 9 -dots problem.

Suppose, however, you suspect that a problem does not have a solution. Then, your task becomes one of proving that the problem does not have a solution. Your proof must be convincing to other people. See if you can prove that the 9 -dots puzzle problem cannot be solved using only two connected line segments.

Hmmm. What is a line? If a line has breadth, then a single "wide" line can pass through all nine dots. In thinking about the difference between a mathematical point and a dot, one might also come to think about the difference between a mathematical line and a line in a drawing. Each discipline makes use of its own precise vocabulary as an aid to representing and solving problems within the discipline.

I suspect that as you thought about this puzzle problem, you forgot about the possibility of the dot pattern being on a sphere, hyperbolic surface, torus, etc. There was no explicit statement in the problem that the nine dots are on a Euclidean plane. The geometry of three dimensions is quite a bit different than the geometry of two dimensions.

Part of thinking outside the box is to think critically and carefully. What do you actually know about the facts of the problem, and what do you make up in your mind? As you work to understand and create meaning in a problem, you may wall yourself into a box within which the problem cannot be solved.

## Problems and Problem Solving

Puzzle problems like these are only one type of problem. A great deal of this book is about problem solving and what we can learn about problem solving through studying and using games.

Problem solving consists of moving from a given initial situation to a desired goal situation. That is, problem solving is the process of designing and carrying out a set of steps to reach a goal. Figure 1.5 graphically represents the concept of problem solving. Usually the term problem is used to refer to a situation where it is not immediately obvious how to reach the goal. An identical situation can be a problem for one person and not a problem (perhaps just a simple activity or routine exercise) for another person.


Figure 1.5. Problem-solving-how to achieve the final goal?
Here is a formal definition of the term problem. You (personally) have a problem if the following four conditions are satisfied:

1. You have a clearly defined given initial situation.
2. You have a clearly defined goal (a desired end situation). Some writers talk about having multiple goals in a problem. However, such a multiple goal situation can be broken down into a number of single goal problems.
3. You have a clearly defined set of resources that may be applicable in helping you move from the given initial situation to the desired goal situation. There may be specified limitations on resources, such as rules, regulations, and guidelines for what you are allowed to do in attempting to solve a problem.
4. You have some ownership-you are committed to using some of your own resources, such as your knowledge, skills, and energies, to achieve the desired final goal.
These four components of a well-defined (clearly-defined) problem are summarized by the four words: givens, goal, resources, and ownership. If one or more of these components is less than fully filled, you have an ill-defined problem situation (frequently called a problem situation or an ill-defined problem) rather than a well-defined problem. An important aspect of problem solving is realizing when you are dealing with an ill-defined problem situation and working to transform it into a well-defined problem.

Consider some problem situations such as sustainability, global warming, globalization of business, terrorism, homelessness, drugs, and the U.S. scoring below some other countries in international tests. These are all problem situations because the givens, guidelines, and resources are not clearly specified. You may or may not happen to care about specific problems that relate to these problem situations.

Notice also that each of the above-mentioned problem situations has a short name. It is easy to memorize a term such as sustainability or global warming. However, the terms mean different things to different people. Part of the difficulty of getting large-scale or worldwide collaboration in attempting to deal with these problem situations is to get wide-scale agreement on the actual problem being addressed.

Nothing in the definition of problem suggests how difficult or challenging a particular problem might be for you. Perhaps you and a friend are faced by the same problem. The problem might be very easy for you to solve and very difficult for your friend to solve, or vice versa. Through education and experience, a problem that was difficult for you to solve may later become quite easy for you to solve. Indeed, it may become so easy and routine that you no longer consider it to be a problem.

People are often confused by resources (number 3) in the definition above. Resources merely tell you what you are allowed to do and/or use in solving the problem. Indeed, often the specification of resources is implied rather than made explicit. Typically, you can draw on your full range of knowledge and skills while working to solve a problem.

However, the rules are different when taking a test and in many other situations such as playing a game. When taking a test or doing a homework assignment, you are not allowed to cheat, steal, or plagiarize. Some tests are open book, and others are closed book. Thus, an open book is a resource in solving some test problems, but is cheating when not allowed (a limitation on resources) in solving other test problems.

Nowadays, people often have access to computers as they work to solve a problem. They draw upon both the capabilities of their own mind/brain and of Information and Communication Technology (ICT) systems.

They may make use of computational thinking (see Introduction) as an aid to problem solving. I like to view this as the think outside the box strategy. My "box" has expanded to include information that I can quickly and inexpensively access from the Web, as well as computerized processes available on the Web for solving a very wide range of problems.

Resources do not tell you how to solve a problem. For example, you decide to create a nationwide ad campaign to increase the sales by at least $20 \%$ for a set of products that your company produces. The campaign is to be completed in three months, and it is not to exceed $\$ 40,000$ in cost. Three months is a time resource and $\$ 40,000$ is a money resource. You can use the resources in solving the problem, but the resources do not tell you how to solve the problem. Indeed, the problem might not be solvable.

Imagine an automobile manufacturer trying to produce a $20 \%$ increase in sales in three months, for $\$ 40,000$ ! At first, this certainly sounds like problem that has no solution. But, think outside the box. We now have the Web. The idea of advertising a product by word of mouth can be viewed in a new light if we think of word of mouth as people using social media to communicate via the Internet. Through use of the Internet, one person can convey a message to quite a few friends and acquaintances. Each of them can convey the message to quite a few more people. In a modest amount of time the original message can reach a significant percentage of the world's population.

Problems do not exist in the abstract. They exist only when there is ownership of the problem. The owner might be a person, a group of people such as the students in a class, or it might be an organization or a country. A person may have ownership assigned by his/her supervisor in a company. That is, the company or the supervisor has ownership, and assigns it to an employee or group of employees.

The idea of ownership can be confusing. In this book, we are focusing on you, personally, having a problem-you, personally, having ownership. That is quite a bit different than saying that our educational system has an achievement problem, our country has a large-scale poverty problem, and the world has a sustainability problem.

The idea of ownership is particularly important in teaching. If a student creates or helps create the problems to be solved, there is increased chance that the student will have a sense of ownership. Such ownership contributes to intrinsic motivation - a willingness to commit one's
time and energies to solving the problem. All teachers know that intrinsic motivation is a powerful aid to student learning and success.

The type of ownership that comes from a student developing or accepting a problem that he/she really wants to solve is quite a bit different from the type of ownership that often occurs in school settings. When faced by a problem presented or assigned by the teacher or the textbook, a student may well translate this into, "My problem is to do the assignment and get a good grade. I have little interest in the problem presented by the teacher or the textbook." A skilled teacher will help students to encounter challenging problems that the students really care about.

Now, what does this formal definition of problem have to do with using games to develop thinking outside the box? Plenty! In a game setting, the rules and regulations are usually carefully stated. Even then, however, there may be exceptions that allow outside the box thinking.

The 9 -dots puzzle certainly illustrates outside the box thinking. Expanding the size of the dots allowed us to see a 3-line solution. Use of a "wide line" in place of a mathematical line gave us a one-line segment solution.

You know that students often develop personal interest in (ownership of) the problem of playing a game well. Now, if only such games had redeeming educational value.... Wouldn't it be nice if students spent time in an intrinsically motivated state, working to learn to solve problems that they have ownership of, but that also tie in well with the contents and goals of the regular school curriculum? I wonder what school would be like if students spent much of their time in such an environment?

Here is one more example of thinking outside the box. The steam engine existed a long time before the internal combustion engine was developed. Imagine being an inventor studying a steam engine, and thinking about how to make a smaller and more fuel-efficient engine. Perhaps the firebox could be made a little smaller and better insulated? Perhaps one could find a fuel that is more concentrated than coal or wood? Thinking outside the box led to using a fuel such as gasoline, and having the "fire" occur right next to the piston, inside the cylinder that contained the piston. What a marvelous example of thinking outside the box!

## Problem Solving Is Part of Every Discipline

In general terms, each discipline or domain of study can be defined by its unique combination of:

1. The types of problems, tasks, and activities it addresses.
2. Its tools, methodologies, and types of evidence and arguments used in solving problems, accomplishing tasks, and recording and sharing accumulated results.
3. Its accumulated accomplishments such as results, achievements, products, performances, scope, power, uses, impact on the societies of the world, and so on.
4. Its history, culture, unifying principles and standards of rigor, language (including notation and special vocabulary), and methods of teaching, learning, and assessment.
5. Its particular sense of beauty and wonder. A mathematician's idea of a "beautiful proof" is quite a bit different from an artist's idea of a beautiful painting or a musician's idea of
a beautiful piece of music-though all feature a firm grasp of the goal(s), economy of means, and eschewing of extraneous intrusions.

Each discipline has its own ideas as to what constitutes a problem to be solved or a task to be accomplished. The following list is not all-inclusive, but is intended to emphasize that we are interested in general ideas of problem solving in all disciplines. We are interested in:

- Question situations: recognizing, posing, clarifying, and answering questions.
- Problem situations: recognizing, posing, clarifying, and solving problems-bridging gaps between initial states and desired future states.
- Task situations: recognizing, posing, clarifying, and accomplishing tasks.
- Decision situations: recognizing, posing, clarifying, and making decisions.
- Using higher-order, critical, creative, and wise thinking to do all of the above. Often the "results" are shared or demonstrated as a product, performance, or presentation.
- Using tools that aid and extend one's physical and mental capabilities to do all of the above. Examples of such tools include reading, writing, math, and computers.

Throughout this book we will be discovering and exploring various strategies for problem solving. The single most important strategy for problem solving is building upon the previous work of yourself and others. We will call this the build on previous work strategy. You may prefer to call it the look it up strategy. The development of the Internet and the Web have made it much easier to retrieve information from online and physical libraries, and from other people. Moreover, tens of thousands of computer programs have been written so that computers can directly solve or help to solve many of the problems that people want to solve.

## Cognitive Maturity

You make routine use of a number of different problem-solving strategies without giving much thought to them. As an example, often when you are about to make a decision, you think about the consequences of this decision. You mentally "play out" what might happen in the future if you make a particular decision or take a particular action.

If you are impulsive-perhaps often acting without thinking of the consequences-you can work to overcome this impulsiveness. Thus, you can become much better at solving problems by merely learning to curb your impulsiveness!

You have had years of informal and formal education in this think before you act strategy. It is now a well-ingrained component of your cognitive maturity. As a parent or teacher, you undoubtedly place considerable emphasis on helping children make progress in this aspect of cognitive maturity.

Another good example is the set of strategies you bring to bear when faced by a challenging learning task. You know a great deal about yourself as a learner. You can self-assess your progress in learning. You can set standards based on how well you have done other learning tasks. Your strategies in dealing with a challenging learning task are an important aspect of your current level of cognitive maturity. You certainly want to help children make progress in learning and using their own set of strategies in this area.

Notice that these aspects of cognitive maturity are not dependent on having learned any specific discipline. Cognitive maturity is a component of every discipline, and it cuts across all disciplines. Games can be used to help create an environment in which children can increase their levels of cognitive maturity. It is easy to see how an adult who has a higher level of cognitive maturity than a student can serve as a teacher and mentor in helping a student increase in cognitive maturity.

## George Polya's General Problem-Solving Strategy

George Polya (1887-1985) was a great mathematician and teacher who wrote extensively about problem solving (California Mathematics Council, n.d.). Polya's six-step problem-solving strategy is useful in math and in most other disciplines. The following version of this strategy has been modified to be applicable in many problem-solving domains. All students can benefit from learning and understanding this strategy and practicing its use for a wide range of problems.

1. Understand the problem. Among other things, this includes working toward having a clearly defined problem. You need an initial understanding of the Givens, Resources, and Goal. This requires knowledge of the domain(s) of the problem, which could well be interdisciplinary.
2. Determine a plan of action. This is a thinking activity. What strategies will you apply? What resources will you use, how will you use them, and in what order will you use them? Are the resources adequate to the task?
3. Think carefully about possible consequences of carrying out your plan of action. Place major emphasis on trying to anticipate undesirable outcomes. What new problems will be created? You may decide to stop working on the problem or return to step 1 because of this thinking.
4. Carry out your plan of action in a reflective, thoughtful manner. This thinking may lead you to the conclusion that you need to return to one of the earlier steps. Note that reflective thinking leads to increased expertise.
5. Check to see if the desired goal has been achieved by carrying out your plan of action. Then do one of the following:
a. If the problem has been solved, go to step 6 .
b. If the problem has not been solved and you are willing to devote more time and energy to it, make use of the knowledge and experience you have gained as you return to step 1 or step 2.
c. Make a decision to stop working on the problem. This might be a temporary or a permanent decision. Keep in mind that the problem you are working on may not be solvable, or it may be beyond your current capabilities and resources.
6. Do a careful analysis of the steps you have carried out and the results you have achieved to see if you have created new, additional problems that need to be addressed. Reflect on what you have learned by solving the problem. Think about how your increased knowledge and skills can be used in other problem-solving situations. Work to increase your reflective intelligence!

## Algorithms and Heuristics

An algorithm is a step-by-step set of instructions guaranteed to solve a specific type of problem or accomplish a specific type of task. You know the algorithms for adding a column of integers or multiplying a pair of integers. A carefully written and tested recipe from a cookbook can be thought of as a different type of algorithm.

Quoting from the Wikipedia (https://en.wikipedia.org/wiki/Algorithm):
Starting from an initial state and initial input (perhaps empty), the instructions [in an algorithm] describe a computation that, when executed, proceeds through a finite number of well-defined successive states, eventually producing "output" and terminating at a final ending state.

A heuristic is a step-by-step set of instructions designed to solve a specific type of problem or accomplish a specific type of task. From a user's point of view, a heuristic succeeds often enough to be useful, but at other times may not succeed. As an example, think about the heuristic a person uses when looking for a parking space in a large parking lot that is nearly full, or a heuristic for avoiding bad traffic.

Quoting from the Wikipedia (https://en.wikipedia.org/wiki/Heuristic):
A heuristic technique, often called simply a heuristic, is any approach to problem solving, learning, or discovery that employs a practical method not guaranteed to be optimal or perfect, but sufficient for the immediate goals. Where finding an optimal solution is impossible or impractical, heuristic methods can be used to speed up the process of finding a satisfactory solution. Heuristics can be mental shortcuts that ease the cognitive load of making a decision. Examples of this method include using a rule of thumb, an educated guess, an intuitive judgment, stereotyping, profiling, or common sense.

Suppose you are trying to remember the name of a person you talked with a couple of weeks ago. It might help to think carefully about when, where, and why you met the person, or what you talked about. It might help to start with the letter a in the alphabet, think about names beginning with $\mathbf{a}$, then proceed to the letter $\mathbf{b}$, and so on. These are heuristics.

In attempting to understand a math problem, a sometimes-useful heuristic is to draw a picture or diagram for the problem situation. If I am stuck in solving a particular problem, it is often helpful to try to divide the problem into several simpler problems.

Many game players learn a variety of algorithms and heuristics to help improve their skill in a particular game. For example, when I was leaning to play chess I learned the heuristic, "When in doubt, push a pawn." In other words, if I cannot figure out a good, productive move, the heuristic suggests I move a pawn. For bridge playing I learned the heuristic, "When in doubt, lead a trump." These heuristics apply to specific games and are only meaningful to a person who knows how to play those specific games.

## Modeling and Simulation

When you were a child, you may well have built and/or played with model cars, model airplanes, and model people (such as toy figures). A model car has some of the characteristics of a "real" car. In general a model, in contrast to the real thing or situation, makes an activity simpler, safer, or more convenient.

Models have long been used as an aid to representing and solving problems. For example, when the Wright brothers were in the process of developing their first airplane, they developed models of components of their airplane (such as a wing) and tested them in a wind tunnel they built.

The development and use of computer-based models is a valuable new addition to the use of models to help solve problems. A computer model of a car or an airplane can be tested in a virtual wind tunnel (that is, in a computer model of a wind tunnel). In biology, chemistry, physics, and other sciences, computer modeling and then running simulations using the models has become a routine aid to research. Indeed, the three standard approaches to research in science are now experimental, theoretical, and computational. The term computational in this case means computer modeling and simulation.

Computational thinking includes thinking in terms of computer modeling and simulation. Building a computer model requires algorithms, which means that assumptions and instructions must be explicit-open to inspection. It also includes thinking in terms of mental modeling and simulation. When you are mentally considering the possible results of various decisions you might make, you are doing mental modeling. That is, you are doing a form of computational thinking.

Spreadsheet software was originally designed for modeling and simulation in business. A spreadsheet model was designed to represent a certain part of a business, such as inventory or payroll. "What if" types of questions could be answered by running the model (that is, doing a computer simulation based on the model) to help answer questions. Spreadsheet models are now a routine tool in business and a number of other fields.

How does this fit in with games? A game can be thought of as a model. Let's take Monopoly as an example. In this game, one buys and sells property, invests in houses and hotels on a property, and travels around the game board. Movement is determined by rolling a pair of dice to produce the number of steps. (As an aside, rolling dice introduces a type of randomness into the game. Can you think of events in your life that seem to have an element of randomness to them?)

The game and its rules can be thought of as a model; playing the game is doing a simulation based on the model. Now, let's carry this one step further. While Monopoly was originally developed as a physical board game, people can now play a computer model of the original game.

Computer models have many advantages. For example, the computer system can help take care of many of the details of playing the game. A computer can furnish a virtual board, virtual playing pieces, and virtual money instead of the physical objects. Thus, none of these objects get worn out, damaged, or lost. A possible disadvantage is loss of sensory pleasure. In the game of Go played with traditional stones and board, the sound made when placing a stone is an aesthetic part of the game experience.

A second advantage of the computer model/simulation is that rules are strictly enforced. A player cannot "accidentally" move one space too far or pay less than the required rent in Monopoly.

A third advantage of the computer model/simulation is the easy setup and take-down of a game. The computer does this for the players.

There are other advantages. Here is a quote from http://www.download-freegames.com/board game download/monopoly3.htm, a website that sells a computerized version of Monopoly:

Monopoly 3 is an exact replica of the traditional board game-only better! Animated tokens and property auctions, talking game announcer, the ability to customize rules and game boards, and online play all improve the classic game to make it even more fun than before.

For additional challenge, choose from 3 different skill levels when playing [against] the computer. Have you always played with a cash bonus on the free parking space? No problem! Just create your own customized rule and you can play Monopoly the way you always have. Overall, Monopoly 3 is a great game for the entire family.

You know, of course, that Monopoly is a game for two or more players. Notice that with the software described above, your opponents can be virtual opponents with the computer playing these roles. Similar types of advantages hold for computerized versions of many traditional games.

Many computerized games have another provision that allows the player to take back or undo a move. For example, suppose that you are playing some version of a solitaire card game on a computer. The computer quickly shuffles the deck and lays out the cards. As you make your moves, you can easily undo a move or a sequence of moves. Indeed, provision is usually made so that a single keystroke allows the player to start over, using the same initial card layout. The undo feature allows you to explore "what-ifs."

Computer modeling and simulation is now one of the most important aids to problem solving. You and your students can learn about uses of this strategy through playing and studying games.

## Games Can be Addictive

There are many different sources or types of addiction. Moreover, the term addiction is often used quite loosely. Thus, a person might say that I am addicted to my morning cup of coffee. The person might then go on to talk about caffeine being an addictive drug and that people experience headaches and other effects as they try to kick the caffeine habit.

Some examples of addictive substances include heroin, morphine, oxycodone, amphetamines, tranquilizers, cocaine, alcohol, nicotine, and caffeine. Some possibly addictive activities include work, shoplifting, gambling, social networking, and games.

Games? When I was in graduate school, one of my friends flunked out of a physics doctoral program because he was addicted to 2-deck games of solitaire. Some of these types of solitaire games are very mentally challenging, requiring deep concentration and careful thinking. The "thrill of victory and agony of defeat" is experienced repeatedly through playing such games. The immediate mental stimulation (the flow of dopamine and other endorphins) can be exhilarating. My friend found that such immediately available rewards overwhelmed the feelings of satisfaction gained through doing physics homework problems and attending physics classes.

I could provide some personal testimonial of the addictive qualities of computer games-but I won't. Interestingly, I find that deep engagement in computer programming or in developing a spreadsheet has-for me-the same characteristics as game playing. Games, computer
programming, spreadsheets, and writing are all environments in which I can immerse myself, finding deep satisfaction in using my creativity and brain power. Here are two examples of thinking outside the box related to addiction.

1. All children growing up in our world will encounter numerous addictive or addictive-like drugs, opportunities, and situations. Part of a good formal and/or informal education is to learn about how to deal with these situations. For some people, games and social networking are sufficiently addictive, or addictive-like, so they provide an opportunity to study themselves in an additive-like setting.
2. For many students, games are intrinsically (internally) motivating. Self-motivation-or the lack thereof-is a very important aspect of education. Teachers work hard to motivate their students; parents work hard to motivate their children. How can teachers and parents take advantage of the intrinsic motivation of games? Undoubtedly you have heard the adage, "If you can't beat them, join them." Outside the box thinking suggests that games be integrated into the ordinary, everyday school curriculum. Our informal and formal educational systems should learn to take advantage of the addictive-like qualities of games.

Quoting James Gee (2004):
For people interested in learning, this raises an interesting question. How do good game designers manage to get new players to learn their long, complex, and difficult gamesnot only learn them, but pay to do so? It won't do simply to say games are "motivating." That just begs the question of "Why?" Why is a long, complex, and difficult game motivating?

Part of an answer is that game designers have researched and implemented profoundly good methods of getting people to learn and to enjoy learning. For a great many students, games and social networking are far more intrinsically motivating than "traditional" components of teaching and learning.

## Final Remarks

Games are a form of play. However, games provide an environment in which game players can learn about themselves. Games provide an environment in which one can interact with other people and develop certain types of social skills. Games provide an environment in which one can develop a variety of thinking and problem-solving skills that are useful in both non-game and game environments. Games provide an environment in which one can gain in mental maturity.

Many games have very long histories. Games that have survived over the years tend to have characteristics that fit well with the needs and interests of children and adults. Well before the advent of computers, many games had addictive-like qualities for some game players.

Computers have added new dimensions to games, and have provided new opportunities for a person to develop an addictive-like dependency on games. At the same time, computers are making possible games that have considerable educational value. The attention grabbing and attention holding characteristics of many of today's computer-based games are a challenge to our traditional formal educational system. At the same time, such games provide an opportunity for some changes that have the opportunity to improve our educational systems.

I recently used the quoted expression think outside the box as a search term in Google. This produced over 7 million results. It is clear that many people think about and write about thinking outside the box. However, our educational system experiences only limited success in developing this type of thinking in students. There is substantial room for improvement.

## Activities for the Reader

Some questions and activities are designed for people who are taking a workshop or course using materials from this book. Readers working alone may also find many of the questions and activities useful.

1. Think of some personal, real-world examples in which you thought outside the box. This book is one of my personal examples. Since I was a young child, I have played games for entertainment. Eventually I thought outside my "box" and began to explore possible educational values of the games I played as a child.
2. Social networking has some game-like characteristics. Do you consider social networking a game? Talk to some students who are "really into" social networking. Do they think of this activity as "sort of" a game?
3. Create a 16 -dots ( $4 \times 4$ grid) puzzle problem akin to the 9 -dots puzzle. Pose various goals associated with the puzzle grid and see if you can achieve these problem-solving goals. Many people enjoy creating puzzle problems and games. This is a different type of intellectual challenge than merely solving puzzles and playing games created by others. In the real world outside of games, problem posing (that is, creating or defining problems) is an important component of each discipline of study.
4. Consider the following quotation:

Comments from a student panel that my school district organized to investigate grading practices further elucidated the problem. Students reported that they see their schoolwork as a game they play for grades-a game that at best treats learning as an incidental, and at worst distracts students from making meaning. One student referred to this grade game as academic bulimia: Students stuff themselves with information only to regurgitate it for the test, with little opportunity for any thoughtful engagement that would produce deep understanding and growth (Winger, November, 2005). [Bold added for emphasis.]

Compare and contrast use of the term game in this quotation with the types of games and educational uses of games being discussed in this book.
5. Do some research on the think outside the box strategy. Develop some ideas on how to improve your own ability to think outside the box, and how to help improve the ability of students to do so.
6. Do some research on the think inside the box strategy. Develop some ideas on how to improve your own ability to think inside the box, and how to help improve the ability of students to do so. Can you tell when you are thinking inside the box and when you are thinking outside the box?

## Activities for Use with Students

This section contains some ideas for use with students. Those making use of these suggestions will, of course, adjust the activities to fit their students' abilities, interests, and needs.

1. What are some games today's students find it fun to play? Ask one or more students to make a long list of games they have played and enjoyed. As the list is being created, divide the games into four categories:
a. Board games, card games, and other types of non-electronic games that are not physical sports games.
b. Electronic games.
c. Physical sports games.
d. Others that do not fit easily into any of the above categories. Perhaps social networking, texting, and other uses of a Smartphone fall into this category.

Use this activity to promote a discussion about whether a game can fit into more than one category, what a game is and is not, whether a puzzle is a game, what makes a game fun, whether and how a game can be fun for one person and not for another, and so on.
2. What does the term problem mean to your students? This question is suitable for use in small group and whole class discussions. Build on your students' responses to help them view the ideas of problem and problem solving much more broadly than they currently do. Make sure that ownership is included in the discussion.
3. Engage students in a discussion about what they have learned and other ways in which they have benefited by playing the various types of games on their list.
4. Have students individually select a game that they have played, and then suggest some changes in the game that would make it more fun, a better social experience, or a better learning experience. Encourage students to think outside the box.
5. Lead your students in a brainstorming session about what it might mean to think outside the box. After your students reach a reasonable level of agreement on what this term means, engage them in discussing the extent to which schools and parents place a lot of emphasis on thinking inside the box, and why they might do so.
6. Talk with your students about cognitive maturity. Try to elicit their insight into the cognitive maturity of older and younger siblings. What do parents mean by the question, "Why don't you just grow up?" Help your students to develop examples in which they display greater cognitive maturity than they did a year or so ago. Do whole class brainstorming on ways to increase one's level of cognitive maturity. To what extent do they agree with the idea that few people grow to full cognitive maturity before their midtwenties? See http://ontheirlevel.org/whats-happening/maturation-of-the-teen-brain/.

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## Chapter 2

## Background Information

Men occasionally stumble over the truth, but most of them pick themselves up and hurry off as if nothing ever happened. (Sir Winston Churchill.)

A pessimist sees the difficulty in every opportunity; an optimist sees the opportunity in every difficulty. (Sir Winston Churchill.)
This book is about some roles of games in informal and formal education. The emphasis is on learning to be better at problem solving. Many people see Games-in-Education as an opportunity to help improve our educational systems. Others see the difficulties and downsides of using or increasing the use of games in education. (See the two quotes from Winston Churchill above.)

For many people, games are intrinsically motivating. Educational research tells us that intrinsic motivation contributes substantially to learning. From an educational point of view, the issues are what one learns through playing games, how this learning relates to helping students achieve agreed upon goals of education, and what roles teachers and other mentors should play in game selection, facilitating game playing, encouraging game playing, and helping "appropriate" learning occur during game playing.

This chapter provides background that will help us explore some possible roles of games in improving our informal and formal educational systems.

## Types of Games Considered in this Book

In this book, game includes electronic and non-electronic games and puzzles. Many games are playable both in a computer mode and a non-computer mode. For example, many solitaire card games and poker games require only a standard 52-card deck. Usually these can also be played on a handheld electronic game device, a game machine, a Smartphone, or a computer. In this book, the term computer game is taken to include all electronic games, whether played on inexpensive battery-powered handheld devices, Smartphones, game machines, supercomputers, or computer networks such as the Web.

The Use of Computer and Video Games for Learning (Mitchell \& Savill-Smith, 2004), is a British government-funded review of the computer game literature. The following quote from this document offers definitions of play and game:

First, play: something one chooses to do as a source of pleasure, which is intensely and utterly absorbing and promotes the formation of social groupings (Prensky 2001, page 112). Fun, in the sense of enjoyment and pleasure, puts us in a relaxed receptive frame of mind for learning. Play, in addition to providing pleasure, increases our involvement, which also helps us learn (Prensky 2001, page 117).

Second, a game: seen as a subset of both play and fun (Prensky 2001, page 118). A game is recognized as organized play that gives us enjoyment and pleasure (Prensky 2001). Dempsey et al. (1996, page 2) define a game as: ... a set of activities involving one or
more players. It has goals, constraints, payoffs and consequences. A game is rule-guided and artificial in some respects.

The Mitchell and Savill-Smith survey of the literature is oriented toward learning and the educational values of games. This may help explain the mention of learning that appears in their definition of play. As I think about this, I am reminded of a statement I have heard many timesthat the "job" of young children is to play.

Here is another definition, from an article by Asgari and Kaufman (2002):
Garris, et al. (2002) define game play as "voluntary, nonproductive, and separate from the real world". On the other hand, Jones (1999) points out that for some people, computer and video games are real and sometimes, they are more engaging than reality. Computer games can be categorized as adventure games, simulation games, competition games, cooperation games, programming games, puzzle games, and business management games (Hogle, 1996, citing from Dempsey et al., 1993; Jacobs \& Dempsey, 1993). During the past 40 years, computer games have been played from a floppy disk, CD-ROM, with the use of email, or online through the Internet. Computer games can be played individually, against the computer, or against other people face-to-face or on-line. [Bold added for emphasis.]

Games are a very large part of the real world of a great many people (McGonigal, 2011) and (McGonigal, February, 2010). Notice the bolded statement in the above definition. For many people, games are attention grabbing and attention holding. They are intrinsically motivating, and they may be addictive. This is an important idea to keep in mind as you explore possible educational roles of games. I am interested in how games can be used to improve education. At the same time, I am fully aware that games can damage a person's education and other aspects of their life. For example, it is well known that addiction to gambling games has seriously damaged or destroyed many lives!

Here is another quite useful way to think about games (Costikyan, 1994):
Games provide a set of rules; but the players use them to create their own consequences. It's something like the music of John Cage: he wrote themes about which the musicians were expected to improvise. Games are like that; the designer provides the theme, the players the music.

A game is a form of art in which participants, termed players, make decisions in order to manage resources through game tokens in the pursuit of a goal.

My doctorate is in mathematics. Thus, it is not surprising that I pay particular attention to games that have been developed to help teach mathematics. For the most part, the examples that I have studied tend to have both poor attention-grabbing characteristics and poor entertainment value. They do not compete well with games that children choose to play for entertainment. A later chapter will explore some educational math games.

## Goals of Education

Education has many goals, and each person tends to have their own ideas as to what constitutes a good education. David Perkins's book, Smart Schools: Better Thinking and Learning for Every Child (1992), contains an excellent overview of education and a wide variety of attempts to improve our educational system. He analyzes these attempted improvements in
terms of how well they have contributed to accomplishing three basic and enduring goals of education. The following list of educational goals is based on his work.

1. Acquisition and retention of basic, important, knowledge and skills. There is considerable agreement that reading, writing, arithmetic, speaking, listening, and information retrieval are basic and important goals for all students. However, there is disagreement about ways to achieve these goals in a cost effective manner that has a very high probability of success. There is less agreement on what students should learn in the fine and performing arts, health, science, social science, physical education, and other commonly taught disciplines-let alone drivers' education and skills needed to maintain a household.
2. Understanding of one's acquired knowledge and skills. Understanding tends to be difficult to define and measure. However, there is considerable agreement nowadays that education must proceed far beyond rote memorization.
3. Active use of one's acquired knowledge and skills. This includes being able to transfer one's learning to new settings, and being able to analyze and solve novel problems. Those who value education highly expect our educational system to:
a. Provide challenging and rigorous programs of study designed to help each student become a literate, responsible, creative adult citizen.
b. Help each student learn to learn, learn to take responsibility for her or his own learning, understand her or his capabilities and limitations as a learner, and to develop persistence and other lifelong habits of learning.
c. Help each student learn to help others learn. In this, it is helpful to think of each student as a teacher. For example, students often help each other and their siblings to learn, and parents spend a lot of time working with their children in "teacher" mode.
d. Help each student learn to cope with technological, social, and other forms of change that will be occurring during her or his lifetime.

This book explores a variety of games in terms of how they contribute to achieving the types of goals listed above. You will note that these goals are quite general-they do not speak to students gaining knowledge and skill in specific disciplines. In that sense, these goals fit in well with a student gaining in general cognitive development and cognitive maturity

However, learning in specific disciplines is an important aspect of getting a good education. There is quite general agreement that students should gain a substantial level of expertise in reading, writing, math, science, and social science. There is steadily growing support of an educational goal that all students should acquire knowledge and skill in using calculators and computers as a general aid to problem solving. However, there are considerable differences of opinion as to what students should learn and how they should demonstrate their knowledge and skills.

## Games-in-Education as a Discipline of Study

The field of education can be divided into many different disciplines. Similarly, the field of games and gaming can be divided into many different disciplines. This book explores some of the overlap between education and games. As illustrated in Figure 2.1, the overlap can be thought of as a discipline called Games-in-Education.


Figure 2.1. Venn diagram illustrating Games-in-Education.
The Games-in-Education discipline gained increased legitimacy in October of 2003 when the Massachusetts Institute of Technology announced an initiative to study educational roles of computer games (MIT, 2001-2003). A number of colleges and universities now offer undergraduate and graduate degree programs in the development of computer games. See http://successfulstudent.org/27-best-video-game-colleges-2015/.

## Expertise

It is useful to think about learning in terms of how it contributes to increasing one's level of expertise in a discipline or in some particular, more-limited area. Figure 2.2 shows a generalpurpose expertise scale.


Figure 2.2. Expertise scale.
The words expert and expertise do not mean the same thing. From an informal and formal education point of view, through training, education, and experience, a person can gain increased expertise in a particular area. People become experts in an area when they have suitable attitudes or dispositions, an appropriate level of natural ability, and work long enough and hard enough at increasing their knowledge and skills to a very high level. In comparing experts in a discipline, we sometimes talk about someone as being a local, national, or world-class expert.

There has been substantial research on the natural ability, education and training, perseverance and determination, time and effort, and so on that it takes for a person to gain a very high level of expertise. As a rough rule of thumb, it takes 10,000 to 15,000 hours of hard work spread out over about ten years to "be all you can be" in a particular, non-trivial area.

For example, consider a six-year-old girl who seems to have the physical ability to become a good gymnast, ice skater, or swimmer. Ten or twelve years later, this girl (now a young woman) competes in the Olympics. She has probably put in more than 10,000 hours-averaging more than three hours per day for many years-achieving her current level of skill. She has had excellent coaches and training facilities for a good part of this time.

Many years ago I earned a doctorate in mathematics and began to write papers accepted for publication in refereed journals and to work with doctoral students. I had probably put in 12,000
hours studying mathematics to achieve my PhD level of mathematical research expertise. By the time I finished my doctorate I was good, but by no means world class.

After completing my doctorate, I became interested in writing books to support my teaching interests. I have authored or co-authored about 65 books and a very large number of other documents. I estimate that I have spent more than 35,000 hours writing-developing, honing, and using my writing skills.

Chess players ranked in the top ten in the world are likely to have put in 15,000 to 30,000 hours or more gaining their chess skills. In 2015, the average age of the top 100 players in the world was about 32 years. (See https://ratings.fide.com/top.phtml?list=men.) People who play chess at this level usually put in well over 2,000 hours a year developing, honing, and maintaining their chess skills.

Benjamin Bloom (best known for Bloom's Taxonomy) was the editor of Developing Talent in Young People (Bloom, 1985). The authors of this book studied 120 people in six different disciplines who rose to world-class levels. The time they had spent in their specialty areas varied somewhat with the specialty. The pianists who were identified and studied had a mean age of about 23 when they achieved world-class stature. On average, they had been begun taking piano lessons at age 6 .

I find it interesting to compare these numbers with the amount of formal schooling that students receive in K-12 education in the U.S., now totaling about 12,000 to 14,000 hours in length. Estimates are that only about $1 / 2$ to $2 / 3$ of this time is actually used productively. However, we can add to this total the time that is productively spent on homework and informal educational activities. Thus, we might conclude that the focused, productive time students spend in K-12 education is about the amount of time it takes for a person to develop a high level of expertise in a narrow discipline such as chess, gymnastics, math, piano, or swimming. But typical 18 year olds have spread their learning time and effort over many different areas.

Bloom's analysis of young people achieving at a world-class level provides many examples of students doing well in school while putting in a thousand or more hours per year in their specialty area. This requires careful scheduling of time and a high level of sticking to the task. The single mindedness of purpose and high standards that these young people deal with tend to be very helpful in later careers-or it can lead to burnout if the achievers fail to keep their balance as they transition to the next phases of their lives.

The research data on how long it takes a person to achieve their potential in a discipline can be compared with data on how much time our K-12 schools are able to devote to teaching various discipline areas. Suppose, for example, that a school system places a very strong emphasis on reading and writing, with two hours per school day just in this area. This means that a student would get about 4,600 hours of formal schooling in this area during a K-12 education. Suppose, at the same time, the school system devotes an hour a day to math. This amounts to about 2,300 hours in total.

We also need to think about the physical and mental maturity needed for a high level of expertise in an area. Very few 18 -year-olds are physically ready to compete at the highest professional levels in football or basketball. The average person reaches cognitive (brain development) mental maturity by about age 25 or 26 .

This sort of analysis suggests why college education is so helpful. It also suggests why schools tend to focus so much attention on the "core" basics, downplaying or eliminating the "frills." I hear many people say:
"How could we possibly let a student who is struggling in math and reading spend any significant amount of school time on art, music, sports, or games? If it isn't on the state or national tests, then we should not be wasting school time on it. We need to spend all of our school time getting students to meet the state and national standards. We need to do a lot better in international competitions in the areas of these standards."
Indeed, there are continuing demands to increase the length of the school day and the length of the school year. We could add thousands of hours to the available formal schooling hours to students prior to their completion of the $12^{\text {th }}$ grade.

