



The Number Rack

Fact Strategies



The MATH LEARNING CENTER

Foundational Facts

LESSON 5

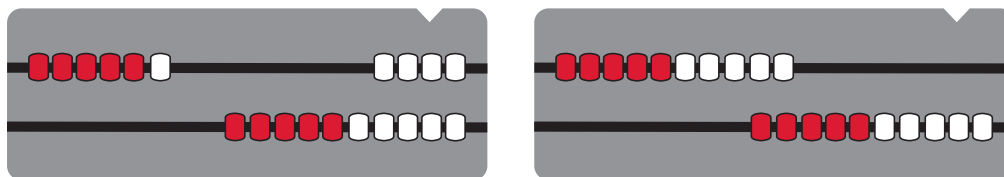
Adding & Subtracting 0, 1, or 2

Students' understanding of counting leads to an introductory set of foundational facts. Most students quickly recognize that adding 1 to a number results in the next number in the counting sequence (e.g., $4 + 1 = 5$). This helps them think about adding 1 as "one more" and subtracting 1 as "one less." Likewise, adding 2 is "two more" and subtracting 2 is "two less." (Though it's possible to extend this idea for adding or subtracting 3, 4, 5, or more, use of other foundational facts or derived fact strategies will be more efficient.) Students also discover that zero, when added or subtracted to any number, does not change the value (e.g., $5 + 0 = 5$ and $5 - 0 = 5$).

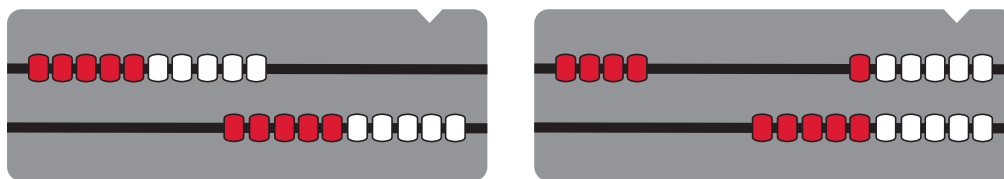
LESSONS 6, 7 & 8

Combinations of 10

The importance of helping young children understand 10 cannot be overstated. After all, we use a base ten number system. Students need to be very familiar with all the two-addend combinations that make 10: 1 and 9, 2 and 8, 3 and 7, 4 and 6, and 5 and 5. The number rack helps students create visual representations of 10 as a foundation.



$$6 + 4 = 10$$



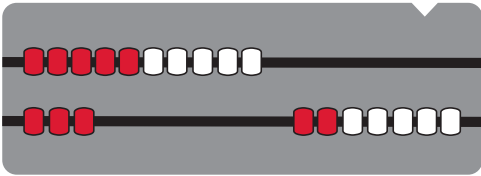
$$10 - 6 = 4$$

10 & More

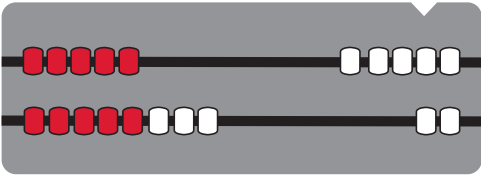
Students need a strong sense of 10 to understand the structure of teen numbers. Without it, a young child just thinks that a “1” and a “6” is how you write 16, without understanding that the 1 actually represents 10 because of its position in the number. Understanding the structure of teen numbers is essential to learning foundational facts, since the 10 & more facts rely on adding or subtracting either the “10” or the “more” (e.g., $10 + 4 = 14$; $4 + 10 = 14$; $14 - 4 = 10$; $14 - 10 = 4$).

When students have opportunities to build teen numbers on the number rack in multiple ways, they see that 10 beads plus some more beads are always needed. This allows students to realize, for instance, that 10 beads plus 6 more beads is 16, that is, that $10 + 6 = 16$. Here is a sample discussion of students sharing:

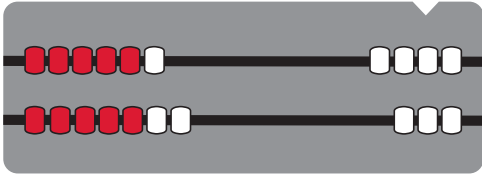
Teacher
Please slide 13 beads to the left side of your rack, using as few pushes as possible. Then let's have a few of you describe your work to the group.



Sondra
I put 10 on the top and then 3 on the bottom because I know that 13 is 10 and 3 more.



