Pattern Block Lessons

to Meet Common Core State Standards

Grades 3–5

The MATH LEARNING CENTER

Excerpts From Bridges in Mathematics

PBLCCSS35

Pattern Block Lessons to Meet Common Core State Standards Grades 3–5

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Prepared for publication on Macintosh Desktop Publishing system.
Printed in the United States of America.

PBLCCSS35 QP1277 P0412

Bridges in Mathematics is a standards-based K–5 curriculum that provides a unique blend of concept development and skills practice in the context of problem solving. It incorporates the Number Corner, a collection of daily skill-building activities for students.

The Math Learning Center is a nonprofit organization serving the education community. Our mission is to inspire and enable individuals to discover and develop their mathematical confidence and ability. We offer innovative and standards-based professional development, curriculum, materials, and resources to support learning and teaching. To find out more, visit us at www.mathlearningcenter.org.

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* Pattern Blocks are the only manipulative required for this activity.

Introduction

Pattern Blocks and the Common Core State Standards

Pattern Blocks are a familiar manipulative available in most elementary schools. We've created this Pattern Block Lessons sampler to help you meet the new Common Core State Standards (CCSS) and organized it in two grade level bands, K–2 and 3–5. The lessons are excerpts from the Bridges in Mathematics curriculum, published by The Math Learning Center. We hope you'll find the free resources useful and engaging for your students.

The Common Core State Standards (2010) define what students should understand and be able to do in their study of mathematics. A major goal of the CCSS is building focus and coherence in curriculum materials. The standards strive for greater consistency by stressing conceptual understanding of key ideas and a pacing the progression of topics across grades in a way that aligns with "what is known today about how students' mathematical knowledge, skill, and understanding develop over time." (CCSSM, p. 4). In addition to the content standards, the CCSSM defines Eight Mathematical Practices that describe the processes—the how teachers will teach, and how students will interact in a mathematics classroom.

Bridges in Mathematics helps teachers meet the challenges of the Content Standards and the Eight Mathematical Practices. During a Bridges lesson, students make sense of mathematics using manipulatives, visual and mental models to reason quantitatively and abstractly. They solve challenging problems daily that develop their stamina to carry out a plan and to present their thinking to their classmates. Students make conjectures and critique the reasoning of others, by asking questions, using tools, drawings, diagrams and mathematical language to communicate precisely. Students develop and use a variety of strategies to become computationally fluent with efficient, flexible and accurate methods that make use of patterns and the structures in operations and properties. They use dimensions, attributes, and transformations to make use of the structures in Number and Geometry. Bridges encourages students to estimate a reasonable answer, and continually evaluate the reasonableness of their solution. This Pattern Block sampler will provide you with examples of lessons from whole group Problems and Investigations and centers called Work Places. In many cases there are suggestions for support and challenge to help you meet the CCSS standards and differentiate your instruction.

Bridges in Mathematics

Bridges in Mathematics is a full K–5 curriculum that provides the tools, strategies, and materials teachers need to meet state and national standards.

Developed with initial support from the National Science Foundation, Bridges offers a unique blend of problem-solving and skill building in a clearly articulated program that moves through each grade level with common models, teaching strategies, and objectives.

A Bridges classroom features a combination of whole-group, small-group, and independent activities. Lessons incorporate increasingly complex visual models—seeing, touching, working with manipulatives, and sketching ideas—to create pictures in the mind's eye that helps learners invent, understand, and remember mathematical ideas. By encouraging students to explore, test, and justify their reasoning, the curriculum facilitates the development of mathematical thinking for students of all learning styles.

Written and field-tested by teachers, Bridges reflects an intimate understanding of the classroom environment. Designed for use in diverse settings, the curriculum provides multiple access points allowing teachers to adapt to the needs, strengths, and interests of individual students.

Each Bridges grade level provides a year's worth of mathematics lessons with an emphasis on problem solving. Major mathematical concepts spiral throughout the curriculum, allowing students to revisit topics numerous times in a variety of contexts.

To find out more about Bridges in Mathematics visit www.mathlearningcenter.org

Activity 1

MAGNETIC BOARD

Pattern Block Fractions

Overview
Students use magnetic pattern blocks to model the relationships between parts and the whole and to find equivalent fractions.

Frequency
Incorporate this routine into your calendar time two days per week.

Skills & Concepts
- ★ Demonstrate an understanding of a unit fraction $1/b$ as 1 of b equal parts into which a whole has been partitioned (e.g., ¼ is 1 of 4 equal parts of a whole) (3.NF.1)
- ★ Demonstrate an understanding of a fraction a/b as a equal parts, each of which is $1/b$ of a whole (e.g., ¾ is 3 of 4 equal parts of a whole or 3 parts that are each ¼ of a whole) (3.NF.1)
- ★ Identify equivalent fractions by comparing their sizes (3.NF.3a)
- ★ Recognize simple equivalent fractions (3.NF.3b)
- ★ Generate simple equivalent fractions (3.NF.3b)
- ★ Explain why two fractions must be equivalent (3.NF.3b)
- ★ Write a whole number as a fraction (3.NF.3c)
- ★ Recognize fractions that are equivalent to whole numbers (3.NF.3c)
- ★ Compare two fractions with the same numerator (3.NF.3d)
- ★ Compare two fractions with the same denominator (3.NF.3d)
- ★ Demonstrate that fractions can only be compared when they refer to the same whole (3.NF.3d)
- ★ Use the symbols >, =, and < to record comparisons of two fractions (3.NF.3d)
- ★ Explain why one fraction must be greater than or less than another fraction (3.NF.3d)
- ★ Partition shapes into parts with equal areas (3.G.2)
- ★ Express the area of each equal part of a whole as a unit fraction of the whole (e.g., each of b equal parts is 1/b of the whole) (3.G.2)

You'll need
- ★ pattern blocks
- ★ magnetic pattern blocks (yellow hexagons, blue rhombuses, green triangles, and red trapezoids, optional)
- ★ magnetic surface (optional)
- ★ erasable marker (e.g., Vis-à-Vis)

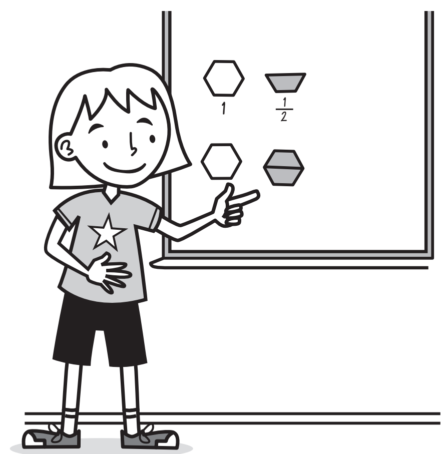
Note This activity can be conducted at a projector if magnetic pattern blocks and surface are not available.

Activity 1 Pattern Block Fractions (cont.)

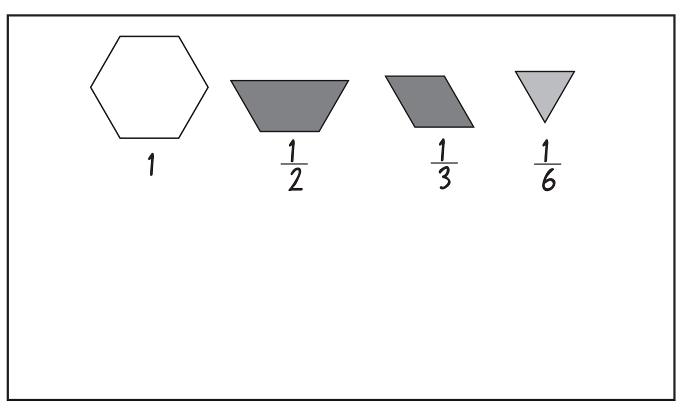
Identifying Fractional Parts of the Whole

Invite students to join you in front of the magnetic board. Place a yellow hexagon on the magnetic board and explain that today, this shape has an area of 1 unit. Write the numeral 1 under the hexagon. Next, display a collection of blue rhombuses, triangles, and trapezoids, and ask students to consider what the area of each of these shapes would be if the hexagon is 1. Invite volunteers to come up to the magnetic board to share their thinking. When students have identified the area of a particular shape, record this information on the magnetic board.

> ***Ginny*** *The red trapezoid is half of the hexagon. I know because when I put two trapezoids together, it's the same as 1 hexagon.*



Once students have determined the fractional parts represented by each shape, leave the labeled shapes on the magnetic board for reference in the coming weeks.



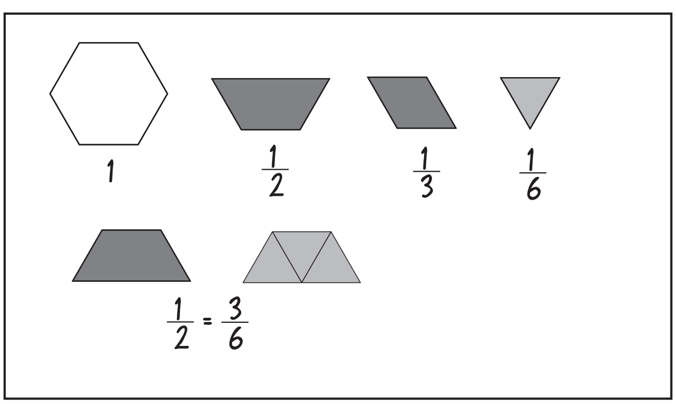
Continuing through the Month

As you continue this workout through the month, invite students to use the magnetic pattern blocks to consider equivalent fractions and determine the fractional value of each shape if the unit is shifted, as described on the next page. Follow your students' lead through the month, and introduce new challenges as they're ready. For many groups of third graders, considering fractional parts of the hexagon whole will be challenging enough to provide rich discussions for the entire month. Make sure students have collections of pattern blocks, if needed.

Identifying Equivalent Fractions & Combinations of Fractions

Invite students to explore equivalent fractional parts by finding a variety of ways to show half (or a third, or two-thirds) of a hexagon, working with the available pattern blocks at the magnetic board. If you have enough pieces, leave these equivalent fractions displayed on the magnetic board so students can consider them at other times.

> ***Teacher*** *Some of you said that when the hexagon is 1 whole unit, the trapezoid is exactly one-half. Are there other ways to show one-half of the hexagon with the other pattern blocks?*
>
> ***Sebastian*** *You can also make one-half with 3 triangles. Look, I'll show you.*
>
> ***Teacher*** *Sebastian, I'd like to write what you've shown as a number sentence. I can write one-half equals. Then what? Any ideas about how to complete the number sentence?*
>
> ***Emma*** *3!*
>
> ***Tom*** *I don't get that, Emma. How can one-half equal 3?*
>
> ***Emma*** *Well, you have 3 triangles. So 3 equals one-half. Hmm, that seems a little funny.*
>
> ***Rosa*** *There are 3 triangles, but each one is one-sixth. So 3 one-sixths is equal to one-half.*
>
> ***Teacher*** *Emma saw 3 triangles, and Rosa explained that each triangle is just one-sixth. So we can say one-half equals three-sixths.*



Activity 1 Pattern Block Fractions (cont.)