Two of the things missing from the above (in my opinion, quite short-sighted) point of view are (1) each individual student's intrinsic motivation, and (2) the importance of striving to meet the individual needs and interests of each student. Intrinsic motivation and striving to meet the individual needs of different students are two of the most important ideas in education. Yes, the basics are important. However, many other important educational goals are not included in the basics and are not on the state and national tests. A few examples include:

1. Learning to learn and to help others learn; learning about one's strengths and weaknesses as a learner.
2. Learning to work both individually and collaboratively with a team on large, long, and challenging projects.
3. Learning for transfer of learning.
4. Learning to improve one's creativity.
5. Learning that helps to increase one's level of cognitive development and cognitive maturity.
6. Learning to make effective use of new aids to solving problems and accomplishing tasks, such as computer modeling and other aspects of Information and Communication Technology (ICT).
7. Learning to make use of all of the above in doing things-developing products; doing performances and presentations; solving challenging, complex problems; and accomplishing challenging, complex tasks.
There are many ways to approach these important educational goals. This book presents ways in which Games-in-Education can help.

## Competition, Independence, Cooperation

Each game can be analyzed from a point of view of its:

- Cooperation/collaboration.
- Independence (not cooperative, not competitive).
- Competition, leading to the determination of winners and losers.

Of course, a game may have components falling into each of these categories. Sometimes, it is not easy to decide which categorization best describes a particular game.

Let's use a crossword puzzle from the morning newspaper as an example. Suppose I work alone (independently) doing a crossword puzzle. I am not competing "head-to-head" with anybody, and I am not cooperating with anybody.

Later in the day, I might talk to a friend who also does the crossword puzzle from the same morning newspaper. We might talk about how hard or easy the puzzle seemed to be. We might talk about how long it took us to complete the puzzle, or how many clues we were unable to decipher. We might even discuss a particular clue, in a cooperative effort to figure it out.

Now, suppose I make use of the Web in doing the crossword puzzle. Does this constitute cheating, or is it a learning experience in which I am cooperating with a computer to solve a problem? In my opinion, learning to work cooperatively with a computer in order to solve a problem or accomplish a task should be one of the goals of education.

Thus, we see how the independence of puzzle solving can be modified to being somewhat competitive and/or somewhat cooperative. Moreover, a social interaction dimension can be added to the overall activity. Often such social interaction can be considered as being cooperative/collaborative in nature.

Some games have a strong social interaction characteristic. This is especially true for young children. Computer games and social networking have substantially changed the nature of social interaction in the way that many people spend their time. Sherry Turkle is a world-class researcher in this area (Turkle, 4/13/2012). She writes and presents talks about the fact that children are not developing face-to-face social skills as they did in the past. You don't need to do or read deep research to understand something has changed as you watch a group of youngsters each engrossed in interacting via Smartphone with other people. Some may be interacting with each other electronically when they could be holding a face-to-face conversation.

The idea of independence in game playing is worthy of further exploration. Suppose I am a recreational bowler. I bowl alone, but I keep a careful record of my scores. Thus, I can tell if I am doing better, about the same as, or worse than I have in the past. This can be thought of as me competing with myself. However, in my opinion that is a poor use of the idea of competition. Competition is more generally a win-lose situation, or being better than an opponent at least in some aspects of a competitive situation.

This is a very important idea in education. Suppose education is considered as a type of game that is designed for independence, rather than for competition or cooperation. As a learner, my goals might be to learn, to get better at learning, to learn to use my learning to better myself, and so on. I take satisfaction in the process of learning, in having learned, and in using what I have learned.

However, it is very helpful to have measures (for my own personal use) of how well I am doing. Am I a better reader than I was last month? Do I understand quadratic equations better than I did a week ago? Can I sight read music and play it on a piano better than I could a year ago?

Moreover, keep in mind that each person is different, and that there are quite large differences in abilities, interests, drive, and so on. I may well want to have some information
about what others are doing and are able to do, but my focus is upon myself as a learner. In some sense, I want to "be all I can be."

As an example, consider learning to keyboard for input to a computer. Personally, I can keyboard much faster than I can hand write or print, and my keyboarded materials are far more legible than my handwriting or printing. Moreover, keyboarding in a word processing environment is a great aid to my writing, as the spell checker and grammar checker find many of my errors, and the word processor aids me in my revision efforts. From time to time, I feel a certain amount of envy of people who can keyboard faster and more accurately than I, or who are better at spelling. In essence, however, keyboarding for me is neither competitive nor cooperative. My (independent) expertise in keyboarding is at a sufficient level to be a great aid to achieving my writing goals.

Contrast this independence model with a competitive model of education. The competition can be with other students, or it can be with "norms" that have been established for various state and national tests. The learner's goal becomes one of winning or being better than other people.

- "I got the top score in our class on this test!"
- "I am the best speller in my school!"
- "I am the fastest keyboarder at my grade level in our school!"

Another type of competition is scoring high enough to meet some specified requirements.

- "I have passed both the reading and math tests required for graduation!"
- "I scored high enough on my SATs to get into an Ivy League school!"

Still another way to look at competition-independence-cooperation is to consider competition versus cooperation. Competitiveness is a genetic characteristic, and all people are competitive. However, people vary considerably in the nature and degree of their competitiveness, and competitiveness is strongly influenced by one's home environment, community environment, and culture.

Moreover, research suggests that males are, on average, more competitively oriented than are females. Put another way, research suggests that females are, on average, more cooperative/collaborative oriented than are males. (Note that this may be a product or how children are raised, a product of male versus female genetics, or both.)

Knowing this, how should we design our educational systems? Research in education supports the cooperative/collaborative approach over the competitive approach. This research indicates that designing schooling along cooperative/collaborative approaches is more effective than designing schooling along competitive lines. Quoting from Clayton Tucker-Ladd's book, Psychological Self-help (Tucker-Ladd, 2003):

It takes Kohn an entire book to summarize the massive data indicating that competition in our society is harmful. Yet, our culture proclaims (without adequate supporting data) just the opposite, that competition is efficient, healthy, and fun. Actually, hard research data documents that people achieve more if they work cooperatively with others (than if they work competitively). We are so brainwashed, we find that hard to believe. (Think of it this way: trying to do your best is very different from trying to beat everyone else.) On the other hand, we can readily accept that a competitive job, school, or social situation,
where someone wins by making others fail, causes dreadful stress, resentment of the winner, contempt for the losers, low self-esteem, and major barriers to warm, caring, supportive relationships. What is the solution? Kohn recommends replacing competition with cooperation, i.e. working together, assuming responsibility for helping each other do our best, and uncritically valuing each other's contributions. We need lots of research to help us to know when and how to reduce our competitiveness. To change our goals in life from competition to cooperation, we need new values and a new philosophy of life (see chapter 3). Competition implies a hierarchy; cooperation implies equality.
Kohn is raising fundamental questions about deeply ingrained American ideas, such as "winning is important," "you should be proud of beating someone who is good," and "you must feel badly since you lost." These beliefs in competition remain strong (although all of us have suffered defeats). Our society is in a slow evolution in which various feelings of superiority are being challenged. For instance, feelings of sexual and racial superiority--chauvinism--have been hot issues for many years (Korda, 1975; Faludi, 1991). But I believe that superiority-inferiority feelings permeate our society, even in many ways we do not commonly acknowledge. Examples: Developed nations feel superior to less developed ones and take pride in beating other countries. Older persons and parents feel superior to youth. Youth feel superior (more "with it") to older persons. Owners and bosses feel superior to workers. The wealthy (even if it was inherited) feel superior to the poor. The smart and/or educated feel superior to the less well trained. Urban dwellers feel superior to persons who live on farms or ranches. The religious feel superior to other religions and non-believers. Women often feel superior to men in terms of morals. Maybe we all strive for some sense of superiority, as Adler suggested. Perhaps this is because we all feel inferior in some ways. Maybe we just grab on to a feeling of superiority whenever we can because it feels good. But, this self-centered I'm-better-than-you attitude causes many interpersonal and societal problems. The good news is: people can and do change their attitudes.

Quoting from an article by Anne Campbell (February, 2004):
Despite a recent surge of popular journalistic books (e.g., Fillion, 1997; Simmons, 2002; Tanenbaum, 2002), academic interest in competition among women was almost nonexistent until the 1980s. Initial research (Gilligan, 1982; Goodwin, 1980; Lever, 1976) found that girls tended to avoid competition in favour of tactics that diffuse conflict and preserve interpersonal harmony. When competition is made inevitable, girls used apologies and excuses to mitigate their behaviour (Hughes, 1988) or "double voicing" to promote their own cases while simultaneously taking into account the positions of their rivals, thereby preserving their relationships (Sheldon, 1992). This attenuation of competition in favour of sustaining positive relationships is thought to reflect socialisation into cultural norms against the overt expression of conflict among females (Miner \& Longino, 1987), (Tracy, 1991), and the greater centrality of intimate friendships to girls than to boys (Brown, 1998).

In brief summary, every person will at times be in situations that require competing for success, or even for survival. People use a variety of competitive techniques.

Therefore, education should prepare one to compete when desirable or necessary, and that there a variety of competitive techniques. In terms of games and gaming:

1. With a little effort, a person can find games that meet his or her interests in and/or orientation toward competition, independence, and cooperation. There are lots of games in each category, and many of these games have overlapping characteristics.
2. If we think about our overall educational system as a game, we can see competitive, independence, and collaborative aspects of this game. In many cases, we can see a mismatch between the characteristics that an individual student desires to develop or learn, and the characteristics that our educational system forces on the student.
3. The field of Games-in-Education can contribute to creating a school environment that better fits the individual competition-independence-collaboration needs of students.

## Learning to Learn

There has been quite a bit of research on how to help students learn faster and better. Somewhat surprising to me is that this is an area in which our educational system has not done a good job of translating theory into practice. You might test this out on yourself. Can you name any research that educators have done in the past two decades that specifically focuses on how to help students learn faster and better? My 11/2/2015 Google of the expression learn faster and better produced over 200 million results. Can you point to specific school-wide and school district-wide curriculum designed to help students learn to make use of the research in this area?

For example, metacognition and other reflective practices are very important in learning. Research indicates that even preschool age children can learn to do metacognition and can learn to reflect on their problem-solving and other activities. Are such metacognitive and reflective practices a routine part of the teaching/learning in schools that are familiar to you?

As another example, consider the fact that the Web is now the world's largest library, and that most students have access to the Web. Just because one has access to the Web does not mean that one has gained the knowledge and skills to make effective use of this electronic global library as an aid to solving problems and accomplishing tasks. Moreover, this electronic library is quite different from a static, print-based library. It is dynamic, with a significant portion of its content changing over the course of a day. This library is interactive, and a significant part of its content is in the form of "I, the computer, can do it for you." The Web provides access to many computer programs designed to solve certain categories of problems.

As a third example, consider computer-assisted learning and distance learning via the Internet. These somewhat individualized modes of teaching and learning are steadily growing in importance. The use of computer-assisted learning and distance learning also are growing in our K-12 schools. But, are the students you work with learning to seek out such materials on the web as aids to learning what they want to learn?

For a final example, consider the idea of self-assessment and of becoming an independent, self-sufficient learner who takes responsibility for her or his own learning. If anything, our current educational system seems to be moving away from this idea. High-stakes, summative assessment tests have become a driving force in our educational system. See my collection of self-assessment instruments available free on the Web (Moursund, 2015).

## Situated Learning and Transfer of Learning

Situated learning and transfer of learning are two important components of the discipline called learning theory. My 11/1/2015 Google search of the expression learning theory produced more than 27 million results. Quoting from the Wikipedia:

Learning theories are conceptual frameworks describing how information is absorbed, processed, and retained during learning. Cognitive, emotional, and environmental influences, as well as prior experience, all play a part in how understanding, or a world view, is acquired or changed and knowledge and skills retained.

The next two subsections cover two important learning theories that fit in well with this book.

## Situated Learning

Brown, Collins, \& Duguid (1989) wrote a seminal article on situated learning. Quoting from the introduction to this paper:

The breach between learning and use, which is captured by the folk categories "know what" and "know how," may well be a product of the structure and practices of our education system. Many methods of didactic education assume a separation between knowing and doing, treating knowledge as an integral, self-sufficient substance, theoretically independent of the situations in which it is learned and used. The primary concern of schools often seems to be the transfer of this substance, which comprises abstract, decontextualized formal concepts. The activity and context in which learning takes place are thus regarded as merely ancillary to learning pedagogically useful, of course, but fundamentally distinct and even neutral with respect to what is learned.

> Recent investigations of learning, however, challenge this separating of what is learned from how it is learned and used. The activity in which knowledge is developed and deployed, it is now argued, is not separable from or ancillary to learning and cognition. Nor is it neutral. Rather, it is an integral part of what is learned. Situations might be said to co-produce knowledge through activity. Learning and cognition, it is now possible to argue, are fundamentally situated. [Bold added for emphasis.]

Situated learning is a learning theory focusing on the situation or environment in which a particular learning activity occurs. For example, suppose that you are walking down a jungle path and you hear a particular sound that your brain/mind does not immediately recognize. You "freeze," carefully look around, and see a large snake.

Your brain/mind recalls that a friend of yours was seriously injured several weeks ago by a snake, and the description the friend gave seems to fit this snake. You immediately learn that the sound you have heard in this jungle trail environment is associated with a dangerous snake. Likely, this learning will last a lifetime. Moreover, the learning occurs very quickly-this is an example of one-trial learning.

Contrast this with being a middle-school student sitting in a classroom. You live in a large city, and there are few or no dangerous snakes within miles of your home. You are viewing a video discussing dangerous snakes. You see and hear video of approximately the same scene as the jungle walker. However, the room you are in is hot and stuffy, you have just had lunch and
you are sleepy, and the audio is turned up too high for your ears. What do you learn, and how long does this learning stay with you?

One of the reasons why a game can be a good learning environment is that the game player is immersed in the environment (the situation) of the game. The game's attention grabbing and attention holding characteristics tend to shut out distractions. The game involves doing thingsusing the knowledge and skills one has learned and is learning.

## Low-Road/High-Road Transfer of Learning

The low-road/high-road theory of learning has proven quite useful in designing curriculum and instruction (Perkins \& Solomon, September, 1992). In low-road transfer, one learns something to automaticity, somewhat in a stimulus/response manner. When a particular stimulus (a particular situation) is presented, the prior learning is evoked and used. The human brain is very good at this type of learning.

Low-road transfer of learning is associated with a particular narrow situation, environment, or pattern. The human brain functions by recognizing patterns and then acting upon these patterns. Consider the situation of students learning the single digit multiplication facts. This might be done via work sheets, flash cards, computer drill and practice, a game or competition, and so on. For most students, one-trial learning does not occur. Rather, a lot of drill and practice over an extended period, along with subsequent frequent use of the memorized facts, are necessary.

Many students find that they have difficulty transferring their arithmetic fact knowledge and skills from the learning environment to the "using" environment. One of the difficulties is recognizing when to make use of the memorized number facts. In school, the computational tasks are clearly stated; outside of school, this is often not the case.

This helps to explain why rote memory is often useful in solving routine, frequently occurring problems, but critical thinking and understanding are essential in dealing with novel and challenging problems. It also supports the need for broad-based practice even in low-road transfer. We want students to recognize a wide range of situations in which some particular lowroad transfer knowledge and skills is applicable.

Math education in schools tries to achieve an appropriate balance between rote memory and critical thinking by making extensive use of word problems or story problems. In word problems, the computations to be performed are hidden within a written description of a particular situation. The hope is that if a student becomes better at reading and deciphering word problems - extracting the computations to be performed and the meaning of the results-that this will transfer to non-school problem-solving situations and solving math problems in other disciplines.

It turns out that it is quite difficult to learn to read well within the discipline of mathematics. Many students have major difficulties with word problems and with learning math by reading math textbooks. Their depth of understanding of math and their ability to read math for understanding stand in the way of their being able to deal with novel, challenging math problems they encounter.

High-road transfer of learning for improving problem solving is based on learning some general-purpose strategies and learning how to apply these strategies in a reflective manner. The build on previous work strategy is an excellent candidate to use to begin (or, expand) your
repertoire of high-road transferable problem-solving strategies. To do this, think of a number of personal examples in which you have used this strategy as an aid to problem solving. Mentally practice what you did in each case. In the near future, each time you make use of this strategy, consciously think about its name and the fact that you are using it. Also, in the future when you encounter a challenging problem, consciously think through your repertoire of high-road transferable problem-solving strategies. Your goal is to increase your ability to draw upon this repertoire of aids to use when faced by a challenging problem.

The break a problem into smaller problems is another example of a high-road transferable strategy. This strategy is often called the divide and conquer strategy, and that is the name that will be used in the remainder of this book. It is helpful to have short, catchy names for strategies. A large and complex problem can often be broken into a number of smaller, more tractable problems. It is likely that many of your students do not have a name for the strategy and do not automatically contemplate its use when stumped by a challenging problem.

Appendix 1 to this book contains a large number of general-purpose problem-solving strategies.

Here is a summary of some key ideas in problem solving. Suppose you are faced by a problem. Then your approach might be:

1. If the problem fits a memorized pattern in which you can apply stimulus/response, lowroad transfer, your mind/body may react automatically and the problem may be quickly solved.
2. If (1) is not successful, think about the domain or general discipline of the problem and whether you have encountered the problem or a quite similar problem in the past. If you have specific knowledge and skills relevant to the problem or problem areas, draw upon this contextual, situational knowledge and skill in a conscious and considered manner to attempt to solve the problem.
3. If (2) is not successful, draw upon your general knowledge and skills in how to attack a new, challenging problem. Here, a large repertoire of high-road transferable problemsolving strategies is helpful.
4. If (3) is not successful, consider what resources, including other people, you might access that will help you get to approaches 1,2 , or 3 .

Figure 2.3 illustrates these three approaches and provides an indication of how fast each response may be in a particular situation.


Figure 2.3. Often-used approaches to problem solving.
In our exploration of strategies, we will emphasize teaching and learning for high-road transfer within the games domain and to other domains. That is, we want students to gain skills in transfer of problem-solving strategies from games to non-games situations, and vice versa.

Here is a general-purpose strategy for such teaching and learning:

1. Identify the generalizable strategy that is being illustrated and used in a particular problem-solving situation.
2. Give the strategy a name that is both descriptive and easily remembered.
3. Working with your students, identify a number of different examples in other disciplines and situations in which this named strategy is applicable.
4. Have students practice using the strategy in a variety of areas in which it is useful, and where students have appropriate general and domain-specific knowledge.
5. In your everyday teaching, you will frequently encounter situations in which a particular problem-solving strategy is applicable and you have previously helped your students gain some initial expertise in using the strategy. Take advantage of such situations by clearly naming the strategy (or, asking your students to name the strategy) and working with your students to refresh their memories on use of the strategy in a variety of situations.

## Learning in a Game Environment

Think about your roles as a parent, teacher, or other adult figure facilitating a child learning to play a game and then playing the game. What might you do to increase the child's cognitive, social, emotional, and kinesthetic growth in a manner that will transfer to other games and to non-game environments? As you think about this, you will realize that Games-in-Education is a challenging discipline!

You know that, for a child, learning to play a game and then playing the game are closely intertwined. Indeed, much of the learning occurs during the playing. This is a good example of a learn-by-doing, hands-on learning environment. The learning is in context (situated learning). The learning is immediately useful, contributing to being able to play the game and/or to play the game better.

This type of learning likely produces immediate gratification. Contrast it with the delayed gratification that is common in most formal schooling. Many students are not intrinsically motivated by statements such as, "You need to learn this so that you can use it in your course next year" or "Learn this because it will be on the test next week."

Teachers understand the gratification issue. They recognize the value of having students immersed in a combination of learning and doing. This ties in closely with the teaching/learning discovery-based learning strategy. In discovery-based learning, teachers try to create learning environments in which students make immediate use of their new learning. My 11/2/2015 Google search of the expression discovery-based learning produced more than 12 million results.

For example, suppose students are learning how to solve a particular type of math problem. Their use of this new knowledge and skill consists of working through many instances of this type of problem. Contrast this with a student learning a new type of chess opening (that is, sequence of initial moves) and then immediately using it in a chess game against an appropriate opponent. The new chess opening is used in the context of playing the overall game, and it adds to the fun of playing the game. It becomes part of the chess player's repertoire of openings. The chess literature contains detailed analyses of thousands of different chess openings. A good chess player is apt to have memorized a large number of opening sequences.

Think for a minute about the opening move in a competitive game such as chess or checkers, and the opening sentence in a piece of writing. There are many different types of writing situations. While rote memorization of a range of first sentences might be helpful and one can learn much from a Web search on the quoted expression opening sentence, a much better approach is to understand the various types of writing situations and what one is trying to accomplish in an opening sentence in these different situations. Thus, you can see that the writing challenge is much more complex than the game-playing opening move challenge in chess.

Moreover, if you teach writing, you may see that we have raised an interesting topic you can discuss with your students. Rote memorization is quite useful in improving one's skill as a chess player. How useful is it in improving one's skill as a writer? When playing a game such as chess, one gets relatively quick feedback on how well one is doing. Contrast this with the feedback situation in writing. This line of thinking suggests to me that it is very important for writers to learn to provide immediate feedback to themselves. As a writer, I also know that delayed feedback from others is also essential to improving the quality of a document that I have written.

## Precise Vocabulary and Notation

You don't need to be a chess player in order to gain some important ideas from this section. However, a surprising number of adults (perhaps more than half) have played a game of chess sometime in their past.

Figure 2.4 shows a chessboard. Notice that the columns (the files) of the $8 \times 8$ board are lettered $a, b, \ldots h$, and the rows (the ranks) are numbered $1,2, \ldots 8$. In chess, the person playing the White pieces always moves first. The lettering and numbering notation used to identify the spaces on the board is convenient and natural from the point of view of the person playing the White pieces.


Figure 2.4. Lettering of columns (files) and numbering of rows (ranks).
The names of the pieces are abbreviated as follows: $\mathrm{K}=$ King, $\mathrm{Q}=\mathrm{Queen}, \mathrm{R}=$ Rook, $\mathrm{B}=\mathrm{Bishop}$, $\mathrm{N}=$ Knight, and $\mathrm{P}=$ Pawn. This board coordinate system and the piece name abbreviations make it quite easy to record all of the moves in a game. For example, here are the first few moves of a game. The game begins with White moving a pawn from e2 to e4. Black responds by moving a knight from g8 to f6.

1. Pe 2 Pe 4
2. Ng8 Nf6

This, and other notational systems that are widely used in chess, allow players to record precisely the moves in a game (Chess Corner, n.d.). Such a written record can be used in writing and talking about a game, and keeping a permanent record of a game.

Keeping a detailed record of one's chess games and studying both one's own and other people's games is a strategy used to improve one's level of expertise in chess.

Is this type of record-keeping strategy applicable to other games? Is it applicable to nongame learning and problem-solving situations? Of course it is. So, let's give this strategy the name. Let's call it the record one's moves strategy. This is what a researcher does when conducting research in any field. Details of the research need to be precisely recorded so that the researcher and/or others can duplicate the experiment. Thus, it should be part of the repertoire of high-road transferable problem-solving strategies that you and your students routinely draw upon.

This is also what is done when a student writes down in detail the steps $\mathrm{s} /$ he carries out in solving a problem or accomplishing a task in math and other disciplines. As another example, think about the slow motion replays of players in an athletic contest. The video is a record of a player's performance or "moves."

The record one's moves strategy helps to explain why each discipline tends to have some special notation and definitions of terms that are unique to the discipline. It is absolutely essential that people working in a discipline be able to accurately record the work they are doing so that it
can be precisely communicated to others and to themselves. A novice in a discipline needs to learn the precise notation and vocabulary in order to take advantage of the accumulated knowledge in the discipline. That is, part of learning a discipline is to learn to read (for understanding) and write (to communicate to self and others) in the content area of the discipline.

## A Few Important Research Findings

My 11/1/2015 Google search of the expression games OR gaming AND research AND education produced about 350 million hits. Obviously, this search needs to be substantially narrowed! However, it suggests that many people are involved in conducting or writing about Games-in-Education.

Some parents and teachers believe that substantial and useful learning from games will occur merely through providing a child with the opportunity to play games. However, Conati \& Klawe (2000) indicate this is not sufficient:

> These results indicate that, although educational computer games can highly engage students in activities involving the targeted educational skills, such engagement, by itself, is often not enough to fulfill the learning and instructional needs of students. This could be due to several reasons.
> One reason could be that even the most carefully designed game fails to make students reflect on the underlying domain knowledge and constructively react to the learning stimuli provided by the game. Insightful learning requires meta-cognitive skills that foster conscious reflection upon one's problem solving and performance, but reflective cognition is hard work. [Bold added for emphasis.]

The Conati and Klawe research helps to make clear important roles of teachers when teaching in a computer game environment. See also Kirschner, et al. (2006). With the aid of teachers, students can learn to be more reflective in such learning environments, and learning goals can be made more explicit. Students can be taught to do metacognition (thinking about their thinking) and to use this reflective practice as an aid to their cognitive development.

The word edutainment is a combination of education and entertainment. There is considerable literature about edutainment. My 11/2/2015 Google search of the expression edutainment produced over a million results. Michael Resnick is a Massachusetts Institute of Technology professor in its world famous Media Lab. Research in this lab focuses in developing media in which users are active learners. Quoting from (Resnick, n.d.):

Let me start with a contrarian point-of-view: I don't like edutainment.
What do I mean by that? Am I a stodgy professor who wants to keep play and fun out of the learning process? Certainly not. In fact, my research at the MIT Media Lab focuses on ways to integrate play and learning. I have found that many of people's best learning experiences come when they are engaged in activities that they enjoy and care about. Based on these ideas, I have helped develop new toys that provide children with opportunities to learn as they play (and play as they learn).
So why don't I like edutainment? The problem is with the way that creators of today's edutainment products tend to think about learning and education. Too often, they view education as a bitter medicine that needs the sugar-coating of entertainment to become palatable. They provide entertainment as a reward if you are willing to suffer through a
little education. Or they boast that you will have so much fun using their products that you won't even realize that you are learning-as if learning were the most unpleasant experience in the world.
I also have a problem with word "edutainment" itself. When people think about "education" and "entertainment," they tend to think of them as services that someone else provides for you. Studios, directors, and actors provide you with entertainment; schools and teachers provide you with education. New edutainment companies try to provide you with both. In all of these cases, you are viewed as a passive recipient. That's a distorted view. In fact, you are likely to learn the most, and enjoy the most, if you are engaged as an active participant, not a passive recipient.
So I prefer to focus on "play" and "learning" (things that you do) rather than "entertainment" and "education" (things that others provide for you). My preference is for "playful learning" rather than "edutainment." It might seem like a small change, but the words we use can make a big difference in how we think and what we do.

## Final Remarks

Games have long been and continue to be an important component of the lives of many children and adults. New computer-based games as well as the computerization of many popular older games present game players with a steadily increasing collection to select from. In recent years, children in the United States have been spending more time playing such games than they have been spending watching television. It is generally believed that the combination of television and electronic games is having a negative impact on education because they compete for student attention and time. However, both television and games (both on and off the computer) can have educational values, so research in this area is not definitive.

The discipline of Games-in-Education is of growing importance in both informal and formal education. The research literature on the design and use of educational games-especially graphics-intense electronic games-is growing. We know that people learn from whatever situation or environment they experience. By combining ideas from situated learning theory and transfer of learning, we can learn how to make better educational games and better educational use of games.

## Activities for the Reader

This section contains some questions and activities for readers of this book. Some are designed for people who are taking a workshop or course using materials from the book. The individual reader working alone may also find many of the questions and activities to be useful.

1. Think back to your own game-playing experiences. Make a list of some things that you learned through this game playing.
2. Give some examples of games you have played that you considered fun. Do the same for games you did not consider fun. Use these examples to explain what, for you, makes a game fun.
3. Are there any games that you have played in both computer and non-computer mode? If so, select one and do a compare and contrast of the playing experience and learning experience.
4. Spend time observing children playing some games. Write a brief report about what you observe going on. The report should include some conjectures about the learning that you think is occurring.
5. This chapter contains a discussion of opening moves in chess versus opening sentences in writing. This discussion illustrates a type of transfer of learning from game playing to writing. Find and discuss another example of transfer of learning from games to a core academic subject.

## Activities for Use with Students

This section contains some ideas for use with students. It is assumed that the teacher, parent, or other person making use of these suggestions will adjust the activities to fit the needs of the students.

1. Transfer of learning is a key idea. Engage students in a discussion about what they have learned by playing one particular game that they have found useful in playing some other game, or that they have found useful in a non-gaming situation. This might begin with an oral discussion and then lead to a written activity in which each student answers the question. During the oral discussion, introduce the terms transfer of leaning and metacognition, and help the students add these important concepts to their vocabulary. Transfer of learning is one of the most important ideas in education, and metacognition (including reflection) is a key aspect of learning.
2. What do your students believe to be some of the major goals in education? This is a good topic for small group and whole class discussion. Work to create a relatively large list of goals, and then work to organize these into a smaller group of "really important" goals. In your teaching and in their learning, it is important that the goals be clear to both the teacher and the students. As students gain in cognitive maturity, they can understand the learning goals they are engaged in and take an increasing personal responsibility for achieving these goals.

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# Chapter 3 <br> Learning by Solving Puzzles (Using Sudoku Puzzles as Examples) 

If at first you don't succeed, try, try again. Then quit. No use being a damn fool about it. (W. C. Fields.)

I could see why Archimedes got all excited. There was nothing finer than the feeling that came rushing through you when it clicked and you suddenly understood something that had puzzled you. It made you think it just might be possible to get a handle on this old world after all. (Jeannette Walls.)

A puzzle is a problem designed to challenge one's brain and to be entertaining. In this book, we consider a puzzle to be a type of game. Many people spend part of almost every day working on crossword puzzles, bridge or chess problems, number or word puzzles, and the other types of puzzles available online or printed in daily newspapers and in a variety of magazines. They enjoy the challenge and the feelings of success as they solve the problem or accomplish the task presented by the puzzle. You can learn about a number of different puzzles at http://en.wikipedia.org/wiki/Puzzle.

Note to Teachers: My belief is that every person is a teacher. Some do it as a profession, while others do it merely as an everyday part of their lives. I am a teacher who writes books. One of my teaching strategies is to try to get readers to take an active part in their own learning. The link to the Wikipedia in the previous paragraph provides an example of this. Why should I spend my writing time and effort trying to duplicate the good work that someone has already done and made available free online? Moreover, suppose you click on the link and begin to read about puzzles. There is a good chance you will find some information that seems particularly interesting to you, and you will follow up on it. Your learning will be driven by intrinsic motivation. You will be learning because you want to learn. Great!

## A Game Without an Opponent

Chapter 2 contains a discussion of competition, independence, and cooperation in game playing. Most puzzles fall into the middle category; they are neither competitive nor cooperative. Of course, if you like to take a competitive view of almost everything, you can think of a puzzle as a game in which you are competing against yourself. You are trying to solve a challenging problem or accomplish a challenging task. Typically, you are doing this for fun. You ask yourself question such as:

- Do I have the knowledge, skills, and persistence to solve this specific puzzle? (For example, perhaps you are looking at a crossword puzzle. Some are much more difficult than others.)
- Am I enjoying spending time solving this puzzle? (Perhaps you are looking at a Rubik Cube and you know that you get no enjoyment in trying to solve such spatial puzzles.)
- Am I getting better at solving this type of puzzle? (If you do jigsaw puzzles or crossword puzzles frequently, you will get better at doing such puzzles.)
- How good am I (in solving this type of puzzle) relative to other people?
- Am I learning anything by solving this puzzle? (Perhaps you wonder if this mental exercise is good for your brain. It is!)
- Why am I spending so much time "playing" with the puzzle, when I could be doing other, more productive work. Am I becoming addicted? Puzzles, like other types of games, can be addictive.


## Introduction to the Game Sudoku

In the remainder of this chapter, the Sudoku puzzle is used to illustrate various aspects of learning to solve a puzzle and increasing one's level of expertise in solving a puzzle. Before proceeding, you might want to reread the first two sections of the Introduction. Sudoku is used in this current chapter because it helps me (the author) to explain to you some important ideas about problem solving, and because it provides an environment in which you can both practice these ideas and learn about yourself as a learner.

Figure 3.1 illustrates the playing board. The coordinate system is similar to that used in chess. It helps us to communicate precisely about the location of each of the 81 spaces on the board. Notice that the board is divided into nine $3 \times 3$ regions, numbered 1 through 9 .


Figure 3.1. Sudoku board grid and nine regions.

Figure 3.2 illustrates an actual puzzle.


Figure 3.2. An example of a Sudoku puzzle.
A Sudoku puzzle is specified by the set of givens (the board is partially filled out - part of the numbers in a solution are given to you) entered onto the board, as illustrated in Figure 3.2. The goal (the problem) is to enter a numerical digit from 1 through 9 in each empty space of the $9 \times 9$ grid so that:

- Each of the nine regions contains all of the digits 1 through 9 .
- Each horizontal row and each vertical column contains all of the digits 1 through 9.

The rules and goal of this puzzle are very simple. Solving the puzzle does not depend on having knowledge of math or any other subject. Indeed, the puzzle might just as well make use of nine different letters from the alphabet or nine different geometric shapes. Sudoku is not a math or a word puzzle.

You can try to find a solution now. However, if you are a beginner to Sudoku you might want to practice on a simpler example. The next section presents a much simpler example. It illustrates the problem-solving strategy of create a simpler problem, an important teaching and learning strategy.

## A 4x4 Example and a High-Road Transferable Strategy

In this chapter, we mainly explore the $9 x 9$ Sudoku puzzle. However, there are $4 \times 4,16 \times 16$, and other variations on this puzzle.

Just for fun, try solving the two $4 \times 4$ Sudoku puzzles given in Figure 3.3. These two puzzles are the same, except that one uses digits and one uses letters. Notice that it is assumed that you can make up a correct goal (an appropriate set of rules) for these puzzles. That is, without any help from your author, you can transfer the rules of this game from a 9 x 9 board to a 4 x 4 board.


Figure 3.3. Two congruent $4 \times 4$ Sudoku puzzles, one using digits, one using letters.
The chances are that you will decide that the $4 \times 4$ Sudoku puzzle is too simple to be much of a challenge for you. However, it might well be a challenge for young children.

In addition, it illustrates a very important aspect in problem solving. If a particular problem seems too difficult for you, try to create a simpler version of the problem or create a closely related problem that is not as difficult. The process of creating and solving a simpler version or a related problem may well give you insights that will help you to solve the more complex problem.

Throughout this chapter we will be looking for general strategies for problem solving that are applicable over a wide range of problems. The goal is to have you add each of these to your repertoire of high-road transferable problem-solving strategies. By the time you finish reading this chapter, you may well have significantly improved your general problem-solving skills. Moreover, you may well have developed some teaching strategies that will be very valuable to your students.

Let's name our newly discovered strategy the create a simpler problem strategy. The strategy has several purposes. It may help you to better understand the original problem. Solving the simpler problem may help you gain insights that will help you solve the more complex problem. If your simpler problem is carefully chosen, solving it will contribute to solving your original problem.

To add create a simpler problem strategy to your repertoire of high-road transfer strategies, you must identify and consciously explore a number of examples that are meaningful to you. High-road transfer involves identifying a number of such examples.

This requires reflective thinking. Here is a personal example. When I write a book-such as this one-I am not able to just sit down and write the whole book in a linear fashion. Indeed, I cannot even produce an outline that stands a decent chance of actually fitting the final product. To get started, I set myself a much simpler problem. I use a word processor to record my ideas as I brainstorm possible goals, audience, and content for the book.

I then set myself the problem of ordering my brainstormed set of ideas into a somewhat logical, coherent order. During this process, I discard some ideas and add some new ideas.

I then set myself another simple problem - to develop a short summary and a set of references for some of the topics that seem particularly important. I can solve this problem off the top of my head using resources I already know, and also by use of the Web. In the process of solving it, I get some new ideas to add to my original brainstormed list. I may well rearrange the order of the brainstormed list, and I may well set aside some items in the list.

Okay, now it's up to you. As you explore your own examples, think carefully about how you will help your students learn this strategy. Make up some examples of the sort that may be especially relevant to them. Think about how you will help them find personal examples. Think about how the sharing of such personal examples in class may help all members of the class find additional personal examples.

## Metacognition

The next two sections are diversions, seemingly leading us away from solving the 9 x 9 Sudoku puzzle of Figure 3.2. However, we will return to this puzzle after the diversions.

Metacognition (thinking about your thinking) is an important aid to learning to solve problems in any discipline. It can be called the metacognition strategy for learning to solve problems and in other learning situations. What can you do, as a teacher, to help your students learn to routinely use the metacognition strategy as an aid to learning both problem solving and other topics?

A puzzle provides a situated learning environment. While some puzzles require considerable knowledge from outside the puzzle environment, others require very little outside knowledge. The Sudoku puzzle requires the player to be able to recognize and distinguish between each of nine different symbols. However, it does not depend on being able to read or to do math.

Even before we begin studying the Sudoku puzzle in some detail, you can do some introspection or metacognition (thinking about your thinking) as you are first faced by this problem-solving puzzle situation. Here are some questions that might help you learn more about yourself:

1. What are your personal feelings and thoughts as you first encounter a puzzle-especially, a puzzle of a type that you have not previously attempted to solve? Are you an "I can't do puzzles" person or an "I'm good at puzzles" person?
2. For you, personally, do you think digits, letters, or geometric shapes would be easiest for you in a Sudoku puzzle? Why?
3. Think about some non-Sudoku puzzle that you have solved or attempted to solve in the past. Was this an enjoyable experience? Did you develop a reasonable level of expertise with this puzzle? How much time and effort did it take you to develop your current level of expertise with this puzzle? Do you feel you are close to your upper limit in how good you can get in solving this type of puzzle?
The metacognitive questions given above are all stated in the context or situation of learning to solve a type of puzzle. However, they are applicable to learning how to solve problems in any discipline. That is, the questions represent a set of ideas applicable to studying problem solving in any discipline. Why do you think some students come to believe, "I can't do math"?