Sherwin
I did kind of the same thing, but I put 10 over with 5 on top and 5 on the bottom, and then I put 3 more in the bottom row.



Logan
Your way is kind of like mine. You have 5 and 5 with the red beads, and then you have 3 whites on the bottom, but I have 1 white in the top row and 2 whites in the bottom row.

Teacher
So Logan, did you also use 10 beads and 3 more beads to make 13?

Logan
Yes, because 5 and 5 make 10, and then 1 and 2 make 3.

Note: Even if Logan had originally thought of 6 beads on top and 7 beads on the bottom without considering a ten, the red beads make a visible ten that he acknowledges in his second comment.

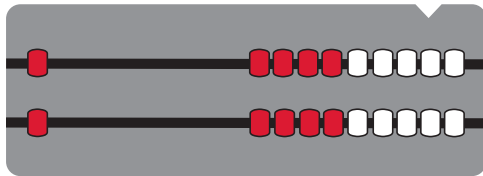
Ten & more facts ask students to consider how the structure of teen numbers relates to written equations of 10 plus some more ones ($10 + 1 = 11$, $10 + 2 = 12$, $10 + 3 = 13$..., $10 + 9 = 19$).

LESSON 13

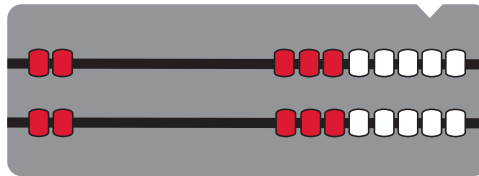
Doubles

Students have a natural understanding that things in the world around them often come in pairs: car wheels, insect legs, their own eyes and hands and feet. Doubles facts draw on this idea of 2 equal groups. A doubles fact is a combination where the two addends are the same (e.g., $1 + 1 = 2$, $4 + 4 = 8$, $9 + 9 = 18$).

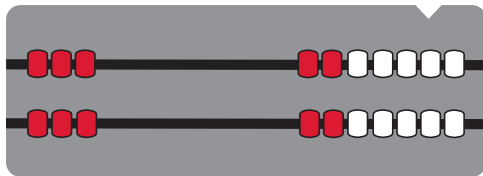
The number rack is a good tool for exploring doubles facts. Students can see that the number of beads in the top row and the bottom row are exactly the same.



$1 + 1 = 2$



$2 + 2 = 4$



$3 + 3 = 6$



$4 + 4 = 8$

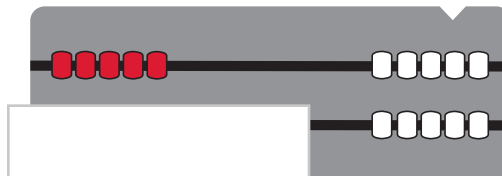


$5 + 5 = 10$

A doubles fact can be thought of as joining 2 equal sets. If students know that $5 + 5 = 10$, then they will also learn that $10 - 5 = 5$ by understanding that addition and subtraction are inverse operations.



$5 + 5 = 10$



$10 - 5 = 5$

Derived Fact Strategies

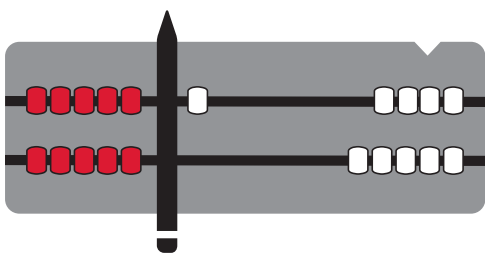
LESSON 15

Near Doubles

Doubles facts can be used to solve other facts with addends (numbers being added) that are nearly the same. Students who are fluent with the doubles facts (e.g., $3 + 3 = 6$ or $5 + 5 = 10$) often recognize and use the near doubles strategy (e.g., $4 + 3 = 7$ or $5 + 6 = 11$).

Students using the near doubles strategy will think about a problem such as $5 + 6 = ?$ in relation to a known doubles fact. Here are some examples:

- A student might think, “I know $5 + 5 = 10$, and 6 is 1 more than 5, so $5 + 6 = 11$.”
- Another student might think, “I know $6 + 6 = 12$, and 5 is 1 less than 6, so $5 + 6 = 11$.”
- Still another student might decompose the 6 by thinking of it as $(5 + 1)$, then rearrange the numbers and think, “ $(5 + 5) + 1 = 10 + 1$, so the total is 11.”