Proving Equivalencies

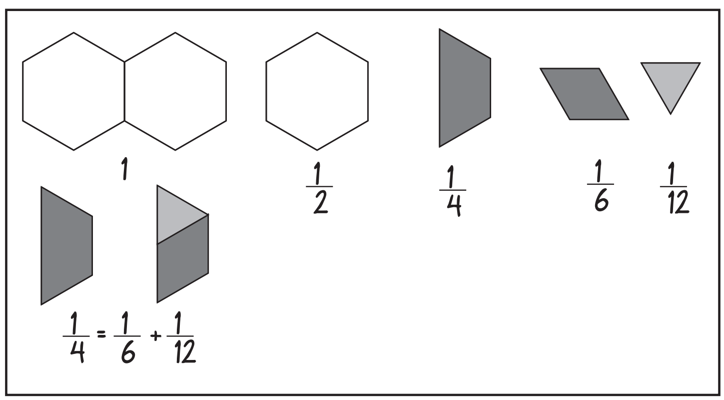
Another way to approach the concept of equivalent fractions with your class is to write the following number sentences on the board one at a time. Then ask students to think about whether or not the number sentence on the board is true. Encourage discussion, and then invite volunteers to use magnetic pattern blocks to prove whether the statement is true.

$$\frac{2}{6} = \frac{1}{3} \qquad \frac{3}{3} = 1 \qquad \frac{3}{6} = \frac{1}{2} \qquad \frac{2}{2} = 1 \qquad \frac{2}{3} > \frac{1}{2}$$

$$\frac{1}{2} < \frac{2}{3} \qquad 1\frac{1}{6} = \frac{7}{6} \qquad 1\frac{1}{3} = \frac{4}{3}$$

Changing the Unit of Area

Later in the month, you could explore with your students what happens if you shift the unit. For instance, what if the hexagon is assigned a value of one-half rather than 1? What would a whole unit look like? What would the values of the other pattern blocks be if the hexagon were one-half? Encourage students to look for different ways to show the same fractions with different pattern blocks, for example, by combining a rhombus and a triangle to make one-fourth.



In their explorations and discoveries, students may combine fractions with unlike denominators, as in the example shown above. Because their explorations are both intuitive and visual, there's no need to do anything but record their findings in fractional terms (e.g., $1/4 = 1/6 + 1/12$). Be sure to express $2/2$, $3/3$, $4/4$, $6/6$ and $12/12$ as a whole as well.

Activity 2

PROBLEMS & INVESTIGATIONS

Creating Symmetrical Snowflakes

Overview

Exploring the natural world provides students with many opportunities to appreciate geometry. Today, you can read a charming book, *Snowflake Bentley*, to introduce students to the symmetry found in snowflakes. Students use white pattern block cutouts to create their own unique snowflakes.

Actions

1. The teacher can opt to read the book *Snowflake Bentley* to introduce snowflake forms.
2. Students use white paper pattern block cutouts to make their own snowflake designs.

Skills & Concepts

- ★ Partition shapes into parts with equal areas (3.G.2)
- ★ Express the area of each equal part of a whole as a unit fraction of the whole (e.g., each of b equal parts is $^1/_b$ of the whole) (3.G.2)
- ★ Identify lines of symmetry (4.G.3)
- ★ Draw lines of symmetry (4.G.3)
- ★ Identify figures with line symmetry (4.G.3)

You'll need

- ★ Snowflake Pattern Blocks, pages 1 and 2 (Teacher Masters 1 and 2, class set run on white paper) or shapes pre-cut on a die cut machine
- ★ *Snowflake Bentley* by Jacqueline Briggs Martin (optional, check the library for availabilty)
- ★ poems about snow and snowflakes (optional)
- ★ 8″ or 9″ squares of black or blue construction paper (class set plus some extra)
- ★ glue sticks

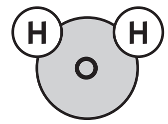
Note Wilson A. Bentley was a Vermont photographer who photographed snowflakes in great detail during the late nineteenth and early twentieth centuries. In addition to the many books available, you can find good sites on the Web to see the beauty of his photographs firsthand. When looking at the photographs, students might notice that most snowflakes are based on a hexagon.

Reading Snowflake Bentley

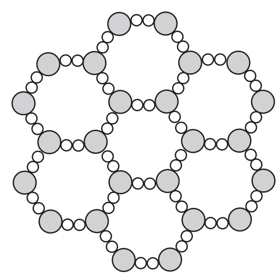
You many wish to begin this lesson by reading the book *Snowflake Bentley*. You may even choose to read it twice, once to capture the essence of his life, and a second time to discuss the scientific insights posted in the sidebars. You might also select a few poems about snow and snowflakes to enjoy with your students.

Activity 2 Creating Symmetrical Snowflakes (cont.)

Note *Snowflakes are six-sided because when water molecules slowly freeze to create snowflakes, they arrange themselves into ice crystals, which are all based on a hexagonal (6-sided) pattern. Why, one might wonder, do ice crystals always take on a hexagonal form? You might remember that water molecules are made up of one oxygen and two hydrogen atoms arranged more or less as shown below.*



When the molecules of water vapor freeze to form crystals of ice, they arrange themselves as shown below. The most basic kind of snow crystal is a simple stack of hexagons (a hexagonal prism), but the hexagonal snow crystals often arrange themselves in more complex structures, tessellating and branching in a wide variety of ways to become beautiful snowflakes.



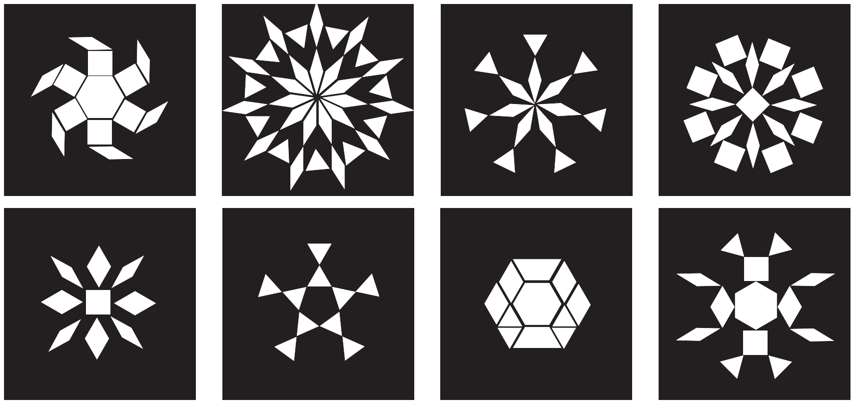
Because of the way these crystals are formed, they have balance, similarity, and repetition—which results in symmetric snowflakes. Most students build and draw with an innate sense of symmetry because it is visually pleasing to them. In today's session, students build on that intuitive design sensibility to create their own snowflakes from pattern block shapes.

Making Snowflake Designs

By third grade, many of your students will have built pattern block designs. Let them know that today's design will need to be a snowflake, like the kind Wilson Bentley would have photographed over a hundred years ago.

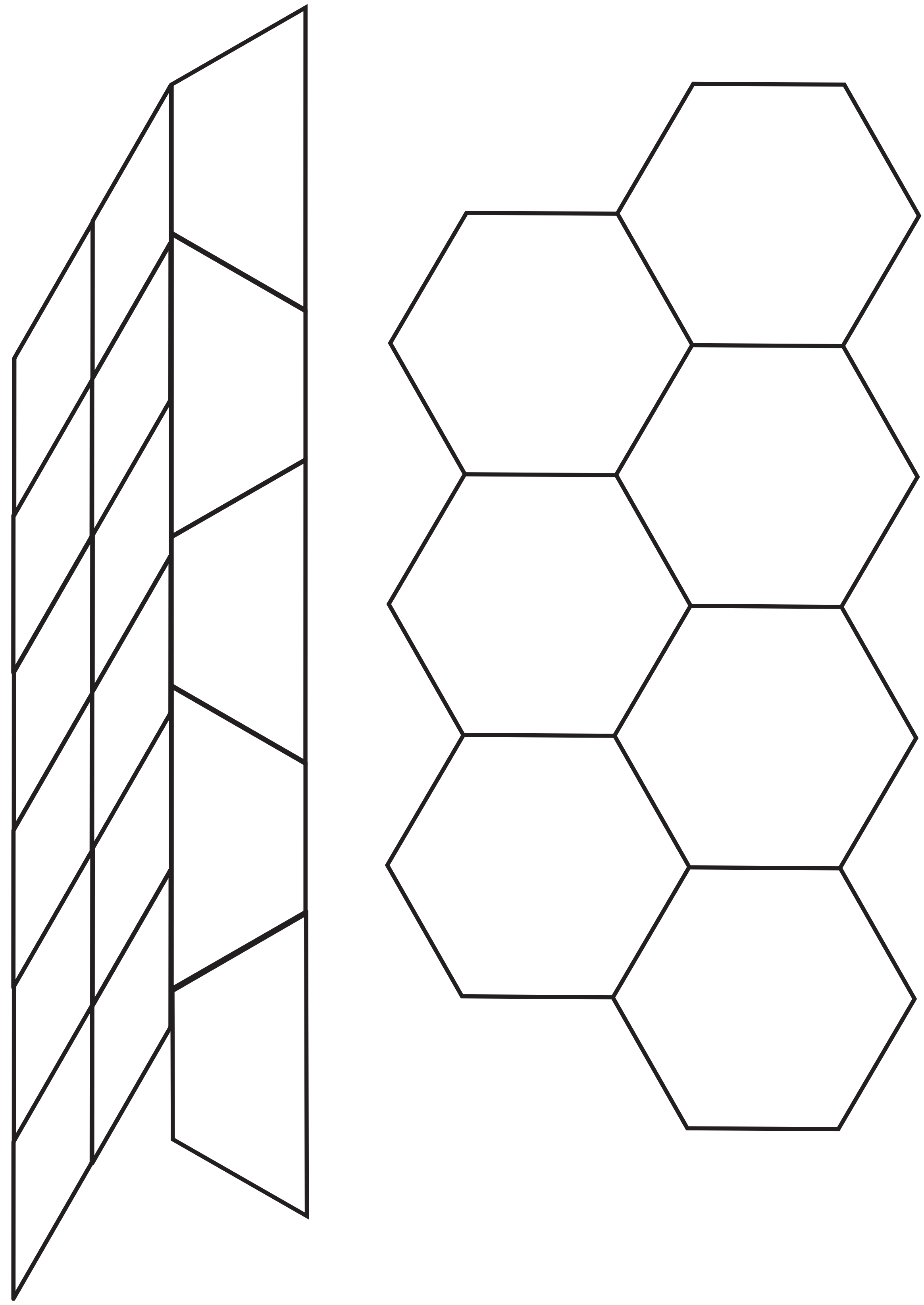
Give each student a pair of scissors, a copy of Snowflake Pattern Blocks, a glue stick, and a square of blue or black construction paper. Have them cut out the pattern block shapes and create snowflakes that have symmetry. Many of our students tried to construct a snowflake with 6 branches after our introduction about how snowflakes are formed. Remind them that each snowflake that falls from the sky is unique, and theirs should be too. Encourage them to use their imaginations and create snowflakes that aren't based on 6 branches, because you'll need snowflakes with a wide variety of lines of symmetry for this session.

Activity 2 Creating Symmetrical Snowflakes (cont.)