This is a very important idea. For many people, recreational puzzles represent a relatively non-threatening learning environment. Within this environment, you can learn about yourself as a learner. You can see yourself making learning gains, moving from an absolute novice to a person with an appreciable level of skill. In many puzzle-solving situations, you can see appreciable gains in your expertise over a relatively short time.

## Is the Puzzle Problem Solvable?

Suppose you are now thinking about how to get started in solving the puzzle in Figure 3.2. Perhaps you spend some time looking at the puzzle, checking to see if the givens in any row, column, or region already violate the solution requirement that each row, column, and region must contain the digits 1 to 9 . If the givens in a row, column, or region already contain two copies of a digit, then these givens cannot be part of a solution to the puzzle. That is, the puzzle that has these givens has no solution.

For many people, the term problem means a math problem, and they believe that a math problem has exactly one correct solution. However, every discipline of study deals with problems. Any given problem may have no solution, one solution, or more than one solution.

Solvability is an important issue in problem solving, and it is usually poorly taught in our precollege educational system. To help illustrate this, it may well be that you believe that every math problem has exactly one solution. Your goal, when faced by a math problem, is to "get the right answer."

Think about each of the following math problem examples:

1. Find a positive integer that, when multiplied by itself, gives the integer 16 . This problem has exactly one solution. (4 $x$ 4)
2. Here is a slight modification of the problem. Find an integer that, when multiplied by itself, gives the integer 16. This problem has exactly two solutions. (Hint: Think outside the box. There are both positive and negative integers.) ( $4 x 4$ and $-4 x-4$ )
3. Next, consider the similar problem: Find an integer that, when multiplied by itself, gives the integer 15. This problem does not have a solution. Just a small change in the wording of a problem or details of a problem can turn a simple problem into quite difficult and challenging. The two different square roots of 15 are irrational numbers. They are neither integers nor fractions of integers. So, exploration of this situation leads to finding that the real number line consists of more than just integers and fractions of integers.
4. Find an integer that is greater than 7 and less than 15 . This problem has 7 solutions. (8, 9, $10,11,12,13,14$ )
5. Here is still another simple math problem. Find two integers that, when added together, give the integer 12. With a little thought, you should be able to convince yourself that this problem has an infinite number of solutions. (Hint: Negative integers are allowed. Think outside the box! What is the sum of -3 and +15 ?)
6. Here is a slight modification of the problem in \#5. Find two integers that, when added together, give the number $11 \frac{1}{2}$. Now the problem has no solution. Can you convince yourself that the sum of two integers is always an integer? (Hint: Start with a definition of integer. Reflect on how "simple" arithmetic problems lead to complex math ideas and the need for math proofs.)

I hope that by now you are convinced that even a quite simple problem may be unsolvable, may have exactly one solution, may have more than one solution (but still a finite number of solutions), or may have an infinite number of solutions. Moreover, I hope that I have stretched your insights into problems and problem solving. You might want to take some time thinking
about problems in non-math disciplines of study, and how seemingly simple problems may not really be so simple.

In summary, this section introduces a problem-solving strategy called the explore solvability strategy. When faced by a challenging problem, think about whether the problem is solvable. Spend some time exploring the idea that the problem might not be solvable, or that it might have one or many solutions. Think about the idea that, if the problem has more than one solution, then perhaps one solution is better in some sense than another solution. (This is certainly the situation that architects routinely face.) What are some criteria for a "good" solution? Work to understand the problem so that you can tell if you are making progress toward developing a solution.

You should spend some time adding the explore solvability strategy to your repertoire of high-road transfer problem-solving strategies. Begin by finding some examples that are personally meaningful to you. Then spend some time developing ideas on how you will go about helping your students learn this strategy. One approach is to frequently expose your students to problems that look like the others they are studying, but that are unsolvable or have more than one solution.

## Getting Started in Solving the 9x9 Sudoku Puzzle

Finally, we are ready to begin solving the Sudoku puzzle given in Figure 3.2. You should now be suspicious that perhaps the puzzle has no solution, or perhaps it has more than one solution. You might want to do a quick check of the givens to see if it is obvious that the puzzle has no solution. However, you should be aware that even if the set of givens do not make a row, column, or region with two copies of one of the digits $1-9$, this still does not tell us whether the puzzle is solvable or whether it has more than one solution.

Let's pretend that I am an absolute novice in solving Sudoku puzzles. I stare at the puzzle for a while. My eyes tend to go to the upper left region, region 7.

Within this region, for some reason my eye stops on the empty space b8. I think to myself: "This empty space needs to contain one of the digits 1-9. Right now, the digit 1 is not in region 7. What happens if I place a 1 into the space b8?" The result is shown in Figure 3.4.


Figure 3.4. The original puzzle and trying a " 1 " in space b 8 .
Placing a 1 into space b8 is a step in the direction of having all nine digits in region 7. However, you can now see that row 8 , into which I have inserted the digit 1 , already contains a 1. Thus, the move is a mistake-a move that cannot lead to solving the puzzle. I have just used the guess and check strategy. I made a guess based on the information that currently the digit 1
does not appear in region 7. I checked the result by looking at the row and column in which I placed the 1 .

In many problem-solving situations, the guess and check strategy can be used mentally, without actually making a move. In Figure 3.4, I find it is easy to make the proposed move in my mind's eye, and then to do the checking in my mind's eye. That is, I don't have to enter a 1 into space b8 in order to "see" that this will make row 8 have two 1 s . Undoubtedly you have heard the expression, "Look before you leap." That is an admonition to do a visual/mental check of possible results before taking an action.

Let's expand the idea given above. As a person's brain grows toward full maturity (which usually occurs at about age 26), its ability to plan ahead, to think about possible consequences of proposed actions, gets better. Some children progress much more rapidly than others in mentally forecasting possible consequences of a proposed action, and then making a logical/rational decision as to whether to take the action. Parents struggle mightily with their children, and from time to time might make the statement, "Why don't you just grow up!" Children are not miniature adults, and this particular growing up task is a very long hard learning task. I believe our entire educational system needs to pay far more attention to this aspect of helping students to "grow up." Most adults still act impulsively.

In addition, the guess and check strategy should be part of your repertoire of high-road transfer problem-solving strategies. This strategy goes by other names, such as engage brain before opening mouth strategy. Please spend some time thinking about how to help your students add this strategy to their repertoire of general problem-solving strategies.

## Persistence and Self-confidence

Our failed first effort did allow us to make slight progress in solving our Sudoku puzzle. We can see that a 1 must go in a7 or c9. This is an important idea. In solving problems, it is routine (and expected) that one will make mistakes. Learning to detect and learn from mistakes is an essential aspect of learning problem solving. So, let's add the learn to detect mistakes strategy and the learn from your mistakes strategy to our list of high-road transferable problem-solving strategies.

We are still examining the space b8. Figure 3.5 shows all possible moves that are not eliminated by a quick consideration of the current entries in row 8 , column $b$, and cell 7 . That is, Figure 3.5 illustrates a start on an exhaustive search approach to filling in space b 8 after making a quick mental elimination of obviously incorrect choices.


Figure 3.5. Three possible moves (separated by commas) in space b8.
Aha! I am beginning to see why a Sudoku puzzle can be a mental challenge. I stare at cell 7, and I mentally contemplate various possibilities. For example, I might mentally contemplate leaving the 7 in space b8, and placing the 2 and 4 as shown in Figure 3.6.


Figure 3.6. Continuing a mental trial.
Now, if my mind's eye (mental image) is working well enough, I see that my contemplated sequence of moves is incorrect, since the situation that has emerged is that I will need to place a 1 into space c 8 , and that will mean that there are two 1 s in row 8 .

If my working memory (short-term memory) is good enough, I might well make my way through this maze of possibilities. In attempting to do so, I will be exercising my working memory and other parts of my brain. With practice, I will get better at this aspect of attempting to solve a Sudoku puzzle.

An alternative is to step back a little. Think of my first trial as being an exploration of cell 7. After putting quite a bit of effort into this exploration, I have experienced slight success.

I could quit right now-just give up, and claim, "I am too dumb to learn to solve Sudoku puzzles. Probably this puzzle does not have a solution. Anyway, who cares?" Alternatively, I can persist, try a different cell to explore, and perhaps discover another strategy that might be helpful.

Think about this situation from a teaching/learning point of view. Many of our students have become convinced that they cannot learn to solve complex problems. They have learned that it is much easier to say, "I can't do it" than it is to persist, continue to learn, and continue to make incremental progress. Saying "I can't do it" may seem to be less painful and less threatening than the alternatives that appear to be available to the student.

Persistence and self-confidence are two important characteristics of good problem solvers. Think about your own levels of persistence and self-confidence as a learner and as a problem solver. What might you do to improve your levels of these two characteristics? What might you do as a teacher to help your students increase their levels of persistence and self-confidence?

Games provide one possible piece of an answer to the question. As a teacher, parent, older sibling, and so on, you can use games to create challenging learning and problem-solving environments in which a learner gets an opportunity to gain in persistence and in self-confidence. With proper help from you, the learner can transfer these gains in persistence and self-confidence to other learning and problem-solving situations.

## The Elimination Strategy

I will not give up! I am ready to select another region to explore in the Sudoku puzzle shown in Figure 3.2. As I explore the board, this time my eye is caught by region 5, and the empty space in the exact center of the board. The combination of region 4 , row 5 , and column e has a lot of givens. Indeed, mentally or with the aid of pencil and paper, I quickly discover that each of the digits 1-9 except the digit 5 is in the set of givens for the combination of region 4 , row 5 , and column e. Thus, e5 has to be a 5. My first success! See Figure 3.7.


Figure 3.7 . Space e 5 correctly filled in.
We have just discovered and used the elimination strategy. In exploring the space e5, we eliminated as many possible moves as we could. It turned out to be easy to eliminate all but one possible move. The elimination strategy is a good one to add to your repertoire of high-road transferable problem-solving strategies.

Here is a personal example of using the elimination strategy. In the morning when I get up I am faced by the problem of what to wear. I have a number of long sleeve shirts, and a number of short sleeve shirts. Thus, depending on the day's expected temperature, I can quickly eliminate about half of my shirts from consideration. I also have a number of dress shirts and a number of non-dress shirts. I can quickly eliminate one of these categories by thinking about whether this is a work or non-work day. These two eliminations greatly simplify my selection problem.

Here is a somewhat more complex example of using the elimination strategy. I am faced by the problem of obtaining some up-to-date information on a topic that will be in the book I am writing. A little thought eliminates from consideration my own personal knowledge and the books in my personal library that I have read. I also quickly eliminate all of the books and journals in the physical library on my campus, since right now I am at home and I want a quick solution to my problem.

This line of thinking leads me to doing a Google search of the expression elimination and thinking. Unfortunately, my search produced about a million hits. That is, Google tells me that it may have found as many as a million sources for the information that I seek.

I definitely need to do some more elimination. I can narrow my search-for example, I can increase the number of terms in my search strategy. However, I thought carefully in developing my original search terms, and so it is not easy to narrow the search.

An alternative approach, one that I most often use, is to explore the brief descriptions of the first half-dozen hits. This uses a guess and check strategy. If one catches my eye as possibly being relevant (a good guess), I go to the website and browse it.

If this website does not meet my needs, I will browse a few more of the top numbered hits. In this guess and check process, I will be gaining information that will help me to narrow or reformulate my search. If none of the hits I browse meet my needs, I may decide to eliminate all million of the hits found by Google, and formulate a new search.

Finally, let's go back to our Sudoku puzzle. Notice that there are now only two blank spaces in column e. Using the elimination strategy, you see that these must contain the digits 3 and 4. By a mental guess and check you easily arrive at Figure 3.8


Figure 3.8. Two more successful moves in column e.
Keep working on this puzzle. (Hint: cell f3 looks like a fruitful cell to explore.) Pay careful attention to the strategies that you use. Each time you use one of the strategies named in this chapter, make note of this fact. This is a good way to solidify strategies in your repertoire of high-road transferable problem-solving strategies. If you use a strategy that has not been discussed earlier in this chapter, explore it for possible inclusion in your repertoire of high-road transfer problem-solving strategies.

## Final Remarks

If you had not previously worked with a Sudoku puzzle, you have now learned the rules and practiced a little in solving a puzzle. You can now see that our goal is to use games and puzzles as a vehicle to help students become better at problem solving and to address other important goals in education.

Using a discovery-based approach, we brought to light some very important strategies such as metacognition that apply to learning in all disciplines. We have also discovered a number of transferable general problem-solving strategies such as:

- Create a simpler problem.
- Explore solvability.
- Guess and check; guess and learn.
- Learn to detect mistakes.
- Learn from your mistakes.
- Look ahead.
- Elimination.

Researchers have found that the typical student has a quite small repertoire of generalpurpose strategies that may be applicable when faced by a new, novel problem. In just a few minutes, we "discovered" a half dozen such strategies (from the bulleted list given above) while exploring the Sudoku puzzle. Through appropriate teaching, students can add these strategies to their repertoire of high-road transfer problem-solving strategies.

Strategies and strategic thinking are part of the more general topic of computational thinking. Examine the list of six bulleted strategies. Can today's computer carry out any of them? At the current time, are humans who have a good education better than computers in a number of these tasks? Can a human learn to work with a computer in making use of these strategies? Reflect on educational implications of your answers.

These are all important questions to hold in mind as you continue to study use of games (especially computer games) as an aid to learning. In subsequent chapters, we will explore computational thinking in more detail.

Many people enjoy learning to solve new puzzles precisely because this provides them with an opportunity to discover strategies that are particularly powerful in solving the puzzle. However, educational researchers tell us that relatively few people automatically transfer such strategies to use in solving other puzzles or to solve real world problems. Explicit teaching is a major help in overcoming this difficulty.

## Activities for the Reader

1. Many popular puzzles and games are available free on the Web. For example, my 11/2/2015 Google search of the expression free Sudoku puzzles produced more than 2.8 million results. My search of the expression free chess puzzles produced more than 1.8 million results. Spend some time browsing the Web for other types of free puzzles.
2. Go to the Web and find a puzzle that you have not previously tried to solve. Explore the puzzle using techniques somewhat like those illustrated in this chapter's exploration of Sudoku. Do metacognition and reflect on the learning experience. If this is a written assignment, keep detailed notes on the overall activity and then use the notes in completing the written assignment.
3. Think about how you, personally, deal with novel, challenging problems that you encounter. Do you have any strategies that you tend to use frequently, and that are often effective? (Have you thought about the possibility of sharing this strategy with your students?) Do you have any strategies that you tend to use frequently, but ones that are unfortunately seldom or almost never effective? Have you thought about the possibility of helping your students to discover some of their own personal ineffective strategies?
4. Suppose that you have a textbook that you have used before, and you want to look up something in it that you are fairly sure is in the book. What strategies do you use? Are these strategies applicable to looking up information in other types of books? Are they applicable to looking up information on the Web?

## Activities for Use with Students

1. Ask several students to describe some general-purpose strategies they use when faced by novel problems. In the process, pay attention to whether they have vocabulary (such as the word strategy) useful in carrying on the conversation and in thinking about how they approach novel problems. Also, focus on problems from many different disciplines-not just math problems or math exercises.
2. Working with a group of students, such as a whole class, determine how many are familiar with Sudoku. If quite a few are familiar with this puzzle, then have the Sudokuexperienced class members teach the game to the others, working in one-on-one or in very small group instruction mode. If few are familiar with the puzzle, teach it to the class. Make use of your Sudoku-knowledgeable students as aides to help the other students as they work on a puzzle.

Then debrief this learning experience with the whole class. Direct the conversation so you gain increased insight into students helping students, students being helped by students, and the overall student experience in learning and playing with this puzzle.
3. Select one of the general high-road transferable problem-solving strategies discussed in this chapter. Use it to explain the meaning of high-road transfer of learning to your students. Engage them in gaining the knowledge and skills to do the high-road transfer of learning strategy. Do whole class brainstorming on types of problems in which this strategy might be applicable.
For example, when trying to write a sentence that contains a word a student does not know how to spell, guess and check (perhaps using invented spelling) might be a useful approach. The "check" might come from looking at the spelled word ("It seems to look right"), from use of a dictionary, or from use of a spelling checker on a computer. Repeat this activity once a week for a number of weeks, teaching other strategies from this chapter or from your own repertoire of high-road transfer problem-solving strategies.

## Chapter 4 More Puzzles

This chapter broadens our exploration of educational puzzles. It includes:

1. Discussion of some educational goals of puzzles.
2. Some good sources and examples of free puzzles.
3. Exploration of some additional high-road transferable, general-purpose aids to problem solving.

## Goals for Using Puzzles in Education

Quoting from http://en.wikipedia.org/wiki/Puzzle:
A puzzle is a game, problem, or toy that tests a person's ingenuity or knowledge. In a puzzle, one is required to put pieces together in a logical way, in order to arrive at the correct solution of the puzzle. There are different types of puzzles for different ages, such as crossword puzzles, word-search puzzles, number puzzles, or logic puzzles.
Puzzles are often devised as a form of entertainment but they can also arise from serious mathematical or logistical problems. In such cases, their solution may be a significant contribution to mathematical research.
Puzzles are used in informal and formal education for many reasons. Here are eight somewhat general goals one might have in mind while introducing a student to a particular puzzle. As you read through this list, pause from time to time to reflect on whether the ideas being presented are supportive of the general educational goals of your school and school district.

1. Historical, cultural. The puzzle may have historical and cultural significance. For example, parents and grandparents may want their children and grandchildren to learn about some of the puzzles that they played during their own childhoods. Teachers may want to share some puzzles from their childhood with their students. Particular puzzles may be common in a town or larger region; for this reason, they might commonly be included in a particular school district's curriculum. In a school setting, students might study the history of a puzzle or set of puzzles; this can include the history and cultural environment in which a puzzle was invented.
It is easy to see how the "Historical, cultural" goal fits in with general goals of education. Indeed, puzzles and games can provide a historical thread that has meaning to children and adults of all ages.
2. Logical thinking and problem solving. Most puzzle solving requires the use of logical thinking and one's problem-solving skills. Solving puzzles often requires strategic and creative thinking. Especially with some mentoring help, students can transfer their increasing puzzle-based logic and problem solving abilities to other situations.
3. Discipline or domain specificity. Many puzzles are discipline specific, and may well require knowledge and skills in a specific domain within a discipline. A word puzzle may be particularly good at "exercising" a student's spelling and vocabulary skills, while a math puzzle may be good for practicing mental arithmetic, and a spatial puzzle may be
useful for improving one's ability to visualize the spatial placement and movement of objects.
4. Persistence and self-sufficiency. Many puzzles require a concentrated and persistent effort. Puzzle solvers are driven by intrinsic motivation and can thus develop confidence in their abilities to face and solve challenging problems. Improving persistence and selfsufficiency are important educational goals.
5. Learning about oneself as a learner. A puzzle environment allows one to explore one's own learning characteristics. Many games and puzzles allow the learner to get started and experience some success after just a little learning, and then to continue to experience much more success through additional learning. Students learn how concentrated effort and practice over a period of time can lead to increased expertise.
6. Peer instruction. Children learn many puzzles and games from other children. Learning to learn from one's peers and learning to help one's peers to learn are both quite important educational goals.
7. Individualization of instruction. Puzzles and games can be used to help create differentiated instruction, where the focus might be independent, cooperative, or competitive activity.
8. Busywork or pure entertainment. Puzzles are often used at school and home to keep students occupied or entertained. The teacher or parent has no particular educational goal such as those listed above, but merely wants to keep the student occupied and out of mischief. Teachers and parents make such uses of puzzles and games as aids to classroom and home child management. Use of ideas from this book can help improve the educational value of such activities.

Teachers considering making an increased use of puzzles in their classrooms should think carefully about the list of possible goals given above. They might well want to add to the list. Many puzzles come in a range of difficulties. Thus, the same general type of puzzle (such as Sudoku) comes in very easy versions and in versions that will challenge the brightest students.

Teachers should also think about the importance of novelty versus allowing students to use the same puzzle repeatedly. A puzzle may well provide a good environment for a student to identify some of his or her capabilities and limitations as a learner. This is a good goal. However, typically it is not appropriate to allow a student to use school time to play/solve the same puzzle or type of puzzle repeatedly, moving toward a very high level of expertise in the puzzle.

For example, consider crossword puzzles. Think about the learning that occurs as a person moves from never having tried to solve a crossword puzzle, to having gained 50 hours of study and practice on such puzzles. What learning will occur in the next 50 hours of solving such puzzles? Might this time be better spent on learning something else?

This same type of questioning can be applied to each major content area that is currently required in a typical $\mathrm{K}-12$ curriculum. When a major change agent comes along, such as the current progress in brain science, genetics, and information and communication technology, our educational system is faced by the challenges of what current curriculum to significantly modify, what to cut, and what new curriculum to add.

## Free Puzzles

Many people generate and/or accumulate puzzles that they make available free on the Web. Some of the Web-based puzzles can be played on a computer, while others can be printed out and used in a paper-and-pencil mode. My 11/4/2015 Google search of free puzzle produced about 160 million results.

Here are three examples that I mentioned in the 2008 predecessor of this book, and that are still available:

- Puzzle Choice: http://www.puzzlechoice.com/pc/Puzzle_Choicex.html. Quoting from the website:

Welcome to PUZZLE CHOICE online and printable puzzles and games. Choose from many different types of printable and online CROSSWORDS plus a daily US style crossword. Play WORD SEARCH puzzles or see the WORDPLAY section for anagrams, brainteasers and other printable word games. Test your trivia knowledge with a QUIZ or give your mind a workout with some original LOGIC or NUMBER PUZZLES. The ONLINE GAMES section has jigsaws, mazes, memory games and a choice selection of classic puzzles and games. Kids young and old should visit the KIDS' CHOICE section for a variety of puzzles and games for all ages.

- Free Puzzles: http://www.freepuzzles.com. Its contents include: quiz for today; geometry puzzles; logic puzzles; math puzzles; weight puzzles; moves puzzles; Sudoku puzzles, and puzzle games.
- Puzzles and Problems: $\underline{h t t p: / / p e r p l e x u s . i n f o / t r e e . p h p . ~ T h i s ~ w e b s i t e ~ u s e s ~ t h e ~ f o l l o w i n g ~}$ categorization terms for puzzles: logic, probability, shapes, general (includes tricks, word problems, cryptography), numbers, games, paradoxes, riddles, just math, science, and algorithms.


## Free Does Not Necessarily Mean Free

Typically, websites that provide free puzzles and games make income to sustain themselves by:

1. Selling ads.
2. Selling games and puzzles.
3. Selling subscriptions or memberships.
4. Selling your email address.
5. In addition, many of today's "free" games can indeed be played at no cost, but are designed so that users gain a considerable advantage in the game by using real money to buy extra features.
Thus, as you browse websites offering free puzzles or free games, use some care to avoid purchasing services or subscriptions that you really don't want to buy.

As you browse, looking for sources of free puzzles and other types of games, from time to time you will encounter excellent websites that offer "no strings attached" free materials. Please share these with your friends. A steadily growing number of people are producing and providing excellent Web-based materials for free. This is a significant trend, and eventually it will have a
major impact on our educational system. Imagine the impact on the educational systems of the world if high-quality computer-assisted instructional materials were available free in many different languages and at all grade levels, to all people of the world!

## A Puzzle a Day...

If you are a teacher who believes in the use of puzzles in your classroom, then you might think about accumulating enough puzzles so you can provide your students with a different one each day, or perhaps only one per week. A good starting point is the collection of puzzles at http://perplexus.info/tree.php listed above. You might want to post a puzzle a day or a puzzle a week on the bulletin board, and offer extra credit to students to solve the puzzle.

During and/or right after you use a puzzle with your students, spend a couple of minutes writing notes to yourself about how well the puzzle was received by the students, if it was at the right level for your students, what the students learned, and how to improve their learning experience. Do this for a year and you will have written a book that you can share with your colleagues, one that will be useful to you for years to come.

This writing suggestion provides a good illustration of the divide and conquer strategy. For most people, writing a book may seem to be an insurmountable task. However, finding and using one puzzle and then writing a few thoughts about the results is an easy task. Do this 180 times, and you are well along to writing a useful and lengthy book.

## Jigsaw Puzzles

Jigsaw puzzles come in many different levels of difficulty. A typical jigsaw puzzle has only one solution, but one can arrive at the solution in many different ways. (Practice thinking outside the box. Can you design a jigsaw puzzle that has more than one solution?)

The mechanized process for producing a jigsaw puzzle consists of printing a picture of some sort onto a piece of cardboard and then using a stamping cutter to cut it into pieces. The pieces are then put into bag or box. If the mechanized process accidently fails to put all of the pieces into the bag or box, the result is puzzle with no solution. If the "lost" piece or pieces are put into the next bag or box, the result may be a puzzle that the puzzle solver finds somewhat puzzling.

## Incremental Improvement

The incremental improvement strategy is very useful in certain situations, such as in putting a jigsaw puzzle together. Each piece that you correctly add to the completed part of the puzzle represents an incremental gain, an incremental improvement. Correctly joining any two pieces together is an incremental step toward completing the puzzle. A jigsaw puzzle can be solved by trial and error, but that tends to be a poor strategy because it takes so long and because it takes away the fun of "seeing" by using color and shape patterns.

Because it is easy to tell an edge piece from a non-edge piece, the divide and conquer strategy is usually the initial jigsaw strategy. Separate in one section all of the edge pieces. Then the original puzzle now consists of an "edge" puzzle and an "interior" puzzle. The edge puzzle contains fewer pieces than the whole puzzle and is almost certainly a simpler challenge. After the edges have been completed, one then begins to assemble the interior, often by directly attaching interior pieces to the completed edge. Usually, the divide and conquer strategy still applies as one looks for pieces with striking colors, lines, or patterns.

However, some jigsaw puzzles have some pieces that are exactly the same size and shape. The coloring and patterns in a puzzle may make it difficult or nearly impossible to decide if two pieces that seem to fit together actually belong together. This may lead to putting together a number of pieces that don't actually belong together. Backtracking (undoing pieces) may well prove to be an essential strategy in solving this puzzle. The backtracking strategy is a good addition to your repertoire of high-road transfer problem-solving strategies.

In essence, backtracking is based on a realization that a mistake has occurred, and then undoing what one has done until it appears that the mistake has been removed. Backtracking is a great topic to explore with your students. In writing a composition, for example, "revise, revise, revise" is one of the key ideas to producing a good product. Revision is a form of backtracking. Similar statements hold for any project-based learning activity that leads to a product, performance, or presentation.

As another example, consider the situation in which you have said something that you did not really mean to say, or have taken an action that you did not really mean to take. In both cases, you want to backtrack-you want to make a revision of what you have done. While an apology or other attempts to undo your actions sometimes work, this is clearly not as easy or effective as making revisions to a paper you are writing.

In many problem-solving situations, incremental improvement is not a successful strategy. Take a look at the two-dimensional hills in Figure 4.1. Starting at A, the goal is to climb to the peak at C. Incremental improvement, by moving in small steps steadily uphill starting at A, will not lead you to C. Instead, you will reach B, the top of a peak that is not as high as C. Indeed, if $B$ is the starting point, one can only reach the goal C by first going downhill.


Figure 4.1. Incremental improvement (hill climbing) starting from point A.
Probably you can quote several adages that are relevant, such as:

- Sometimes you have got to break it before you can fix it.
- Things may get worse before they get better.

Making incremental improvements can solve some problems. However, it is by no means a foolproof strategy for solving all problems. Figure 4.2 given below is the same as Figure 3.6 from the Sudoku game presented in chapter 3. The three moves a7: 2; b8: 7; and c9: 4 can each be considered to be an incremental improvement. Each increases the total number of spaces that have been filled in, and none produces a region, row, or column with a duplicate digit entry. Unfortunately, this sequence of moves is a dead end. The only remaining possible move into region 7 is c8: 1 . This means that Row 8 would then have two 1's. We must use the backtracking strategy in order to move forward.


Figure 4.2. Incremental improvement in Region 7 leads to a dead end.
You should have little trouble identifying problem situations in your life or problem situations in the world where incremental improvement does little to solve a problem, and is often a waste of time and other resources. Most attempts to improve our educational system focus on only one or a few possible approaches, and they tend to not allow for the possibility of things getting worse before they get better. This is true of many other big problems, such as poverty and homelessness.

## Online Jigsaw Puzzles

My 11/4/2015 Google search of the expression free online jigsaw puzzles produced well over 2 million results. I was curious as to what I might find through this search, so I browsed a few dozen of these hits. See, for example, http://www.jigzone.com/. This was at the top of the results list when I did this search in 2008, and the site still exists. Quoting from the site:

- A new Puzzle-of-the-day every day.
- Puzzle sizes from 6 to 247 pieces.
- Upload your photos and make them into jigsaw puzzles that you can share with others.
- Embed jigsaw puzzles into your own web pages and blogs.
- Send a Puzzle Postcard.

Many online jigsaw puzzles are designed to be solved just by sliding puzzle pieces right/left or up/down to their correct location, without any rotations. All of the pieces are initially displayed in their correct rotation for insertion by sliding without rotating. This greatly reduces the complexity of a puzzle.

A slider puzzle can be thought of as a special type of jigsaw puzzle in which there is a limited amount of open space into which pieces may slide. My 11/4/2015 Google search of the expression free slider puzzles produced a little less than 200 thousand results.

Figure 4.3 is an example from $\underline{h t t p}: / / m y p u z z l e . o r g / s l i d i n g . ~ T h e ~ g o a l ~ i s ~ t o ~ s l i d e ~ p i e c e s ~ u p, ~$ down, right, and left, staying within the $3 \times 3$ box containing pieces, so that the final result is the
first row is $1,2,3$; the second row is $4,5,6$; and the third row is 7,8 , and blank. Solving this type of puzzle requires the use of two dimensional special visualization skills and planning ahead. Spatial intelligence is one of the nine categories of Multiple Intelligences identified by Howard Gardner (Moursund, 2015).


Figure 4.3. A $3 \times 3$ slider puzzle.
A major advantage of online jigsaw puzzles is that the pieces do not get lost-for example, they do not get chewed up by a pet or sucked up by a vacuum cleaner. Another advantage is that the same puzzle picture is often available at a number of different difficulty levels. A very young child may enjoy working with a six-piece version of the puzzle, while an older child may enjoy the challenge of a puzzle containing hundreds of pieces.

A disadvantage of online jigsaw puzzles is that many people like to work together with others when doing a jigsaw puzzle.

## Complexity of a Puzzle or Other Problems

Complexity is an interesting topic. What makes one puzzle more complex or more challenging (harder) than another? More generally, what makes one problem more complex or challenging than another? This is a good topic for discussion in any discipline that you teach. What makes one poem harder to understand than another? What makes one idea in science harder to understand than another? What makes one math problem harder than another? Unfortunately, this topic is usually not covered very well in most courses. Have you discussed it with the students you teach? Notice that "complexity" and "hardness" are not identical concepts. This is especially true when solving a problem involves thinking outside one's box.

Before considering problem complexity in general, let's look at the simpler issue of jigsaw puzzle complexity. What makes a jigsaw puzzle relatively easy or relatively difficult?

In discussing this problem, we will surely come up with the idea that having more pieces tends to make a puzzle more difficult. When a puzzle has a very small number of pieces, the guess and check strategy can be used quite effective. You may have noticed this when watching very young children try to solve quite simple jigsaw puzzles.

If we are mathematically oriented, we might gather data on how long it takes a typical person to solve jigsaw puzzles of various sizes. For example, does it take four times as long to do a typical 200-piece puzzle as doing a typical 100-piece puzzle, and four times as long to do a typical 400-piece puzzle as doing a typical 200-piece puzzle? Or, perhaps the difficulty level
triples for each doubling in size? The point is, one can conduct empirical research on this question.

We might well come up with the idea that if the pieces are easy to orient correctly (so that, after orientation, then can be placed into position without rotation) the puzzle is much easier than one where the proper rotation of each piece is a challenge. If it is easy to orient the pieces, then one way to approach the solution task is to first orient each piece, and then continue. This is an example of breaking a big problem into two smaller problems.

After further discussion, we might decide that the coloring or pattern of a puzzle makes a lot of difference in its difficulty. In some puzzles, the colors or patterns make it quite easy, provided the solver is not color blind, to sort pieces into groups that must fit near each other. This makes the puzzle much easier (because now one can solve smaller, simpler problems) than if such sorting is difficult or impossible.

Okay, we have now made good progress in studying the complexity of jigsaw puzzles. Next, the mental challenge is to take information about solving jigsaw puzzles and apply it to studying the complexity of other types of problems. One problem is harder than another if it cannot readily be broken into smaller sub-problems. One problem is harder than another if it has many more choices-many more possibilities to try if one is using a guess and check approach. One problem is simpler than another if it can be solved by incremental improvement, while the other cannot.

Here is a quote that I thoroughly enjoy. In essence, it says that it is easier to write a long document than to write a short document:
"I have made this letter longer than usual, only because I have not had the time to make it shorter." (Blaise Pascal, almost 400 years ago.)

Abraham Lincoln's Gettysburg Address provides an excellent example of shorter being much more effective than longer.

## Water-measuring Puzzles

Here is an example of a water-measuring puzzle:
Given a 5-liter jug, a 3-liter jug, and an unlimited supply of water, how do you measure out exactly 4 liters?
Notice that the same problem can be stated using a different unit of measure.
Given a 5-gallon jug, a 3-gallon jug, and an unlimited supply of water, how do you measure out exactly 4 gallons?
My 11/5/2015 Google search of the expression puzzle problem water measuring produced about 7.5 million results. There are many different water-measuring problems. According to Ivars Peterson, such problems date back to the $13^{\text {th }}$ century (Peterson, May, 2003). Peterson's article gives additional examples and discusses some of the underlying mathematics of how to solve this type of problem.

Many problems can be solved by starting at a solution and working backward. Let's try this idea with the water-measuring problem given at the beginning of this section, where the goal is to measure out four liters. What are some ways to make the integer 4 that might be relevant to this problem?
a. $4=2+2$
b. $4=1+3$
c. $4=5-1$

From a working backward point of view, $4=2+2$ tells me that if I manage to get two liters into each jug, the problem is solved. The representation $4=1+3$ tells me that if I can get one liter into one of the jugs and three liters into the other, the problem is solved.

Suddenly, an aha! strikes me. One of the jugs holds exactly three liters. So if I can just figure out how to get one liter into the other jug, the problem is solved.

However, before thinking about how to do that, let's think about $4=5-1$. I know how to get five liters, but how do I get minus one liter? Maybe I need to think outside the box? My mind gets confused as I try to think of a jug containing -1 liter of water. However, I can understand pouring one liter out of a jug, thus decreasing its contents by a liter. Pouring is like subtraction. Aha! If the 3-liter jug has two liters in it, then I could fill the 5-liter jug and pour from it until the 3-liter jug (that contains two liters) is full, thus leaving four liters in the 5 -liter jug.

My two aha! moments give me two approaches to solving the puzzle. In the first, I strive to get one liter into the 5 -liter jug. In the other, I strive to get two liters into the 3 -liter jug. Thus, by working backward using some simple arithmetic and keeping my brain in gear, I have formulated two new problems. If I can solve either one of them, I can then solve the original problem.

How do I measure out exactly one liter or exactly two liters? Using simple arithmetic skills, I see that 5-3 $=2$. With a flash of insight, I see that if I fill the 5 -liter jug and pour into the empty 3 -liter jug, I will end up with two liters in the 5 -liter jug. I have now found a pathway to solving the problem.

The work backwards strategy is a powerful aid to solving many different kinds of problems. You will want to add it to your repertoire and your students' repertoires of high-road transferable problem-solving strategies. You and your students may at first find it challenging to find problems that are often solved by working backwards. Here is a hint of one source of such problems. You need to be at work at 7:30 in the morning. What time should you set your alarm for?

## Multiple Intelligences

Almost all teachers are aware of Howard Gardner's work on Multiple Intelligences (Gardner, 2003). His first book on this topic was published in 1983. Nowadays, many teachers pay attention to Gardner's work as they design and present instruction. The nine types of intelligences that Gardner identified are (Moursund, 2015):

- Linguistic intelligence ("word smart").
- Logical-mathematical intelligence ("number/reasoning smart").
- Spatial intelligence ("picture smart").
- Bodily-kinesthetic intelligence ("body smart").
- Musical intelligence ("music smart").
- Interpersonal intelligence ("people smart").
- Intrapersonal intelligence ("self smart").
- Naturalist intelligence ("nature smart").
- Existential intelligence ("spiritual intelligence" or "cosmic smarts"). See http://thesecondprinciple.com/optimal-learning/ninth-intelligence-existential-cosmic-smarts-2/.

When I was graduating from high school, I took a variety of vocational aptitude tests. My spatial intelligence tested well below 100 on an IQ-type scale with a mean of 100. I want to share three parts of this story. First, I was advised that I should not attempt to major in math, as many people believe that math requires having good spatial sense. It turns out, however, that I had little trouble in undergraduate and graduate work in mathematics-I achieved a straight A average in math courses as I earned a doctorate in this area. In my math studies, my strong logical/mathematical intelligence more than overcame my weak spatial intelligence.

Second, I am terrible at finding my way when walking or driving around a city. Indeed, I can easily get lost in a large building! Even though I pay careful attention to this situation, I haven't improved. I partially overcome this difficulty by making careful maps and/or by carefully planning and using maps. I have a younger sister with a doctorate in physical chemistry, and she suffers the same spatial intelligence challenge.

Third, a number of years ago, my wife and I began doing jigsaw puzzles together as we listened to audio books. At first, I was very poor at putting jigsaw puzzles together, and I was embarrassed by my ineptitude. Eventually, however, I got a lot better. I developed some jigsaw puzzle-solving strategies that fit well with some of my strengths, and my spatial abilities in the jigsaw domain improved with practice.