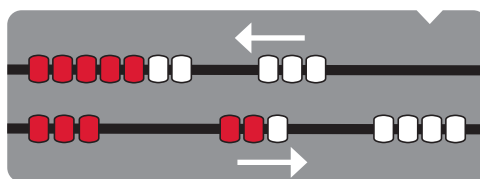
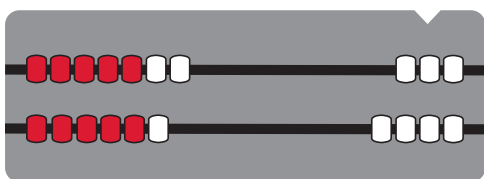
The number rack provides great visual support for the near doubles strategy, as it shows two rows with identical numbers of beads. The relationship between doubles facts and facts that are nearby can be highlighted by using a pencil or other divider to find the related doubles fact, as shown.

Making 10

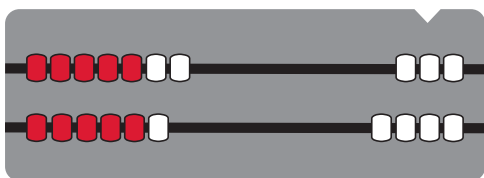
Making 10 is a derived fact strategy for addition that involves decomposing one addend to make 10, and then adding the rest to the 10 to get the total.

Students who are confident with their foundational facts—combinations of 10 and 10 & more—often approach unknown problems using the making 10 strategy. Here are some examples for solving $7 + 6 = ?$ when using this strategy:

- A student might take 3 from the 6 and add it to the 7 to make 10 ($7 + 3 = 10$). Once at 10, they add the remaining 3 to the 10 to get 13 ($10 + 3 = 13$).



- Another student might take 4 from the 7 and add it to the 6 to make 10 ($6 + 4 = 10$) and then add the remaining 3 to the 10 to get 13 ($10 + 3 = 13$).
- A student might see the number 7 as a combination of 5 and 2 more, and the 6 as a combination of 5 and 1 more. They combine the two 5s to make 10 ($5 + 5 = 10$) and add the 2 and the 1 to get 3 ($2 + 1 = 3$). Finally, they add the 10 and the 3 to get 13 ($10 + 3 = 13$).



$$7 + 6 = ?$$

$$(5 + 2) + (5 + 1) = ?$$

$$(5 + 5) + (2 + 1) = ?$$

$$5 + 5 = 10$$

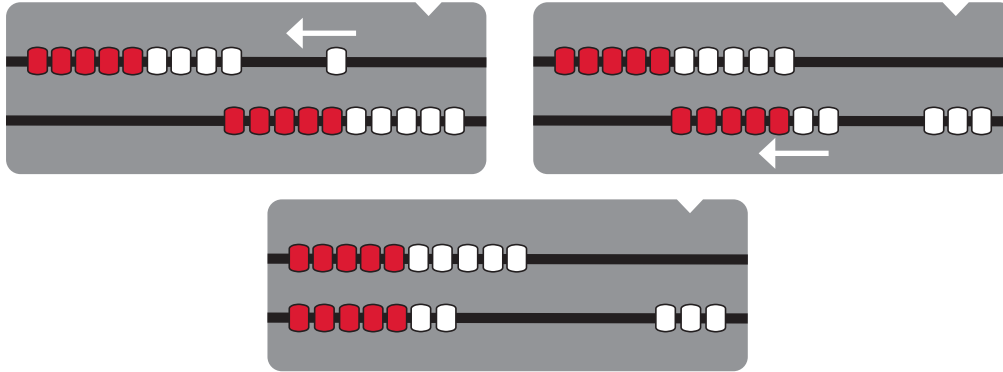
$$2 + 1 = 3$$

$$10 + 3 = 13$$

LESSON 19

Up Over 10

Up over 10 is a derived fact strategy for subtraction where students think about the subtraction problem as an addition problem and work their way over 10 to identify the missing addend. For example, in solving $17 - 9 = ?$, students would think of the related equation $9 + ? = 17$. They'd add 1 to the 9 to get to 10, and then they'd add 7 more to get to 17. The difference is $1 + 7$, or 8.



Solving $17 - 9$ by starting with 9 and building up to 17.

This strategy is an efficient way to find the difference when the minuend (the number from which another number is subtracted) is in the teens and the subtrahend (the number being subtracted) is less than 10 (e.g., $13 - 9$ or $16 - 7$).