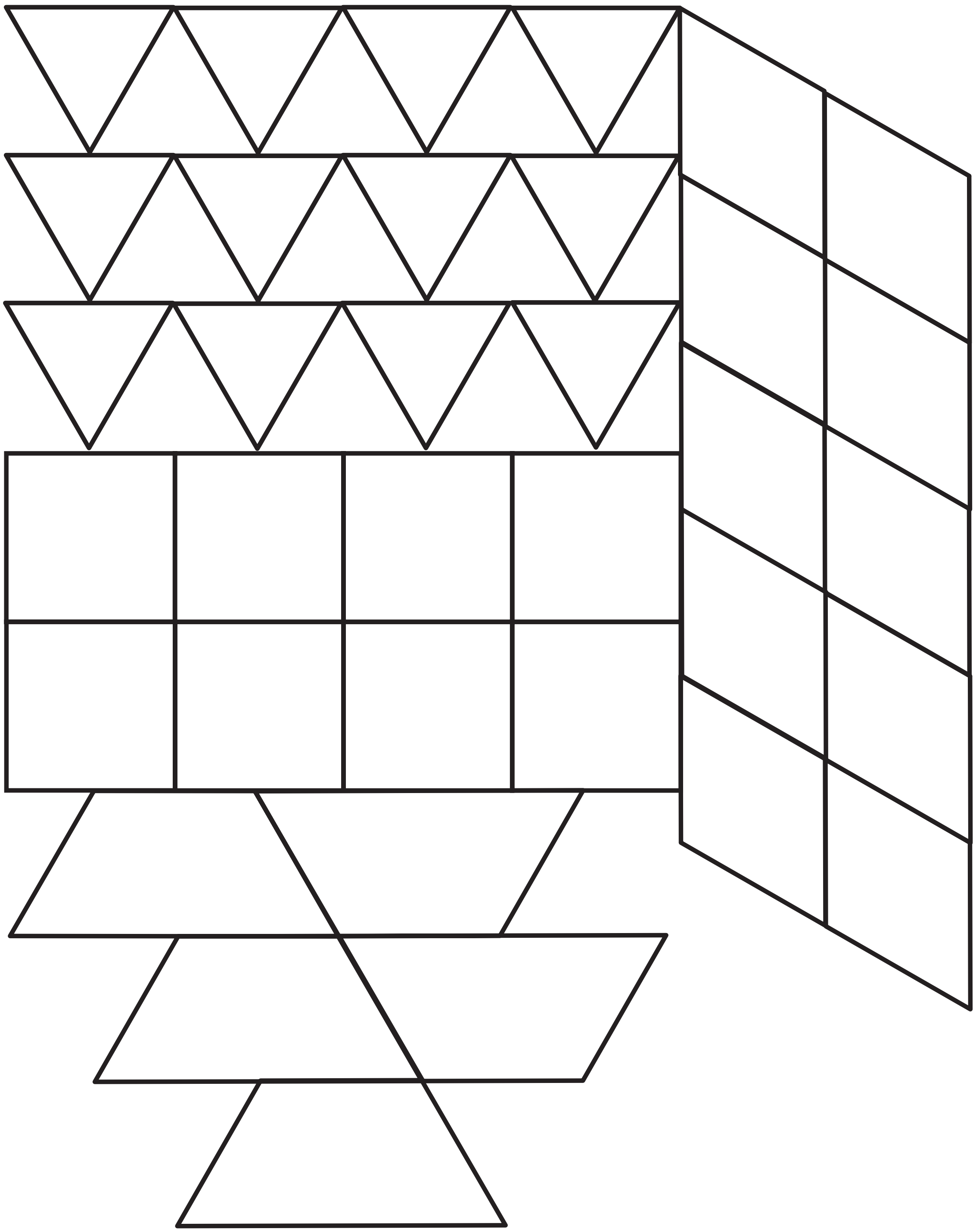


Have students set their completed snowflakes off to the side to use in Activity 3. They can write their names on the backs of their papers if they wish, but many of them will simply recognize their own work because it is unique.

Snowflake Pattern Blocks page 1 of 2



Snowflake Pattern Blocks page 2 of 2



Activity 3

PROBLEMS & INVESTIGATIONS

Sorting Snowflakes by Symmetry

Overview

Students compare the snowflakes they created in Activity 2. Then they sort and classify the snowflakes by the kinds of symmetry and the number of lines of symmetry they have. These snowflakes may be organized in the form of a bar graph for a wall display or put together to make a paper quilt with student comments attached.

Actions

1 In small groups of 4 and then as a class, students compare the snowflakes they created in the previous session.

2 The teacher and students define *line of symmetry.*

3 Together as a class, students count the lines of symmetry on a snowflake or two.

4 Student pairs count the lines of symmetry in their own snowflakes.

5 The class sorts their snowflakes by the number of lines of symmetry they have, discuss rotational symmetry, and then post observations about their class collection of snowflakes.

Skills & Concepts

★ Partition shapes into parts with equal areas (3.G.2)

★ Express the area of each equal part of a whole as a unit fraction of the whole (e.g., each of b equal parts is 1/b of the whole) (3.G.2)

★ Identify lines of symmetry (4.G.3)

★ Draw lines of symmetry (4.G.3)

★ Identify figures with line symmetry (4.G.3)

You'll need

★ Word Resource Card (line of symmetry, see Note)

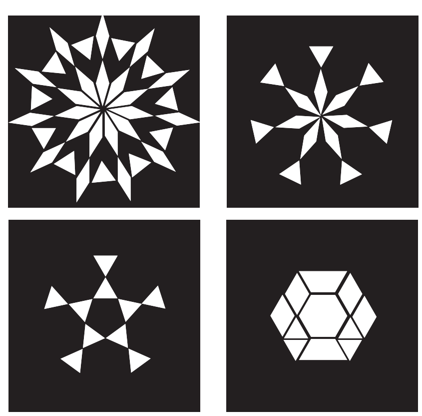
★ straws or pencils

★ students' snowflakes from Session 5

★ about thirty 3″ × 5″ index cards

★ erasable marker (e.g., Vis-à-Vis)

Note Teacher Masters to make Word Resource Cards are located at the end of this packet. Locate the necessary term. Run on cardstock and fold in half, tape edges.



Activity 3 Sorting Snowflakes by Symmetry (cont.)

Comparing Snowflakes

Invite students to examine their snowflakes in groups of 4. What's different and what's the same about their designs? Reconvene as a class and take a moment to celebrate all the beautiful snowflakes. Ask students to think quietly about what they notice about the class set of snowflakes. Then ask them to share with a partner, and ask a few volunteers to share with the whole group.

Students *They're all different.*
Some of them are bigger than the others.
Some people used lots of hexagons, and others used lots of triangles and rhombuses.
Some kind of look more like flowers or snowflakes, and others look like different kinds of designs.

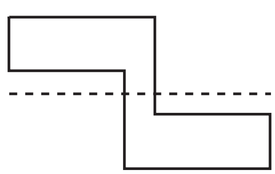
Defining Lines of Symmetry

Display the *line of symmetry* Word Resource Card in your pocket chart. Invite students to speculate about what this term means by looking at the pictures on the card.

Teacher *Based on these pictures and your past experiences, what do you think a line of symmetry is?*

Students *I think it's a line that cuts a shape in half.*
And those shapes are the same, the halves are, I mean.

Teacher *What about this shape? What if I draw a line like this? Is it a line of symmetry? Think to yourself, talk it over with a neighbor, and then we'll see what everyone thinks. … What did you decide? Is it a line of symmetry or not?*



Sara *We said yes. Because look, you cut it in half. The top is the same as the bottom, but just turned different.*

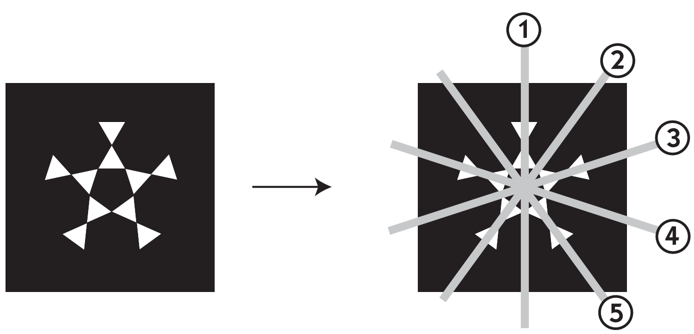
Jamal *We didn't think so. It is cut in half, but we think you have to be able to fold it so the halves overlap. So if it was a piece of paper and you folded it like that, the halves wouldn't match up.*

Teacher *Jamal's talking about what we call a fold test. That means that if you can fold a shape in half and those halves match up exactly, then the fold is a line of symmetry. So this line would not be a line of symmetry. The shape does have a special kind of symmetry, though, called rotational symmetry, which we'll come back to in a little while. Teacher Today we're going to find out how many lines of symmetry are in your snowflakes.*

Identifying Lines of Symmetry on Snowflakes as a Class

Select an example or two from the class set of snowflakes that have lines of symmetry. Place one where everyone can see it (preferably on the floor in the middle of a discussion circle), and ask students to think quietly to themselves for a minute. Can they see any lines of symmetry?

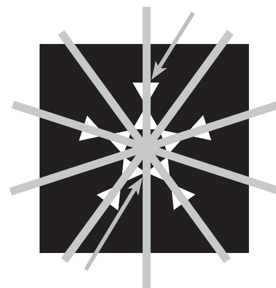
Then invite students to show the lines of symmetry, placing a straw or pencil on top of the snowflake to identify each one. In the diagram below, we've numbered the straws so it's easier to see that the snowflake has 5 discrete lines of symmetry. Once all lines of symmetry have been identified, it can be difficult for students to determine how many separate lines of symmetry there are. We find it helps to either keep a running tally of the number of straws they've placed or to simply let students place the straw and then count them all up at the end. Ask students to share what they notice about the lines of symmetry.



Omar *There were only 5 lines of symmetry, but it looks like more than that when they're all there at once. It's kind of confusing to look at.*

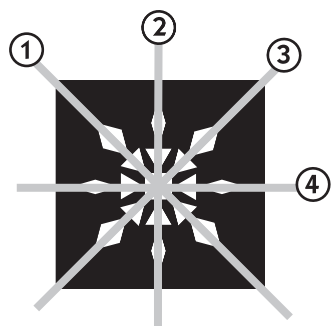
Yoshiko *All those lines of symmetry divide the snowflake in half the same way. See? They all go through one of those triangle parts that stick out and then through a dent on the other side.*

Activity 3 Sorting Snowflakes by Symmetry (cont.)



Identifying Lines of Symmetry on Snowflakes in Pairs

Next, ask students to pair up and work together to find the lines of symmetry in their own snowflakes. You can give them each some pencils or straws, which they can use to split their snowflakes in half to find lines of symmetry. If students are having trouble knowing where to start, you might want to suggest they look toward the center of the snowflake. In shapes with multiple lines of symmetry, the central point of the figure is the intersection of those lines of symmetry.



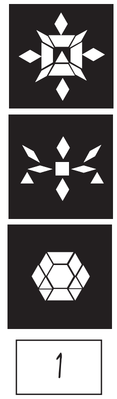
Sorting Snowflakes by Lines of Symmetry

When students are done, gather them back together as a group, and ask anyone whose snowflake had just one line of symmetry to show his snowflake to the group and identify the line of symmetry.

Then place all snowflakes with 1 line of symmetry above an index card with the number 1 written on it. If you have a group area space, you may prefer to lay them out on the floor. If not, consider taping the index cards and snowflakes onto the whiteboard.

> ***Teacher*** *As I was walking around listening to your conversations, I noticed that your snowflakes have many different lines of symmetry. Let's sort them according to how many lines of symmetry they have. First, who made a snowflake with just 1 line of symmetry? Let's arrange them over here by the 1 card.*

Activity 3 Sorting Snowflakes by Symmetry (cont.)



Continue to have students group their snowflakes by the number of lines of symmetry they have. Make the index card labels as you group students' snowflakes. As students share their snowflakes, ask them to show everyone where they see the lines of symmetry.

Discussing Snowflakes with Rotational Symmetry Only

Your class's snowflakes may have no examples for certain numbers of lines of symmetry. You may also have some snowflakes that have no lines of symmetry, either because they are not symmetrical or because a student has created a snowflake with rotational symmetry. In rotational symmetry, there is a central point that is the center of rotation.