The third piece of the story is particularly relevant. One's expertise in an area can be increased by study and practice. If you have a researcher-oriented mind, perhaps your first question would now be, "Did my improvement in jigsaw puzzle spatial expertise transfer to other spatially oriented problem-solving domains?"

I don't know, as I did not gather data before beginning the jigsaw puzzle "experiment." My guess, however, is that I am as bad as ever at finding my way around in a city or large building.

What I do know, however, is that there has been considerable research on this general topic. Indeed, one of my doctoral students worked on this topic in the mid 1980s. She was interested in whether playing spatially oriented computer games would help improve girls' general spatial abilities more than it improved boys' general spatial abilities. In her particular study, both girls and boys improved, but the girls did not improve more than the boys.

Figure 4.4 illustrates a spatial puzzle named Assemble the Square that is suitable for use by students of all ages. The puzzle provides you with a number of pieces that can be dragged without rotation onto a $4 \times 4$ square, to exactly cover the square. The puzzle is available at http://www.vemix.com/GlFlashGm.php. Click on Assemble the Square and then click on Begin to generate a new puzzle.


Figure 4.4. Five pieces to be dragged without rotation to form a 4 x 4 square.

## Tower of Hanoi Puzzles

The Tower of Hanoi puzzle consists of three pegs and a number of disks of different sizes that slide onto the pegs. The puzzle starts with the discs neatly stacked in order of size on one peg, smallest at the top, thus making a conical shape. See Figure 4.5.


Figure 4.5. Tower of Hanoi puzzle.
See http://en.wikipedia.org/wiki/Tower_of_Hanoi.
The object of the game is to move the entire stack of disks to another peg, obeying the following rules:

- Only one disc may be moved at a time.
- No disc may be placed on top of a smaller disc.

You can play this game online, for free. See http://www.softschools.com/games/logic games/tower of hanoi/.

Mathematicians consider this as a mathematical game. They state and prove theorems about the solvability of this and similar puzzles. Children with no knowledge of the underlying mathematics enjoy the game.

Most people find the Tower of Hanoi puzzle somewhat overwhelming the first time they face it. Indeed, only those who are quite persistent do not give up after exploring (trial and error) for a few minutes.

However, this puzzle provides an excellent opportunity to try out one of most important general problem-solving strategies. It is called the explore a simpler case strategy. The idea is to create a simpler version of a problem that is close enough to the original so that solving the simpler problem gives one some useful insights into the original problem.

For an example, consider a Tower of Hanoi puzzle that has exactly three disks, and set yourself the goal of ending up with the three disks moved to the middle peg. See Figure 4.6.


Figure 4.6. Three-disk Tower of Hanoi puzzle.
The 3-disk puzzle is still somewhat of a challenge. Figure 4.7 shows an intermediate position in a sequence of moves leading to solving the puzzle.


Figure 4.7. Position achieved after three moves.
Notice that I did not give you all of the details for solving the 3-disk puzzle. Instead, I pointed you in a right direction and left you to fill in the details. This is the way that many math books are written. The author discusses a particular theorem, aims you in a right direction, and leaves you to fill in the details. This approach is used because one of the goals is to help the reader get better at making proofs. One way to get better at making proofs is to fill in the steps that an author leaves out in presenting an outline of a proof. This is an example of a learn to fill in the details strategy.

I am hoping for an aha! from you, my reader. One way to teach is to provide students with all of the details of how to solve a particular type of problem or accomplish a particular type of task. Students are expected to memorize the details, and then to practice over and over again to develop speed and accuracy. A different approach is to present the general ideas and an outline of an approach. Students are expected to figure out the details for themselves. Notice the advantage of the second approach when a student later makes a tiny error in remembering a procedure (a set of steps). Rote memory is useful, but it is a poor approach in many educational situations.

After completing the 3-disk puzzle, you might want to try 4-disk puzzle. Figure 4.8 illustrates a possible intermediate goal that you might work toward in attempting to solve this puzzle.


Figure 4.8. An intermediate goal in solving the 4-disk puzzle.
This sequence of examples illustrates another very powerful general problem-solving strategy. It is called the look for patterns strategy. Perhaps Figures 4.6, 4.7, 4.8, and 4.9 suggest to you the possibility of setting an intermediate goal of moving all but the bottom disk onto the third peg. In essence, this pattern shows how a problem with a certain number of disks can be solved by solving the problem with one less disk.


Figure 4.9. A possible intermediate goal in solving the 5 -disk puzzle.
By now you should have seen the overall strategy. You use the "how to solve the 3-disk problem" to solve the 4-disk problem. You use the "how to solve the 4-disk problem" to solve the 5 -disk problem.

## Bridge Crossing Puzzle Problems

My 11/5/2015 Google search of the expression bridge crossing puzzle problem produced about 750,000 hits. Here is a typical bridge crossing puzzle:

Four people have to cross a bridge at night. The bridge is old and dilapidated and can hold at most two people at a time. There are no railings, and the people have only one flashlight. In any party of one or two people cross, one must carry the flashlight. The flashlight must be walked back and forth; it cannot be thrown, etc. Each person walks at a different speed. One takes 1 minute to cross, another 2 minutes, another 5 , and the last 10 minutes. If two people cross together, they must walk at the slower person's pace. There are no tricks - the people all start on the same side, the flashlight cannot shine a long distance, no one can be carried, etc. What is the fastest they can all get across the bridge?

This puzzle can be played (free) on the website https://nrich.maths.org/5916.
The story (perhaps apocryphal) is often told that many years ago such a puzzle was given during interviews of programmers applying to work at Microsoft. Many people like to play with this type of puzzle and make up variations. For example:
a. Suppose that in the 4-person puzzle, the 5-minute person is changed into an 8-minute person. Does that change the total time needed to get all four across the bridge?
b. Suppose that there are only three people needing to cross the bridge: the 1-minute person, the 5 -minute person, and the 10 -minute person. What is the fasted the three can get across?
c. Suppose that in the three-person puzzle (b), the 5-minute person is changed into an 8minute person. What is the fasted the three can get across? How is it possible that the answer to (a) is smaller than the answer to (c)?
This type of puzzle can be approached by using the bottleneck strategy. The bottleneck strategy is applicable in analyzing lots of different kinds of problems in which a number of different activities need to be accomplished in a timely fashion. A team of people may be able to accomplish such a task faster than one person, provided one can identify situations in which more than one person can be working at a time in a productive manner.

The bottleneck in the bridge example is the two slower walkers. In the original version of the puzzle, if each walks accompanied by a faster walker, then it takes 18 minutes just to get these two across. If they walk together, it takes only 10 minutes for the two to cross. These two constitute the bottleneck. Figure out how to have them walk together, and you (may) have made a good step toward solving the puzzle.

You will want to add the bottleneck strategy to your repertoire and your students' repertoires of high-road transferable problem-solving strategies.

## Brain Teasers

Many people like brain teasers. My 11/5/2015 Google search of the expession free online brain teaser produced about 1.4 million results. The website http://www.puzz.com/iqteasers.html contains a large number of what it calls IQ Brain Teasers. Here is an example:

At Parkview High School, the science club has 11 members, the computer club has 14 members, and the puzzle club has 25 members. If a total of 15 students belong to only one of the three clubs, and 10 belong to only two of them, how many students belong to all three clubs?

Notice that this is a type of logic puzzle that requires significant reading skill. Many brain teaser puzzles require good reading skills and good use of logic. In addition, math skills are often helpful. Math people can solve this particular math puzzle mentally, using only elementary school arithmetic. If you are not able to figure out a direct way to solve the puzzle, think about using trial and error-perhaps in conjunction with a Venn diagram picture of the situation.

## Symmetrical Word Box Puzzles

The website http://www.rinkworks.com/brainfood/ contains a large number of different types of brain teasers. For example, there are a number of different Symmetrical Word Box puzzles. Quoting from the website:

Word Boxes are like miniature crossword puzzles, except that each word is filled in across and down the grid. That is, the answer to 1 across is the same word as the answer to 1 down; 2 across is the same as 2 down; etc. Can you solve these Word Boxes?


## 1. matter unit <br> 2. lion handler's task <br> 3. warning sign <br> 4. fix

Figure 4.10. An example of a $4 \times 4$ Symmetrical Word Box puzzle.
The four words for the above puzzle are ATOM, TAME, OMEN, and MEND. On 11/5/2015, the website contained 84 of these $4 \times 4$ puzzles, each with a hint and with a solution. It also contained a number of $3 \times 3,5 \times 5$, and $6 \times 6$ puzzles. If you want to give your students a different word puzzle each day, this website will get you off to a good start.

## Logi-Number Puzzles

The website http://www.rinkworks.com/brainfood/ also contains a large number of LogiNumber puzzles. Quoting from the website:

Logi-Number Puzzles are a cross between logic problems and mathematical puzzles. In each, you must determine what values the variables are equal to, using the rules of the game and the given clues. The rules are: (1) all the variables are equal to integer values between one and the number of variables in the puzzle, and (2) none of the variables are equal to each other. For example, if there are six variables, each will equal a number from 1 to 6 . Since no variable equals another, all six values will be used.

1. $\mathrm{A}+\mathrm{B}=\mathrm{D}$
2. $\mathrm{B}+\mathrm{F}=\mathrm{E}$
3. $\mathrm{C}+\mathrm{D}=\mathrm{E}$
4. $B \neq 1$


Figure 4.11. An example of a 6x6 Logi-Number puzzle.
Personally, I find the "box" representation of this puzzle problem to be confusing. There are six variables, each having a different positive integer value, and the possible values are $1,2,3,4$, 5 , and 6 . How might one go about solving such a puzzle? The following represents my thinking as I worked to solve this puzzle. I have never previously encountered this type of math puzzle.

To begin, I thought about the explore solvability strategy. Unless the person posing this puzzle tells me explicitly that this puzzle has a solution, then I don't know if it does or doesn't. Also, unless the person tells me that it has exactly one solution, I don't know if it has more than one possible solution. However, I assumed that the puzzle has exactly one solution. As I worked
to solve the puzzle, however, I kept an open mind on the possibility that the puzzle does not have a solution.

My next thought was that perhaps I could use my knowledge of algebra to solve the puzzle. The puzzle uses algebraic notation and some algebraic equations, so perhaps I can easily solve it using $8^{\text {th }}$ grade or $9^{\text {th }}$ grade algebra. However, there are 6 unknowns, with 3 equations and 1 inequality. From my study of algebra, I remember how to solve 2 linear equations in 2 unknowns, or 3 linear equations in 3 unknowns, and so on. However, I do not know an algorithm for solving 3 linear equations and one linear inequality in 6 unknowns. This situation is unlike any that I have previously encountered.

Hmmm. Since E is the sum of two different integers in the range 1 to 6 , the very smallest it could be is 3 . Under the constraints of this puzzle, the only way to get 3 by adding two of the unknowns, is $1+2$. However, E must be larger than 3 since if $\mathrm{B}+\mathrm{F}=3$, than $\mathrm{C}+\mathrm{D}$ will certainly be larger than 3 . Similarly, if $C+D=3$, than $B+F$ will be larger than 3 .

Aha. A small insight. Perhaps I can make some progress by eliminating some possible solutions. That is consistent with the given elimination information that $\mathrm{B} \neq 1$.

Can $E=4$ ? No. In this puzzle, the only way to get a 4 by adding together two of the unknowns is by adding 1 and 3 . However, we need two different ways to get E .

Can $E=5$ ? I see that $5=1+4$ and that $5=2+3$. Thus, I cannot immediately rule out the possibility that $\mathrm{E}=5$.

Can $E=6$ ? I see that $6=1+5$ and that $6=2+4$. Thus, I cannot immediately rule out the possibility that $\mathrm{E}=6$.

This use of the elimination strategy does not seem to be going very well. By elimination, I conclude that $\mathrm{E}=5$ or $\mathrm{E}=6$, but I can't tell which is correct. I have made some progress with this approach, but it may be that it is not a good way to go. For a moment, I feel stuck.

Then I see that I can make use of a combination of the first and third equations and conclude that $\mathrm{A}+\mathrm{B}+\mathrm{C}=\mathrm{E}$. Aha. I have used some algebra. Why didn't I try that earlier? It must be that $\mathrm{E}=6$, since that is the smallest possible sum of three different integers in the range of 1 to 6 .

My initial elimination efforts to determine a value for E were not too fruitful. I did gain some information through this use of the guess and check strategy. Now, however, I feel a small sense of satisfaction because I am making some progress. It has taken quite a bit of "messing around," exploring, making trials, and getting a feel for the problem.

During my elimination approach, I found that there are exactly two possible (legal, following the rules of this puzzle) ways that E could be $6: 1+5=6 ; 2+4=6$. From this, using elimination, I conclude that $\mathrm{A}=3$. This is because the values for $\mathrm{B}, \mathrm{C}, \mathrm{D}$, and F must come from (and use up all of) the four integers $1,2,4$, and 5 , and I know $\mathrm{E}=6$.

From the first equation, it is now evident that $\mathrm{B}=2$. Why? Because I know that B cannot be either 1 or 3 , and the first equation tells me the $B$ cannot be larger than 3 . From this point, it is quite easy to complete the puzzle.

Here are five educational values that I see in this type of puzzle:

1. The puzzle makes use of algebraic notation and some basic algebra ideas taught before students take an algebra course.
2. The puzzle requires use of numbers and simple arithmetic that one can do mentally.
3. The puzzle illustrates use of the elimination strategy and requires persistence.
4. The logical arguments used are much like those one uses in solving other math problems and in constructing math proofs. It looks to me like there is the possibility of quite a bit of transfer of learning to these aspects of doing math.
5. Writing, and explaining one's math/logic thinking and processes, can be built into use of this activity in a school setting. Such writing and explaining are important components of learning math. From a math book authoring point of view, my discussion of how I solved the puzzle problem allowed me to leave some gaps to be filled in by the reader. That is a standard technique used in writing math books.

## Cryptograms

The website http://www.rinkworks.com/brainfood/ also contains Cryptogram puzzles and a Cryptogram puzzle maker. (See the Word Puzzles section.) As an example of using the Cryptogram puzzle maker, I provided the sentence:

## DAVID MOURSUND HAS WRITTEN MANY DIFFERENT BOOKS

I received the following encryption:

## BETCB VZMAPMXB FEP SACYYGX VEXO BCJJGAGXY LZZIP

I then used the same sentence as input a second time, and got:

## GVFEG SYRTMRJG XVM NTEWWCJ SVJK GEUUCTCJW QYYLM

These Cryptogram puzzles are based on simple letter substitutions. Here is a challenge for you and your students. If I use an encrypted sentence as input, thus encrypting the encryption, will the result be a harder puzzle than the original?

My 11/5/2015 Google search of the expression cryptogram produced over a million results. My search of the expression cryptogram puzzles produced over 300 thousand results. My search of the expression cryptography produced over 12 million results.

Cryptography is an important discipline with a long and interesting history. Nowadays, computers play a major role in this discipline. Quoting from http://en.wikipedia.org/wiki/Cryptography:

Modern cryptography exists at the intersection of the disciplines of mathematics, computer science, and electrical engineering. Applications of cryptography include ATM cards, computer passwords, and electronic commerce.

Modern cryptography is heavily based on mathematical theory and computer science practice; cryptographic algorithms are designed around computational hardness assumptions, making such algorithms hard to break in practice by any adversary.

## Miscellaneous Additional Examples of Puzzles

There are many different types of puzzles. This section lists a few that can be accessed from and used on the Web.

## Peg Puzzles

Peg Puzzles are mental, spatial puzzles that involve jumping pegs over each other, with a jumped peg being removed, to achieve a particular goal. My 11/5/2015 Google search of the expression free online peg puzzles produced over 300 thousand results. Here is an example:

The goal is to remove pegs from the board by jumping over each peg with another peg; this removes the "jumped" peg (similar to Checkers jumps). The game is over when no more jumps are possible. You win the game by removing all the pegs except one from the board. A perfect game would leave one peg in the center position.


Figure 4.12. A Peg puzzle.
Peg puzzles are now available both in physical format (a playing board with holes, and pegs) and on the computer. When I was a child, people often made their own peg puzzles, and this is still a fun activity for children of all ages. It is easy to drill holes in a board and use golf tees for pegs.

## Traditional Crossword Puzzles

Many people are hooked on certain types of puzzles. For example, some people cannot start the day without spending time on the crossword puzzle in their morning newspaper. In some sense, they have a type of addiction to crossword puzzles. The fun is in meeting the challenge of the puzzle -making some or a lot of progress in completing the puzzle.

Crossword puzzles draw upon one's general knowledge, recall of words defined or suggested by short definitions or pieces of information, and spelling skill. Through study and practice, a person learns some useful strategies and can make considerable gains in crossword puzzlesolving expertise. Doing a crossword puzzle is like doing a certain type of brain exercise. In recent years, research has provided evidence that such brain exercises help stave of the dementia and Alzheimer's disease that are so common in the elderly.

From an educational point of view, it is clear that solving crossword puzzles helps to maintain and improve one's vocabulary, spelling skills, and knowledge of many miscellaneous tidbits of information. Solving crossword puzzles tends to contribute to one's self esteem. For many people, their expertise in solving crossword puzzles plays a role in their social interaction with other people.

You can access free crossword puzzles in many newspapers and magazines. Many websites offer free online crossword puzzles. My 11/6/2015 Google search of the expression free online crossword puzzle produced over 2.5 million results. For example, see http://www.boatloadpuzzles.com/playcrossword. An example is given in Figure 4.13.


Figure 4.13. Puzzle \#3 of 40,000 free puzzles at this site.
Note the Hide Errors menu at the bottom of a puzzle. You can set it so that each incorrect letter you enter is displayed in red, or you can turn this feature off. Beginners may find this feature a useful learning aid.

Many websites provide free access to a computer program that can generate a crossword puzzle that is based on words and clues you provide. My 11/6/2015 Google search of the expression free crossword puzzle maker produced more than 550 thousand results. For example, see Instant Online Crossword Puzzle Maker at http://www.puzzle-maker.com/CW/.

Compared to traditional crossword puzzles, these teacher-plus-computer-generated puzzles are of very modest quality. However, many teachers make use of computer software to develop such puzzles for their students. An alternative approach - perhaps educationally more sound-is to have students develop such puzzles for use by their fellow students, siblings, parents, and so on. While this might be done through the use of a computer, a more challenging task is to develop such puzzles by hand, working to achieve rather compact puzzles with a large number of crossings.

## Math Computation Puzzles

My 12/6/2015 Google search of the expression free math computation puzzle produced over 25 million results. For example, Figure 4.14 shows a brain teaser from the site http://www.mathinenglish.com/PagePL1P26to30.php. The goal is to find positive integer values for each of the three shapes so that the sums of the rows and columns are the integers shown in the figure.


Figure 4.14. A math computation puzzle involving integer addition.
This is a relatively simple puzzle. The fourth row provides the information that a "star" has the value 5 . From this you can use the third row to determine that a circle has the value 3 .

Figure 4.15 is a somewhat similar puzzle. The goal is to use the integers 1 to 5 in the $Z$ shape so that the sums of the integers in the three sides (top, diagonal, and bottom) each have a sum of 7. In the $V$ shape, the goal is for the sum of the integers in each of the two lines of the $V$ to have a sum of 8. The V puzzle is trickier because one integer (at the point of the V ) is in both of the two "sides."


Figure 4.15. A "trickier" math puzzle involving integer addition.
My 12/19/2015 Google search of the expression math square puzzle produced nearly a million results. The example in Figure 4.16 comes from the website http://mentalfloss.com/article/33409/brain-game-math-square-143. Notice that there are nine empty squares. The goal is to fill in these empty squares with the digits 1 through 9 , using each digit exactly once, so that the three horizontal and three vertical equations are satisfied.

In Figure 4.16 you can see that this is puzzle \#143. You can return to this same puzzle on the website by use of its puzzle number. The site provides both puzzles and their solutions. Most
students will find that a paper copy of such a puzzle, plus pen or pencil, are exceedingly useful aids to solving the puzzles.


Figure 4.16. A math square puzzle.
Spend some time trying to solve this puzzle. While doing so, think carefully about what you (or a student) might learn by working on this type of puzzle. For example, it provides examples of equations, and of doing addition, subtraction, multiplication, and division. Although the puzzle looks like it only deals with integers, a division might produce a non-integer answer, and then a subsequent multiplication might produce an integer answer. So, some knowledge of fractions can also be required in such puzzles.

Think about approaching this puzzle with an exhaustive search strategy-trying every one of the possible arrangements (the permutations) of the digits 1 through 9 . There are $9 \times 8 \times 7 \ldots \times 2$ x 1 possible permutations that would need to be tried. This is nine factorial, which is 362,880 . That approach seems like a very poor use of a person's time! However, this might be an interesting problem for a math-oriented student who knows how to program a computer.

This type of puzzle does require some trial and error, but the number of trials needed can be greatly reduced by careful thinking. The time to solve the puzzle can also be reduced by doing the needed trials mentally.

So, let's begin with the top horizontal equation. Call the three unknowns A, B, and C. There are three different digits from the set 1 to 9 . Hmm. What can A and B be so that when I divide by the integer C the quotient is 9 ? Mentally I start thinking $9 / 1=9 ; 18 / 2=9 ; 27 / 3=9 ; 36 / 4=$ 9 ; etc. So, The integers A and B must give me one of: $\mathrm{A} \times \mathrm{B}=9$, or $\mathrm{A} \times \mathrm{B}=18$, or $\mathrm{A} \times \mathrm{B}=27$, etc. My mental arithmetic (plus using pencil and paper because my memory isn't what it used to be) gives me the two possibilities $\mathrm{A}=3, \mathrm{~B}=6$, and $\mathrm{C}=2$, or $\mathrm{A}=6, \mathrm{~B}=3$ and $\mathrm{C}=2$. Great! I have found one of the unknowns, and I have only two possible values for two more of the unknowns. A great start!

Next I look at the right most vertical equation. The first unknown has the value 2. So, this equation must be one of:

$$
\begin{aligned}
& 2 \times 1+5=7 \\
& 2 \times 3+1=7
\end{aligned}
$$

But, the first horizontal equation contains a 3 , so the right most vertical equation must be 2 x $1+5=7$. I have now filled in two more of the unknowns! With this start, you will likely experience little difficulty in completing the puzzle. A complete solution in given in Figure 4.17 on the next page.

## THE SOLUTION:



Figure 4.17. Solution to puzzle given in Figure 4.16.
My colleague Bob Albrecht and I have written a book, Using Math Games and Word Problems to Increase Math Maturity. It is available free on the Web (Moursund \& Albrecht, 2011). The games explored in this book have all been tested with children and have considerable math education value. All can be played using inexpensive materials such as dice, spinners, paper, and pencil.

## Final Remarks

A huge number of puzzles are available for free use from the Web and other sources, and/or are ones that people can easily construct for themselves. For the teacher or parent who cares to make a little effort, students can be provided with one or more new puzzles each day throughout the school year. Alternatively, students can learn just a few general types of puzzles, and these can be used multiple times throughout the school year. For example, students can learn to do crossword puzzles, and there are plenty of such puzzles available to meet the needs of students throughout the school year.

Many puzzles come in a range of difficulty levels. This means that a teacher can make use of a particular type of puzzle (such as a crossword puzzle) and select versions that fit the various capabilities of students in the class. Students can find a level of the puzzle that they find appropriately challenging and progress to more difficult versions as they increase their knowledge and skill in solving the type of puzzle.

Puzzles are inherently educational. However, some puzzles have much more educational value than others. A "good" puzzle is cognitively challenging (but solvable) by a student and helps teach important components of the curriculum. Appropriate teaching and mentoring can substantially increase the educational value of puzzles. A teacher who is interested in puzzles should have no difficulty justifying the routine integration of puzzles into the curriculum.

## Activities for the Reader

1. Select a goal of education that you feel is quite important and that you can help your students to achieve. Find a puzzle that is suitable for use by your students and that helps to support achieving the educational goal. Discuss how the game supports the goal. Note that in this activity, you start with an educational goal and look for a puzzle that helps to achieve the goal.
2. Select a puzzle. Analyze it from the point of view of how it supports various general goals in education. Note that in this activity, you start with a "solution," (a puzzle) and you look for a problem (an educational goal) that this puzzle helps solve.
3. What makes one Sudoku puzzle easier or harder than another? Each day the website http://www.monterosa.co.uk/sudoku/ publishes three computer-generated, guaranteed to be solvable, different levels of difficulty, Sudoku puzzles.

## Activities for Use with Students

1. Ask your students who are "into" puzzles to bring some interesting examples to class. Then ask for volunteers to do two things:
a. Demonstrate and talk to the whole class for a couple of minutes about a puzzle.
b. Teach a person (another volunteer from the class) more details about the problem or task represented by the puzzle, and how to solve or accomplish it.

Meanwhile, the rest of the class serves as observers. They keep notes on the teaching and learning process. They are looking for what works, what doesn't work, and how the teaching/learning might be improved. This information is shared in a whole class discussion and debriefing. The activity can be used over and over again, with different puzzles and different participants.
2. A variation on $\# 1$ above is to divide the class into smaller groups, so that each constitutes a "small" class. This allows many more students to be presenters.
3. Spend some time talking to your students about what they can learn from puzzles. Then:
a. Involve the whole class in working together to analyze a puzzle that most or all are familiar with.
b. Have each student select a puzzle that he or she is familiar with, and analyze it from an educational point of view. The results are to be written into a report to hand in to the teacher.
4. Have individual students or teams of students explore mechanical puzzles. A good starting point is the Wikipedia website http://en.wikipedia.org/wiki/Mechanical_puzzle. The following examples are from that website.


Figure 4.18. Examples of mechanical puzzles.

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## Chapter 5 <br> One-Player Games

Golf is like solitaire. When you cheat, you only cheat yourself. (Tony Lema.)
Life is like a game of solitaire. (Many writers have used this statement. My 1/10/2016 Google search of this expression found over 2 million results.)

Probably you are familiar with one or more versions of the type of card game called solitaire. There are a great many different solitaire card games. Most often solitaire games are played by a person playing alone, using one or more standard decks of playing cards, or playing electronically. The website https://shopper.mycommerce.com/checkout/product/1265-3 sells a book containing rules for 476 different solitaire games. John McCloud's website, Card Game Rules, provides rules for and links to a very large number of two-player games (McCloud, n.d.).

This chapter discusses some one-person solitaire games that can be played with physical cards or electronically. It also contains a brief discussion of Tetris, a one-player computer game that does not make use of a deck of cards. Tetris is very popular but unusual game, because the human player always loses.

My 11/6/2015 Google search of the expression solitaire card game produced more than 3.5 million results. Many websites allow a person to play a variety of solitaire games for free. My 11/6/2015 Google search of the expression free online solitaire card game produced well over 2 million results.

## Learning to Play a Game

The process of learning any game consists of:

1. Learning some vocabulary so that you can communicate about the game. It is useful to think of a particular game as a self-contained sub-discipline of the overall discipline of games. Thus, each game has its own vocabulary, notation, history, culture, and so on. Precise vocabulary is important in order to understand the rules and to facilitate communication among people playing the game.

Note how this same idea applies to solving real-world problems. Suppose your computer is not working right. Do you know precise vocabulary to describe the problem? If not, you will have difficulty using information retrieval to find help or to talk to a person to get help. Getting help from stored information and from people is a very important strategy in problem solving. It requires effective communication between you and the information source.

More generally, consider reading across the content areas. To read with understanding within a discipline content area, you need to know how to read, you need to have an understanding of the special vocabulary and notation used in the discipline, and you need to have some understanding of the discipline.
2. Learning the legal moves (plays). Each game has a set of legal moves. Notice that this is consistent with the formal definition of the term "problem." One can create a variation of a game (in essence, a new game) merely by changing the rules.
3. Gaining knowledge and skill that help one to make "good" moves. Often this knowledge is in the form of strategies that help to govern one's overall play as well as one's decisions on individual moves or small sequences of moves (tactics).
4. Gaining speed and accuracy at making good moves.

## Solitaire (Patience/Klondike)

Quoting from the Wikipedia https://en.wikipedia.org/wiki/Patience \% 28 game $\% 29$ :
Patience, or solitaire as it is known in the US and Canada, is a genre of card games that can be played by a single player. Patience games can also be played in a head-to-head fashion with the winner selected by a scoring scheme.
In the US, the term solitaire is often used specifically to refer to solitaire with cards, while in other countries solitaire specifically refers to Peg solitaire. Both Solitaire and Patience are sometimes used to refer specifically to the Klondike form of Patience.

There are many different solitaire games. The most commonly played one is called Klondike. For many years, Microsoft has provided a free electronic version of Klondike in its Windows operating system. Thus, it is probably the most widely played electronic game in the world.

It is assumed that you are at least somewhat familiar with Klondike. It uses a standard 52card deck of playing cards. The card deck is shuffled and then dealt out, as illustrated Figure 5.1. If you are not familiar with the game, you might want to read a little about its rules at http://en.wikipedia.org/wiki/Klondike solitaire.


Figure 5.1. The start of a game of Klondike solitaire.
(The Klondike solitaire screen shots used in this section were made from my old electronic version of Eric's Ultimate Solitaire; see the current website at http://www.deltatao.com/ultimate/.)

The top row of the layout in Figure 5.1 is the computer representation of the four foundation stacks where cards will be built up in sequence, starting with the ace. Collectively, these four
stacks are called the foundation. There are no actual cards in the foundation at the beginning. The game is won by getting all 52 cards onto the foundation.

Below the foundation are seven piles of cards, containing $1,2, \ldots 7$ cards, respectively. The top card of each pile is exposed, while the remaining cards are face down. Finally, the remaining cards in the deck (shown in the lower left corner) are face down and are called the reserve or the stock. To their right of the reserve is an empty space for the waste pile.

Figure 5.2 shows the results after the game player has taken the top three cards from the reserve, turned this set of three cards over in a manner that does not display the first and second cards, and placed the three cards on top of the waste pile. The player has also moved the 9 of spades onto the 10 of hearts, and then turned up the card that was beneath the 9 of spades.


Figure 5.2. The display early in playing the game.
Figure 5.3 shows the game after the player has made several moves. Three foundation stacks now have cards in them.


Figure 5.3. Later in the game...

## Winning and/or Playing Well

Klondike is fairly easy to learn to play. You win the game if you get all 52 cards into the foundations. However, that does not occur very often. If you have played Klondike a large number of times, you probably have a sense of how often you win.

If you are mathematically inclined, you might wonder what percentage of deals are winnable, or what percentage of the time a good player wins. It turns out that the first question is a math problem that has not yet been solved.

However, using the simpler problem strategy of posing a simpler but related problem has led some people to explore a variation of this problem. Suppose that the player can see all of the cards (that is, all the cards are face up). Then a computer program has been developed that wins about $70 \%$ of the time, and good human players win about $35 \%$ of the time (Yahoo! Answers, n.d.).

To answer the second question, one merely has to keep statistics for a number of good players, as they play a large number of games. In addition, Klondike is sometimes used as a gambling game in casinos, so that data should be available on the odds of wining. After I spent a good deal of time searching the Web, I found Aldous and Diaconis (July 21, 1999):

Rabb simulated a common form of Klondike in which cards are turned over three at a time (with only the top card exposed) and where one can cycle through the deck indefinitely. She found that the computer won about $8 \%$ of games whereas she won about $15 \%$. [Note from David Moursund. This result suggests to me that the computer program that Rabb used was not very good. It reminds me of early computer programs written to play chess. Good human chess players could easily beat these programs. Eventually, however, the best chess-playing computer programs became better than the best humans.]

Koller studied modern game-playing heuristics applied to Klondike. Preliminary results suggest a win probability around $15 \%$.

Personally, I sometimes cheat when playing Klondike, and yet my win percentage is probably less than $10 \%$.

Many players also use a measure of how well they did when they did not win-namely, the number of cards added to the foundation stacks. Note, however, this may not be a very good measure of how well you have played in a particular game. You have no idea of how many cards a good player would have added to the foundations when playing the exact same deal.

I find it interesting to think about the intrinsic motivation that drives so many people to play Klondike repeatedly. The possibility of winning is somewhat motivational. However, winning perhaps $10 \%$ of the time or less is not very encouraging-rather, I find this to be discouraging. Still, I feel somewhat good when I an able to play a large number of cards onto the foundation stacks.

What holds my attention and keeps me motivated, however, seems to be the overall process. My mind/brain seems to interpret the process as one in which I am accomplishing something that it deems worthwhile.

As I play the game, I am continually involved in doing something. I am turning up cards from the reserve and remembering the location of some of these cards. I am thinking about possible moves, trying to figure out good moves. I am following the rules as I make moves. New cards are displayed because of my moves. In summary, my mind and body are engaged, small rewards are occurring all of the time, and occasionally I win.

The previous paragraph reminds me of the P.T. Barnum statement, "You can fool most of the people most of the time." A mind/brain is a complex thing. However, in a mind/brain, pleasure can come from quite simple things. Playing Klondike stimulates my mind/brain in a manner that brings me pleasure. The same holds true for many other games. Over the years, I have come to understand this. I have also come to understand that, from time to time, I fall into an addictivelike behavior of playing games rather than doing other things that have greater "redeeming" values but provide less immediate rewards.

I have talked to a number of people about this type of game-playing experience. They tell me about how they have learned to carefully restrict (ration) their game playing. Their level of addiction is not so strong that it overwhelms their determination to use their time for other, more productive activities.

## Applications to Other Games and General Problem Solving

In Klondike, as well as in many other games and problem-solving situations, there are possible moves, plays, or actions. While you would like to make a good move, you often fail to do so. As you play, you can learn more about problem solving by reflecting on your play. For example:

- You do not "see" (discover, recognize) a possible move, so it receives no consideration. This can be through carelessness and oversight, or it can be caused by just not spending enough time in careful thinking and searching for possible moves. While some real world problem-solving situations require very quick decision-making, the majority allow time for reflection and for consideration of consequences of moves. However, one may have a blind spot and need appropriate external stimulus to be able to "see."
- You find a possible play or move, give it consideration, but make a relatively obvious mistake in this consideration. Immediately upon making the move you recognize the mistake and want to take the move back. (Undoubtedly you have made such "mistakes" in conversations and/or electronic communications.) The message is clear-use the look before you leap strategy and the think before you act strategy.
- You find two or more possible legal moves and do a compare/contrast consideration of the moves. If one is clearly better than the others, you make it. However, quite likely you will not know for sure which is the best move. It may turn out that the one you think is best isn't, because you lack information on what will happen because of your move. (In Klondike, for example, you may not know what card will turn up. In negotiating a business deal, you do not know for sure how a person will react to your proposal.) This type of uncertainty literally petrifies some people. They just seem unable to make decisions in the face of uncertainty. Practicing in a game-playing environment, where it is easy to take back moves, may help such a person get better at making decisions under uncertainty.


## The Solitaire Game Eight Off

Eight Off is my favorite one-deck solitaire game. However, I am not trying to force you to learn to play the game. Rather, as I did with Sudoku earlier in this book, I use Eight Off as an excellent vehicle for illustrating some very important ideas useful in problem solving. So, bear with me, and I hope you will appreciate the increased problem-solving skills you will gain through this learning experience.

Eight Off is available in many commercially available bundles of computer solitaire games. It can be played free at a number of Websites. All but one of the screen shots used in this section are from the Eight Off game \#31853 from http://www.acecardgames.com/en/.

Click on the first screen to bring up a screen of free games. Click on Eight Off to produce a dealt-out game and you will see the words Eight Off in the top left corner. Click on this and you can key in the number 31853 to be playing the exact deal discussed in this section. Figure 5.4 shows the layout for the specific example of Eight Off \#31853 that will be used to illustrate the rules and playing of the game.


Figure 5.4. The start of a game of Eight Off.

A regular deck of 52 playing cards is shuffled. The first 48 cards are then dealt face up in eight columns (called Main Stacks) of six cards each. The remaining four cards are dealt face up into four of the eight Free Cells. In Figure 5.4, the eight free cells are in two columns of four each, with one column on the left and one column on the right.

Above the Main Stacks is space to build four Object Stacks. An Object Stack is built up in a suit, starting with the Ace and continuing with 2, 3, 4... Jack, Queen, and King of the suit. The object of the game is to build all four Object Stacks until they contain the entire 52-card deck.

The rules for playing are:

1. Cards are played one card at a time.
2. The lowest, fully exposed card in each Main Stack (in the example of Figure 5.4, these include the 5 of spades, the 3 of spades, the 6 of clubs, and so on) and each card in the Free Cells is available to play.
3. Cards that are available to play may be played as follows:

- If the card is an Ace, it is played in an empty Object Stack.
- The card may be added to an Object Stack, provided that it is the next-higher card in rank of that suit and Object Stack. Ranks are 2-10, J, Q, K.
- The card may be played to any empty Free Cell, of which there are eight with four occupied initially, or to any empty Main Stack.
- The card may be played by adding it to any Main Stack whose top card is of the same suit and is the card immediately above it in rank. In Figure 5.4, for example, the 2 of spades can be played on the 3 of spades. However, the 5 of spades cannot be played on the 6 of clubs.
The set of rules is relatively simple. Some people can read and memorize such a set of rules quite quickly, while others will find they need to refer back to the rules from time to time until all become familiar. This situation gives us some important insights into schooling. Often, schools expect students to memorize information in advance of when they will need to use it. Or the students may be tested over the memorized information outside of the context in which they might eventually use the memorized information.

However, most people learn best when they immediately find a use for what they are memorizing. The memorization is interspersed with the using. The learner eventually memorizes what needs to be memorized through frequently looking it up and using it.

This can be summarized in the problem-solving memorize through use strategy. A different name for the strategy in the only memorize if quite useful strategy.

This strategy is very applicable in education. Suppose that a child is learning to write and is writing on a word processor. The word processor provides feedback on possible misspelled words and errors in grammar. The child's spoken vocabulary is far larger than the child's "I know how to spell it" vocabulary, especially in non-phonetically spelled languages such as English. Research on memorizing lists of spelling words versus invented spelling strongly favors invented spelling. See https://www.heinemann.com/shared/onlineresources/08894/08894f4.html.