> ***Maria*** *Mine doesn't work. It doesn't look the same when I put the strip down the middle. See? It spins around, but it isn't the same on both sides. I think it looks kind of like a pinwheel that you blow on and it spins around and around.*

> ***Teacher*** *Your snowflake has a special kind of symmetry. It's called rotational symmetry. When you turn or spin the shape a certain amount, it still looks the same. If we begin with this central point, here in the middle of Maria's snowflake, you can see how this square and rhombus*

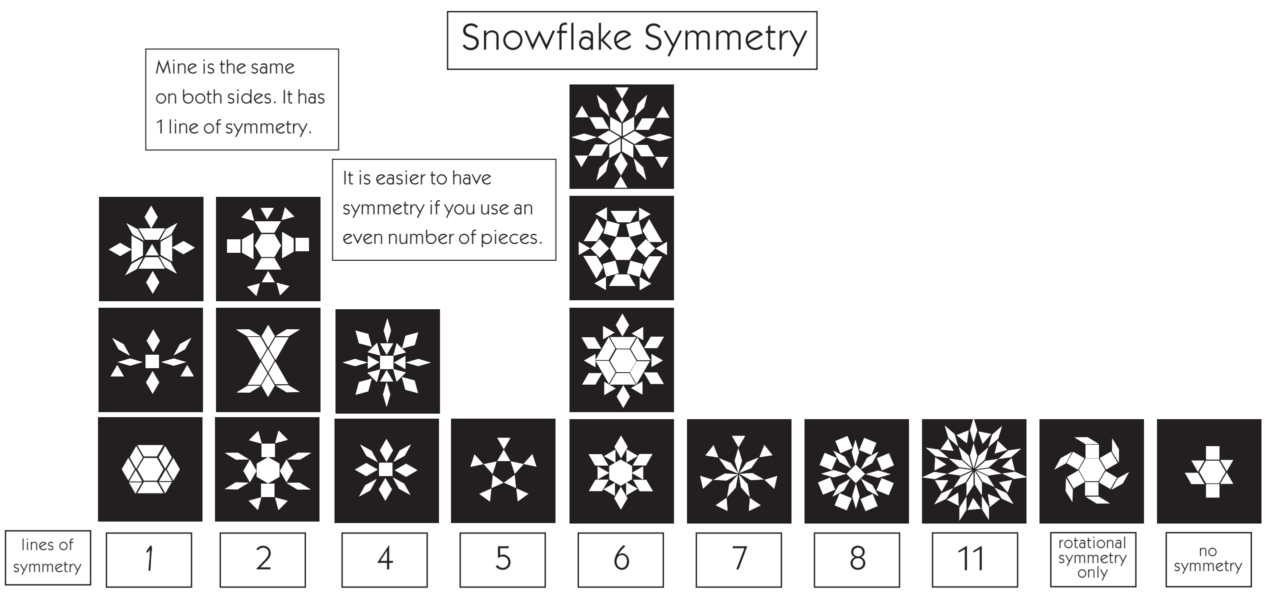
Activity 3 Sorting Snowflakes by Symmetry (cont.)

> *repeat in a series around the hexagon. Maria, let's make a special card for your snowflake. I'll label this card* rotational symmetry only. *Does anyone else have a snowflake with rotational symmetry?*

Posting Observations about the Snowflakes

When you've completed the display, take a few minutes to have students share some observations. Have them record them now on index cards and then keyboard their comments later during computer time so that spelling and grammatical errors can be edited before you post them on the display.

> ***Jamal*** *I used just 1 hexagon, 6 rhombuses, and 6 triangles. I think if you use even numbers, it's easier to get symmetry.*
>
> ***Kaiya*** *That's what I did, but I used different shapes.*
>
> ***Andre*** *I used some odd and some even numbers of pieces, and mine still had symmetry if you split it down the middle.*



Activity 1

PROBLEMS & INVESTIGATIONS

Pattern Block Symmetry

Overview

Students discuss the isosceles trapezoid and explore how they can use frames to identify its lines of reflective symmetry and rotational symmetries. Students then work in pairs to identify the reflective and rotational symmetries of all 6 pattern block shapes.

Actions

1 The class uses the idea of a frame to explore reflections and symmetry.

2 Students determine the symmetry in pattern blocks.

3 Students share their discoveries with the class.

Skills & Concepts

- ★ Classify 2-D figures based on the presence or absence of parallel lines, perpendicular lines, angles of a specified size. (4.G.2)
- ★ Identify right triangles (4.G.2)
- ★ Identify lines of symmetry (4.G.3)
- ★ Draw lines of symmetry (4.G.3)
- ★ Identify figures with line symmetry (4.G.3)

You'll need

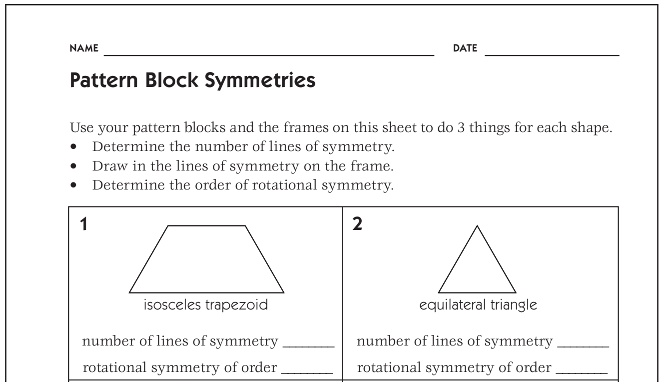
- ★ Pattern Block Symmetries (Teacher Master 1, run a class set plus one for display)
- ★ Word Resource Cards (line of symmetry, rotational symmetry. See Note)
- ★ pattern blocks
- ★ class set of rulers
- ★ erasable marker (e.g., Vis-à-Vis)
- ★ paper to mask sections of the Teacher Master on display

Note Teacher Masters to make Word Resource Cards are located at the end of this packet. Locate the necessary term. Run on cardstock and fold in half, tape edges.

Using Frames to Identify Symmetry in a Trapezoid

Ask students to get out their journals and pattern blocks, and display the top left section of the Pattern Block Symmetries Teacher Master. Ask students to find the next available page in their journals and trace around one of the trapezoid pattern blocks to create a frame like the one at the overhead. Then ask them to use the trapezoid pattern block and frame to explore some ways that a frame could be used to help see lines of symmetry.

Activity 1 Pattern Block Symmetry (cont.)

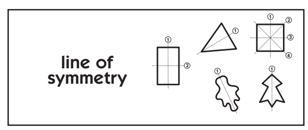

NAME ______________________ DATE ____________

Pattern Block Symmetries

Use your pattern blocks and the frames on this sheet to do 3 things for each shape.
- Determine the number of lines of symmetry.
- Draw in the lines of symmetry on the frame.
- Determine the order of rotational symmetry.

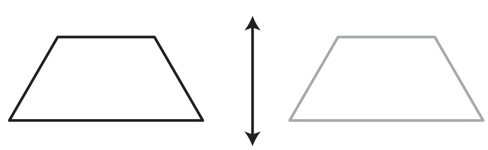
1	2
isosceles trapezoid	equilateral triangle
number of lines of symmetry _______	number of lines of symmetry _______
rotational symmetry of order _______	rotational symmetry of order _______

Before they begin, you may want to review the term *line of symmetry* with the class. What is a line of symmetry? How can they prove that a line is a line of symmetry? From past experiences, most fourth graders will know that a line of symmetry divides a figure into two congruent halves that are mirror images of each other, although they may not define it in such formal terms. Many may be familiar with the fold test, in which a figure is folded along a line to test its symmetry. If the fold results in one half covering the other exactly, the fold line is a line of symmetry. You can have students use a piece of scratch paper to demonstrate the fold test.



After they have had a minute to work independently with the trapezoid, have students demonstrate in pairs how they would use their frames to find reflections. Then have a pair of students come up to the overhead to share their thinking with the class. Be alert for opportunities to pose counter-examples, as the teacher does in the discussion below.

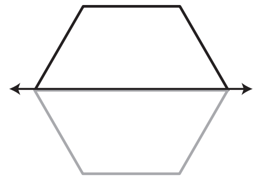
> ***James*** *We said that you can use this frame for a reflection, because you can flip over the pattern block and it fits right into the frame again.*
>
> ***Teacher*** *Hmm, if I flip the trapezoid over this line, though, it ends up here. So where would the reflection line be for what you just showed us?*



no line of symmetry

> ***Alec*** *If you do it that way it ends up outside of the frame. But there has to be a way to do it so it lands up in the frame, because it fits that way.*
>
> ***Nicole*** *I have an idea, use the bottom. … No, that doesn't work.*

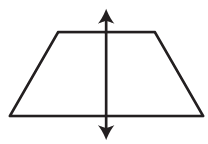
Activity 1 Pattern Block Symmetry (cont.)



no line of symmetry

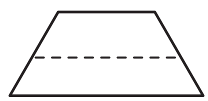
Antoine *I think you need to use the symmetry line. Here let me come up. If you flip it over the symmetry line, then this half goes here and this half goes there and it works.*

James *That's what we were trying to tell you at the beginning!*

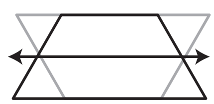


line of symmetry

Teacher *What about this line?*



Keith *No. That's like Nicole's idea. The trapezoid ends up upside down and it doesn't fit in the frame.*



no line of symmetry

Rafael *Also, if you cut out a paper one and you folded it on that line, the sides wouldn't match up. If it's a line of symmetry, they have to match up if you fold it.*

Bring the discussion to a close with a look at the Word Resource Cards for symmetry, line of symmetry, and rotational symmetry. Give students several minutes to write and draw their current understandings of these terms.

Determining Symmetry in Pattern Blocks

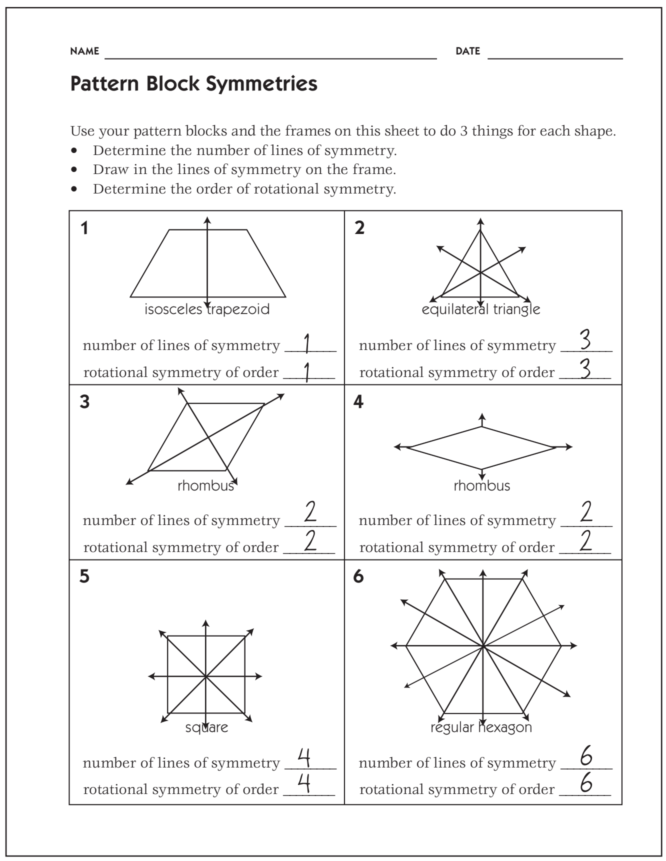
Distribute copies of the Pattern Block Symmetries Teacher Master and display a copy. Give students time to read the instructions on the worksheet. Then ask what they notice and if they understand what to do. You may need to let students know that an isosceles trapezoid is a trapezoid in which the non-parallel sides are equal in length. Invite students to complete the worksheet in pairs or small table groups. Remind them that if there are disagreements,

Activity 1 Pattern Block Symmetry (cont.)

they should try to work them out. If they cannot work them out, ask them to make a note on the worksheets and bring up the disagreement during the whole-class discussion that will follow.

Sharing Discoveries

After they have had time to work in pairs or small groups, ask volunteers to share their findings with the class. How many lines of symmetry and orders of rotational symmetry did they find for each figure? Does everyone agree? Did they notice anything interesting or surprising during their investigations? Many interesting questions and disagreements are likely to arise. Encourage students to justify their reasoning, using pattern blocks at the projector to show their thinking, and know that the conversation may end with some students still unconvinced or a bit confused. Rest assured that they will have many opportunities to make sense of these ideas in future sessions, units, and grade levels.


NAME ____________ DATE ________

Pattern Block Symmetries

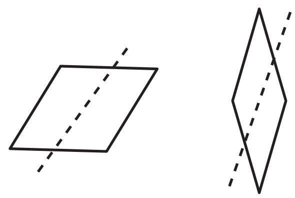
Use your pattern blocks and the frames on this sheet to do 3 things for each shape.
- Determine the number of lines of symmetry.
- Draw in the lines of symmetry on the frame.
- Determine the order of rotational symmetry.