I believe that far too much time is spent in school memorizing "stuff" that one will seldom or perhaps never use in the future. Moreover, most of what one memorizes will have long been
forgotten before a need arises to make use of it. In my opinion, well-designed curriculum can be described by:

- Learn it and use it-both in the short run and in the longer run.
- Learn to learn as one learns. (Metacognition and reflection are critical learning aids.)

For convenience is discussing the game, I have lettered the eight Main Stacks a through h. See Figure 5.5.


Figure 5.5. The 8 Main Stacks are lettered a through h.
Eight Off is a solitaire game that requires thinking in sequences of moves. Notice the Ace of spades is the second card in Main Stack b. If I move the 3 of spades to an empty Free Cell (currently there are 4 empty Free Cells), this will expose the Ace of spaces, so that it can be played in one of the Object Stacks. The result is shown in Figure 5.6. This moves me in the direction of building the Object Stacks, which is the goal of the game.


Figure 5.6. A sequence of 2 moves is completed.
You can see that the 2 of spades in Main Stack g and the 3 of spades in the Free Cells can now be played on the Ace of spades in the Object Stack. My "look ahead" thinking saw that this would be the situation if I uncovered the Ace of spades in my first move. So, the 2 and 3 of spades can be moved to the Object Stack.


Figure 5.7. A sequence of 2 more moves is completed.
Well, that is enough of this game for now. Try playing it with a deck of cards or on the computer. Practice using the plan ahead strategy. You will get better at this endeavor through practice.

As you go about your daily life, look for situations in which it is useful to plan ahead. Become consciously aware of times in which you do this. Help your students to improve their use of the think before you act strategy.

## Strategies Used So Far in Our Eight Off Game

We started out by designing a sequence of moves. This is an important strategy that we will call the sequence of moves strategy. The idea is to think in terms of multi-step sequences of moves or actions when attacking a complex problem. Depending on the problem being attacked, steps may be done sequentially, they may be done in parallel (all at the same time), or they may be done in a combination of sequential and parallel steps. Large problems that are being worked on by a team of people are attacked using the sequence of moves strategy. Thus, this strategy should be part of your repertoire and your students' repertoires of high-road transferable problem-solving strategies.

The first sequence of moves that was used was designed to get an Ace into the Object Stacks. Just for the fun of it, lets call this the getting an Ace into the Object Stacks strategy. This strategy is useful in playing Eight Off. Indeed, it is a strategy useful in many different solitaire card games.

However, it is not a general-purpose problem-solving strategy that we will want to add to our repertoire of high-road transferable problem-solving strategies. Most real world problems do not involve getting a card called an Ace onto a space called an Object Stack.

## Score, and Then Work to Improve Your Score

However, there is something akin to "getting and Ace into the Objects Stacks." Consider track events such as the long jump, discus, and the shot put. A contestant gets three tries, with only the best one counting. A foul in a try counts as a distance of zero. Many contestants will focus heavily on not fouling on their first trial, not trying to get as great a distance as they are capable of. That is, the athlete has two goals: 1) get a relatively good distance, and 2) get as long a distance as possible. Often an athlete decides to focus on the first goal in the first try. If the athlete does not foul in this try, than the second and third tries are all-out efforts to achieve the greatest distance possible.

So, we have another general-purpose strategy that is suitable for adding to one's repertoire of high-road transfer problem-solving strategies. Let's call it the score, then improve your score strategy.

For example, suppose that the problem a person faces is to complete a short answer or objective test. The score, then improve your score strategy might lead the student to browsing through the test, answering the questions that he or she is confident about. Then it's time to go back and spend time on the other questions.

For another example, consider being faced by a complex problem, but one that can readily be broken into a number of smaller or somewhat easier sub-problems. Use the break a problem into smaller problems strategy to first solve some of the easier sub-problems. This assumes, of course, that the sub-problems are independent of each other, so can be solved in any order. Progress on the easier sub-problems is somewhat like first answering the easier questions on a test. This is a good strategy for students to learn if their life (their schooling, at least) requires taking multiple-choice tests. However, it also has the advantage that solving the easier subproblems may provide one with insights that will help in solving the more difficult subproblems.

For another example, consider writing an essay. One can write a few paragraphs and edit them over and over again, polishing them so they are perfect. This may take all of the available time. A different approach would be to do a quick rough draft of the whole document, and then begin polishing it. Using a word processing program rather than pencil and paper has made this strategy more effective and more pleasurable (or less painful).

## Returning to the Eight Off Solitaire Game

The moves that we have made so far can all be viewed as using an incremental improvement strategy toward the goal of having all 52 cards in the Objects Stacks. However, it may well be that this particular Eight Off solitaire game cannot be solved by just "any old" collection of incremental improvement sequences. For example, look back at the start of the game given in Figure 4.5. Consider the sequence of moving the 2 of spades onto the 3 of spades, moving the King of diamonds into an empty Free Cell, and moving the Ace of diamonds into an empty Object Stack.

This sequence of three moves results in an incremental improvement, just as did the sequence of moves that we actually made. Which of these two sequences of moves is better? Might one be
a good start on winning the game, while the other be a start on losing the game? Remember the incremental improvement picture in Figure 4.1, where the choice of starting point determines whether incremental improvement moves you to the highest peak.

The message is, even if you see a sequence of moves that looks good, continue looking to see if you find a better sequence. If you find two or more sequences that seem equally good, look still farther ahead to determine whether one is better in a still longer sequence. Or, consider the idea of the mobility strategy in the section below.

## Mobility: An Important New Strategy

Probably you have heard the adage, "Don't paint yourself into a corner." It is applicable in many game-playing and non-game situations.

I have played Eight Off many times, winning more often than I lose. I tend to lose when I fill up my Free Cells, thereby cutting down on my freedom to make sequences of moves that involve the use of empty Free Cells. Having quite a few empty Free Cells gives me lots of options that can be carried out in a sequence of moves.

In games such as chess and checkers, mobility describes having options. A high level of mobility of one's collection of pieces means that one has many possible moves; a low level of mobility means that one's possible moves are severely restricted.

Let's use the same term in discussing Eight Off. Having lots of empty Free Cells and empty Main Stacks gives one a high level of mobility in developing a sequence of moves. In many games and in many real world problem-solving situations, it is desirable to keep one's options open - to maintain or increase one's mobility. Let's call this the mobility strategy. Another name for it is the don't box yourself into a corner strategy. This is an important strategy to add to your repertoire and your students' repertoires of high-road transferable problem-solving strategies.

## Returning to the Eight Off Game Example

Now, finally, back to our Eight Off game. The mobility strategy helps me to decide between the opening sequence of moves that I actually made, and the sequence that would have led to getting the Ace of diamonds into the Object Stacks. This latter choice would have decreased my mobility.

Here is the "working" of my mind. I examine the current situation given in Figure 4.7. I think in terms of incremental improvement, but I hold in mind the mobility strategy. An obvious incremental improvement would be to use the sequence of 2 moves that ends with the Ace of diamonds being played in the Object Stacks. However, this sequence of moves decreases my mobility. Therefore, I spend some more time analyzing the current situation. Soon I see that a three-move sequence will add the 4 and 5 of spades to the spade Object Stack. This sequence does not decrease my mobility, so I make it, producing the position shown in Figure 5.8.


Figure 5.8. Two more cards added to the spade Object Stack.

## A Warning about Using Strategies

This illustrates an important concept in game playing and in more general problem solving. As you employ various strategies to decide on actions that seem to help move toward winning a game or solving a problem, you may well be working yourself into a hole from which there is no recovery except backtracking. In some games and in some real-world problems it is easy to back track. In others, it isn't easy, and it may be impossible. An increasing level of expertise in a game or in a real-world problem-solving situation allows one to avoid some of the dead end, losing sequences of actions that various strategies suggest might be helpful. Having a large repertoire of problem-solving strategies is helpful, but it does not guarantee success.

Another important concept that has been illustrated but not made explicit enough is to look as far ahead as your mind can handle when considering the outcome of a sequence of moves. Think about how to accomplish a particular intermediate goal, and think about the consequences of the situation created by working to accomplish this intermediate goal. This look ahead strategy works both in one-person games and in two-person games.

Here is the type of thinking one uses in a two-person game: "If I go/move here, and my opponent chooses (or is forced) to move there, then I can (or must) move there," and so on. The superior solver or player in games and in addressing "real world" problems can consistently keep a problem in mind, contemplate actions to accomplish an intermediate goal, and analyze the consequences of the resulting situation if the intermediate goal is achieved by the proposed approach. Again, this is using the look ahead strategy (also known as the look before you leap strategy).

The one-deck solitaire game named Fortress (available for free play at http://www.acecardgames.com/en/) has some characteristics that are somewhat similar to Eight Off. If you have learned to play Eight Off and not learned to play Fortress-or, vice versa-this provides a good opportunity to analyze transfer of learning between the two games.

## Tetris

There are relatively few computer games that are more popular with women than with men. Tetris is one of these. It is a solitaire (one-player) game, but it is not played using a deck of cards. Tetris (sometimes called Penta) is available on a huge range of handheld, game machine, and computer platforms. Quoting from http://en.wikipedia.org/wiki/Tetris:

Tetris is a puzzle game invented by Alexey Pazhitnov (last name sometimes transliterated Pajitnov) in 1985, while he was working for the Academy of Sciences in Moscow, Russia during the days of the Soviet Union. Pajitnov has cited pentominoes as a source of inspiration for the game. Its name is derived from the Greek word "tetra" meaning four, as all of the blocks are made up of four segments.

While versions of Tetris were sold for a range of 1980s home computer platforms as well as arcades, it was the hugely successful handheld version for the Game Boy launched in 1989 that established the game as one of the most popular ever. Electronic Gaming Monthly's 100th issue had Tetris in first place as "Greatest Game of All Time." In 2007, Tetris came in second place in IGN's " 100 Greatest Video Games of All Time." In January 2010, it was announced that the Tetris franchise had sold more than 170 million copies, approximately 70 million physical copies and over 100 million copies for cell phones, making it the best selling paid-downloaded game of all time.

My 11/6/2015 Google search of the expression free Tetris game produced about 19 million results. There are many places where one can play the game free on the Web.

## Tetris Game Play

Seven randomly rendered tetrominoes or tetrads-shapes composed of four blocks each-fall down the playing field. See Figure 5.9 for the seven tetrominoes.


Figure 5.9. The seven tetrominoes shapes.
The object of the game is to manipulate these tetrominoes (by rotation and horizontal movements) as they are falling, with the aim of creating a horizontal line of blocks without gaps. When such a line is created, it disappears, and the blocks above (if any) fall down one row. As the game progresses, the tetrominoes fall faster, and the game ends when the stack of tetrominoes reaches the top of the playing field. See Figure 5.10 for a game in progress.


Figure 5.10. A Tetris game in progress.
Playing this game requires hand-eye coordination, as well as quick recognition of figures in two dimensional space and quick decision-making. I am relatively poor in all of the abilities that it takes to become good at this game. Thus, it is not surprising that I do not enjoy playing Tetris.

However, I find it interesting to introspect as I play the game, and I find it interesting to see how practice makes me better at the game. At a beginner's level, the game can be set so that the pieces fall very slowly and one can experience success. One's mind/brain/body adjusts to the demands of the game (learns).

I also find it interesting to see/sense this learning occurring and to discover that I get better with practice. Through playing this game, I have gained increased appreciation for the learning capabilities of my mind/brain/body.

## Final Remarks

I thinkthat meditation and one-player games have certain things in common. For me, when I am playing a game such as Klondike, I shut out the outside world. The cares of the outside world and the cares inside my head go way.

For me, shutting out the outside world and being deeply involved in a game provides an example of what Mihaly Csikszentmihalyi calls "flow." See https://en.wikipedia.org/wiki/Mihaly_Csikszentmihalyi. Quoting from the reference:

In his seminal work, Flow: The Psychology of Optimal Experience, Csíkszentmihályi outlines his theory that people are happiest when they are in a state of flow- a state of concentration or complete absorption with the activity at hand and the situation. It is a state in which people are so involved in an activity that nothing else seems to matter. The idea of flow is identical to the feeling of being in the zone or in the groove. The flow state is an optimal state of intrinsic motivation, where the person is fully immersed in what he is doing. This is a feeling everyone has at times, characterized by a feeling of great
absorption, engagement, fulfillment, and skill-and during which temporal concerns (time, food, ego-self, etc.) are typically ignored.

Many people experience flow when playing engrossing games.

## Activities for the Reader

1. If you have had experience with meditation, think about some of the similarities and differences between meditation and playing one-player games.
2. Select a solitaire game that you have not previously played. Introspect, and then write notes to yourself about these introspections, as you learn to play the game. For example, you might think about the challenge of learning the rules, and how this challenge compares with learning the "rules" of appropriate social behavior in a particular situation, or learning the "rules" for carrying out some job or task. You might think about how you get better at playing the game as the rules become internalized-sort of automatic, governing your behavior with little conscious thought. You might think about how the game is similar to and different from other solitaire games you have played.
3. Suppose that you are playing a game of solitaire, and you "take back" a move or "peek" at a hidden card, in violation of the rules of the game. Is "cheating" an appropriate word to describe this situation? (For example, you may do this as part of the process of learning to be a better player of the game.) Compare and contrast this activity with that of cheating on a test or cheating on one's income tax return. Can you think of a more appropriate term than "cheating" to use in a one-player game-playing situation?

## Activities for Use with Students

1. Playing cards and card games have a very long history. Select some aspect of this history that interests you, study it, and write a report on your findings. To the extent possible, tie your findings in with other historical events and people. For example, it is speculated that Napoleon Bonaparte developed the solitaire games Napoleon at St. Helena and Napoleon's Square.
2. Find a game of solitaire that you have not played before. Learn to play it. Then do a personal compare and contrast with a solitaire game that you already know how to play. Which game is more fun, more challenging, more attention-holding, and so on. Which game would you recommend to a friend, and why?
3. In your own words, explain the difference between a puzzle and a one-player game. Which do you like better, and why?
4. Have you every experienced "flow" when playing a game, engaging in social networking via the Internet, or in some other situation? If yes, describe the situation and how it personally felt to you.

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## Chapter 6

## Two-Player Games

You have to learn the rules of the game. And then you have to play better than anyone else. (Albert Einstein.)
Each player must accept the cards life deals him or her: but once they are in hand, he or she alone must decide how to play the cards in order to win the game. (Voltaire.)
There are many different kinds of two-person games. You may have played a variety of these games such as chess, checkers, backgammon, and cribbage. While all of these games are competitive, many people play them mainly for social purposes. A two-person game environment is a situation that facilitates face-to-face communication and companionship. However, it can also facilitate anger and hostility.

Two major ideas are illustrated in this chapter:

1. Look ahead: Learning to consider what your opponent will do as a response to a move that you are planning.
2. Computer as opponent: In essence, this makes a two-player game into a one-player game-a solitaire game with a computer opponent.
In addition, we will continue to explore general-purpose, high-road transferable, problem-solving strategies.

## Tic-Tac-Toe

To begin, we will look at the game of tic-tac-toe (TTT). TTT is a two-player game, with players taking turns. One player is designated as $X$ and the other as $O$. A turn consists of marking an unused square of a $3 \times 3$ grid with one's mark (an X or an O ). The goal is to be the first to get three of one's marks in a vertical row, horizontal row, or diagonal row. The board contains eight such rows. Traditionally, X is the first player. A sample game is given below.


Figure 6.1. Example of a Tic-Tac-Toe game.

TTT provides a good environment in which to explore how a computer can play a game. I assume you are familiar with the idea of random numbers. For example, if you designate one side of a "true" coin as 0 , and the other side as 1 , then repeatedly flipping this coin generates a random sequence of 0 's and 1 's. If you repeatedly toss a "true" six-sided die, you will generate a random sequence of integers in the range [1,6]. In many games, a spinner is used to generate random moves. The process of shuffling a deck of cards is a process of randomizing the locations of the cards in a deck.

Randomness is a complex area of study. For example, suppose you throw a pair of "true" dice, and add up the total. You will get an integer in the range [2,12]. However, a sequence of such throws will not produce a random sequence of integers in this range. Some numbers, such as 2 and 12, will occur much less frequently than others. The number 7 will occur far more frequently than 2 or 12 . The study of this and related situations is part of the field of probability.

It turns out that random numbers are quite important in many non-game settings. For example, an educational researcher is conducting an experiment in which one set of students receives a certain treatment, and a different set receives a different treatment. To decide which student gets a particular treatment, each student's name is written on an identical small piece of cardboard and placed in a box. The contents of the box are thoroughly stirred. Then names are drawn out one at a time, alternately placing the name into the group to receive the first treatment and the group to receive the second treatment.

Computer programs have been developed that generate pseudorandom numbers. (See http://en.wikipedia.org/wiki/Pseudorandomness.) Quoting from the website:

A pseudorandom process is a process that appears to be random but is not. Pseudorandom sequences typically exhibit statistical randomness while being generated by an entirely deterministic causal process. Such a process is easier to produce than a genuinely random one, and has the benefit that it can be used again and again to produce exactly the same numbers-useful for testing and fixing software.

You know that computers can play some games quite well. Perhaps you believe that if a computer has been programmed to play a game, the computer will never lose. However, that is not the case. Let's use TTT for an example. Suppose that a computer has been programmed to make random moves when playing against a human opponent. When it is the computer's turn to make a move, it selects one of the legal available moves at random, and makes that move. As you might expect, a human player can often beat such a computer program.

Your students may enjoy creating a "by hand" simulation of this situation. Prepare nine small pieces of paper that are numbered $1,2, \ldots 9$, respectively, and place them in a small box. Number the spaces of a TTT board with the nine digits as follows:


Figure 6.2. A TTT board with its squares numbered 1 to 9 .
Let us suppose, as an example, that X is going to play first and that X 's moves will be randomly generated. You play O against the random mover. Start at step 1.

1. To generate $X$ 's move, stir up the pieces of paper in the box and draw one out. Its number will be the space into which X moves. Then one of the following 3 situations occurs:

1a. If this move completes a row (vertical, horizontal, or diagonal) with three X 's, X wins and the game ends.

1b. Otherwise, if this is the ninth move in the game, the game ends and is a draw.
1c. Otherwise, go to step 2.
2. You (O) make a move. If this produces a row with three O's, you win and the game ends. Otherwise, look into the box and remove the slip of paper that contains the number corresponding to the move you just made. Then go to step 1.

The set of steps can easily be written as a computer program. The set of steps is an algorithm that generates moves for X and determines who wins or if the game is a draw. An algorithm is an unambiguous set of instructions. See https://en.wikipedia.org/wiki/Algorithm for more detail. It should be evident to you that just because a computer has been programmed to play a game it does not follow that the computer wins all the time or will always play well. Indeed, the random number player will play poorly. However, it will occasionally best a child who is just learning to play the game.

The idea of using random numbers in a computer program adds an unpredictability dimension to what a computer can or cannot do. People often argue about limitations of computers by stating that a computer can only do what it is told to do in a computer program. However, when this "telling the computer" includes making use of random numbers, the programmer or the program user may not be able to accurately predict the results.

## A TTT Algorithm that Never Loses

Next, we will explore a TTT algorithm that can be followed by a person or programmed into a computer, and that plays quite well. This algorithm is specifically designed to produce moves for X , who moves first.

Begin by numbering the nine squares on the grid as follows:


Figure 6.3. TTT board numbered to help specify a game-playing algorithm.
The first player (X) uses the following 4-part algorithmic procedure to determine what move to make at each turn:

1. Examine the grid and carry out the following sub-steps:

1a. If there are one or more rows that contain 2 X's and no O's, play in the one that contains the lowest numbered blank space. Thus, you win the game, and the game ends.

1b. Otherwise, if there is only one blank square remaining, play in it. The game then ends as a draw.

1c. Otherwise, go to step 2.
2. If there is a row containing 2 O 's and no X , play in that row. Otherwise go to step 3 .
3. Consider each possible remaining legal move, from the lowest numbered one to the highest numbered one. For each, see if making that move would result in the creation of two or more distinct rows each containing two X's and no O's. If (and as soon as) such a possible move is discovered, make it. Otherwise:
4. Move into the lowest numbered unused square.

Through some careful thought, you should be able to convince yourself that X (playing first) never loses, and X's first move is in the center of the board. This algorithm that never loses is dependent on X going first, on the board being numbered as shown, and on the "look ahead" feature in step 3 .

Use of the look ahead strategy is a key feature in writing a program that plays a good game of chess, checkers, or other somewhat similar games. On a more general note, look ahead is a process of considering the consequences of possible actions-before taking an action. In essence, look ahead allows one to study possible outcomes of an action. This is important in computer game playing, but it is also an important and routine aspect of functioning as a responsible human being. A game-playing environment (various types of games and computer simulations) can be used as an aid to helping students learn to use the look ahead strategy to gain increased responsibility for their own actions.

The $3 \times 3$ TTT game is not much of an intellectual challenge-some chickens have been trained to play well. There are a variety of games that can be considered as modifications of TTT (Boulter, 1995). At Boulter's website, you can play on boards of size $3 \times 3$ up to $7 \times 7$, and the boards need not be square. You can set your own rules for how many squares in a row are needed for winning. Games that are somewhat similar to TTT in that one wins by getting n-in-arow include Connect Four, Gobblet, Nine Men's Morris, Pente, Three Men’s Morris, Gomoku, and Quarto. There are also 3-dimensional variations of TTT.

## Some Other Popular 2-Player Games

## Gomoku

Quoting from http://www.springfrog.com/games/gomoku/:
Gomoku was introduced to Japan by the Chinese in 270 BC. The full name of Gomoku is actually Gomoku Narabe, which is Japanese for "five points in a row." Japanese chronicles show that at the time of the late 17th and early 18th Century Gomoku Narabe was at its height of popularity, being played by young and old alike. By 1880 the game had reached Europe where it is also known as Gobang.

Gomoku can be thought of as a much more challenging version of Tic-Tac-Toe. (It should not be confused with the game of Go. See https://en.wikipedia.org/wiki/Go \%28game\%29.) The goal is to get exactly five markers in a horizontal, vertical, or diagonal file. Many people enjoy playing this game. It is also a game that can be used as a programming challenge in an Artificial Intelligence course. See, for example, http://web.cs.wpi.edu/~ruiz/Courses/cs4341_A97/project.html. This (writing a program to play a game) illustrates a somewhat common and very valuable use of games in education.

My 11/6/2015 Google search of the expression free Gomoku produced close to 400 thousand results. Figure 6.4 illustrates the end of a game in which I played black, and won.


Figure 6.4. Black won this Gomoku game.
Gomoku is a strategy game that most people find quite challenging. Many computer implementations of this game allow the human player to select the level of his or her (computer) opponent. When first learning to play the game, you can select a very weak computer opponent. In that environment, you can experiment with strategies and perhaps develop a winning strategy against the weak opponent.

Notice the two dimensionality of the game play. To play well, you must consider threats and opportunities throughout the 2-dimensional board. You must use a careful look ahead strategy to thwart threats and develop possible winning positions.

## Connection Games

My 11/6/2015 Google search of the expression free connection games produced more than 200 million results. In Hex, players take turns coloring in one of the empty cells with their color, or placing a token of their color on the board. There are many variations of 2-player games in which a player attempts to form a connection between two borders. Figure 6.5 shows an empty $11 \times 11$ cell Hex game board and a completed game in which blue has won by completing a connection between the two blue borders.


Figure 6.5. Hex board and a completed game won by blue.
Mathematicians have proven that the player moving first can always win-if the player avoids making a mistake that then puts the second player into the position of having a winning strategy.

For more information about connection games, see Pegg (March 28, 2005). His interest in games in education is broader than just connection games. Quoting from Pegg's short article:

At a family gathering a few years back, I taught the game of Go to a young nephew. In the space of a few hours, he learned the rules, various tools of strategy, and applications of that strategy. He recognized the traps I set for him in a series of quick demonstration games. Then we played a 13 -stone handicap game on a 9 x 9 board, and he beat me.
In math instruction, one learns the rules, various tools of strategy, and applications of that strategy. Students learn to recognize common traps within a series of demonstrated problems. Notice the parallels here. Could abstract games work as part of a school curriculum? Historically, outside of Go and Chess, there haven't been many books on abstract games. That started changing in 1959 with Martin Gardner's column "The Game of Hex." [Boldface added for emphasis.]

The above quote asks the question, "Could abstract games work as part of a school curriculum?" Think about the discipline of math. Might one consider math as an abstract game, and learning math as learning to play the math game? Hmm.

Math has rules. A math problem to be solved or a math theorem to be proved can be thought of as a goal. Feedback in solving a problem or proving a theorem can come from the "player," a teacher, or others. Finally, there is the issue of voluntary participation. Certainly mathematicians do math (play the game of math) because they want to and enjoy doing so. Unfortunately, many students do not voluntarily learn to play the game of math and to play for fun. A good math teacher working with good math education materials can help students to voluntarily and successfully play the game of math.

## Chess

In some two-person games, there are ranking systems that rank the best players in a country or worldwide. For example, look at the chess site of the World Chess Federation, http://ratings.fide.com/toplist.phtml. It lists the top 100 players in the world.

Interestingly, chess remains a widely popular two-person game even though an inexpensive chess program running on an ordinary desktop computer or handheld device can readily defeat the great majority of human chess players.

I have found the history of computer chess to be quite interesting. As electronic digital computers were first being developed and then became commercially available, a number of people decided that computer chess was an interesting and challenging problem. Eventually, computer versus computer chess tournaments were held, and then computer versus highly ranked human players became of interest. In 1997, a computer developed by IBM especially for chess playing beat Garry Kasparov, the reigning world chess champion. See http://www.research.ibm.com/deepblue/.

Since that experience, Garry Kasparov has taken a leadership role in developing Super Chess, in which a team consisting of a computer and a human play against another team consisting of a human and a computer, or against a computer program that is ranked higher than the human (individually) and the computer program (individually) in the team. This effort is part of ongoing research on how to take advantage of human and machine knowledge and capabilities in working together to solve complex problems. In many situations, a human and a computer working together can do better than either alone.

There are several important ideas involved in writing a computer program that will play good chess. The first idea is having the computer memorize a large number of good opening sequences of moves and also memorize the moves needed to win or avoid losing when there are only a small number of pieces remaining on the board. Well-qualified human chess players have analyzed many such situations, and the results have been recorded in books and in other chess publications. Can you imagine the challenge of memorizing many thousands of different sequences of moves, with some of the sequences being 10 or more moves by you and an equal number by your opponent? This is a trivial task for a computer, but is an overwhelming challenge for most humans.

The second important idea is evaluating a board position. At any particular time in a game, how can one tell how well each player is doing? It is easy to count pieces remaining on the board. It turns out that mobility of one's pieces is an important consideration, and this can be measured. (Remember, we talked about the mobility strategy in chapter 5.) However, there are many other important considerations, such as the quality of one's defensive structure or offensive thrust, progress toward getting a pawn to the eighth rank (and thus turning it into a Queen or another piece of the player's choice), and so on. Much chess research has gone into developing computable schemes for rapidly evaluating a board position. A good human chess player is able to mentally evaluate about two or three one-move-ahead board positions per second. This speed is accomplished mainly by the quick recall of memorized patterns from one's brain $/ \mathrm{mind}$. It is not done by quick analysis of all possible moves of individual pieces. The computer program that defeated Kasparov could evaluate 200 million board positions per second.

The third requirement is to have an efficient, fast implementation of the look ahead strategy. This requires generating huge numbers of sequences of possible moves and evaluating the resulting board positions.

A huge amount of people's time and effort has gone into developing good computer-playing chess programs. A number of early researchers in Artificial Intelligence selected this game and other games as vehicles in which to explore how to develop intelligent-like computer programs. Games were a popular choice of such researchers both because they were an appropriately difficult challenge (because the rules were fixed) and because the results could easily be communicated to non-computer people. I am impressed to learn that computer programs have
beaten the world's best players of checkers, backgammon, chess, and so on (but not in the game of Go). In the past year or so, considerable progress has occurred in developing computer programs that play Go well.

Such accomplishments have significant educational implications. In many jobs or professions people work with relatively well-defined sets of rules to solve relatively difficult problems. Over the years, computers or a combination of people and computers handle more and more of these problems. Our educational system needs to understand this situation and to educate students accordingly.

Understanding the computerization of games and the uses made of artificial intelligence will help you to understand computational thinking. (See http://iaepedia.org/Computational Thinking.) When you think about solving problems and accomplishing tasks outside the world of games, consider the following two facts about computers and game playing:

1. Computer programs have been developed to facilitate the play of most of the games considered in this book. The program "knows and understands" the rules sufficiently well to check whether a player's moves (by either a human player or a computer) are legal. The computer system "knows and understands" the rules for scoring and winning.
2. For many of the games considered in this book, computer programs have been written that can play the game. These programs can serve as an opponent in two-person games, and as a player in games involving larger numbers of players. In some cases, computer programs have been developed so that they can defeat, or never lose to, the best human players in the world.

I have used quotation marks around "knows and understands" because the computer does not know and understand in the way a human does. However, a computer can follow and enforce the rules. Similarly, most computer programs that play games do not do this by imitation of how a human knows, understands, and plays a game. Computers can, however, be programmed to make effective use of their high speed and large storage capacity to play quite well.

As you think about facilitating the education of today's children, consider how these computer capabilities and limitations-despite not knowing and understanding in the same way that a human brain/mind knows and understands-can compete with and can augment humans as they solve problems and accomplish tasks. If a problem situation requires human understanding and working with the complexities of human-to-human interaction, humans far exceed the capabilities of computers. If a problem situation requires knowing and precisely following a fixed set of rules, making use of large amounts of memorized information, and functioning very rapidly, then computers are apt to be able to far exceed the capabilities of humans.

Moreover, keep in mind that computer capabilities are going to continue to rapidly increase. Not only will the computers of the future be faster and have larger storage capacity, they will also be "smarter." It may take a large team of human computer programmers several years to develop a computer program that has some useful, new, "intelligent-like" capabilities. Nevertheless, once this programming task is completed, the program can be installed on billions of computers. This rapid and accurate dissemination is not at all like our educational system trying to integrate a new idea into teaching. Part of computational thinking is understanding the accumulation and rapid dissemination of computer-based capabilities.

## Checkers

Checkers is a board game played on the same board as chess, but it is a much simpler game. Some of the early research with computerization of checkers involved studying rote learning. In a very simple game such as TTT, it is easy to have a computer memorize every possible sequence of moves that can occur in a game. The computer can then be programmed to use this information in a manner to never lose a game.

Suppose you are a researcher interested in learning the value of memorizing a hundred different opening sequences of moves in checkers. You develop a game-playing program and gather baseline data by having the computer play against itself by using the program. Then you make a copy of the program and add 100 memorized openings to one of these programs before having the two programs play against each other a number of times. You will perhaps learn that this makes little difference in which computer wins, or you may see that the computer with the memorized openings has a distinct advantage.

You can then continue your research, adding to the number of memorized openings, adding memorized sequences to use near the end of the game, and so on. You can experiment with what happens when one computer has only a modest number of memorized openings and end game sequences, while the other has many more. You can test out various board evaluation procedures.

In checkers, as in chess, the number of possible opening sequences is immense. It turns out that brute force memorization of all possible games is infeasible with current computers. Indeed, checkers and chess games move relatively quickly into the middle game in which memorization of sequences of moves is no longer of value. As the play proceeds and many pieces are lost, then in chess and checkers one moves into the end game. There, the number of pieces on the board becomes small enough so that memorized sequences of moves are again valuable.

Many real world problems can be thought of in terms of opening, middle game, and end game. Rote memory (perhaps aided by looking up information in a book or online, referring to one's notes, or use of a computer) can be very valuable in the opening and end game. Thus, a good education focuses on preparing students to handle the "middle game" of the types of problems they will encounter when using their education. In my opinion, choosing rote memory when other learning options are feasible, is a very poor approach to education.

## A Machine Learning to Improve Its Artificial Intelligence

Suppose you have two identical computer programs that can play checkers. Call these programs P1 and P2. Each uses the same combination of measures to determine how good a particular board position is. You then add to P 1 a set of directions that makes some changes to the board evaluation procedure. These might be random changes or some specific pattern of changes you want to explore in the board evaluation procedure one of the programs is using, perhaps counting mobility more strongly, and advancement of pieces less strongly.

You then have the two programs play against each other a number of times. If P1 is now significantly better than P2, then make a new P2 that has the same board evaluation function as P1 and repeat the whole process. If the two programs remain approximately equal in playing strength, then make another set of changes to P1's board evaluation process and a new round of game play occurs.

Voila! You have developed a computer program that learns how to play a better game. This technique can be used in many types of computer-based problem-solving situations. Examples
include improving programs to handle voice input to a computer, developing better programs to make money buying and selling stocks, and developing better medical diagnosis programs.

This type of machine learning, and the underlying research and programming, all fits into the general field of Artificial Intelligence. It involves and is an example of computational thinking. Often people and computers work together on this type of machine learning. The best of the improvement ideas developed by humans are combined with the "ideas" that the computer comes up with. The results may well be better than either the humans or the computer can do alone.

## Hangman

Many children learn to play the 2-person game named Hangman. One player-the Game Master-thinks of a word, indicates the length of the word, and perhaps provides a clue, such as whether the word is a noun or a verb. The other player-the Word Guesser-attempts to guess the letters in the word. As illustrated in Figure 6.6, correct guesses are entered into their correct location in Target Word. Incorrect letters are added to the Bad Guess list, and each incorrect letter leads to adding one piece to the gallows picture. (In the version pictured, the horizontal line at the bottom was the first piece of the gallows.)


Figure 6.6. Completed example of a Hangman game. The initial clues were "eight letter verb." The seven incorrect guesses are indicated by the seven pieces of the scaffold and person.
Nowadays, some people consider the name of this game and its gallows to be inappropriate for children. It is a simple matter to rename the game (for example, to Guess My Word) and to establish a rule such as " 10 incorrect guesses and the Word Guesser loses."

In the traditional version of the game, the Hangman diagram grows with each incorrect guess. A large number of incorrect guesses leads to the gallows and the hanged person being completely drawn; the Game Master wins, and the Word Guesser loses.

In a teacher-led setting, the teacher may want to write a large number of suitable words and their clues on pieces of paper, and place them in a box. The Game Master draws one of these pieces of paper from the box, reads the clues to the Word Guesser, and draws the boxes for recording correct and incorrect guesses.

Note that a certain type of intelligence is needed by each player. It is relatively easy to understand how the Game Master can be a computer. The role of the Word Guesser can also be played by a computer. The computer makes use of a large dictionary, information about the frequency of letter usage in words, placement of vowels in words, and so on. Thus, the game of Hangman can help us to learn a little bit about artificial intelligence.

## Relative Frequencies of Letters Use

If you want to improve your level of expertise in Hangman and many other games that involve forming or guessing words, then a good strategy is to memorize information such as the relative frequency of letter use in written English or other language in which you are playing the game. . Let's call this the letter frequency strategy. There are other lists that can be memorized and are useful in similar games. The Wikipedia site http://en.wikipedia.org/wiki/Letter_frequencies contains English language letter use frequencies, top ten beginnings of words, top ten endings of words, most common bigrams (pairs of letters occurring in words), and most common trigrams (triples of letters).

I have previously mentioned the strategy, memorize when personally effort-effective. Suppose that you have a personal goal of getting better at playing word games. Then you might well make significant progress by memorizing some of the information about letter combination frequencies. As a specific example, consider that in Scrabble it is very helpful to know all of the two-letter words.

Even a small amount of this memorization will increase your expertise in Hangman and the TV game, Wheel of Fortune. For example, it is not too hard to memorize the two nonsense words ETAOIN SHRDLU. As the frequency table given below indicates, this gives you the 12 most frequently used letters in English language writing, in their order of use.

| Letter | Frequency |
| :--- | :--- |
| E | 0.12702 |
| T | 0.09056 |
| A | 0.08167 |
| O | 0.07507 |
| I | 0.06966 |
| N | 0.06749 |
| S | 0.06327 |
| H | 0.06094 |
| R | 0.05987 |
| D | 0.04253 |
| L | 0.04025 |
| U | 0.02758 |

Figure 6.7. Frequency estimate for the 12 most used letters in written English.
This table suggests that the letter E may be your best first guess in Hangman or Wheel of Fortune, since about $1 / 8$ of all letters in typical English language written material are the letter E.

## Othello (Reversi)

The title "Othello" is a registered trademark of Anjar Corporation. The game Reversi was developed in 1883, and is essentially the same as Othello. See https://en.wikipedia.org/wiki/Reversi.

This is a two-person game. See Figure 6.8 for the board setup for the beginning of a game. I have played it many times, but I don't recall ever playing against a human opponent. It is a game that lends itself to playing against a computer or against another person, when both are playing on a computer. Some of the advantages of playing it on a computer include:

- There is no need to have the special playing board and pieces.
- Computer catches all illegal moves.
- Computer keeps track of score.
- Computer does all of the "physical" movement of the playing pieces.
- If playing against a computer, one can set the skill level of the computer. (The best computer programs play better than the best humans players in this and many other twoperson games.)
A free version of the game can be played on the Web at http://www.checkers.ws. The figures shown in this section are from that website. Figure 6.8 shows the setup at the beginning of the game. The $8 x 8$ board is shown with two black pieces and two white pieces placed on the board. The flip side of a black piece is white, and the flip side of a white piece is black. Computer versions of the game allow one to set the level at which the computer is to play.


Figure 6.8. Setup at the beginning of an Othello game. Graphics from http://www.freegames.ws/games/boardgames/othello/othello.htm.
Brief summary of rules.

- A legal move by black is one that causes one or more of the white pieces that are connected together in a row, column, or diagonal segment to be surrounded by having a black at each end. The result of such a move is that all white pieces that are in the connected segment are flipped from white to black. Note that a move may surround more than one segment, and that all of the white pieces in all of the surrounded segments are flipped from white to black.
- If black has one or more legal moves, black must make one of them. Otherwise, black passes.
- A similar set of rules apply to white.
- The game ends when both players pass, one right after the other, or when the board is completely full. Each player's score is the number of pieces of the player's color that are on the board when the game ends. A game may end in a win for either player, or in a tie.
The X's in Figure 6.9 shows the four legal moves that black can make from the starting board position of Figure 6.8. (If one thinks about the symmetry of the situation, in essence, the four moves are the same.)


Figure 6.9. The four possible legal first moves of black.
Figure 6.10 shows the results of black playing in d3 and then white playing in c5.


Figure 6.10. Black plays in d3 and then white plays in c5.
At the end of the moves illustrated in Figure 6.10, both black and white have three points. Figure 6.11 shows black's next legal moves.


Figure 6.11. Black's possible moves.
Figure 6.12 shows the results of black making the move f 6 and white responding with the move e3.


Figure 6.12. Black moves into f6 and white responds with e3.
Othello is a two-person game with simple rules, but with a high level of complexity. If you don't believe this, try playing against the computer when it is set at a relatively high level of expertise.

The website http://radagast.se/othello/Help/strategy.html contains an analysis of some strategies. In earlier parts of this book, I have tended to focus on strategies that might be useful in many different game and non-game problem-solving situations. Research in problem solving
suggests that general-purpose problem-solving strategies tend to be somewhat weak as compared to strategies that are quite specific to a particular game or type of problem.

A good example of this is the mobility strategy discussed earlier in the previous chapter. In many different games and in many non-game problem-solving situations, maintaining or increasing one's options (one's level of mobility) may be helpful. This is a wait until something better comes along strategy. One first looks for a good move that may contribute significantly toward achieving a winning position. If no such move can be found, than a fall back approach may well be to make a move that maintains or increases mobility.

Here are some suggestions for improving your level of play:

1. Memorize some common (good) opening sequences.
2. In the early part of the game, don't grab too many of your opponent's discs.
3. Try to avoid placing discs in the three squares adjacent (horizontal, vertical, diagonal) to the corner squares.
4. Try to play in the corner squares.

If you have not developed any strategies specific to Othello, then learning just a very few strategies can greatly improve your level of play. This same concept holds in most problemsolving situations. There is usually considerable advantage to learning some strategies that are specific to the type of problem one is attempting to solve.

Learning domain-specific strategies for problem solving is an important aspect of increasing one's level of expertise in a specific problem domain. Knowledge and skill in using even a small number of good strategies in a particular game can move a novice player into being a moderately competent player.

For a math example, when doing arithmetic calculations either by hand or with a calculator think about the check your answer strategy. This can often be done by a mental estimation, common sense, or redoing the calculation. And, of course subtraction can be checked by addition, the addition of two numbers by subtraction, multiplication by division, and division by multiplication.

## Dots and Boxes

Dots and Boxes is quite easy for a child to learn to play. Two children can play against each other, or one can play against a computer. Figure 6.13 illustrates a game played on a $2 \times 2$ board. (Personally, I find the vocabulary confusing. The $2 \times 2$ board is two boxes by two boxes. However, it is played on a three by three grid of dots.) The game is more complex and challenging when played on larger boards.

Starting with an empty board of dots, players takes turns adding a single horizontal or vertical line between two unjoined (horizontal or vertical) adjacent dots. A player who completes the fourth side of a box earns one point and takes another turn. The game ends when no more lines can be placed. The winner of the game is the player with the most points. For details and strategies, see http://en.wikipedia.org/wiki/Dots and boxes.


Figure 6.13. Example of $2 \times 2$ dots and boxes game ( 2 boxes by 2 boxes) won by player A.
See http://en.wikipedia.org/wiki/Image:Dots-and-boxes.png.
Very young children have trouble learning to plan ahead-to think about how their opponent may respond to a certain move. Dots and Boxes is a good game in which to practice this plan ahead strategy. But, here is an important reminder. A human teacher is often very helpful in learning to plan ahead. As a teacher, you will want to emphasize planning ahead over and over again in various decision making and problem solving environments.

## Cribbage

Cribbage is a popular 2-person game. The game is played using a standard 52-card deck of playing cards and a cribbage board, which is used to show the score accumulated by each player. Nowadays, one can also play cribbage against a computer, or people can play against each other online. Figure 6.14 shows a cribbage board.


Figure 6.14. Cribbage board.
Quoting from http://en.wikipedia.org/wiki/Cribbage:
According to John Aubrey, cribbage was invented by Sir John Suckling, a British poet, in the early 17 th century. It was derived from an older card game called Noddy. It has
survived, with no major changes, as one of the most popular games in the Englishspeaking world.

Cribbage holds a special place among American submariners, serving as an "official" pastime. The wardroom of the oldest active submarine in the United States Pacific Fleet carries the personal cribbage board of World War II submarine commander and Medal of Honor recipient Rear Admiral Dick O'Kane on board, and upon the boat's decommissioning the board is transferred to the next oldest boat.

The rules and play in cribbage are moderately complex. Details are available at http://www.cribbage.org/rules/default.asp and at the Wikipedia site mentioned above. Here is the gist of the game. The two players are each dealt six cards. Each player keeps four cards and discards two cards face down into a crib. The four cards in the crib belong to the dealer and are scored after the playing of the cards in each player's hand.

A starter card is turned up from the remaining deck of cards, and players take turns playing a card. Detailed rules govern this playing of the cards, and either player can score points during this playing process.

After each player has played her or his four cards, each player determines the point value of their four cards plus the starter card. Then the dealer determines the score of the crib plus the starter card.

In brief summary, the game:

1. Involves randomness in using a shuffled deck of cards.
2. Involves strategy in deciding which two cards to discard into the crib. The strategy a player selects depends on whether the player will get the crib (that is, score the crib for her- or himself) or the player's opponent will get to score the crib. Randomness is involved because the scoring is based on the four cards in the crib plus the starter card that has not yet been turned up.
3. Involves strategy in the play of the cards.
4. Involves counting the value of one's hand and the crib.

Figure 6.15 shows some of the rules for scoring during play, scoring one's hand, and scoring the crib. Notice that two of a kind (pairs), three of a kind (triples) and a variety of other combinations score points. There is a strong resemblance to various poker games. People who frequently play cribbage have usually memorized this data.

|  | Points Earned |  |
| :--- | :--- | :--- |
| Cards | During <br> Play | Hand or <br> Crib |
| Two of a kind (pair) | 2 | 2 |
| Three of a kind (triple) | 6 | 6 |
| Four of a kind (quadruple) | 12 | 12 |
| Straights of three or more cards: per <br> card | 1 | 1 |
| 15-count (sum of any combination of <br> cards) |  | 2 |


| Four-card flush (only in the hand) |  | 4 |
| :--- | :--- | :--- |
| Five card flush |  | 5 |

Figure 6.15. Some of the rules for scoring in cribbage.
The board is used for keeping score. The game can just as well be played with pencil and paper being used to keep score.

Cribbage is a competitive game that includes some randomness, a variety of strategies, and certain aspects of "bluffing." The Wikipedia site http://en.wikipedia.org/wiki/Cribbage\#Tactics contains examples of some strategies (tactics) that are specific to cribbage. Through learning and appropriately using some of these strategies, you can significantly improve the level of your play. However, these strategies tend to not transfer to other games or to non-game problemsolving situations.

## Final Remarks

Two-person games have been with us for a very long time. Perhaps the largest changes in recent years are that one can play such a game in real time with a human opponent located thousands of miles away, a computer can serve as your opponent, or you and another person (sitting with you or far away) can play as a team against the computer.

When you play such games face-to-face with a human opponent, you are engaged in a social activity. Not only are you honing your skills in playing the game, you are honing your skills in interacting with people in one-on-one settings. Some of this is lost when playing with a human opponent who is located far from you. Still there can be "banter" and relationship building between you and a distant opponent.

Playing against a computer opponent certainly provides the opportunity to hone your playing skills. Typically you can set the playing level of your computer opponent, so that you can stretch your skills without being completely overwhelmed by a far superior opponent. Perhaps more important, however, is that playing against a computer helps you to learn about the current and steadily growing capabilities of computers. An important component of a modern education is for a student to learn:

- What can a computer do better than I?
- What can I do better than a computer?
- What can a computer and I working together do better than either one of us working alone?

These are difficult-might we say profound-questions. A good human teacher or mentor can help a student to ponder and explore these questions.

## Activities for the Reader

1. Describe exactly how to make a peanut-butter-and-jelly sandwich. This is a fun and educational activity that has been widely used in schools. The website http://everything2.com/?node_id=1179139 offers what the author claims is a complete description. Think about how a child learns to make a peanut butter and jelly sandwich. Compare this learning process versus a process of memorizing a very detailed set of instructions.
2. While some authors still use pencil (or pen) and paper to write books, I write at a computer keyboard. As I write, I take advantage of the spelling checker, grammar checker, software to measure readability level, dictionary, and thesaurus built into the word processor. I generate the table of contents entries and index entries, and do the detailed layout for desktop publication as I write. I also make frequent use of the Web, and occasionally use email to get information and ideas from colleagues. My computer is my partner as I write. Compare/contrast this with the idea of super chess developed by Kasparov, or with some other example of human and machine working together to solve a complex problem or accomplish a complex task.

## Activities for Use with Students

1. If some of your students know how to play Dots and Boxes, have them teach the rest of the class. If you don't have any Dots and Boxes players in your class, then provide whole class instruction or take some other approach so that all of your students know the rudiments of the game. After all have had an opportunity to play the game quite a few times, give the following writing assignment:

Explain how to play Dots and Boxes. Your explanation should include both the rules of the game and some of your ideas on how to make good moves. You may also want to write about how well you currently play the game and how you might go about getting better at playing the game.
2. Repeat (1) above for some other game, such as Hangman. This activity can be used many times. It gives students practice in organizing their knowledge and understanding of a game. It is a writing challenge to write the rules of a game in a manner that others can read and understand. It is a challenge to figure out how to make good moves, and then to tell (in writing) how to make good moves. It is important to learn to do metacognition and self-assessment to figure out one's level of competence and how to improve.

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## Chapter 7 <br> Games for Small and Large Groups

Some of my happiest memories are of my family sitting around the table playing games. We didn't have a lot of money, but we had a lot of fun playing games together. (Anonymous.)

Cards are war, in disguise of a sport. (Charles Lamb.)
This chapter explores some multiplayer games. In some of the games, each player plays as an individual, competing against other individuals playing the game. This is typical in board games such as Monopoly or in card games such as hearts and poker.

In other games, such as pinochle and bridge, teams compete against other. The rules of these games allow some sort of communication among the two or more members of a team. Often the allowable communication is quite limited, and part of the process of learning the game is learning how to communicate effectively subject to severe restrictions.

## Monopoly



Figure 7.1. Monopoly board.
Quoting from http://en.wikipedia.org/wiki/Monopoly (game):
Monopoly is a board game that originated in the United States in 1903 as a way to demonstrate that an economy which rewards wealth creation is better than one in which monopolists work under few constraints and to promote the economic theories of Henry George and in particular his ideas about taxation and women's rights. The current version was first published by Parker Brothers in 1935. Subtitled "The Fast-Dealing Property Trading Game", the game is named after the economic concept of monopoly-the domination of a market by a single entity.
A huge number of board games have some of the characteristics of Monopoly. Monopoly has a square board, with each side having 10 squares on which a player's piece can land. Each player
has a marker-perhaps a figurine - to mark his or her position of the board. A player rolls a pair of dice and lands on the space the total of the two dice ahead of the current position, moving clockwise. Further randomness is involved in the two shuffled decks of cards called Chance and Community Chest.

The play of the game involves making a variety of decisions, such as: buying or not buying property, houses, and hotels; mortgaging property; making trades, and so on. Players are actively engaged in receiving and paying out money in buying and selling, and keeping track of their money and property. A player gradually learns effective strategies useful in becoming a better player. See, for example, http://boardgames.about.com/cs/monopoly/a/monopoly101.htm.

Many other board games have some of the same characteristics. Players deal with a set of rules, with money, dice, making decisions, and striving to win the game. A substantial amount of learning occurs in such environments, with more experienced players serving in the role of teachers and individual tutors. There is transfer of learning both among such games and to nongame settings.

For example, a young child learning to play Monopoly rolls the dice and laboriously counts the dots. A more proficient player or mentor helps the child learn to recognize without counting the number of dots corresponding to $1,2,3,4,5$, and 6 . The young child might then learn to count on (count upward) from the recognized number on one die, to add in the value of the second die. With still more instruction and practice, the child learns to mentally add the values of the two dice.

Somewhat similarly, the young child laboriously counts out each space of a move. Eventually a child's number skills advance to a level where a dice total of 10 leads to moving 10 spaces (one fourth of the length of the board) without counting. A dice throw of 12 might be moved as 10 and two more, while a move of 9 might be made as a 10 and one less. Gradually the child develops a mental model of numbers used in moving around the board.

The money to be counted, paid out, and received provides a good environment for learning to deal with integer amounts of money-up to $\$ 2,000$ for the rent on the Boardwalk property with a hotel-and the wealth that the player has accumulated.

The game involves some reading, such as following a simple instruction "Go To Jail," and following more complex instructions on the Chance and Community Chest cards.

Finally, the game involves following rules, taking turns, and interacting in a civil manner with other players.

When all of these learning and socialization opportunities are combined, the result is a very good learning opportunity. Learning how to play Monopoly has an added value of learning a game that your parents and grandparents played. In some sense, Monopoly is part of the culture in many families and communities.

## Oh Heck: A Relatively Simple Bidding-based Trick-Taking Card Game

Quoting from https://en.wikipedia.org/wiki/Trick-taking_game:
A trick-taking game is a card game or tile-based game in which play of a "hand" centers on a series of finite rounds or units of play, called tricks, which are each evaluated to determine a winner or "taker" of that trick.

Playing Oh Heck involves bidding (trying to make a good estimate of how many tricks you will take) and playing to take or not take tricks.

Bidding is an important and challenging dimension in those trick-taking games that have bidding. It takes considerable knowledge and experience to become accurate at bidding. This topic is discussed more in the section on bridge.

There are many different variations of the rules for Oh Heck. The following common set of rules is adapted from https://en.wikipedia.org/wiki/Oh_Hell.

## The Deal and Play

Oh Heck can be played with almost any number of players, although 4-7 is considered optimal. The game is played using a standard 52 -card deck, with ace being the highest rank and two the lowest.

The first hand is played with one card dealt to each player. On each succeeding deal, one more card is dealt out to each player, until there aren't enough cards for another round. After this, the number of cards per player decreases by one every round. The game is complete when the last round (with one card per player) has been played. For example, a four-player match of Oh Heck consists of twenty-five deals, from hand size 1 up to 13 and back down to 1 .

The dealer deals out the cards one by one, starting with the player to the left, in a clockwise direction, until the required number of cards has been dealt. After the dealing is complete, the next card is turned face up, and the suit of this card determines the trump suit for the deal. (If there are no unused cards, the hand is played without a trump suit.)

Each player is now obliged to bid for the number of tricks he or she believes can be won. The player to the left of the dealer bids first. Bidding is unrestricted except that the total number of tricks bid cannot equal the number available. That is, the last bidder (the dealer) must make a bid so that the total number of bids is different from the number of cards each player has received. For example, if five cards are dealt to each of four players, and the first three bids are two, zero, and one (equaling three), then the dealer may not bid two. However, if five cards are dealt, and the first three bids are three, one, and two, then the dealer is free to make any bid.

When every player has made a bid, the player to the left of the dealer places the opening lead. Play then proceeds as usual in a trick-taking game, with each player in turn playing one card. Players must follow suit, unless they have no cards of the lead suit, in which case they may play any card. The highest card of the lead suit wins the trick unless ruffed (trumped), when the highest trump card wins. The player who wins the trick leads for the next trick.

## Scoring

There are a variety of ways to score the game of Oh Heck. A simple way to score the game is by use of a single scoring rule, such as:

Each player scores the number of tricks he or she takes. A player that wins the exact number of tricks bid receives 10 points for making the contract.
A variation on this is:
A player who makes the exact number of tricks bid receives 10 for making the contract plus the amount bid. Players who overbid or underbid score nothing.

It turns out that -on average- the easiest bid to make is a bid of zero. Thus, sometimes the following scoring rule is added to the one of the two rules given above that is being used:

Making a zero contract scores only five points.

## Hearts

Quoting from http://www.pagat.com/reverse/hearts.html:
Hearts is a trick-taking game in which the object is to avoid winning tricks containing hearts; the queen of spades is even more to be avoided. The game first appeared at the end of the nineteenth century and is now popular in various forms in many countries.

Hearts and many other multiplayer card games start with the shuffling of a 52-card deck, and involve each player playing from his or her "hand" of cards A full game consists of a number of rounds that involve a shuffle, deal, and a sequence of plays of individual cards. Rules for what cards can be played and who wins the "trick" vary with the card game. Hearts is a good card game for learning about this type of card playing and trick taking. It is a game of both strategy and luck, but is far less complex than bridge.

Hearts is usually a four-person game, although it can be played with fewer or more players. It is played with a standard 52 -card deck, with the deuces being the lowest ranked cards and Aces the highest ranked cards. In Hearts, suits have no ranks. Note that these rules tend to hold in many different card games, such as in pinochle, bridge (though suits have ranks), and many versions of poker.

In many games, the goal is to get as high a score as possible. There are some exceptions, such as in Hearts and golf. In both of these games, the goal is to get as low a score as possible. In Hearts, if you win a trick, then each heart in the trick adds one point to your score. If the trick contains the queen of spades, it adds 13 points to your score. Since your goal is to get as low a score as possible, the usual strategy is to avoid taking tricks that contain hearts or the queen of spades.

The rules of Hearts are simple enough so that it doesn't take very long to learn to make legal moves and thus to participate in playing the game. Hearts, as in may other competitive games, admits of a number of strategies or tactics specific to the game. Often several strategies are applicable in a particular situation as you do the thinking required to make a good decision. Over time, as you gain experience in playing the game, you will gradually develop insights into when a particular strategy is apt to prove fruitful. Your mind/brain, working at a subconscious level, will begin to learn patterns and then recognize the patterns in a manner that helps you to make good moves. This general occurrence is part of increasing your level of card sense expertise.

## Card Sense

My 11/8/2015 Google search on the expression card sense produced over 40 thousand results. Quoting from http://bizarrebridgeworld.blogspot.com/2010/04/what-is-card-sense.html:

Top class [card game] players often attribute their success to their "card sense. " Indeed, these players may well possess an innate organic ability to understand the game of bridge and all its facets. But what does card sense really mean?

Some commentators define it as "an acute awareness of the totality of what is going on." Others include within the definition the notion that a player must have "a fantastic sense of seeing how the play of a hand will develop, immediately after the opening lead."

Some aspects of card sense readily transfer from one card game to another. For example, a person learns some of the probabilities of occurrence of various combinations of cards, and these are applicable in many different card games. Other aspects of card sense are relatively specific to a particular card game. The two terms card sense and intuition, when applied to playing a card game, are closely related. Card sense or intuition within a card game comes from long hours of careful thinking (reflective analysis) while playing and studying a game-though some persons develop card sense in a particular game quickly and others never do.

The game of hearts, like other relatively challenging games discussed in this book, helps to illustrate the overall process of learning and developing an increasing level of expertise within a particular domain. There is the initial challenge of learning some of the rules-enough of the rules so that one can participate in a game. There is the challenge of dealing with playing poorly relative to one's opponents who have had much more experience and have achieved a higher level of expertise. There is the rapid growth in expertise level that comes through the first few hours of study and playing the game. This growth is aided by having helpful mentors (friendly opponents) who help you by sharing their insights and by teaching you some initial strategies.

Many games are complex enough so that a player will continue to gain in expertise even after hundreds or thousands of hours of playing and studying the game. Here is a personal example. By the time I began college, I had lots of experience playing many different card games. My dad taught me how to play poker, and I played a lot of poker with my siblings and friends. I thought I was a good poker player.

In college, I became friends with a fellow student through a sequence of physics and math courses we took together. He told me that he sometimes played poker for "real money" at a gambling place where this was legal. With my usual self-confidence in academic things (being a math and physics major), I challenged him to a game of poker. It was fortunate that we were not playing for real money. He massacred me! Although I was at least as smart as he in physics and math, he had a far higher level of expertise in poker playing. There is much more to being a good poker player than just knowing the rules and being good at math. If you are interested in one person's pathway to becoming a successful professional poker player, see Diary of a Card Shark at http://www.bostonphoenix.com/archive/features/98/08/06/CARD SHARK.html.

## Whist: A Trick-Taking Card Game

The Horatio Hornblower seafaring books by C.S. Forester have provided me with a great deal of reading enjoyment. These stories take place starting about 200 years ago and include many exciting sea battles and other adventures. The protagonist, Horatio Hornblower, enjoys playing a card game named Whist. I have included the game here just for my own edification.

Quoting from Wikipedia https://en.wikipedia.org/wiki/Whist:
Whist (a trick-taking game) is a classic card game that was played widely in the 18th and 19th centuries and was a development of an older game Ruff and Honours. Although the rules are extremely simple, there is enormous scope for scientific play and since the only information known at the start of play is the player's own thirteen cards (plus possibly the turned up trump card from the dealer's hand), the game is difficult to play well.

In its heyday, a large amount of literature was written about how to play Whist. Edmond Hoyle, of "according to Hoyle" fame, wrote an early popular and definitive textbook. By the late 19th century an elaborate and rigid set of rules detailing the laws of the game, its etiquette and the techniques of play had been developed that took a large amount of study to master. In the 20th century, Bridge, which shares many traits with Whist, has displaced it as the most popular card game amongst serious card players. Nevertheless, Whist continues to be played in Britain, often in local tournaments called "whist drives."

The following set of rules is condensed from Wikipedia at https://en.wikipedia.org/wiki/Whist.

Whist is a four-player game played with a standard 52-card deck of cards. The cards in each suit rank from highest to lowest: A K Q ... 432.

The four players play in two partnerships, with the partners sitting opposite each other. Players cut or draw cards to determine partners and the first dealer.

The deck of cards is shuffled and cut. The dealer deals the cards one at a time so that each player has thirteen cards. The final card, which belongs to the dealer, is turned face up to indicate the trump suit. This card remains face up on the table until it is dealer's turn to play to the first trick.
The player to the dealer's left leads to the first trick; any card may be led. The other players, in clockwise order, each must follow suit by playing a card of the suit led if they have one. A player with no card of the suit led may play any card from his or her hand. The trick is won by the highest card of the suit led, except if a trump is played. In that case, the highest trump wins. The winner of the trick leads to the next trick.
After all 13 tricks have been played, the side that has won the most tricks scores one point for each trick in excess of six. Thus, only one team scores for the play of a hand, and the most points a team can score is seven.

All of the skill in whist is shown in the play of the cards, attempting to take as many tricks as possible. Since there is no bidding, the only information available to players before the play of the first card is the trump suit and one trump card that the dealer has. The dealer has the advantage of having at least one trump card (some players may have none), and the other three players gain a slight advantage by knowing one of the cards in the dealer's hand.

As the play proceeds, the players gain information from each card played. A good whist player will remember every card played and the order in which each card is played. If a player is talented in this task, then the memorization occurs with little or no conscious effort. This information will be combined with card sense to help make good decisions during the play.

## Bridge: A Trick-Taking Card Game

Bridge is a four-person card game in which a team of two players competes against another team of two players. Bridge was derived from whist and uses a standard 52-card deck of playing cards. While the taking of tricks is the same as in whist, bridge includes a sophisticated system of bidding that occurs before the play.

The bidding process begins with the dealer. It leads to the determination of the trump suit (or No Trump when there will not be a trump suit) and a goal of meeting or exceeding the number of
tricks specified in the highest bid. In brief summary, quoting from
http://www.infoplease.com/encyclopedia/society/bridge-card-game-basic-rules.html:
The cards in contract bridge rank from ace down to two; in bidding, suits rank spades, hearts, diamonds, and clubs. After all cards are dealt, so that each player holds 13 cards, the dealer begins the auction, which proceeds in rotation to the left. Each player must bid, pass, double (increase the value of the previously stated contract), or redouble (only after a double, further increasing the point value of the contract). A bid is an offer to win a stated number (over six) of tricks with a named suit as trump or with no trump. The lowest bid is one, the highest seven. Each bid, i.e., "one diamond," "one no-trump," "four hearts," must be higher than the preceding bid, with no-trump ranking above spades. Artificial bids are those that convey certain information to a partner and are not meant to be taken literally. The highest bid of the auction becomes the contract after three consecutive passes end the bidding. The player who first named the suit (or no-trump) specified in the winning bid becomes the declarer. The player to the left of the declarer leads any card face up, and the next hand, that of the declarer's partner, is placed face up on the table, grouped in suits. This is known as the dummy, and the declarer selects the cards to be played from this hand. The object of the game for both partnerships is to win as many tricks as possible, a trick being the three cards played in rotation after the lead. Suits must be followed, but a player who has no cards in the suit led may play any card. Highest trump or, if no trump card is played, highest card of the suit led wins. Points are awarded for the number of tricks won.
The general rules of bridge require a team to explain their bidding system to their opponents. If a team is using a widely used bidding system, they can convey this information by merely naming the system. If they have developed a variation of a widely used system, or if they have developed an entirely new system, then they must provide the details to their opponents. Needless to say, a good memory and the ability to learn the meaning and implications of the bidding system used by one's opponents are essential to playing bridge well.

The scoring in bridge is more complex than in whist. In whist, each trick won in excess of six tricks counts one point. In bridge, each trick won in excess of six may count 20 points (if the contract is in diamonds or clubs) or 30 points (if the contract is in spades or hearts) -with the exception of a no-trump contract in which the first trick above six counts as 40 points and each subsequent trick counts as 30 points. Scoring is also affected by whether a contract has been doubled, or doubled and redoubled, and by other considerations. In total, learning to score in bridge is a significant challenge.

Bridge is far more complex and challenging than whist because of the bidding and because of differences in scoring. The game is played competitively at local, regional, national, and international levels. A ranking system has been developed so that a player can achieve a competitive rank through the accrual of points in certain sanctioned bridge-playing events. For some details on this, read about American Contract Bridge League (ACBL) master points, see http://www.acbl.org/masterpoints-results/masterpoint-ranks/.

## Massively Multiplayer Online Games (MMOGs)

Quoting from
https://en.wikipedia.org/wiki/History_of_massively_multiplayer_online_games:

The history of massively multiplayer online games spans over thirty years and hundreds of massively multiplayer online games (MMOG) titles. The origin and influence on MMO games stems from MUDs, Dungeons and Dragons and earlier social games.
In 1974, Mazewar introduced the first graphic virtual world, providing a first-person perspective view of a maze in which players roamed around shooting at each other. It was also the first networked game, in which players at different computers could visually interact in a virtual space. The initial implementation was over a serial cable, but when one of the authors began attending MIT in 1974, the game was enhanced so that it could be played across the ARPAnet, forerunner of the modern Internet.
If you own and use a credit card, you are a participant in a massively multi-user online financial system. If you use email or do social networking on the Web, you are a participant in a massively multi-user online communication system. If you search the Web for information, you are a participant in a massively multi-user online virtual library system. If you make online purchases from Amazon or other large online businesses, you are a participant in a massively multi-user online business.

Nowadays, it is no big deal for many thousands of people to be making simultaneous use of a computer system that processes business transactions, communication transactions, or game moves. In such a MMOG, there can be tens of thousands of simultaneous players.

In many online games, players from around the world can organize themselves into teams. A team, consisting of cooperating humans each running an individual character within the game, carries out activities that may include fighting or in some other way competing against other teams being run by human players, against teams being run by a computer, or perhaps just in overcoming major challenges being generated by the computer system.

There has been considerable research on MMOGs. The following quoted paragraphs from Young, et al. (April, May, 2006) provide an early insight into MMOGs in education:

Yes, video games are mainly for play and fun. But video games are educative as well as interesting and engaging-something that we all hope that more classrooms could be. Many of today's students spend more time playing video games than they do watching television, reading books, or watching films. Massively multiplayer online games (MMOGs)-long and surprisingly complex gaming environments that normally require over forty hours to get beyond novice levels (Squire 2004)-represent the latest development in the history of video game technology (Exhibit 1).
Success in a MMOG requires developing new literacies, understanding intricate and intersecting rule sets, thinking creatively within constraints, collaborating with other participants towards shared goals, and perhaps most importantly, taking on new identities as players (via their avatars) inhabit game spaces (Gee 2003). Such properties offer significant potential for educational contexts, as indicated by the emergence of MMOGs specifically designed to enable student interactions and centered on instructional topics (e.g., Quest Atlantis, AquaMoose 3D, and RiverCity). [Bold added for emphasis.]

Notice the "forty hours" in the bolded part of the quoted material. Research suggests that many game players enjoy the challenge, the many hours of learning, and the resulting level of expertise that results from such dedication. Players of such games become thoroughly immersed
in the game. Many may talk about characters in the game (such as their own characters) in the same way they talk about other people in their lives.

It is easy to draw parallels between this and a team of workers in a company competing against workers from other companies and participating in the overall world of business to develop products that capture market share and make profit for the company. It is now common for a team of researchers, located throughout the world, to work together on a project. Indeed, it is now common for certain types of jobs to be filled by telecommuters located thousands of miles from their employers and customers.

MMOG designers have worked diligently to reduce the time it takes for a beginning player to develop a usable but novice level of competence in playing a new game. Quoting from a 2013 statement in http://www.pantheonmmo.com/content/forums/topic/1681/ea-thinks-games-are-too-hard-these-days/view/post id/20201:

Our games are actually still too hard to learn," [Richard] Hilleman [a leading developer of games] said during an on-stage interview with other developers. "The average player probably spends two hours to learn how to play the most basic game."
"And asking for two hours of somebody's time-most of our customers, between their normal family lives...to find two contiguous hours to concentrate on learning how to play a video game is a big ask," he added. [Bold added for emphasis.]

Hmm. Compare this "two hours to learn to play the most basic game" with the hours of schooling students spend before they can use their schooling to solve problems and accomplish tasks of interest to them. I can imagine a conversation between a student and the student's parents:

Parent: Hi Pat. You just got home from six hours of school. What did you learn today?"
Pat, with a somewhat dejected and defensive tone of voice: Not much. We had a test in math, and we started on a new experiment in our science class. Our English teacher gave us a writing assignment that is due next week. We have to look up some information on the Web and use it in an essay.

Now, contrast this with:
Parent: Hi Pat. How did you enjoy your six hours at computer game camp today?
Pat, with great enthusiasm: I learned to play DragonVale. It is neat $\ldots$. all those beautiful dragons, getting more dragons, getting money, building habitats for the dragons. I am up to level 10 already! And, we also started to learn the Scratch programming language and how to build a game in this language. I got my first game to work, and one of my friends enjoyed playing it!
The point is, in carefully designed computer game and computer programming environments, learners can experience considerable success early on, and they can actually "do something" with their new learning as they continue learning.

## Star Trek's Holodeck

I have been a Star Trek fan since its early days. I am particularly enamored by the Holodeck, because it provides an interesting vision of the future of education. In Star Trek science fiction, a Holodeck creates a virtual reality in which one can interact with virtual people, places, and
things. For example, in a Holodeck, a person can talk with Albert Einstein, take piano lessons from Ludwig van Beethoven, be a player on a sports team made up of great players from the past, and so on.

There are two major aspects of a Holodeck. One is the three-dimensional animated graphics, and the other is "live" interaction and communication with people and other objects in the Holodeck. Substantial progress is occurring in creating and projecting three-dimensional animation. We now see this routinely in movies, where it is becoming nearly impossible to tell a computer-generated person or animal from the real thing. However, we still have a very long way to go to achieve live, intelligent communication and other interactions among people in a Holodeck.

## Final Remarks

While this book focuses on learning to solve the problems and accomplish the tasks in game playing, the larger goal is to use games as a vehicle for getting better at solving the problems and accomplishing the tasks that one faces in the "real" world outside of games.

Here is some advice applicable to becoming a better player in games involving two or more competing and/or collaborating players. As you read and analyze this advice, think about the extent to which it is applicable in learning to interact with people in dealing with real world (outside of games) situations.

A good problem solver:

1. Draws upon and cultivates the ability to "read" people, to collaborate, and to compete in problem-solving situations that involve working both with or in competition with other people.
2. Knows his or her problem-solving strengths and weaknesses. Draws upon the strengths and circumvents the weaknesses while being open to learning to improve and diversify capabilities.
3. Brings to bear general knowledge as well as general problem-solving strategies and experience.
4. Brings to bear domain-specific knowledge as well as domain-specific problem-solving strategies and experience.
5. Draws upon and develops an ability to quickly assess the problem situation and begin gathering relevant information.
6. Draws upon and develops an ability to acquire new information during the problemsolving activity and to integrate it with all of the above.
7. Recognizes the need for and value of experience in all phases of problem solving and in many different problem-solving situations and environments. This experience, along with reflective thinking, helps to build intuition (card sense, common sense, horse sense, having and possibly following hunches, etc.).

As a teacher, you will want to help your students make progress in all of the above areas, and to use their increasing capabilities in learning and using their learning.

## Activities for the Reader

1. What does intuition mean to you? Think about the ideas of card sense, common sense, horse sense, having and possibly following hunches, etc. They are all related to using intuition decision-making where there is uncertainty. Reflect on the times you made decisions under uncertainty, and the role these ideas play in your life.
2. Take another look at Moursund's seven ideas listed in Final Remarks above. Identify one topic that you feel is one of your strengths and another one that you feel is one of your weaknesses as a problem solver. Suggest some reasons why one of these seems to be so much more useful or relevant to you than does the other.

## Activities for Use with Students

1. Find out from your students which ones have seen a Star Trek episode in which the Holodeck was used. Then have these students explain what a Holodeck is, and possible uses of a Holodeck in learning and entertainment.
2. Have the whole class work together to develop a list of card games that various members of the class have played, and how many have played each of the games. Then select one of the more popular games. Lead a whole-class discussion on why this game is popular and what one learns by playing the game. Repeat for a second game and a third game as time permits.

## Reference

Young, M., Schrader, P.G., \& Zheng, D. (April/May, 2006). MMOGs as learning environments: An ecological journey into Quest Atlantis and The Sims Online. Innovate. Retrieved 11/8/2015 from https://zhengd.wordpress.com/research/.

## Chapter 8 <br> Game-based Lesson Planning and Implementation

Learning means making sense of experiences and developing new capabilities to act in and on the world. (Geoffrey and Renate Caine.)

Before you start anything, know (or learn) how to finish it. (Unknown.)

A lesson plan is a strategy designed to help solve a teaching and learning problem. This chapter presents some ideas on developing and implementing game-based lesson plans. Such lesson plans will likely have several different goals. For example, a lesson may be designed to teach some general problem-solving strategies, to teach a specific game, and to help students experience the process of gaining an increased level of expertise in a subject area. Whatever the goals in such a lesson, they should be made explicit both in the lesson plan and to the students. They should help a student to be able to give a good and meaningful answer to a parent's question, "What did you learn in school today?"

## Roles of a Teacher

With the background you have gained by reading the previous chapters, you can now better understand the roles of a teacher in helping students to improve their problem-solving abilities. Students can discover strategies on their own, read about them in a book or on the Web, or learn them from a fellow student, parent, or teacher. However, without explicit instruction, few students will attempt to generalize such strategies for possible inclusion in their repertoire of high-road transfer strategies.

For an informal environment to be fully effective as a learning activity, it often must be augmented by tutorial guidance that recognizes and explains weaknesses in the student's decisions or suggest ideas when the student appears to have none. This is a significant challenge requiring many of the skills analogous to those of a coach or laboratory instructor. The tutor or coach must be perceptive enough to make relevant comments but not so intrusive as to destroy the fun inherent in the game (Burton \& Brown, 1982).

The teaching technique emphasized in this book is a combination of seizing the teachable moment and teaching for high-road transfer. Whatever you are helping students to learn, keep problem solving in mind. Each teaching/learning situation is an opportunity for students to get better at problem solving:

- Within the specific domain, discipline, or activity being studied.
- In a manner that cuts across many domains, via high-road transfer of learning.

You want to help your students to increase their repertoire of domain-specific and domainindependent problem-solving strategies. High-road transfer to increase a student's repertoire of general-purpose problem-solving strategies consists of:

1. Identifying a strategy and giving it a short, descriptive, easy to remember name.
2. Helping students to understand the strategy in the context of the learning and problemsolving situation in which they are currently engaged.
3. Helping each student to identify personal applications of this strategy to other problemsolving situations. The goal is to help each student to develop (construct) a personally relevant understanding of uses of the strategy in a variety of situations. This is an example of constructivist teaching.
4. Repeat steps 1-3 frequently, both for new strategies and for strategies already introduced. Whatever you are teaching, use this approach as an opportunity to reinforce student understanding and use of underlying problem-solving strategies. Provide students with multiple opportunities to reflect on strategies they are using. Get your students used to the idea of identifying, learning, and then explicitly, reflectively using such strategies.
Some teachers' thoughts will immediately jump to the approach of having students memorize a long list of strategies. A teacher may contemplate making use of fill-in-the-blank, matching, and other short answer techniques to assess this memorization. Please do not do this!

A memorize and test approach misses the whole idea of the reflective practices of thinking, situated learning, personal construction of knowledge and understanding, and high-road transfer of learning. Consider instead teaching and assessing knowledge and skills that might make a significant, lasting contribution for a student. Suppose a student really masters one new strategy per month, or nine during a school year. For many students, this would more than double their repertoire in a single year!

A starting point might be to prepare and present a lesson or unit on problem solving across the curriculum, somewhat akin to the idea of reading across the curriculum. Encourage students to be aware of times when they are doing problem solving. When teaching any lesson or unit of study, help students to recognize when they are doing problem solving. (Remember, every discipline is defined by the problems it has solved and is attempting to solve.)