1 isosceles trapezoid number of lines of symmetry 1 rotational symmetry of order 1	2 equilateral triangle number of lines of symmetry 3 rotational symmetry of order 3
3 rhombus number of lines of symmetry 2 rotational symmetry of order 2	4 rhombus number of lines of symmetry 2 rotational symmetry of order 2
5 square number of lines of symmetry 4 rotational symmetry of order 4	6 regular hexagon number of lines of symmetry 6 rotational symmetry of order 6

Note *Students often have a hard time recognizing lines of symmetry that are not horizontal or vertical. For instance, for the equilateral triangle, students may record the vertical line of symmetry and miss the other two. Some students will have similar trouble finding the lines of symmetry in the rhombuses and may propose*

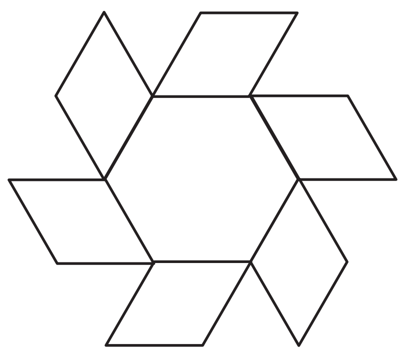
Activity 1 Pattern Block Symmetry (cont.)

symmetry lines that do not work. Some students are likely to argue that the lines shown below, for example, are symmetry lines.



mistaken for lines of symmetry

Students may have noticed that the order of rotational symmetry matches the number of lines of symmetry for each of the pattern blocks. If so, ask them to consider whether this will be true for any shape. It will not, as illustrated by the figure below, which has no lines of symmetry and rotational symmetry of order 6. Encourage students to debate the question, but don't insist that they come to a consensus on the matter today.



no lines of symmetry
rotational symmetry of order 6

Students may also have noted that for the triangle, square, and hexagon, the order of rotational symmetry and the number of lines of symmetry match the number of sides. Students may also note that these figures are all regular polygons (polygons in which all sides and all angles are equal).

Teacher Master 1 Run a class set plus one for display.

NAME ______________________________ **DATE** ______________

Pattern Block Symmetries

Use your pattern blocks and the frames on this sheet to do 3 things for each shape.

- Determine the number of lines of symmetry.
- Draw in the lines of symmetry on the frame.
- Determine the order of rotational symmetry.

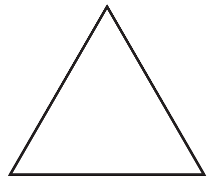
1

isosceles trapezoid

number of lines of symmetry _______

rotational symmetry of order _______

2

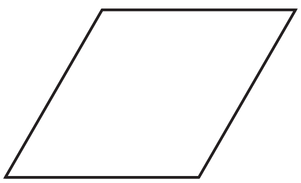


equilateral triangle

number of lines of symmetry _______

rotational symmetry of order _______

3

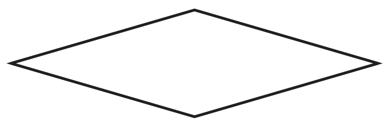


rhombus

number of lines of symmetry _______

rotational symmetry of order _______

4

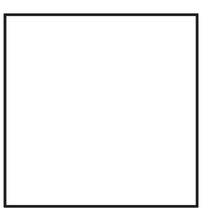


rhombus

number of lines of symmetry _______

rotational symmetry of order _______

5

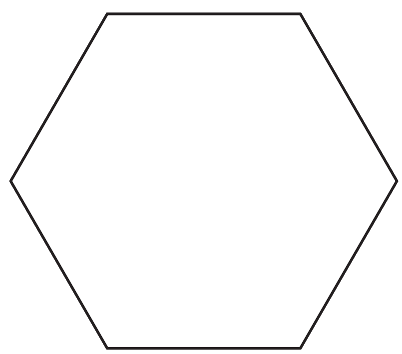


square

number of lines of symmetry _______

rotational symmetry of order _______

6



regular hexagon

number of lines of symmetry _______

rotational symmetry of order _______

Activity 2

WORK PLACE

Mosaic Game

Overview

Students roll a die to determine which 6 patten blocks they will use use to build a simple design. Player earn a point for the lines of symmetry and order or rotational symmetry found in their design. The student with the most points wins.

Skills & Concepts

- ★ Classify 2-D figures based on the presence or absence of parallel lines, perpendicular lines, angles of a specified size (4.G.2)
- ★ Identify right triangles (4.G.2)
- ★ Identify lines of symmetry (4.G.3)
- ★ Draw lines of symmetry (4.G.3)
- ★ Identify figures with line symmetry (4.G.3)

Each pair of students will need

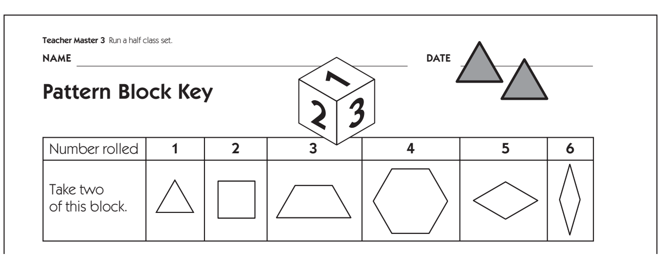
- ★ Work Place Instructions (Teacher Master 2, run a half class set.)
- ★ Pattern Block Key (Teacher Master 3, run a half class set.)
- ★ Mosaic Game Record Sheet (Teacher Master 4, run one and a half class sets.)
- ★ Mosaic Game Challenge (Teacher Master 5, run a half class set. Optional.)
- ★ 1–6 die
- ★ pattern blocks
- ★ pattern block templates, optional
- ★ tape

Note This Activity can be introduced as a game-teacher versus students and then played in pairs for guided practice.

Instructions for the Mosaic Game

1. Take turns rolling the die to see who will go first.

2. Roll the die 3 times. For each roll, take a pair of pattern blocks. The chart on Teacher Master 4 shows which pair of pattern blocks to take for each number on the die.


Teacher Master 3 Run a half class set.

NAME DATE

Pattern Block Key

Number rolled	1	2	3	4	5	6
Take two of this block.						

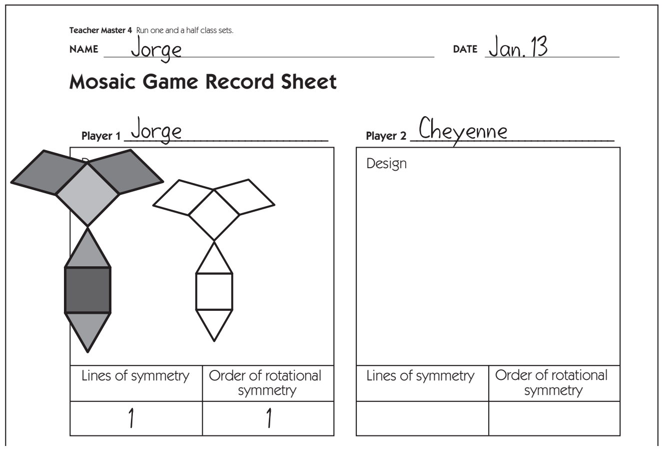
I rolled a 1, so the chart says to take 2 triangles.

3. Make a design with the 6 pattern blocks. You will get a point for every line of symmetry and order of rotational symmetry in your design. Tape the pat-

Activity 2 Mosaic Game (cont.)

tern blocks together if you need to rotate your shape to determine its order of rotational symmetry.

4. Draw the design on your record sheet. Use the pattern block template if you need to. Write the number of lines of symmetry and the order of rotational symmetry your design has.



Teacher Master 4 Run one and a half class sets.

NAME Jorge DATE Jan. 13

Mosaic Game Record Sheet

Player 1 Jorge Player 2 Cheyenne

Design

Lines of symmetry	Order of rotational symmetry	Lines of symmetry	Order of rotational symmetry
1	1		

5. Take turns until you and your partner have gone twice. Record the designs and scores for you and your partner. After 2 rounds, add together all your numbers. The player with the highest total score wins.

Instructional Considerations for the Mosaic Game

Encourage students to help each other and confirm that they have each identified all the lines of symmetry and rotational symmetries in each figure.

If you notice that students are finding it cumbersome to sketch their own and their partner's designs, invite them to sketch only their own designs. If students are creating ambitious designs and need more room to sketch, invite them to record their designs in their journals, instead of on the record sheets.

If a player rolls 3 different numbers, it is hard to get a score above 2. If a player rolls 2 of the same number (thereby getting 4 of the same pattern block), his or her ability to create a multi-symmetric design is enhanced, as shown here. Some students may notice this after playing a few rounds.

Activity 2 Mosaic Game (cont.)

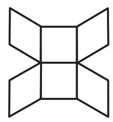
Teacher Master 4 Run one and a half class sets.

NAME Becky DATE Jan. 13

Mosaic Game Record Sheet

Player 1 Becky **Player 2** Lilly

Design



Player 1: Lines of symmetry	Player 1: Order of rotational symmetry	Player 2: Lines of symmetry	Player 2: Order of rotational symmetry
2	2		

CHALLENGE

Students who are interested in extending their opportunities to build and score can use the challenge sheet, which directs them to use combinations of different numbers of each pattern block. You might also invite them to decide with their partner whether they want their shapes to be symmetrical with regard to both color and shape, or only to shape.

Teacher Master 5 Run a half class set. Optional.

NAME ______ DATE ______

Mosaic Game Challenge

Number rolled	**1**	**2**	**3**	**4**	**5**	**6**
Take this many of this block.	4	2	2	1	3	4

Player 1 ______

Design

Lines of symmetry	Order of rotational symmetry

Player 2 ______

Design

Lines of symmetry	Order of rotational symmetry

Work Place Instructions

WORK PLACE

Mosaic Game

Each pair of students will need

★ Work Place Instructions (Teacher Master 2)

★ Pattern Block Key (Teacher Master 3)

★ Mosaic Game Record Sheet (Teacher Master 4)

★ Mosaic Game Challenge Sheet (Teacher Master 5)

★ 1–6 die

★ pattern blocks

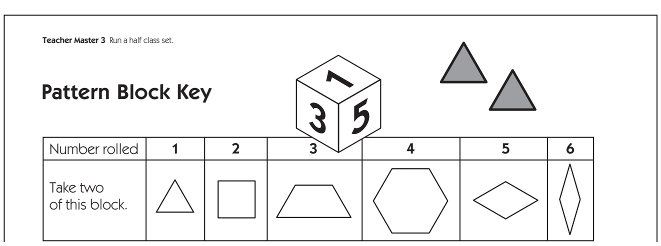
★ pattern block stencils, optional

★ tape

Instructions for the Mosaic Game

1 Take turns rolling the die to see who will go first.

2 Roll the die 3 times. For each roll, take a pair of pattern blocks. The Pattern Block Key on page 66 shows which pair of pattern blocks to take for each number on the die.

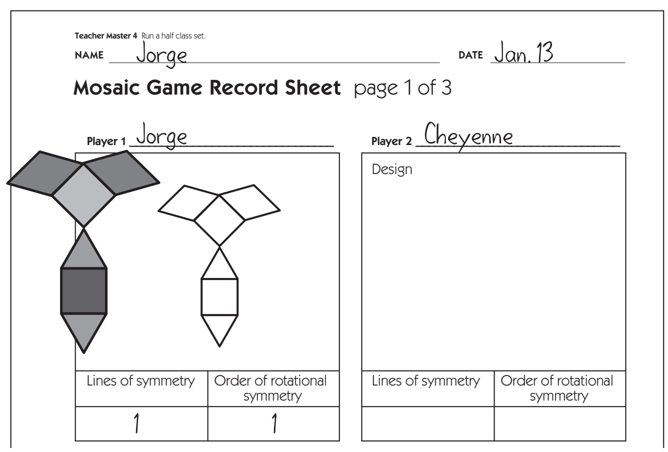

Teacher Master 3 Run a half class set.

Pattern Block Key

Number rolled	1	2	3	4	5	6
Take two of this block.						

3 Make a design with the 6 pattern blocks. You will get a point for every line of symmetry and order of rotational symmetry in your design. Tape the pattern blocks together if you need to rotate your shape to determine its order of rotational symmetry.

4 Draw the design on your record sheet. Use the pattern block stencil if you need to. Write the number of lines of symmetry and the order of rotational symmetry your design has.


Teacher Master 4 Run a half class set.

NAME Jorge DATE Jan. 13

Mosaic Game Record Sheet page 1 of 3

Player 1 Jorge Player 2 Cheyenne

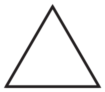
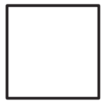
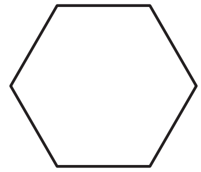
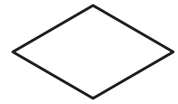
Design

Lines of symmetry	Order of rotational symmetry	Lines of symmetry	Order of rotational symmetry
1	1		

5 Take turns until you and your partner have gone twice. Record the designs and scores for you and your partner. After 2 rounds, add together all your numbers. The player with the highest total score wins.

NAME ______________________ **DATE** ______________

Pattern Block Key

Number rolled	**1**	**2**	**3**	**4**	**5**	**6**
Take two of this block.						

NAME ______________________________ **DATE** ______________

Mosaic Game Record Sheet

Player 1 ______________________________

Design	
Lines of symmetry	Order of rotational symmetry

Design	
Lines of symmetry	Order of rotational symmetry

Player 1 total points __________

Player 2 ______________________________

Design	
Lines of symmetry	Order of rotational symmetry

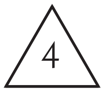
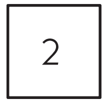
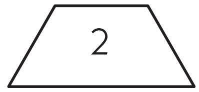
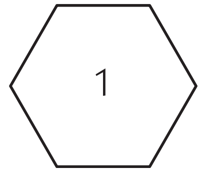
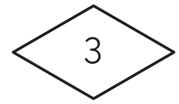
Design	
Lines of symmetry	Order of rotational symmetry

Player 2 total points __________

Teacher Master 5 Run a half class set. Optional.

NAME ______________________________ **DATE** ______________

Mosaic Game Challenge

Number rolled	**1**	**2**	**3**	**4**	**5**	**6**
Take this many of this block.	 4	 2	 2	 1	 3	 4

Player 1 ______________________________

Design

Lines of symmetry	Order of rotational symmetry

Player 2 ______________________________

Design

Lines of symmetry	Order of rotational symmetry

Activity 1

PROBLEMS & INVESTIGATIONS

Pattern Block Angles

Overview
This session begins a multi-session exploration of angle measure as it relates to both geometry and measurement. In this session, students review different kinds of angles. Using the fact that a straight angle is 180°, they determine all the angle measures in the pattern block shapes. They will continue this investigation and discuss their strategies and findings in Activity 2.

Actions
1 Students review five kinds of angles as a whole group.

2 Students are introduced to the idea of measuring angles by degrees.

3 Students discuss the first problem on the worksheets.

4 Students investigate angle measure further by completing the worksheets.

5 Students who finish may begin a 2-page challenge worksheet.

Skills & Concepts
The skills and concepts listed below are addressed throughout Activites 1–3.

★ Identify an angle as a geometric figure formed where two rays share a common endpoint (4.MD.5)

★ Demonstrate an understanding that an angle that turns through 1⁄360 of a circle is called a “one-degree angle” (4.MD.5a)

★ Demonstrate that a “one-degree angle” can be used to measure other angles (4.MD.5a)

★ Measure angles by identifying the fraction of the circular arc between the points where the two rays forming the angle intersect the circle whose center is at the endpoints of those rays (4.MD.5a)

★ Identify the measure of an angle by identifying the total number of one-degree angles through which it turns (4.MD.5b)

★ Sketch an angle of a specified measure (4.MD.6)

★ Decompose an angle into non-overlapping parts (4.MD.7)

★ Express the measure of an angle as the sum of the angle measures of the non-overlapping parts into which it has been decomposed (4.MD.7)

★ Demonstrate an understanding that angle measure is additive (4.MD.7)

★ Identify points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular lines in 2-D figures (4.G.1)

★ Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular lines (4.G.1)

★ Solve problems involving finding the unknown angle in a diagram, using addition and subtraction (4.MD.7)

★ Demonstrate an understanding that attributes of a category of 2-dimensional figures also belong to all subcategories of that category (5.G.3)

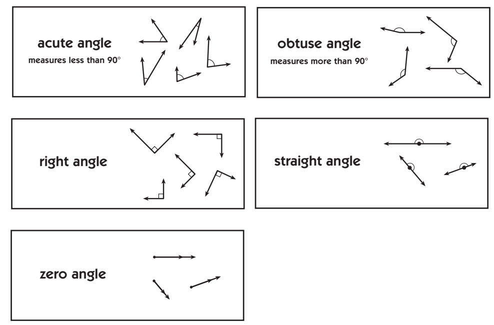
★ Use facts about supplementary, complementary, vertical, and adjacent angles to write and solve simple equations for an unknown angle in a figure (7.G.5)

Activity 1 Pattern Block Angles (cont.)

You'll need

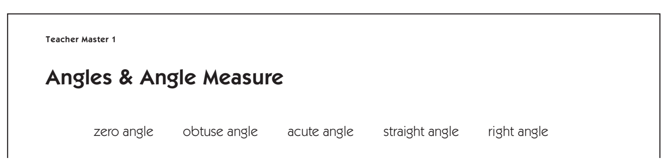
- ★ Angles & Angle Measure (Teacher Master 1, 1 copy for display)
- ★ Pattern Blocks & Angle Measure, pages 1 and 2 (Teacher Masters 2 and 3, run a class set plus 1 for display)
- ★ Pattern Block Angle Puzzles, pages 1 and 2 (Teacher Masters 4 and 5, run a class set)
- ★ Word Resource Cards (acute angle, obtuse angle, right angle, straight angle, zero angle). See note.
- ★ class set of pattern blocks
- ★ world map (optional)
- ★ 2 new, sharpened pencils
- ★ erasable marker (e.g., Vis-à-Vis)
- ★ piece of paper for masking portions of the Teacher Master on display

Note Teacher Masters to make Word Resource Cards are located at the end of this packet. Locate the necessary term. Run on cardstock and fold in half, tape edges. Place the Word Resource Cards listed above on display before the session starts.



Reviewing the Definitions of Angles

Display the top section of the Angles & Angle Measure Teacher Master. Using the Word Resource Cards you have on display, have students briefly share what they remember (and can ascertain from the illustrations) about each kind of angle.

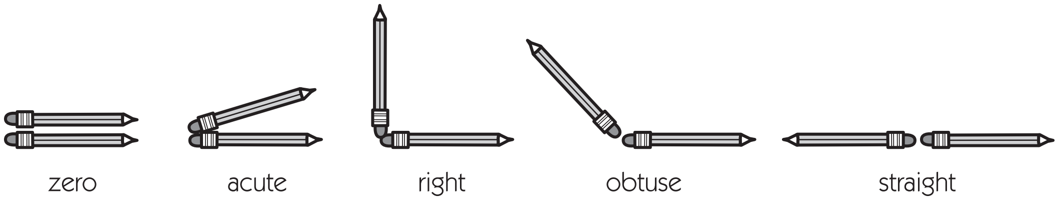

Teacher Master 1

Angles & Angle Measure

zero angle obtuse angle acute angle straight angle right angle

Use two new, sharpened pencils to demonstrate these five kinds of angles. Hold the pencils together to form a zero angle. Then slowly open them to form, in order, an acute angle, a right angle, an obtuse angle, and finally, a straight angle. For this session and some that follow, it is important that students understand what a straight angle is. Use the tips of the pencils to help students see the difference between a zero and a straight angle. Show that for a zero angle, the tips will be pointing in the same direction, while for a straight angle they will be pointing in different directions. After demonstrating these angles, you might go through the sequence one more time and ask students to make the different kinds of angles with their own pencils.

Activity 1 Pattern Block Angles (cont.)

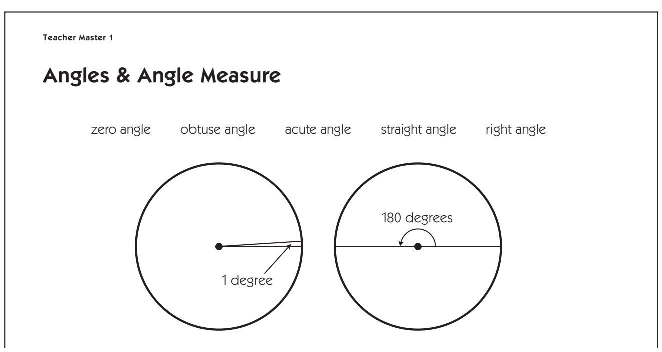


Introducing the Idea of Measuring Angles

In sharing what they know about the different kinds of angles, one or more students will probably have mentioned that a right angle measures exactly 90 degrees. Revisit this observation as a way to review what a degree is. If pressed to explain what a *degree* is, many students will express some confusion.

> ***Teacher*** *You mentioned that a right angle is an angle that measures 90 degrees, no more and no less. So what is a degree?*
>
> ***Rian*** *Mmm. It's how much the angle is.*
>
> ***Ichiro*** *I thought degrees said how warm or cold it is. I'm not sure why an angle has degrees.*

Explain that when we talk about angles (as opposed to temperature), a degree is a unit used to measure an angle between two line segments or rays. Degrees can also measure rotation, how far something has been rotated from a starting point. You can use the middle portion of the Angles & Angle Measure Teacher Master while you read the passage below to provide some historical background about where the degree came from.


Teacher Master 1

Angles & Angle Measure

zero angle obtuse angle acute angle straight angle right angle

> The idea of measuring angles in degrees was invented about 3700 years ago in ancient Babylonia. The Babylonians, worked in a base sixty system. (One of the reasons we work in base ten is because we have 10 fingers. No one is quite sure why the Babylonians worked in base sixty. Can you think of any good reasons to use that number?)
>
> The Babylonians divided the circle into 360 parts, calling each tiny part a degree. How does 360 relate to 60? (6×60 is 360) Half of 360 is 180, so it takes 180 degrees to rotate halfway around a circle. Along with degree measure, we have inherited from the Babylonians two other measures

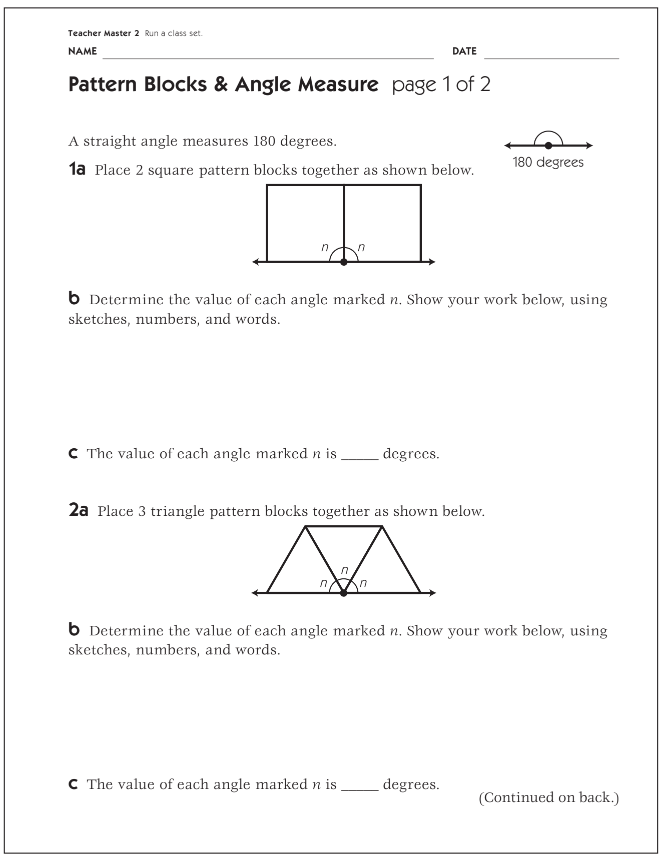
Activity 1 Pattern Block Angles (cont.)

that use 60. Can you think of what they are? (60 minutes in an hour and 60 seconds in a minute)

Note *Babylon, the capital of ancient Babylonia, was located on the Euphrates River, about 110 kilometers south of modern Baghdad in Iraq. You might ask students to find the approximate location on a world map. You might also explain that there are a variety of theories as to why the Babylonians utilized a base sixty system. If you count the 3 parts of the 4 fingers on your right hand with each of the 5 digits on your left hand, you arrive at 60. Also, 60 is the smallest number divisible by 1, 2, 3, 4, and 5. None of the theories put forth so far has been deemed conclusive.*

Discussing the First Problem on the Pattern Blocks & Angle Measure Worksheet

Ask students to get out their pattern blocks and distribute copies of the Pattern Blocks & Angle Measure Teacher Masters.