Your initial lesson or unit of study needs to introduce the idea of strategy and of high-road transfer of learning. Pick two or three different strategies you feel your students already know something about, ones that are really important. Make these strategies explicit and use them to explore the idea of high-road transfer of learning. Then for a number of weeks, when appropriate teachable moments arise, reinforce the ideas of strategies useful in problem solving and highroad transfer of learning.

After students are quite comfortable with the ideas of strategies of high-road transfer of learning, and with the initial two or three strategies used in teaching these ideas, then introduce a new strategy. Remember, the focus is on students mastering a strategy-making it part of their everyday repertoire of aids to problem solving across the curriculum. A small number of strategies that are thoroughly understood and regularly used is much better than a large number of memorized strategies that are poorly understood and seldom used.

## Learning to Learn

Your mind/brain knows how to learn. Learning is an ongoing, automatic process. When we talk about learning to learn, we are talking about how to improve the learning process. One of the goals of education is to help students learn to learn more efficiently and effectively.

The totality of human knowledge is huge and is growing very rapidly. A person has no hope of learning everything. Indeed, it is now a major challenge just to develop a high level of expertise in one or two disciplines.

Thus, as you work to educate yourself and others, you need to think carefully about what is to be learned and how to use learning time and effort efficiently and effectively. Let's use science as an example. Science encompasses many different disciplines, such as biology, chemistry, geology, physics, and so on. In any of these disciplines, it is possible to earn a doctorate, specializing in a small part of the discipline.

Science is all around us. Thus, each of us learns a lot of science at a subconscious level, just by functioning in the world and processing the steady stream of input to our senses. One of the interesting things that educational researchers have found is that each person constructs their own mental models and theories of various aspects of science. Some of these models are correct enough so that they require little change over time, as we learn more and more about the science aspects of the world we are growing up in. Other models do not fit well with what we observe as we grow older and with what is being taught to us in school.

Piaget used the terms assimilation and accommodation to describe how some new information can be assimilated into (incorporated and merged into) the models and theories we have already developed, while assimilating other information and ideas requires accommodation by developing new mental models. Thus, one important aspect of learning science is to develop general mental models that are robust enough to assimilate and accommodate the science we will encounter in our future experiences and more formal learning opportunities.

As a very young child, you certainly did not have an inherent understanding that a key aspect of science is developing very accurate descriptions (for example, of things that one sees in nature) and predictive/descriptive theories (for example, that the moon rotates around the earth and appears to shine in the night sky because it reflects light from the sun). Our informal and formal educational systems have helped you to develop internal constructs (collection of mental models) that you bring into play as you think about the meaning of science, what scientists do, and so on.

Our educational system faces the challenge of helping each student develop a general understanding of science, learn some specific science, learn to learn science, and learn to apply scientific strategies. In the past, our educational system has tended to place considerable emphasis on the first two challenges, and less emphasis on the third and fourth. Even as our understanding of the theory and practice of learning has grown, we have tended to expect that students will figure out on their own how to learn a particular discipline. After all, each person is unique; each brain/mind is unique. Each brain/mind knows how to learn and can assimilate new learning challenges into its repertoire of learning skills.

However, each discipline of study presents its own learning challenges. The processes of learning math, music, and basketball share some aspects in common, but also have many aspects specific to the discipline being studied.

While each student will make some progress in learning to learn whatever we attempt to teach in school, many students will develop quite ineffective and inefficient methods for learning. Moreover, they will not even be aware that they are developing these methods. Here is a three-part approach to solving this teaching/learning problem:

1. Incorporate the best of our learning to learn theory and practice into teaching in each discipline area. Explicitly teach this to students.
2. Actively engage all students in the study of their own interests, capabilities, and limitations. Over the years of formal schooling, help each individual student gain a steadily increasing understanding of herself or himself as a learner and how to become a more effective and efficient learner.
3. Stress aspects of learning that cut across disciplines. For example, every discipline involves problem solving. Problem solving begins at understanding the problem to be solved. So, help students to learn what it means to "understand a problem" and strategies one can use to gain a personally useful understanding of a problem.

A lot or research and practice literature addresses learning to learn. My 11/12/2015 Google search of the expression learning to learn produced over 650 million results.

Games provide a good environment in which to help students learn about learning and learn about themselves as learners. This is one of the justifications for making use of games in education. As you help students learn a game, you can make it clear that a game has rules that must be learned. You can make it clear that there are a number of learning and playing strategies useful both in many different game settings and in many different non-game settings. You can make it clear that each game tends to have some specific strategies that make a significant contribution toward increased expertise in playing the game. You also can make it clear that the same "specific strategies" situation holds for developing an increased level of expertise in each discipline one studies.

One of the advantages of a game environment is the relatively short period of time required to move from being a person first being exposed to a game to being a person with a reasonable amount of expertise in playing the game - a person who can play for enjoyment and for learning while playing. Unfortunately, this is in marked contrast to much of traditional learning in school. Delayed gratification describes much of what we are expecting of students in school.

## Lesson Plan Ideas

Each of the preceding chapters includes some activities for teachers and others using this book in a course or for independent study. In addition, each contains activities to be used with students. Thus, by browsing these chapter materials, you may come up with a number of ideas for lesson plans.

Another starting point is to browse a modest length of lists of goals of education, looking for big ideas that you feel need increased emphasis in your curriculum.

The research on the use of games in education strongly supports the value of having clear learning goals in mind and of specifically teaching to these goals. Here is a short list of possible goals for making educational uses of games in a classroom setting.

1. To help students learn more about themselves in areas such as:
a. Learning to learn and understanding how concentrated, reflective effort over time leads to an increasing level of expertise.
b. Learning about one's cooperative versus independent versus competitive inclinations both in learning and in demonstrating or using one's learning.
c. Learning about oneself as a giver of feedback to others and as a receiver of feedback from others. This includes learning to complete and to make use of both selfassessment and peer assessment.
2. To help students better understand problem-solving strategies and to increase their repertoire of and use of problem-solving strategies. This includes:
a. Learning about low-road (essentially, rote memory) and high-road transfer of learning, especially as they apply to problem solving.
b. Learning how to recognize/identify a problem-solving strategy and explore its possible use across many different problem domains.
c. Learning how to do high-road transfer of learning of problem-solving strategies that cut across many domains.
d. Increasing fluency in making effective use of one's repertoire of domain-independent problem-solving strategies.
3. To help students learn some games and increase their understanding of historical and current roles of games and game playing in our society. This includes:
a. Learning games as an aid to social interaction in small and large groups.
b. Learning games as part of the culture and history of a family or community.
c. Learning games as environments that facilitate communication, collaboration, and peer instruction.
d. Learning how to help other people learn a new game. (Think of the idea that every student plays both learning and teaching roles in life.)

Development of a game-based lesson plan can begin with the selection of an age-appropriate game, or it can begin with the selection of some specific learning goals such as those listed above. Thus, for example, you may think of a game that you thoroughly enjoyed playing as a child. You may decide that your students or children would likely enjoy learning to play this game. Before introducing the game to your students or children, think about your educational goals. If you are a parent, for example, you might decide the main goal is to help your children understand a bit of your childhood and the culture/environment that you grew up in.

If the beginning point is a learning goal, such as learning about a particular strategy and highroad transfer of this strategy, then the next step is to identify an appropriate game or set of games that employ the strategy. Be sure to think about how you will use this teaching/learning situation to help students learn to recognize strategies, make use of strategies, and learn high-road transfer of domain-independent strategies.

In all teaching/learning situations, a great deal of incidental learning goes on. As a teacher or parent, when you see a good example of such incidental learning, make it explicit to the learners. For example, suppose that a teacher is making use of the following instructional plan:

Begin the lesson by role modeling the idea of thinking out loud as you solve a puzzle problem. Then divide your class into groups of two or three, and provide each group with a variety of developmentally appropriate puzzle problems. One member of the group thinks out loud while attacking a puzzle problem. The other member or members of the
group are silent observers, perhaps taking notes on what they observe. If the puzzle solver successfully solves the puzzle, the observers then lead a debriefing interaction, talking about what they observed and what they learned in the overall process. If the puzzle solver gets stuck, a short debriefing occurs, and then another member of the team takes over and works to solve the puzzle problem. Remember, only one person works on the puzzle at a time. The other members of the team are silent observers, possibly taking notes on what they are observing and their own personal thoughts about what is going on.

As you (the teacher) wander around the classroom observing the various teams at work, you may happen to see an observer taking good notes and writing down a suggestion of a strategy he or she would use. That would be a good time to interrupt the class. Point out the good activity you observed, and suggest that observers may want to think about and write down strategies that they see being used and their suggestions of other strategies that they feel would be useful.

Note that the same set of ideas can be used in any problem-solving activity. That is, the problem need not be a puzzle problem. The activity could be based on a science problem, a social science problem, a math problem, and so on.

Note to teachers: Probably you have already thought of this. If not, here is a good chance to practice high-road transfer of learning. The teaching/learning strategy illustrated can be called the think out loud strategy. It is useful in group problem-solving settings, and it is a useful component of many lesson plans.

## More Specific Educational Goals

The table in Figure 8.1 lists some of the possible specific learning goals that might be applicable to a student engaging in playing a game. As you explore learning goals, keep in mind one of the overriding principles of good educational practices: Lessons should be challenging and rigorous. Thus, as you think about a specific goal, think about how your lesson will approach this goal in a challenging and rigorous manner. The next section of this chapter provides additional ideas on challenge and rigor.

| Possible Goals in a Game-Playing Lesson Plan | Points |
| :--- | :--- |
| 1. Declarative knowledge about the game-rules, vocabulary, objectives, history. |  |
| 2. Knowledge and understanding of algorithmic and heuristic procedures relevant to <br> making good moves. Both algorithms and heuristics are detailed step-by-step <br> instructions. An algorithm is "guaranteed" to accomplish a specified task, while a <br> heuristic is a rule of thumb that is considered to be useful, but is not guaranteed to <br> work. |  |
| 3. How to learn a game. How expertise increases through gaining improved <br> declarative and procedural knowledge, through practice, through metacognition, <br> and through reflective analysis. |  |
| 4. How to obtain and use feedback from one's self, the computer (if playing in a <br> computer environment or a one-player game), and one's fellow players (if playing <br> with and/or against others). Often this requires careful record keeping and then <br> reflective analysis of moves made in a game. |  |
| 5. To practice the high-road transfer of learning heuristics of developing an overall <br> long-range strategy and making use of the look ahead strategy. |  |
| 6. How to interact appropriately with fellow players and opponents. This includes <br> learning the culture and social skills of game playing in general, as well as for the <br> particular game being played. |  |
| 7. How to deal appropriately with the thrill of victory and the agony of defeat when <br> playing a competitive game. |  |
| 8 How to help others learn to play the game; how to be a teacher/mentor in a game- <br> learning and game-playing environment. |  |
| 9. Self-assessment and peer assessment. Receiving and giving feedback from oneself <br> and others that can be used to improve the level of one's expertise. |  |
| 10. Etc. Add goals that you feel are appropriate. | The total of the points must add up to $\rightarrow$ | 100 

Figure 8.1. Possible goals in a game-playing lesson plan.
You may think it was rather silly of me to provide a column for points. The purpose is to encourage some quantitative thinking. Suppose, for example, you are developing a lesson plan with four goals. Are all goals of equal importance from your point of view or from your students' points of view? If you don't provide your students with information about the relative importance of various goals, how will the students self-assess or do peer assessment?

It is useful to analyze some of the possible goals and to think about the nature of learning that might occur in a gaming environment.

1,2. A gaming environment tends to have less clutter and complexity than the school environment typically available as one studies a traditional school discipline. Thus, a gaming environment may be well suited to helping a student learn about declarative and procedural knowledge, and procedural thinking. A procedure can be classified as an algorithm (proven to solve a specific type of problem or accomplish a specific type of
task), or heuristic (such as a rule of thumb that is designed to solve or help solve a problem, but is not guaranteed to be successful). Learn more about procedures and procedural thinking in Moursund (2004).
3. If you are a person who is apt to be learning many different games during a lifetime, it is beneficial to learn how to learn games. In addition, many non-game problems have much in common with game problems. Thus, there can be considerable transfer of learning from the process of learning a game to the process of dealing with such real world problems.
4. Feedback is important in any learning and problem-solving situation. Part of the learning process is to develop skill in obtaining and using feedback from self, others, the problemsolving situation, and so on. This is the basis for an important high-road transfer of learning strategy. When attempting to solve a problem, make a list of possible sources of feedback that can provide information on the progress you are making in solving the problem.
5. Game playing often lends itself to developing and using a long-range strategy. The long range planning strategy is so widely useful that it should be part of the problem-solving repertoire of all students. A similar statement holds for the look ahead strategy. Such strategies can be learned in a widely applicable manner through appropriate teaching for high-road transfer. (Note also that a person can learn to recognize opportunities for highroad transfer and then do the steps needed to help increase high-road transfer-a teacher's presence may be helpful but certainly is not a necessary requirement. Talented/gifted students tend to be good at this.)
6. There are many different learning theories. Lev Vygotsky's learning theory is called social constructivism (Social Constructivist Theories, n.d.). It is a social development theory, quite a bit different from Piaget's cognitive development theory. The social aspects of a learning situation are a key aspect of social constructivism. Many people play games because they enjoy being in the social setting that the environment provides. In an educational setting, metacognition might help game players learn more about their own social skills, likes, and dislikes.
7. Some games are competitive, some are collaborative, some are both (a team working together, competing with another team), and so on. Many games provide an opportunity to experience the thrill of victory and the agony of defeat - and to learn about how one deals with winning and losing.
8. I believe that all people are "naturally" both learners and teachers. Through informal and formal education, we all get better at learning and teaching. Game-playing environments tend to provide a good opportunity to practice helping others learn. For example, a game player might be engaged in both teaching and learning when asking another player a question such as, "Can you explain to me what you had in mind when you...? (question asker describes a particular move or action by the other player).
9. Learning to self assess is an important goal in education. Games provide an environment in which one can practice self-assessment, peer assessment, and giving and receiving feedback based on such assessments.

The types of learning goals listed in Figure 8.1 are not automatically achieved by students who play games. Quoting from an early version of Conati \& Klawe (2000):

These results indicate that, although educational computer games can highly engage students in activities involving the targeted educational skills, such engagement, by itself, is often not enough to fulfill the learning and instructional needs of students. This could be due to several reasons.

One reason could be that even the most carefully designed game fails to make students reflect on the underlying domain knowledge and constructively react to the learning stimuli provided by the game. Insightful learning requires meta-cognitive skills that foster conscious reflection upon one's problem solving and performance, but reflective cognition is hard work.

## Goals of Education: Rigor on Trial

As a teacher, I often have trouble thinking about my lessons from a student's point of view. Thus, I was impressed by the work being done by Tony Wagner and others as they explored lesson plans from a student point of view (Wagner, January 11, 2006). The emphasis in this work was on exploring the rigor of classroom instruction and learning. Quoting from Wagner's article:

We began to realize that rigor has less to do with how demanding the material the teacher covers is than with what competencies students have mastered as a result of a lesson. We were able to agree on this because, in our journey, we had gone from creating a series of teacher-centered observations to reaching consensus on a set of questions we would ask students.

The seven questions that emerged from this work are the following:

1. What is the purpose of this lesson?
2. Why is this important to learn?
3. In what ways am I challenged to think in this lesson?
4. How will I apply, assess, or communicate what I've learned?
5. How will I know how good my work is and how I can improve it?
6. Do I feel respected by other students in this class?
7. Do I feel respected by the teacher in this class?

One of the most important goals in education is to help students learn to take a steadily increasing level of personal responsibility for their own education. The list of seven questions given above is a piece of an overall strategy for doing this. When you (as a teacher) develop and use a lesson plan, it is helpful to do a whole class debriefing at the end of the lesson. Engage your students in questions such as those given in the list. Help them to learn that these are good questions to use any time - at the beginning of a lesson, during a lesson, or at the end of a lesson. Use this activity to learn to become a better teacher!

## Rubrics

Quoting from http://rubistar.4teachers.org/index.php?screen=WhatIs:

Rubrics have become popular with teachers as a means of communicating expectations for an assignment, providing focused feedback on works in progress, and grading final products. Although educators tend to define the word "rubric" in slightly different ways, Heidi Andrade's commonly accepted definition is a document that articulates the expectations for an assignment by listing the criteria, or what counts, and describing levels of quality from excellent to poor.
Rubrics are often used to grade student work but they can serve another, more important, role as well: Rubrics can teach as well as evaluate. When used as part of a formative, student-centered approach to assessment, rubrics have the potential to help students develop understanding and skill, as well as make dependable judgments about the quality of their own work. Students should be able to use rubrics in many of the same ways that teachers use them - to clarify the standards for a quality performance, and to guide ongoing feedback about progress toward those standards.
Almost all teachers make use of rubrics as they assess student work and provide feedback to students. Typically, a rubric is aligned with goals in a unit of study, a lesson, or a specific assignment. Sometimes a rubric is designed just for the use of the teacher. At other times, a rubric is designed for use both by the teacher and by the teacher's students.

In the latter case, it is essential that the students be able to understand the rubric in order to use it to self-assess. There is considerable literature on the value of involving students in the development of rubrics. Such involvement may increase student motivation and can serve as a good approach to helping students understand a rubric.

Teachers know that students vary widely in their backgrounds and interests. In teachercentered education, the teacher and higher-level components of the school system are apt to specify rubrics and relative weights to be given to the assessment of various components of a graded activity.

Contrast this with assessment in student-centered education. There, one might facilitate students in developing their own rubrics. Within a range of goals decided upon by the teacher (or, teacher and students working together), each student might develop a specific rubric or a part of a rubric to fit individual interests and needs.

## Final Remarks

Although human brains are designed so that they "naturally" learn from their environments, the totality and complexity of human knowledge overwhelms all learners. Schools and other instructional vehicles help students to deal with the overwhelming learning challenges they all face.

As a teacher or parent, you routinely think carefully about what you can do to help your students and/or children learn. You recognize that you can help, but ultimately it is the students and children who are doing the learning, and they must learn to help themselves. Thus, for example, you can provide them with a game that can help to create an excellent learning environment, and you can help them learn to play the game. But, there is much more that you can do.

Here are the first five of the seven things that Tony Wagner (January 11, 2006) believes all students should do/use as they participate as learners in a lesson:

1. What is the purpose of this lesson?
2. Why is this important to learn?
3. In what ways am I challenged to think in this lesson?
4. How will I apply, assess, or communicate what I've learned?
5. How will I know how good my work is and how I can improve it?

As a teacher and/or parent, your efforts in helping learners should be strongly guided by helping them to understand and answer for themselves these five questions for the content they are being expected to learn.

## Activities for the Reader

1. Brainstorm goals of education that you think might be approached through use of games. Do this either individually or in a small group.
2. Working individually or in small groups, compile a list of games, each accompanied by an appropriate educational goal that might fit well in a lesson based on the game.
3. In a small group, share your thoughts on allowing students to help develop rubrics and perhaps even to individualize a teacher-developed or whole class-developed rubric to better fit her or his interests and needs.
4. Introspect about how you, personally, create and maintain rigor in your teaching.

## Activities for Use with Students

1. Reread the section Goals of Education: Rigor on Trial earlier in this chapter. Then try out the ideas of this section with your students.
2. Experiment with the idea of involving your students in developing rubrics, and with the emphasis on the individualization of rubrics to fit the interests and needs of individual students.

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## Chapter 9 <br> Miscellaneous Other Topics

If life doesn't offer a game worth playing, then invent a new one. (Anthony J. D'Angelo.)
When I watch children playing video games at home or in the arcades, I am impressed with the energy and enthusiasm they devote to the task.... Why can't we get the same devotion to school lessons as people naturally apply to the things that interest them? (Donald Norman.)

When I write a book on a particular topic, I spend a lot of time reading and talking to people about the topic. Eventually, I begin to get an idea for the specific aspects I want to include in the book. As I work on organizing the material and ideas, I continue to search for other possible content.

This chapter is a collection of topics that were identified as being relevant and important to the themes of the book, but that are not included in previous chapters. My goal and plan in writing this book was to keep it relatively short. Therefore, I have decided to lump all of these topics together into a final chapter. One way to think about this chapter is that it consists of a large number of mini-chapters. Alternatively, think of this chapter as a future writing challenge to the author. Each section of this chapter could be developed into a complete chapter.

## Dangers of Too Much Game Playing

It is clear that computers, cell phones, digital cameras, video games, and other aspects of ICT are "here to stay" and are now routine parts of the everyday lives of several billion people. Moreover, it is clear that children growing up in this environment tend to be more comfortable with it than are many of today's adults. ICT has already substantially changed the day-to-day life patterns of many people. For example, as I watch college students moving from class to class, I am beginning to wonder if using a cell phone or a music player is now an integral component of walking! Rhea Kelly's article, Study: Millennials Spend More Than 3 Hours a Day on Mobile Phones, indicates a major change in the way 16-30 year olds spend their time (Kelly, 12/1/2015).

Video games are steadily moving in the direction of having the video and story line quality of broadcast television, along with steadily improving interactivity that allows the viewer to be an active participant in the story. In that sense, a video game can be thought of as video plus interactive participation. It is not surprising that large numbers of children and young adults spend more time playing video games than they do watching (non-interactive) television.

There is a substantial and growing literature on the actual and possible harm to children and adults who spend so much time playing video games and making other uses of ICT. My 11/13/2015 Google search of the expression danger of too much computer game playing produced over 330 million results. For example, see Sherry Turkle's video, Connected, But Alone? (Turkle, February, 2012). Quoting from the video:

Over the past 15 years, I've studied technologies of mobile communication and I've interviewed hundreds and hundreds of people, young and old, about their plugged in lives. And what I've found is that our little devices, those little devices in our pockets, are so psychologically powerful that they don't only change what we do, they change who we
are. Some of the things we do now with our devices are things that, only a few years ago, we would have found odd or disturbing, but they've quickly come to seem familiar, just how we do things.

So just to take some quick examples: People text or do email during corporate board meetings. They text and shop and go on Facebook during classes, during presentations, actually during all meetings. People talk to me about the important new skill of making eye contact while you're texting. People explain to me that it's hard, but that it can be done. Parents text and do email at breakfast and at dinner while their children complain about not having their parents' full attention. But then these same children deny each other their full attention. This [a picture of some people in a small group, all texting] is a recent shot of my daughter and her friends being together while not being together. And we even text at funerals. I study this. We remove ourselves from our grief or from our revery and we go into our phones.

Why does this matter? It matters to me because I think we're setting ourselves up for trouble -trouble certainly in how we relate to each other, but also trouble in how we relate to ourselves and our capacity for self-reflection. We're getting used to a new way of being alone together. People want to be with each other, but also elsewhereconnected to all the different places they want to be. People want to customize their lives. They want to go in and out of all the places they are because the thing that matters most to them is control over where they put their attention. So you want to go to that board meeting, but you only want to pay attention to the bits that interest you. And some people think that's a good thing. But you can end up hiding from each other, even as we're all constantly connected to each other.

## Women and Gaming

Women are very under-represented in employment in the Information and Communication Technology (ICT) areas of employment (Moursund, 2015). However, this is no longer the case in online game playing. My 11/13/2015 Google search of the expression women and computer games produced over 200 million hits. Quoting from a Washington Post article by Drew Harwell (10/17/2014):

The stereotype of a "gamer"-mostly young, mostly nerdy and most definitely male-has never been further from the truth. In the United States, twice as many adult women play video games as do boys, according to the Entertainment Software Association, the industry's top trade group. Male gamers between ages 10 and 25 represent a sliver of the market, only 15 percent, according to Newzoo, a games research firm.
Yet America's 190 million gamers, 48 percent of whom are women, still play in a harsh frontier. About 70 percent of female gamers said they played as male characters online in hopes of sidestepping sexual harassment, according to a study cited by "Hate Crimes in Cyberspace" author and law professor Danielle Keats Citron.

This huge switch from online game players being mainly males has also occurred in the UK. Quoting from The Guardian (UK) newspaper article by Meg Jayanth (9/18/2014):

A study published on Wednesday [9/17/2014] by the Internet Advertising Bureau reveals that $52 \%$ of the gaming audience [in the UK] is made up of women. That's right-the majority of people playing games are women.

Does this surprise you? It shouldn't. Three years ago that figure was $49 \%$, which is hardly a trifling minority. Women have always played games, and in recent years the growth of the mobile games industry in particular has been driven by a female consumer base.

Yet the stereotype that games are a pastime for adolescent boys is an enduring one, and one that is perpetuated by the aggressive marketing of many big-budget games.

What do we think when we think of a videogame? Most likely a multimillion dollar console title dripping with machismo and bristling with weaponry. Yet the reality is that the most popular gaming device today is the Smartphone, and the most popular genres are puzzle, trivia and word games. Less Call of Duty and more Words With Friends.

There are a number of organizations and conferences for women in gaming. Women In Games International was founded in 2005 in response to a growing demand around the world for the inclusion and advancement of women in the game industry (WIGI, n.d.) Quoting from the website:

Women in Games International (WIGI), made up of both female and male professionals, works to promote the inclusion and advancement of women in the global games industry.

WIGI promotes diversity in video game development, publishing, media, education, and workplaces, based on a fundamental belief that increased equality and camaraderie among genders can make global impacts for superior products, more consumer enjoyment and a stronger gaming industry.
Women In Games International stands as strong advocates for issues crucial to the success of women and men in the games industry, including a better work/life balance, healthy working conditions, increased opportunities for success and resources for career support.

## Computer Programming and Student Creation of Games

As you watch small children at play, you see that they are adept at creating games they find entertaining and attention holding. Many students enjoy creating games for themselves and others. We have long known this can be a valuable educational experience. Quoting from The Educational Potential of Electronic Games article by Yasmin B. Kafai (October 27, 2001):

We have only begun to build a body of experience that will make us believe in the value of game activities for learning. Obviously, the image of children building their own games is as much a "knee-jerk reflex" for constructionists as making instructional games is for instructionists. In the case of instructional games, a great deal of thought is spent by educational designers on content matters, graphical representations, and instructional venues. The greatest learning benefit remains reserved for those engaged in the design process, the game designers, and not those at the receiving end, the game players. [Bold added for emphasis.]
For a number of years, Computer and Information Science Departments in higher education have sought ways to increase enrollment of women in their programs. Now, many such programs offer an introduction course for majors that includes a strong emphasis on game development, computer graphics, and animation using programming languages designed for such activities (Moursund, 5/12/2011). As an example, see David Intersimone's article, Once More into the

Code, for brief introductions to some of these languages (11/23/2009). A language named Scratch is now widely used by children. Quoting from Intersimone's article:

The precursor to the Scratch programming language was work that MIT did for Lego to create the Lego Mindstorms programming system. Scratch was created to allow young (and young at heart), beginning programmers to create applications, games, animations and interactive stories. It also was created to help teach math and computational lessons. Scratch was also designed to foster collaborative work. Three words define the goals of Scratch: imagine, program, share.
Scratch provides visual programming constructs and blocks for building program logic and flow. To create programs, you just snap together blocks. The blocks have different shapes and colors so that syntax and data type errors are avoided. Programs are created from stacks of the blocks. Along the way, Scratch helps programmers learn about loops, conditionals and event-driven programming. [Bold added for emphasis.]
The website http://www.ambrosine.com/resource.html provides many examples of free software available for creating computer games.

Here is another quote from Yasmin B. Kafai's article (October 27, 2001):
We know that as many children enjoy playing games according to given rules, they are also constantly modifying rules and inventing their own. Piaget (1951) claimed that these modifications reflected children's growing understanding of the world. The process of game construction represented for Piaget the ultimate effort by children to master their environment in creating their representations of the world. Turkle (1984) pointed out an interesting parallel between the attractions of playing games and of programming computers. She saw programming as a way for children to build their own worlds. Within this context, children could determine the rules and boundaries governing the game world and become the makers and players of their own games. In contrast, when children play a video game, they are always playing a game programmed by someone else; they are always exploring someone else's world and deciphering someone else's mystery. Turkle saw that what she called the holding power of playing purchased video games could be applied to the making or programming of video games.

## Tools for Creating and Constructing

In computer programming, one designs and constructs a computer program that is intended to solve a particular type of problem or accomplish a particular task. There are many computerbased tools and environments that can facilitate users in creation and construction.

For example, you are familiar with electronic musical instruments. Versions of some of these are available free on a computer. Figure 9.1 comes from the website http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/music/piano/. Notice the range of instruments that are available through this music synthesizer.

Computerized music synthesizers have been available for a great many years. Perhaps 30 years ago I heard a presentation by a music teacher in Wisconsin who had been funded to spend a year going to various grade schools where he taught students to compose and play electronic music.

That is an interesting approach to teaching music. Grade school students are capable of learning to compose. However, they do not know how to notate (write) music, and most do not play a musical instrument well enough to adequately render their compositions. Computer technology, with the help of a human music teacher, overcomes these challenges.


Figure 9.1. A virtual (electronic) keyboard.
For another example, consider computer graphics. My 1/10/2016 Google search of the expression free computer graphics software for children produced over 9 million results. Tux Paint was first on the list. See http://www.tuxpaint.org/. Quoting from the website:

Tux Paint is a free, award-winning drawing program for children ages 3 to 12 (for example, preschool and K-6). Tux Paint is used in schools around the world as a computer literacy drawing activity. It combines an easy-to-use interface, fun sound effects, and an encouraging cartoon mascot who guides children as they use the program.
Kids are presented with a blank canvas and a variety of drawing tools to help them be creative.

Tux Paint is also available free on tablet computers that use an Android operating system.
3D printing provides another excellent example of computer systems that students can use to design and create. See a video at https://www.youtube.com/watch?v=8aghzpO_UZE. 3D printers have been available for more than 20 years, and the price has now declined to a level that some secondary schools and many colleges make such facilities available to students. Quoting from https://en.wikipedia.org/wiki/3D printing:

3D printing, also known as additive manufacturing, refers to various processes used to synthesize a three-dimensional object. In 3D printing, successive layers of material are formed under computer control to create an object. These objects can be of almost any shape or geometry, and are produced from a 3D model or other electronic data source. A 3D printer is a type of industrial robot.

Futurologists such as Jeremy Rifkin believe that 3D printing signals the beginning of a third industrial revolution, succeeding the production line assembly that dominated manufacturing starting in the late 19th century. Using the power of the Internet, it may
eventually be possible to send a blueprint of any product to any place in the world to be replicated by a 3D printer with "elemental inks" capable of being combined into any material substance of any desired form.
My 1/6/2016 Google search of free building block computer games and applications produced over 7 million results. These games and applications provide a virtual environment in which players can build a wide variety of objects. For example, the company Fun Games for Free provides a variety of such games for tablet computer.

## Games and the Aging Brain

I have long been quite interested in brain science and how it relates to lifelong education. Also, as I have continued to grow older, I have developed an interest in the (changing) capabilities of the aging brain. You may enjoy reading my free book, Brain Science for Educators and Parents (Moursund, 2015).

Brain research has progressed substantially over the past two decades. This has been greatly helped by use of non-invasive brain scanning equipment. This equipment depends heavily on computer hardware and software. The steady increase in the speed and the cost-effectiveness of computer systems has been a major factor in the improvement of brain imaging equipment.

As an example of research progress, we have learned much about dementia. Quoting from a Web MD article, Best Memory-boosting Games by Seri Harrar (n.d.):
"The decline [in memory] you're noticing is real - and it starts before age 30 ," says Michael Merzenich, Ph.D., a professor in the Keck Center for Integrative Neurosciences at the University of California at San Francisco." A 60-year-old brain takes in information two to three times slower than a 20 -year-old brain. As a result, what's stored in memory is two to three times less clear and detailed. And by age 80 , you may be five to eight times slower. That's a big difference!"

The good news? Old brains can learn new tricks. "We used to think that with age, brain cells shriveled up, died, and that was that," says Paul Laurienti, M.D., Ph.D., a brain researcher at Wake Forest University School of Medicine. "Now we know that even older brains can grow new, stronger connections."

In a 2007 study that scanned the brains of 23 elderly people, Dr. Laurienti found that those who'd gone through a brain-training program were better able to focus - a plus because aging brains become more distractible. Growing evidence suggests that a lifetime spent using your noodle - in your day job as an astrophysicist or mom, or after hours playing Monopoly, tooting the clarinet in your local chamber group, or doing crossword puzzles - may build extra brain connections (a kind of mental savings account called cognitive reserve) and slow the symptoms of dementia.
In brief summary, the regular use of your brain in dealing with cognitively challenging tasks builds a reserve of dendrite connections that will serve you well in the future as you age.

However, there is much more to the challenging puzzle of dementia. Diet and exercise are very important. See the 17 minute video at http://www.alz.org/research/video/video pages/diet and exercise in alz.html.

## Artificial Intelligence

Throughout my professional career, I have been interested in artificial intelligence. The following is quoted from my free book, Introduction to Educational Implications of Artificial Intelligence (Moursund, 11/3/2007).

Artificial intelligence (AI) is a branch of the field of computer and information science. It focuses on developing hardware and software systems that solve problems and accomplish tasks that-if accomplished by humans-would be considered a display of intelligence. The field of AI includes studying and developing machines such as robots, automatic pilots for airplanes and space ships, and "smart" military weapons. Europeans tend to use the term machine intelligence (MI) instead of the term AI.

The theory and practice of AI is leading to the development of a wide range of artificially intelligent tools. These tools, sometimes working under the guidance of a human and sometimes without external guidance, are able to solve or help solve a steadily increasing range of problems. Over the past 60 years, AI has produced a number of results that are important to students, teachers, our overall educational system, and to our society.
Nowadays, most computer games make use of some aspects of AI. For example, when you are playing a computer game, you decide on a move and communicate this to the computer. You might do this by use of a touchscreen, keyboard, mouse, joystick, gesture, verbal command, or mental thought. In some sense, the computer "understands" your specification of a move and checks to see if it is a legal move. If it is not a legal move, the computer tells you so. If it is a legal move, the computer makes the move. It takes a certain amount of computer intelligence (a very low amount) to receive a specified move, decide if is a legal move, and then take appropriate action.

Many computer games make use of considerably more AI. For example, in computer games that require two or more players, the computer may serve as some (or all) of these players. If you like to play games such as checkers and chess, you can play them against a computer opponent. The chances are that this computer opponent has enough checker-playing or chess-playing intelligence to defeat you.

You may have noticed that the definitions of AI usually do not discuss the computer's possible sources of knowledge. Two common sources of an AI system's knowledge are:

- Human knowledge that has been converted into a format suitable for use by an AI system.
- Knowledge generated by an AI system, perhaps by gathering data and information, and/or by analyzing data, information, and knowledge available to it.

While most people seem to accept the first point as being rather obvious, many view the second point only as a product of science fiction. People may find it scary to think of a machine that in some sense "thinks" and thereby gains increased knowledge and capabilities. However, my 11/13/2015 Google search of machine learning produced over 64 million results. Quoting from the Wikipedia https://en.wikipedia.org/wiki/Machine_learning.

Machine learning is a subfield of computer science that evolved from the study of pattern recognition and computational learning theory in artificial intelligence. Machine learning explores the study and construction of algorithms that can learn from and make
predictions on data. Such algorithms operate by building a model from example inputs in order to make data-driven predictions or decisions, rather than following strictly static program instructions.
Many real world problems or problem situations are very large, complex, and interdisciplinary. The translation of speech from one natural language to another provides a good example. While considerable progress has been made in this area, bilingual humans are still much better at such translation than are artificially intelligent computer systems.

Here is an interesting and educational language translation game to play. Google Translate https://translate.google.com/ is free and can translate between about a hundred languages. Write a short paragraph in a language you know, have the computer translate it into a language you don't know, and then have the computer translate the result back into the language you know. The results are sometimes amusing, and they certainly provide insight into the current capabilities of Google's language translation system. Researchers and developers at Google continue to work on improving their system and to increase the number of languages it encompasses.

The point is, AI is an increasingly important use of computers that affects everyday life in our society. Thus, it is important that students learn some of the characteristics, capabilities, and limitations of AI systems. Games can be a useful part of an environment in which to study and experiment with AI. My 11/13/2015 Google search of games and artificial intelligence produced over 40 million results.

## Static and Virtual Math Manipulatives

Math educators often make use of math manipulatives in helping their students to better understand mathematics. Many of these manipulatives have game-like characteristics. A brief discussion of computer-based math manipulatives (virtual manipulatives) and links to a number of virtual manipulative websites are available at CT4ME (n.d.). Quoting from that website:

In What are Virtual Manipulatives?, Patricia Moyer, Johnna Bolyard, and Mark Spikell (2002) define a virtual manipulative as "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge." Static and dynamic virtual models can be found on the Web, but static models are not true virtual manipulatives. ... The key is for students to be able to construct meaning on their own by using the mouse [or stylus/finger] to control physical actions of objects by sliding, flipping, turning, and rotating them.

Many virtual manipulatives are computer simulations of physical manipulatives. This situation provides a good example of the "computational" in sub-disciplines such as computational math, computational biology, and computational physics. It also helps to illustrate computational thinking. If I can develop a computer model of a problem situation I am thinking about or some project I am doing, I can take advantage of the computer model in doing the thinking and the project. Part of computational thinking is to think about use of computational modeling when faced by challenging problems and tasks. Note that making use of virtual manipulatives denies one the sensory experience of physical manipulatives.

Tangram serves as a nice example of a physical and virtual manipulative. This is a Chinese puzzle consisting of a square cut into five triangles, a square, and a rhomboid, to be reassembled into different figures with no overlapping pieces (Tangram, n.d.). Figure 9.2 shows the seven
pieces and the pieces arranged into a running person. Tangram is available for free online play at http://www.addictinggames.com/puzzle-games/tangramgame.jsp. A number of examples are shown. Use the Help and Settings button for directions.