Teacher Master 2 Run a class set.
NAME ______ DATE ______

Pattern Blocks & Angle Measure page 1 of 2

A straight angle measures 180 degrees.

1a Place 2 square pattern blocks together as shown below.

b Determine the value of each angle marked *n*. Show your work below, using sketches, numbers, and words.

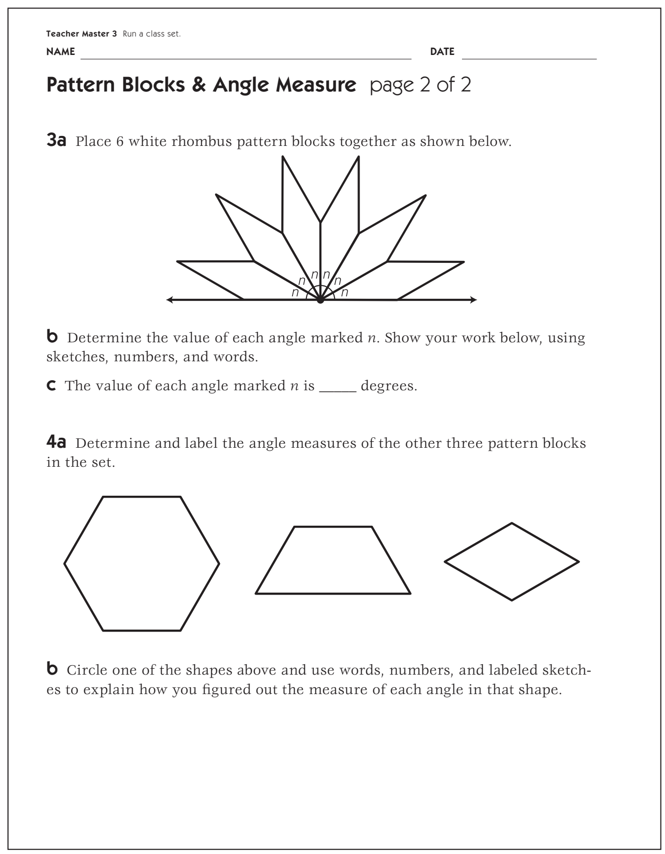
c The value of each angle marked *n* is _____ degrees.

2a Place 3 triangle pattern blocks together as shown below.

b Determine the value of each angle marked *n*. Show your work below, using sketches, numbers, and words.

c The value of each angle marked *n* is _____ degrees.

(Continued on back.)


Teacher Master 3 Run a class set.
NAME ______ DATE ______

Pattern Blocks & Angle Measure page 2 of 2

3a Place 6 white rhombus pattern blocks together as shown below.

b Determine the value of each angle marked *n*. Show your work below, using sketches, numbers, and words.

c The value of each angle marked *n* is _____ degrees.

4a Determine and label the angle measures of the other three pattern blocks in the set.

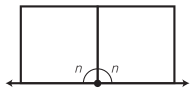
b Circle one of the shapes above and use words, numbers, and labeled sketches to explain how you figured out the measure of each angle in that shape.

Display the first question at the bottom of the Angles & Angle Measure Teacher Master. Ask students to work in pairs to come up with at least two different ways to solve the problem. While some students will already know that a square has 4 right angles, each of which measures 90º, they might be surprised that an unknown angle measure can be represented with a variable.

Activity 1 Pattern Block Angles (cont.)

A straight angle measures 180 degrees.

1 Place 2 square pattern blocks together as shown below.



2 Determine the value of each angle marked *n*. Show your work below, using sketches, numbers, and words.

3 The value of each angle marked *n* is ______ degrees.

Nick *I know that* n *is 90 degrees because those are right angles in the squares.*

Blanca *And 90 + 90 = 180.*

Jon *I don't understand what that has to do with the problem. We already know that a right angle has 90º.*

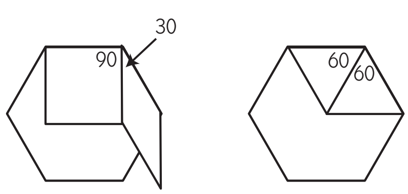
Blanca *The angles together add up to 180 degrees because they make a straight line together. So since 90 + 90 = 180, each angle must be 90.*

Morgan *I sort of thought of it like Blanca, but instead of adding I divided. The 2 angles put together make a straight line, so I divided 180 by 2 to get the answer: 90º.*

Investigating Angle Measure Further by Completing the Two Worksheets

Have students spend the remainder of the session completing the worksheets. Problems 2 and 3 are similar to the first problem, and students might use the same sorts of methods to solve all three. Problem 4 asks students to find the measures of the angles in the hexagon, trapezoid, and blue rhombus. You might encourage them to use what they discovered in questions 1–3 to help, although many will do this on their own. They can use various combinations of the angle measures they have already determined to figure out the angle measures in the hexagon, trapezoid, and blue rhombus.

CHALLENGE



Activity 1 Pattern Block Angles (cont.)

If necessary, let students know that they will have a little more time to complete the pages at the beginning of next session, and you'll ask volunteers to share their solutions and strategies then.

Pattern Block Angle Puzzles

Although some students may need additional time at the beginning of Activity 2 to complete the two pages assigned today, others may finish today with time to spare. Invite them to start working on Teacher Masters 4 and 5, Pattern Block Angle Puzzles. Although even the most capable probably won't have time to complete all of the problems right now, they can return to these problems as they have time.

Teacher Master 4 Run a class set.

NAME ______________________ DATE __________

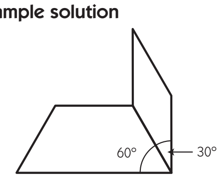
Pattern Block Angle Puzzles page 1 of 2

CHALLENGE

If possible, put 2 different (noncongruent) pattern blocks together to make the angles listed below. For each one, trace around the pattern blocks to make a labeled sketch of your solution. If you can find more than one way to make the angle, show it.

1 right angle (90°)

sample solution



$60° + 30° = 90°$

your solution

2 an obtuse angle (greater than 90° and less than 180°)

first solution

second solution

(Continued on back.)

Teacher Master 5 Run a class set.

NAME ______________________ DATE __________

Pattern Block Angle Puzzles page 2 of 2

CHALLENGE

3 a straight angle (180°)

first solution

second solution

4 a reflex angle (greater than 180°)

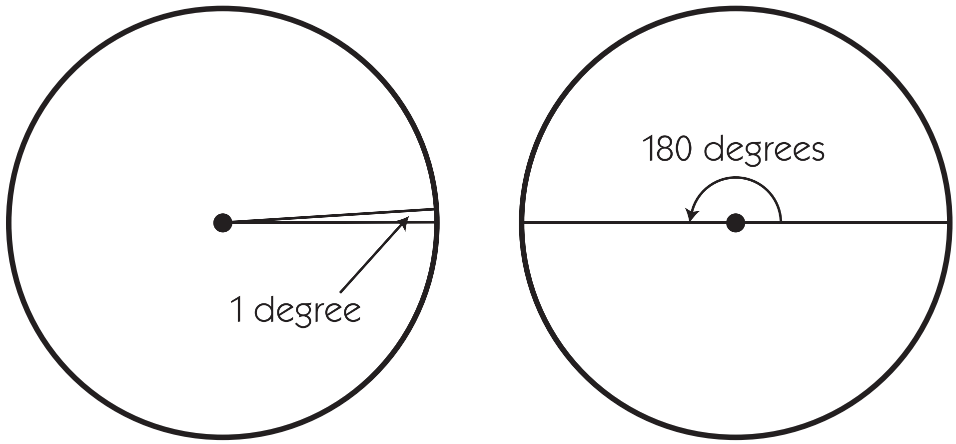
first solution

second solution

5 Mai says it's impossible to put 2 different pattern blocks together to make an acute angle (an angle less than 90°). Do you agree or disagree with her? Explain your answer.

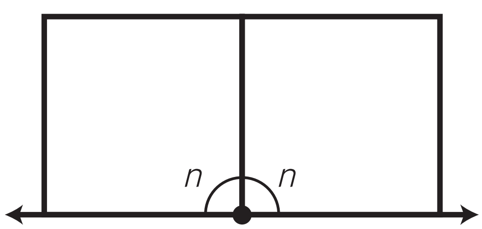
Angles & Angle Measure

zero angle obtuse angle acute angle straight angle right angle



A straight angle measures 180 degrees.

1a Place 2 square pattern blocks together as shown below.



b Determine the value of each angle marked n. Show your work below, using sketches, numbers, and words.

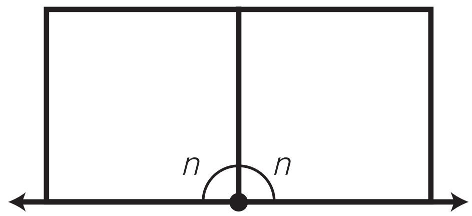
c The value of each angle marked n is ______ degrees.

NAME ______________________________ **DATE** ______________

Pattern Blocks & Angle Measure page 1 of 2

A straight angle measures 180 degrees.

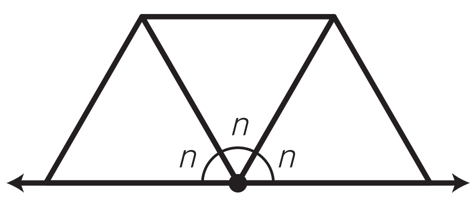
1a Place 2 square pattern blocks together as shown below.



b Determine the value of each angle marked *n*. Show your work below, using sketches, numbers, and words.

c The value of each angle marked *n* is _____ degrees.

2a Place 3 triangle pattern blocks together as shown below.



b Determine the value of each angle marked *n*. Show your work below, using sketches, numbers, and words.

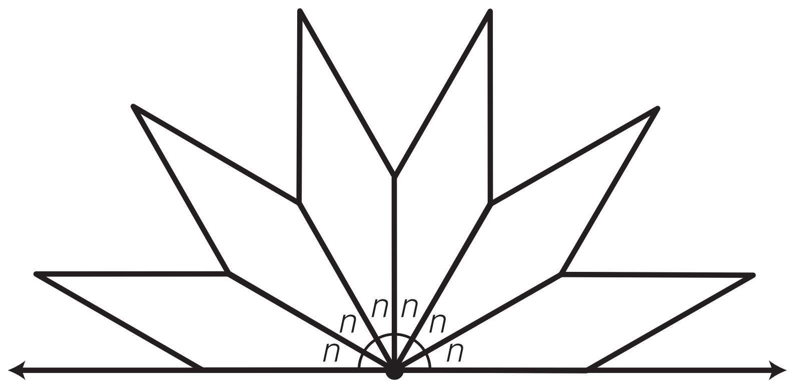
c The value of each angle marked *n* is _____ degrees.

(Continued on back.)