Figure 9.2 The seven Tangram pieces and a running person.
The Math Learning Center (MLC) provides a number of free online math manipulatives. (Full disclosure: David Moursund provided funding to get this free math manipulatives project started.) See http://free.funeducationalapps.com/2015/04/8-free-educational-math-apps-by-the-math-learning-center-.html. Quoting from the website:

Press Release - April 15, 2015
The Math Learning Center ("MLC") is celebrating the widespread success of their innovative educational apps for visualizing mathematical concepts. MLC's growing collection of free apps reflects its commitment to inquiry-based learning using effective instructional strategies and practices supported by research. The apps are the product of the organization's collaboration with Clarity Innovations, a Portland-based strategy and development services firm focused on the K-12 education industry. The apps are used on their own or in conjunction with MLC's K-5 mathematics curriculum, Bridges in Mathematics. This month, total downloads reached one million worldwide.

## Research on Games and Gaming

Many people view computer games as an opportunity to help improve our educational system. James Paul Gee is a professor at Arizona State University and a world leader in educational uses of computer games. He notes that computer games are often quite complex and can present a serious learning challenge. Quoting from his website (Gee, 2004):

For people interested in learning, this raises an interesting question. How do good game designers manage to get new players to learn their long, complex, and difficult gamesnot only learn them, but pay to do so? It won't do simply to say games are "motivating". That just begs the question of "Why?" Why is a long, complex, and difficult game motivating? I believe it is something about how games are designed to trigger learning that makes them so deeply motivating.

Substantial progress has occurred since Gee wrote his 2004 article. A good summary of this progress is available in Jane McGonigal's book, Reality Is Broken (2011). She begins with this warning about the changes that games are causing in the people of our world:

Anyone who sees a hurricane coming should warn others. I see a hurricane coming.

Over the next generation or two, ever-larger numbers of people, hundreds of millions, will become immersed in virtual worlds and online games. While we are playing, things we used to do on the outside, in "reality," won't be happening anymore, or won't be happening in the same way. You can't pull millions of person-hours out of a society without creating an atmospheric-level event.
If it happens in a generation, I think the twenty-first century will see a social cataclysm larger than that caused by cars, radios, and TV, combined.... The exodus of these people from the real world, from our normal daily life, will create a change in social climate that makes global warming look like a tempest in a teapot.

## Serious Games

The term serious games is now used to describe games that are designed for educational purposes. Quoting from the Wikipedia at https://en.wikipedia.org/wiki/Serious_game:

Serious games are simulations of real-world events or processes designed for the purpose of solving a problem. Although serious games can be entertaining, their main purpose is to train or educate users, though it may have other purposes, such as marketing or advertisement.

My 11/14/2015 Google search of the expression serious games produced more than 99 million results. While serious games have received increasing attention in recent years, they are by no means new on the scene. Here is a bit of history. Quoting from the course description of a graduate-level course developed by Katrin Becker (7/21/2006):

The use of computer and video games for learning is an emerging area of research, and interest is growing rapidly. As a sub-field of Serious Games, digital game-based learning poses some unique problems and challenges. As more and more young people grow up with digital games as one of their primary forms of entertainment, it behooves us to become familiar with this genre, how it affects people, and how we might use it for educational goals. Computer technology has advanced to the point where it is feasible (we now have the horse-power to accomplish this) to use games in a classroom setting. "Computer pioneer Alan Kay (DARPA in the '60s, PARC in the '70s, now HP Labs) declares 'The sad truth is that 20 years or so of commercialization have almost completely missed the point of what personal computing is about.' He believes that PCs should be tools for creativity and learning, and they are falling short."

## My Three Currently Favorite Computer Games

This book devotes a full chapter to solitaire games. My current favorite solitaire game is Eight Off. I play it on a computer, but I first learned to play it using an ordinary deck of playing cards. Playing this game well requires careful planning ahead, and that is why I like the game. If I think carefully enough, I can win most of the time. So, I have both the joy of winning and the joy of my brain being able to do the careful planning that leads to winning. However, I don't see much transfer of learning from repeatedly playing the game to problem solving in my everyday life.

Currently, my favorite computer game is DragonVale. This game is available free on the Web, but the player is strongly enticed to spend some money to buy gold, food, dragons, and other objects to speed up one's progress toward high levels in the game. This "free, but with ingame purchases available" approach to the design of computer games is now commonplace.

In DragonVale, one raises beautiful and playful dragons that can be purchased using game money or by breeding one's dragons. One acquires: food, gold, and gems; islands and habitats for the dragons; buildings, farms, and decorations; and experience points. Dragons can participate in races and in coliseum events. There is no fighting or combat in the game, and one cannot lose the game. It is possible to move to higher and higher levels, and to acquire more habitats and dragons. While it is possible to rise to the highest level and acquire one of every type of dragon, this goal is very far above all but the most dedicated players. For all practical purposes, it is not possible to "win" the game in a traditional sense.

Playing this game provides me with pleasure, but I don't consider it to be particularly useful in improving my problem-solving knowledge and skills. It does exercise my brain, which is good.

I like to play Scrabble on a computer. The computer environment provides me with the ability to easily access a Scrabble dictionary, to try out words to see if they are allowable, and to take back words that I have tried to use but that are not in the Scrabble dictionary. I can set the difficulty level of my (computer) opponent. I don't have to worry about spilling the pieces on the floor or pieces getting lost.

With practice, I get a little better at playing, and I learn a few new words. However, I mainly play for fun and to exercise my brain.

## Final Remarks

The pace of change in the world is accelerating. Very roughly:

1. Humans were hunter-gathers until about 10,000 years ago.
2. The Agriculture Era lasted nearly 10,000 years, up until the start of the Industrial Age began about 250 years ago.
3. The Information Age started in the mid 1950s. The Internet began in 1983, the Web began in 1989, and the first Smartphones were produced in 2007. Research into the study of genes has led to the development of rapid and accurate means for gene splicing (CRISPR) in the current decade. Robots are getting smarter, more capable, and more cost effective-contributing to major changes in human employment.
Of course, these are by no means all of the important changes that are occurring. The world has become smaller (improvements in transportation and communication) and much more crowded (high increase in population.) The planet is heating up and the climate is changing. We have made some amazing breakthroughs in science and medicine. The world now attempts to deal with large numbers of people living in poverty and with diseases that readily jump country boarders and threaten people throughout the world.

In these and other problem situations, more and better education is proposed as part of the solution. However, our current educational systems were certainly not designed for a fast pace of change. While the industrial might of the world is now capable of providing every person on earth with a Smartphone having good connectivity, our educational systems have not begun to deal effectively with the ICT that is now commonplace in the more wealthy countries.

Computer games are both part of the problems faced by our schools, and quite likely part of the solution. This book has provided you with some insights and tools for making effective use
of computer games as an aid to improving the teaching and learning of problem solving. Please make use of your new knowledge and skills!

## Activities for the Reader

1. Engage your students in a discussion about why boys and girls don't necessarily like the same types of games. You might want to do whole class or small-group brainstorming about what might make a game more appealing to girls than to boys, or vice versa.
2. Chapter 5 of Marc Prenzsky's book, Digital Game-based Learning, contains a list of 12 characteristics of games (Prenzsky, 2001).
3. Games are a form of fun. That gives us enjoyment and pleasure.
4. Games are a form of play. That gives us intense and passionate involvement.
5. Games have rules. That gives us structure.
6. Games have goals. That gives us motivation.
7. Games are interactive. That gives us doing.
8. Games are adaptive. That gives us flow.
9. Games have outcomes and feedback. That gives us learning.
10. Games have win states. That gives us ego gratification.
11. Games have conflict/competition/challenge/opposition. That gives us adrenaline.
12. Games have problem solving. That sparks our creativity.
13. Games have interaction. That gives us social groups.
14. Games have representation and story. That gives us emotion.

Select some traditional school academic discipline or topic that you teach. Analyze it in terms of the 12 items. One way to do this would be to develop a 5 -point scale, ranging from very low to very high. Select a game that you know well, and a school discipline or topic that you know well. Rate the game and its use in teaching/learning that school discipline or topic on each of the 12 items in the list.
3. Think about your own education and how it has prepared you to help deal with the current rapid pace of change. If you were in charge of education in your city, state, or country, what changes would you suggest we should be implementing?

## Activities for Use with Students

1. Provide your students with paper, color crayons, dice, spinners, paper clips, and miscellaneous other materials. Working in teams of two or three, students are to create a game. One possible starting point is to make a list of characteristics that make a game "fun" to members of the team, and then to design a game that has a number of these characteristics. During the game developmental process, a team may want to try out some of their ideas with members of other teams. After each team has created a game, the teams can demonstrate and teach their games to the whole class.
2. Here is a multi-day project. Each student in your class is to select a subject area (a discipline of study) suitable for students at their grade level. The goal is to find or create a
free game that is useful in learning some of the important content and methodology of that subject area. Then divide the class into small teams, with each team being given a subject area that they are studying in school. Each team is to analyze their subject and the way it is being taught in terms of the characteristics that make a game fun.
3. Engage your students in a discussion of changes that are going on in today's world. What are their insights into these changes? What roles do they see their schooling is playing in helping them to understand and deal with these changes?

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## Appendix 1 <br> Summary of Problem-solving Strategies and Some Key Terms

We all make use of strategies as we attempt to solve problems and accomplish tasks. The research literature in problem solving indicates that most people have a relatively limited repertoire of general-purpose problem-solving strategies. This research also suggests that it is helpful to increase one's repertoire. Teaching for high-road transfer of learning is an effective method of helping students to accomplish this goal.

However, increasing the collection of one's problem-solving strategies is only one part of increasing one's level of expertise in solving problems. Problem solving in a particular domain requires knowledge that is specific to that domain. Therefore, increasing one's expertise in problem solving in a domain requires substantial cognitive effort. It does little good to memorize a number of strategies. One must consciously practice using the strategies and reflect on the results over a large range of problems and a long period of time.

The teaching of widely applicable strategies and strategies specific to a particular domain of study can be integrated throughout the daily curriculum, and integrated into a wide range of lesson plans. For ideas on how to accomplish this teaching, see chapter 8.

The following alphabetical list contains problem-solving strategies that cut across many domains. Most are discussed and illustrated in this book; the number in parentheses indicates the chapter where that term is located.

The list also contains some important ideas that are not strategies, but are quite useful aids to learning to become better at solving problems.
act before you think. Some problem-solving situations require that immediate (stimulusresponse; intuitive) actions be taken. There is no time to think consciously. We certainly see this in sports such as football, basketball, tennis, etc. Of course, the immediate actions that well-trained athletes take are based on a huge amount of training and practice. See also think before you act.
algorithm. $(1,6,8)$ A finite step-by-step set of instructions that is guaranteed to solve a specified problem or accomplish a specified task. In grade school, students are taught pencil and paper algorithms for adding and multiplying integers, as well as other mathematical algorithms. See also heuristic.
backtracking. $(4,5)$ Sometimes you recognize a mistake immediately after making it. Taking back or undoing one or more moves that one has made in playing a game or in attempting to solve a problem. This is especially easy to do when the steps being taken are "virtual" steps, working with a computer representation of the problem and the steps being taken. This is a useful aid to learning. See also learn from your mistakes.
bottleneck. (4) Identify components of a problem-solving task that severely impede progress toward solving the problem. Particularly useful in problems where certain resources such as time or materials are severely restricted or a goal is to minimize their use. See also divide and conquer; don't box yourself into a corner.
brain aids. Reading and writing are such important brain aids that they are considered basics of education. A computer program can be considered a brain aid. It is now commonplace for a problem solver or game player to think about having the computer aid in playing the solving a problem or helping to determine good moves in a game. There are many articles about the nature and extent of the artificial intelligence (AI) built into various games. In some instances, such uses of AI as an aid to problem solving in games illustrate or are somewhat parallel to uses of AI to help solve non-game types of problems.
break a problem into smaller problems. $(2,5)$ Solving a complex problem often proceeds by breaking the original problem into smaller, less complex problems that are easier to solve and that perhaps one has solved before. Indeed, it is quite useful to have a repertoire of "smaller" problems that one knows how to solve and/or knows how to look up a solution process. See also create a simpler problem; divide and conquer.
build on previous work. $(1,2)$ Don't reinvent the wheel. Stand on the shoulders of people who have come before you. Much of formal schooling emphasizes learning information that is considered to be especially important from the collected knowledge of the human race. See also look it up.
check your answer. (6) You are used to the idea that problem sets in many books include answers to every other problem. This is an aid to learning, but does not contribute much to learning to check your answers when an answer key is not available. A very important aspect of learning problem solving is learning to detect when one makes errors. Does the result one has produced make sense in the context of the problem being solved and in the context of what one knows about the world? For example, if you are solving a problem to determine a person's height, and the answer you get is minus four feet, you should be able to recognize that this cannot be correct.
collaboration and cooperation. In many problem-solving situations "two heads are better than one." Indeed, many problems and tasks require the collaborative work of large teams of people working together over a period of years. Nowadays such teams may contain members located long distances apart.
collect data. Think of playing a game or attempting to solve a problem as a research process. Think of yourself as a "scientific" researcher, carefully gathering data about the moves you are making or thinking about making and the strategies you are using, and then analyzing the results that are obtained from a particular move. You can see that this is essentially the same process as the scientific method that researchers use. Thus, this is an excellent opportunity for high-road transfer of learning. Often a pattern will emerge as one collects and analyzes data. See also information retrieval; patterns.
computational thinking. (Preface, 1, 3, 6) Incorporating (integrating) potential uses of computers as one thinks about possible ways to solve a problem or accomplish a task.
computer modeling and simulation. $(1,7)$ The development and use of computer models (virtual models) -as distinguished from the development and use of physical models-as an aid to understanding, representing, and solving problems. See also modeling and simulation.
create a simpler problem. (3) When faced by a challenging, complex problem, create a simpler but closely related problem and attempt to solve it. The goal is to gain insight into the original problem. For example, instead of thinking about how to reduce hunger in the

United States, think about reducing hunger in your state, or in your city, or in one small area in your city, or the hunger that you know exists for one student in a class you are teaching. For another example, consider the problem of learning the rules of a complex game. Set yourself the simpler problem of learning the rules for making your first move. See also break a problem into smaller problems; divide and conquer; simpler problem.
discovery-based learning. (2) This is a technique of inquiry-based learning or learning by doing, and is considered a constructivist based approach to education.
divide and conquer. $(2,4)$ Divide a large problem into smaller sub-problems that are more manageable. Do this in a manner such that once the sub-problems are solved, it is relatively easy to put the pieces together to solve the original problem. Note the value of having a large repertoire of "sub-problems" that one can readily solve. Often, some of the subproblems can be solved by a computer or other machine. See also break a problem into smaller problems; create a simpler problem.
domain-specific. $(2,4,6,7,8)$ Most of the strategies listed in this appendix are applicable in many different game and non-game problem-solving situations. However, within any problem-solving or game domain, there are some strategies that are quite specific to the domain. These are called domain-specific problem-solving strategies or tactics. For example, play in the center square if you are the first player in a Tic-Tac-Toe game. This is a good TTT strategy because if your opponent responds by playing in the center of any of the four edges, you can then force a win. If your opponent plays in a corner, you can easily avoid losing. When learning a new game or studying a new type of problem, pay attention both to the domain-specific and to the domain-independent strategies you are learning.
don't box yourself into a corner. (5) Think about what might happen as a consequence of your action. For example, it is usually not a good idea to start out a conversation with a potential friend by insulting the person and the person's friends. You practically force a rebuttal. Read your hastily written email message before sending it. See also bottleneck; engage brain before opening mouth; look before you leap; mobility.
draw a picture or diagram. (1) In many problem-solving situations it can be helpful to represent the problem graphically. A related idea is picturing the problem in your "mind's eye." See also heuristic.
elimination. $(3,4)$ In many problems, it is possible to eliminate certain categories of potential solutions or approaches relatively quickly and easily. This narrows the things that one needs to think about or try out in an attempt to solve the problem. As a simple example, suppose your goal is to find all possible pairs of positive even integers that add to a dozen. You can eliminate from your search any pair that contains an odd integer, any pair that contains zero or a negative integer, and any pair that contains an integer above 12.
engage brain before opening mouth. (3) This is another way of saying "curb your impulsive behavior." See also look before you leap; think before you act.
exhaustive search. $(3,4)$ Many problems can be solved by trying out all possible allowable moves or sequences of moves. If the number of possibilities is relatively small, a person or team of people might be able to carry out such an exhaustive search in a timely fashion. If the search process can be carried out by a computer, it may be possible to explore many millions of possible solutions or sequences of moves. See also explore solvability.
explore a simpler case strategy. (4) We frequently take a big problem and modify it into a more manageable problem. For example, hunger is a major problem throughout the world. Rather than trying to solve this problem, I might work on reducing the number of children attending the school in which I teach who come to school hungry. See also create a simpler problem.
explore solvability. $(3,4)$ Many of the situations that people call problems are actually not clearly defined and understandable problems. Rather, they are problem situations. One of the first steps to take when faced by a problem situation is to explore whether it is actually a clearly defined problem (given initial situation, clear goal, resources, ownership). One does not solve a problem situation; one solves a problem. Next, spend some time exploring whether you actually understand the problem. If you don't understand the given initial situation, the goal, and the resources, you are not in a good situation to attempt to solve the problem. One way to increase your understanding of a problem is to consider whether the problem might not have a solution. Ask yourself: "How would I recognize a solution if I happened to find one?" See also check your answer.
extrinsic motivation. (1) Teachers work hard to motivate their students; parents work hard to motivate their children. Reward systems are designed to motivate students. The offer of a high rate of pay and good working conditions might motivate a person to accept a particular job offer. A good teacher will create learning environments and requirements that are extrinsically motivating but that will lead students to becoming intrinsically motivated. See also intrinsic motivation.
feedback. (2) Feedback is essential to learning. When playing a game, one often makes a move designed to gain information about or from one's opponent(s). In face-to-face games, one can often obtain useful feedback by reading body language. See also collect data; guess and learn; metacognition.
getting an Ace into the Object Stacks. (5) This is an example of a domain-specific strategy applicable to the solitaire game of Eight Off. However, it illustrates the idea that in many game-playing and problem-solving situations there are a small number of domain-specific sub-goals that may be particularly valuable. So, the suggestion is in learning a game or how to solve a category of difficult problems, it can be quite helpful to learn about relevant subgoals. See also break a problem into smaller parts; create a simpler problem.
good start. (1) Quoting a proverb cited by Aristotle, "Well begun is half done." Quoting Lao Tzu, "A journey of a thousand miles must begin with a single step." In problem solving, a good start or a good first step is one that is likely to make a significant contribution to solving the problem. In competitive two-player games such chess, many thousands of person-hours of effort have gone into analyzing opening sequences of moves. Knowledge of and use of "good" openings can give a player a substantial advantage over an opponent who is less familiar with this form of improving one's performance. Initial instruction/learning in how to solve a particular new category of problem may include a focus on "what to do first." A general rule of thumb is, "First understand the problem." Think about the goal, guidelines, resources, and your ownership. See also explore solvability; think before you act.
guess and check. $(3,4)$ Many problems can be approached by making a guess (sometimes called an "educated guess") at a solution or a possible approach to obtaining a solution. If the
guess provides a correct solution or a correct pathway to obtaining a solution, that is well and good. If it doesn't, then one still gains useful information about the problem. For example, if one makes a guess of a solution and the guess is incorrect, one learns that the guess is incorrect. However, in many problem-solving situations, one gains additional information that helps in making a better guess or helps in developing a better plan. Generally speaking, increasing one's expertise in problem solving in a particular domain includes getting better at making educated guesses that are useful aids to learning more about how to solve the current problem as well as how to solve other problems in the domain. See also explore solvability; learn from your mistakes.
guess and learn. (3) One way to explore a new problem is to make a guess at an answer and then figure out why it is correct or incorrect. If you can determine that your guess is incorrect, the process you have gone through may give you insights that will help you to solve the problem. See also guess and check; learn from your mistakes.
heuristic. (1) A heuristic technique, often called simply a heuristic, is any approach to problem solving, learning, or discovery that employs a practical method not guaranteed to be optimal or perfect-and indeed, not guaranteed to be successful. A heuristic is sometimes called a rule of thumb. See also algorithm.
high-road transfer of learning. $(2,3,4,5,8)$ This is one of the most important ideas in the book. In whatever you are studying, think about how whatever you are learning might be applicable to further study of the domain you are studying, and also applicable in other domains. For example, math is a useful aid to representing and helping to solve problems in a great many domains. So, as you learn something new in math, you should bounce this new knowledge against your current understanding of a number of other domains. Reading is a general-purpose aid to learning. However, there are many domains in which reading may be of relatively little use in gaining a high level of expertise in the domain. (Being illiterate does not mean being dumb!) See also low-road transfer of learning.
hill climbing. (4) Some games and problem-solving situations have a goal that can be measured by how high (you or your score) is. In such a situation, you may think that any move or effort that raises your score is desirable. However, it is possible to box yourself into a corner that you cannot get out of. Also, if you use a metaphor of climbing a hill or mountain, you sometimes have to go downhill in order to get to a spot where you can go uphill. Of course, hill climbing can also apply to trying to get down a hill. See also incremental improvement.
incremental improvement. $(4,5)$ Some problems can be solved through a sequence of incremental improvements. This is somewhat akin to walking to the top of a mountain by making sure each step moves you uphill. However, many real world problems cannot be solved by incremental improvement. In our efforts to improve our educational system, we often make very small improvements that fail to address the larger problems that are the root cause of schools not doing as well as we would like. Thus, incremental improvement is often a poor strategy, wasting time and other resources, and contributing little to actually solving a major, complex problem. See also score, then improve your score.
information retrieval. The word information is often used to include data, information, and knowledge. Information can be stored in your brain, in books, in computers and other storage devices, online, and so on. In solving a problem or accomplishing a task you may
draw on (retrieve) information stored in your brain, but you may also draw on information stored in other people's brains, in books, in computers, online, and so on. See also collect data.
intrinsic motivation. (1, 5) For many students, games are intrinsically (internally, self) motivating. Self-motivation-or the lack thereof-is a very important aspect of education. See also extrinsic motivation.
learn from your mistakes. (3) We all make mistakes from time to time. When you recognize that you have made a mistake, think carefully about what led to the mistake. How will you avoid this mistake in the future? Is there anything you can do to rectify the mistake. (In some cases, a heartfelt apology might help.) See also backtracking; engage brain before opening mouth; look before you leap; mistakes.
learn to detect mistakes. (3) This, of course, is a useful skill when playing competitive games against other people. As another type of example, listen to what a politician is saying in a speech. Train yourself to detect an incorrect statement and to make use of the Web to do fact checking. See also check your answers.
learn to fill in the details. (4) A powerful alternative to rote memory is to learn/understand general approaches to solving certain types of problems, accomplishing certain types of tasks, and making correct arguments. With such general understanding, one can then fill in the details (figure them out) when it is useful to do so. This is a common teaching technique in math and is applicable to any problem-solving instruction. See also memorize though use; memorize when personally effort-effective.
letter frequency. (6) Knowledge of the frequency of occurrence of the various letters, or pairs of letters, and so on is useful in many different games that involve words. Data has been collected on the frequency of use of each letter of the English alphabet in typical writing. Also, data has been collected on the most frequent beginnings of words and endings of words, most common bigrams, most common trigrams, and so on. A person can memorize such detailed data, and it can be incorporated in computer programs. The data is useful in cryptography, in working to identify the author of a manuscript, and in a variety of games. Use of letter frequency is a good example of building on the previous work of others. The same idea can be used in analyzing customer purchasing patterns. See also collect data; patterns.
long-range planning. (8) This is often called long-range strategic planning. It refers to developing a broad, strategic plan that provides a good sense of direction of where one is heading in trying to solve a particular problem or accomplish a particular task. (Sometimes people use the expression, "Keep your eye on the ball.") Often a long-range strategic plan is accompanied by short-range plans and strategies, and by detailed tactics that are designed to accomplish the short-range plans. See also look ahead; plan ahead.
look ahead. $(3,5,6,8)$ Typically, solving a problem involves a sequence of steps or moves. When there is an opponent or another player involved, then your move is followed by a response that might well affect your next move. In attempting to solve real-world problems, each step or action makes a change in the problem situation. Failure to anticipate major changes often leads to failure to solve a problem or to the creation of other problems. See also long-range planning; look before you leap; plan ahead.
look before you leap. (5) One of the hardest things for young children (and older children, and young adults, and adults) to learn is that actions have consequences. One of the advantages of learning in "friendly" game environments is that the consequences of poor planning are minor-perhaps you lose a game. We try to teach young children to "look both ways before starting to cross a street." Failure to do so might result in serious injury or death. See also look ahead; plan ahead; think before you act.
look for patterns. (4) Perhaps a better way to state this important idea is to "look for possible cause and effect situations." Math is sometimes described as the study of (mathematical) patterns. Statistics is a branch of math that can help to detect quite complex possible cause and effect relationships. People who are trying to be careful in their spending on groceries are told to not shop when they are hungry. The suggestion is that shopping for groceries when hungry is related to over-spending. See also collect data; patterns.
look it up. (1) Computer technology, including wearable technology and mobile technology (like a GPS in a car) has changed the nature of problem solving. Our educational system is facing the problem of what data, knowledge, and information students should carry around in their heads, and what should they retrieve (from a computer system) when needed. In a game such as Scrabble, playing with "open Scrabble dictionary" is quite a bit different from playing exactly according to the rules. But if solving difficult Scrabble-like problems were an important part of real life, then people encountering such problems should have access to aids such as computers and instruction manuals to help them to solve the problems. David Moursund's parable, Chesslandia, provides an excellent example. See also build on previous work; collect data; information retrieval.
low-road transfer of learning. (2, 3) A theory of transfer of learning based on rote memory and repeated practice. Here is an excellent example of low-road transfer of learning. When I was a child, I had great difficulty in learning to tie my new brown shoes that had brown shoelaces. Now, I can tie my different pairs of shoes that have different colors of shoelaces, and I can do it in the dark with little thought. See also high-road transfer of learning.
memorize through use. (5) For many students, school consists of "memorize, regurgitate on a test, and forget." This is a very ineffective type of schooling. In learning a new game, a beginner learns just a little bit and almost immediately begins using this knowledge. More is learned as needed-when situation arise where the added knowledge is immediately useful. You have heard the expression, "Use it or lose it." Both teachers and students should follow that maxim. See also memorize when personally effort-effective.
memorize when personally effort-effective. (6) Here "effort-effective" means that the effort to memorize is effective and worthwhile. Memorized information and thoroughly practiced processes can be thought of as solutions to specific sub-problems or problems. People vary considerably in terms of how quickly and accurately they can memorize a particular set of materials, and how long and accurately they retain the memorized information. A rule of thumb is to memorize information that one needs to use frequently enough, and in a timedependent manner, to make the memorization effort worthwhile. Keep in mind the capabilities of a computer system to store the full contents of millions of books, and the abilities of search engines to aid in almost instantaneous retrieval of information stored in a computer. See also only memorize if quite useful.
mental aids. Reading, writing, arithmetic, books, the Web, and computers are all examples of mental aids. They help to overcome limitations of one's brain. They are resources that can be applied to problems in every domain. See also modeling and simulation.
metacognition. (3) Metacognition is thinking about—analyzing, reflecting on-one's own thinking. It is a highly effective technique or strategy for improving one's problem-solving and learning skills.
metaphor. A metaphor is an attempt to provide a written or oral representation of something, where the words and sounds are not the actual thing being represented. In some sense, most written and oral language is metaphorical. When describing and thinking about a problem, metaphors can be a powerful aid to understanding or constructing understanding. See also thinking outside the box.
mistakes. A mistake is an error, a wrong choice, an incorrect judgment, and so on. Our educational systems tend to emphasize that it is bad to make mistakes. However, many educators believe that this represents poor teaching. A great deal of creativity is based on trial and error-making mistakes, analyzing the mistakes, and doing better the next time. See also learn from your mistakes.
mobility. $(5,6)$ As you work to solve a challenging problem, keep open options that may later prove to be fruitful. Don't paint yourself into a corner where you have very few or no options. See also don't box yourself into a corner; look before you leap.
modeling and simulation. $(1,9)$ The development of models and then the use of these models (for example, develop a model of an airplane and test it in a wind tunnel) has long been used in problem solving. Computer modeling and simulation is such a powerful aid to problem solving that it has added a new dimension to how problems in science, engineering, architecture, and many other fields are approached. Nowadays, science is done experimentally (designing and carrying out experiments), theoretically (developing theories, such as Einstein's theory of relativity), and computationally (developing and using computer models). See also computer modeling and simulation.

Moursund's 7-step advice. (7) This is a seven-part list of advice that can be used to become better at solving a wide range of problems. It summarizes ideas such as learning general knowledge and strategies, learning domain-specific knowledge and strategies, and learning during the process of solving a problem.
only memorize if quite useful. (5) For most people, it takes considerable effort to memorize a relatively small amount of material. Moreover, without frequent use, such memories tend to fade and become imprecise. Nowadays our educational system is increasing its focus on learning for understanding and decreasing its emphasis on rote memorization. However, the types of test questions students encounter often emphasize rote memorization. See also memorize when personally effort-effective.
patterns. I find it helpful to think of randomness as being the absence of patterns. The human brain can be thought of as an organ for the input, storage, processing, and use of patterns. (That statement is quite similar to the statement that an electronic digital computer is a machine for the input, storage, processing, and output of data and information.) The identification and use of patterns is a key aspect of problem solving in every discipline. See also collect data; look for patterns; random and randomness.
plan ahead. (Introduction, 5, 6) As a human brain matures, it becomes increasingly able to think about a goal it would like to accomplish, plan how to achieve the goal, and then direct the mind and body in carrying out actions to achieve the goal. This capability separates humans from all other creatures on earth. See also long-range planning; look ahead; look before you leap.
Polya, George. (1) George Polya was a leading $20^{\text {th }}$ century mathematics researcher who wrote and lectured extensively about problem solving. His 6 -step procedure (think of it as a heuristic) for problem solving is widely taught and used.
problem situation. (1) Many of the things that people describe as problems are actually problem situations. They lack one or more of the characteristics (givens, goal, resources, ownership) that clearly define problems of personal interest. Polya's six-step strategy begins with understanding the problem-determining if one actually has a clearly defined problem that he or she is interested in solving. See also explore solvability.
random and randomness. (6) Somewhat surprisingly, the use of random moves or random activity can be a useful approach in many different problem-solving situations. Of course, many games make use of randomness. For example, one shuffles the cards in solitaire and in many other card games. One makes use of a spinner or dice to generate moves in many different games. For a more mundane example, imaging a person playing a game such as Tic-Tac-Toe by making completely random moves. At a deeper level, randomness can be used in modeling and simulation as an aid to solving a wide range of problems in science and other areas. The results can be used to establish baseline data on how well a person plays the game before developing or learning any strategies that can lead to an improved level of play. See also look for patterns; modeling and simulation.
record one's moves. (2) This reminds me of the Chinese proverb: "The weakest ink is stronger than the strongest memory." In math instruction, students are taught to "show your work." A computer program is a written representation of the steps a computer is to follow to solve a particular problem or accomplish a particular task. A careful representation of the steps one has followed in playing a game, solving a problem, and so on, can be studied afterwards as an aid to learning what went right and what went wrong. See also collect data; mental aids.
reinvent the wheel. This strategy takes two forms: 1) don't reinvent the wheel; 2) do invent a better wheel (or that which takes the place of the wheel). In the first instance, the idea is to build upon work that you and others have done in the past. Use the Web and other resources of stored information-do library research-to find out what is already known about how to solve a particular problem. In the second instance, the idea is to not be boxed in by conventional approaches to the problem. This approach is also a key in learning how to solve problems with the goal of improving one's level of expertise in solving novel, challenging problems. See also build on previous work; collect data.
rubric. (8) Scoring or grading criteria. Teachers make use of rubrics as they assess student work and provide feedback to students. Typically, a rubric is aligned with goals in a unit of study, a lesson, or a specific assignment. Sometimes a rubric is designed just for the use of the teacher. At other times, a rubric is designed for use both by the teacher and by the teacher's students. Some teachers engage their students in developing rubrics for an assignment.
score, then improve your score. (5) There are many real-world problem-solving situations in which a score of zero is explicitly or implicitly given for not making a reasonable attempt, or for completely failing in one's attempts. In many tests, one can get partial credit for a good start, even if one fails to actually solve the problem. Even complete failure will give oneself, a coach, or a teacher a baseline from which to obtain or provide feedback for improvement. See also good start; learn from your mistakes.
sequence of moves. (5) In many card and board games and in many puzzles, it is important to think in terms of sequences of moves. Through training and experience, one can become quite skilled at mentally (in one's mind's eye) examining a sequence of possible moves. See also look ahead; plan ahead.
simpler problem. (5) Creating and solving a related but simpler problem is often a useful strategy. This is a very important idea in teaching. As a teacher, you want to provide your students with problems that stretch their capabilities without being completely beyond them. This is quite difficult to do when working with students with widely varying levels of knowledge, experience, and capabilities in one classroom. See also create a simpler problem.
strategy. $(5,6,8)$ The high-road transfer problem-solving strategies illustrated in this book are deigned to help you to become better at developing and using problem-solving strategies. A strategy can be thought of as a plan of action to be used in attempting to achieve a goal. Some strategies are general purpose, useful over a wide range of problems. However, it typically takes a considerable level of domain-specific knowledge and skills to solve a challenging problem within a specific domain. As one develops such domain-specific knowledge and skills, one develops specific strategies (or, fine-tunes general strategies) to better fit the problem-solving requirements of the domain.
think before you act. $(1,5)$ Many problem-solving situations do not require immediate, splitsecond responses and actions. In these situations, there is time to mull over possible actions, to think before taking an action. See also act before you think; engage brain before opening mouth; look before you leap; plan ahead.
think inside the box. (1) A well-defined problem can be thought of as a box in which one must act to solve the problem. If the givens, goals, and resources are very carefully specified, and you have ownership, you are restricted by the actions that you can take. However, it is important to fully understand exactly what is inside the box in such situations. See also explore solvability; look before you leap; plan ahead.
think out loud. (8) When a team of two or more people is working on a problem, it is often helpful to have one member of the team think out loud about the problem, while the other team members merely listen and perhaps take notes. A different approach is to have two or more members of the team thinking out loud, interchanging possible strategies and ideas, as they explore and work on the problem, often using a whiteboard. See also explore solvability.
think outside the box. (1) When faced by a problem, most people have a strong tendency to use the approaches and problem-solving steps familiar and comfortable to them. If this does not work, a standard next step is to seek help from others, perhaps directly from other people or through library and/or online research. There are many problems where these approaches
do not work. Solving the problem may require developing new ideas, new ways of thinking, or new inventions. It may involve deliberately ignoring or tabling ideas and approaches that first come to mind, or that others have developed. Individual and group brainstorming can sometimes be an effective aid to thinking outside the box. See also explore solvability; think out loud.
Wagner's 7-step strategy for students. (8) Tony Wagner (January 11, 2006) has produced seven questions that students should routinely ask themselves and answer as they study a lesson. Here are the first four:

1. What is the purpose of this lesson?
2. Why is this important to me?
3. In what ways am I challenged to think in this lesson?
4. How will I apply, assess, or communicate what I've learned?
wait until something better comes along. (6) In many game-playing situations, as well as in debates or arguments among people, you may gain an advantage by merely not making a disastrous error. This is sometimes called "playing a waiting game." See also engage brain before opening mouth; think before you act.
work backwards. (4) Start at a solution and move back one or more steps in a manner such that it is easy to see how to move forward to a solution. In essence, the strategy is to create a new problem to solve, with the new problem having the characteristic that once it is solved, it is easy to solve the original problem. If you work backward to the original problem, or to a simpler one that you can solve, you can then solve the original problem See also create a simpler problem; simpler problem.

## Reference

Wagner, T. (January 11, 2006). Rigor on trial. Education Week. Retrieved 11/12/2015 from http://www.tonywagner.com/238.

## Appendix 2 <br> Games \& Puzzles Discussed in the Book

| Name | Chapter(s) |
| :--- | :--- |
| 9-dots Puzzle | 1 |
| Assemble the Square Puzzle | 4 |
| Backgammon | Preface, 7 |
| Brainteaser Puzzle | 1,4 |
| Bridge | 7 |
| Bridge Crossing Puzzle | 4 |
| Checkers | 6 |
| Chess | 2,6 |
| Computer Programming | Preface, 9 |
| Connection Games | 6 |
| Cribbage | 6 |
| Crossword Puzzle | Introduction, |
| Cryptogram Puzzle | $1,2,4$ |
| Dots and Boxes | 6 |
| DragonVale | $1,7,9$ |
| Draughts | 6 |
| Eight Off | 5,9 |
| Fortress | 5 |
| Go | 6 |
| Gomoku | 6 |
| Hangman | 6 |
| Hearts | 7 |
| Hex | 6 |
| Klondike | 5 |
| Jigsaw Puzzle | 1,4 |
| Logi-Number Puzzle | 4 |
| Massively Multiplayer Online <br> Games (MMOGs) | 7 |
| Math Computation Puzzle | 4 |
| Math Square Puzzle | 4 |
| Mechanical Puzzle | 4 |
| Monopoly | 1,7 |
| Napoleon at St. Helena | 5 |
| Napoleon's Square | 5 |
| Oh Heck | 7 |
|  |  |
|  | 4 |


| Othello | 6 |
| :--- | :--- |
| Patience | 5 |
| Peg Puzzle | 4 |
| Penta | 5 |
| Pinochle | 7 |
| Poker | Preface, 7 |
| Reversi | 6 |
| Rubik Cube Puzzle | 1,4 |
| Scrabble | 9 |
| Slider Puzzle | 4 |
| Solitaire | 5 |
| Sudoku Puzzle | 3 |
| Symmetrical Word Box <br> Puzzle | 4 |
| Tangram Puzzle | 9 |
| Tetris | 5 |
| Tic-Tac-Toe | 6 |
| Tower of Hanoi Puzzle | 4 |
| Water Measuring Puzzle | 4 |
| Whist | 7 |
| World of Warcraft | Preface |