NAME ______________________________ **DATE** ______________

Pattern Blocks & Angle Measure page 2 of 2

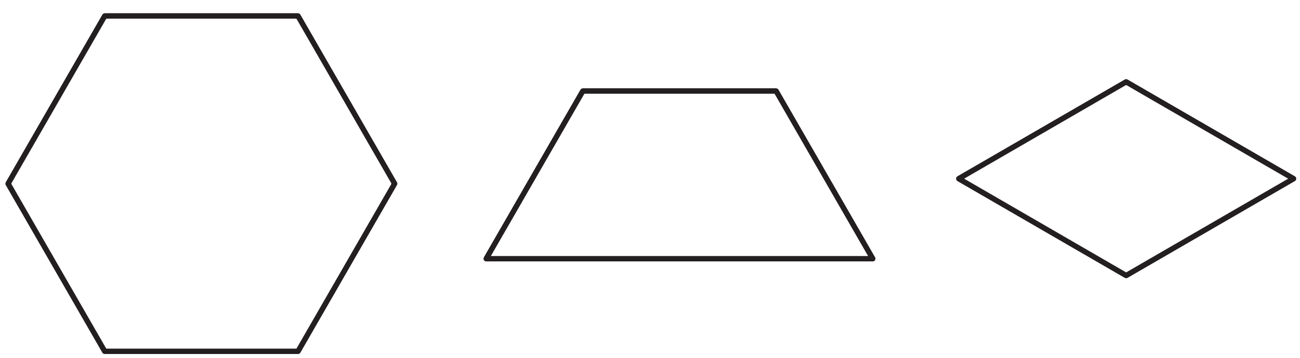
3a Place 6 white rhombus pattern blocks together as shown below.



b Determine the value of each angle marked *n*. Show your work below, using sketches, numbers, and words.

c The value of each angle marked *n* is _____ degrees.

4a Determine and label the angle measures of the other three pattern blocks in the set.



b Circle one of the shapes above and use words, numbers, and labeled sketches to explain how you figured out the measure of each angle in that shape.

NAME ______________________ **DATE** ____________

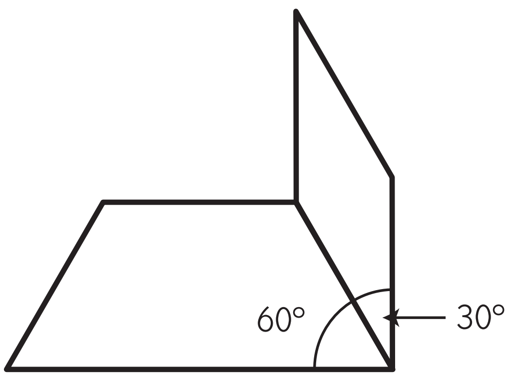
Pattern Block Angle Puzzles page 1 of 2

CHALLENGE

If possible, put 2 different (noncongruent) pattern blocks together to make the angles listed below. For each one, trace around the pattern blocks to make a labeled sketch of your solution. If you can find more than one way to make the angle, show it.

1 right angle (90°)

sample solution



60° + 30° = 90°

your solution

2 an obtuse angle (greater than 90° and less than 180°)

first solution

second solution

(Continued on back.)

NAME ____________________ **DATE** ____________

Pattern Block Angle Puzzles page 2 of 2

CHALLENGE

3 a straight angle (180º)

first solution

second solution

4 a reflex angle (greater than 180º)

first solution

second solution

5 Mai says it's impossible to put 2 different pattern blocks together to make an acute angle (an angle less than 90º). Do you agree or disagree with her? Explain your answer.

Activity 2

PROBLEMS & INVESTIGATIONS

Angle Measures in Triangles & Quadrilaterals

Overview

Students use paper cutting activities to illustrate that the sum of the angle measures in any triangle is 180° and that the sum of the angle measures in any quadrilateral is 360°. First, students draw a scalene triangle on construction paper and cut it out. Then they tear off all the vertices and piece them together to discover that they form a straight angle. They repeat the process with a quadrilateral to discover that the sum of the vertices is 360°.

Actions

1 Students complete and discuss the Pattern Blocks & Angle Measure worksheets from the previous session.

2 After watching a demonstration by the teacher, students piece together the vertices of different triangles and discuss their findings.

3 Students piece the vertices of a quadrilateral together.

4 Students sketch and describe the results of their explorations in their journals.

Skills & Concepts

Please see Activity 1 for the list of skills and concepts addressed in this lesson.

You'll need

★ Angle Measures in Figures (Teacher Master 6, run a class set, plus 1 for display)

★ Pattern Blocks & Angle Measure, pages 1 and 2 (Teacher Masters 2 and 3, from Activity 1)

★ Pattern Block Angle Puzzles, pages 1 and 2 (Teacher Masters 4 and 5, from Activity 1)

★ student journal or paper

★ class set of pattern blocks

★ erasable marker (e.g., Vis-à-Vis)

★ class set of scissors

★ class set of rulers

★ transparent tape for student use

★ class set of 9″ × 12″ light-colored construction paper, plus extra

Advance Preparation Draw a large scalene triangle (all sides different lengths) on 9″ × 12″ construction paper and cut it out before today's session.

Completing and Discussing the Pattern Blocks & Angle Measure Worksheets

If some students need additional time to finish Teacher Masters 2 and 3, have them get out their work sheets and pattern blocks, and let them know how long they have to work. Students who have finished these two pages can work on Teacher Masters 4 and 5, Pattern Block Angle Puzzles. (If students completed Teacher Masters 2 and 3 yesterday, move on to the class discussion below.)

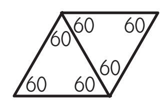
Activity 2 Angle Measures in Triangles & Quadrilaterals (cont.)

When most students have completed Teacher Masters 2 and 3, ask them to share their answers to Problems 2 and 3, but plan to devote the majority of the discussion to problem 4. Begin by asking students to pair-share how they determined the angle measures in the blue rhombus and how they verified their solutions. As students are talking with one another, circulate and watch for strategies you'd like them to share with the whole group. Keep an eye out for strategies that will lead to the correct answers, as well as ones that will stir up a bit of puzzlement. When they have had a few minutes to talk in pairs, invite specific students to share their strategies with the class.

Best Practice Tip If you select a student who has made an error that will be instructive for the class, be sure the student is confident, and take care to handle the discussion with sensitivity.

Teacher *Raven, would you share how you thought about the measure of each angle in the blue rhombus?*

Raven *I made the rhombus out of 2 green triangles. Each angle of the triangle is 60º, so the small angle of the rhombus is 60º and the larger one is 120º because it is 2 of those angles put together.*

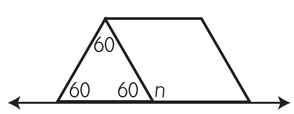


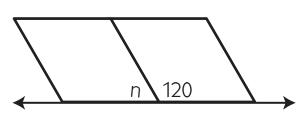
Teacher *Did you find a different method of verifying your solution?*

Raven *No, I didn't think about doing that, but my partner got the same thing, so I figured we were both right.*

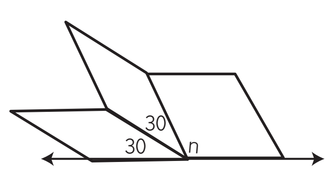
Teacher *That's okay. Did anyone use a different way to find the measure of those angles?*

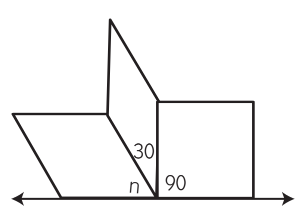
Maria *I put the triangle next to the rhombus to form a straight angle and I could see that the obtuse angle on the rhombus is 180º minus 60º and that's 120º. Then I put 2 rhombuses together like this and figured out that the acute angle is 180º minus 120º and that's 60º.*





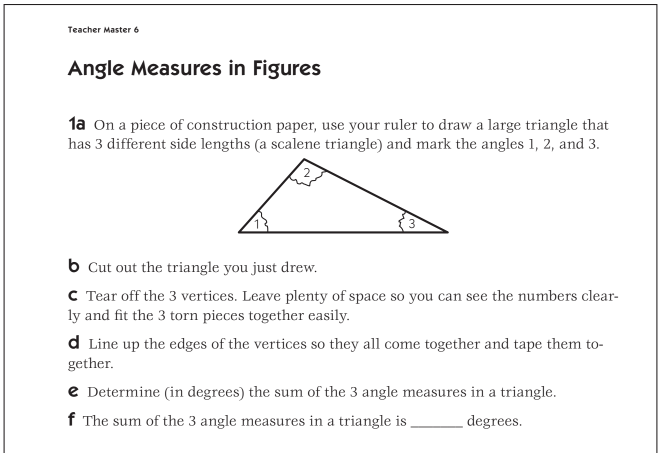
Justin *I used 2 white rhombuses and 1 blue one to make a straight angle. Since the white ones are each 30º, the big angle on the blue one must be 120º. Then I had fun and used a square and a white rhombus. Those angles add up to 120º. So 60 more makes 180º.*





Piecing the Vertices of a Triangle Together

Now display the top portion of the Angle Measures in Figures Teacher Master. Read the directions out loud, and then model each step as students watch. (If you cut out a triangle already, you can number the 3 angles and move on to step c.)



Teacher Master 6

Angle Measures in Figures

1a On a piece of construction paper, use your ruler to draw a large triangle that has 3 different side lengths (a scalene triangle) and mark the angles 1, 2, and 3.

b Cut out the triangle you just drew.

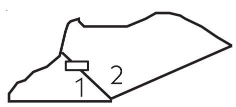
c Tear off the 3 vertices. Leave plenty of space so you can see the numbers clearly and fit the 3 torn pieces together easily.

d Line up the edges of the vertices so they all come together and tape them together.

e Determine (in degrees) the sum of the 3 angle measures in a triangle.

f The sum of the 3 angle measures in a triangle is _______ degrees.

When you get to step d, demonstrate how to line up and tape two of the vertices together, but don't add the third vertex; doing so will deprive students of the opportunity to discover that the three angles together form a straight angle.



Now give each student a piece of construction paper while they get rulers, scissors, and tape. Encourage them to make triangles that are distinctly different from the ones the people around them are making. You might also encourage students to experiment with isosceles and equilateral triangles, in addition to scalene triangles. Let students know that they will have an easier time with this activity if they rip out each angle instead of cutting it, because it will be easier to see the original angle. They will also find it easier to piece the angles together if they rip out larger pieces.

Discussing the Results

If some students finish well ahead of their classmates, ask them to meet with one another and share their results. Some may even have time to try the exercise with a different triangle. After most or all of the students have finished, reconvene the class and ask students to share their results. You can use the questions below to guide the discussion. During the discussion, ask several volunteers to show their taped vertices at the overhead and explain their results.

- What happened when they taped the three vertices together?
- Did it work that way for everyone?

Activity 2 Angle Measures in Triangles & Quadrilaterals (cont.)

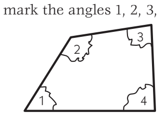
- How would they describe the sum of the three angles they've taped?
- Why do they think it works that way?

After students have had a chance to share and compare their results, fill in the sum of the angles (180°) at the bottom of the first section of the overhead with input from students.

Piecing the Vertices of a Quadrilateral Together

Display the lower portion of the Teacher Master. Read the instructions out loud and then invite students to share and explain some predictions about what the results will be.

2a On a piece of construction paper, use your ruler to draw a quadrilateral with 4 different side lengths and mark the angles 1, 2, 3, and 4.



b Cut out the quadrilateral you just drew.

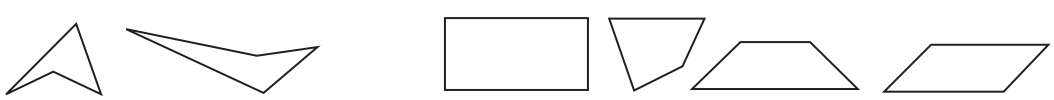
c Tear off the 4 vertices. Leave plenty of space so you can see the numbers clearly and fit the 4 torn pieces together easily.

d Line up the edges of the vertices so they come together and tape them together.

e Determine (in degrees) the sum of the 4 angle measures in a quadrilateral.

f The sum of the 4 angle measures in a quadrilateral is _______ degrees.

After a bit of discussion, have students begin working. Circulate as students work, giving assistance as needed and listening in as they share their discoveries with one another. Encourage them to draw large quadrilaterals that are quite different from the ones their neighbors are making. If necessary, ask them not to make concave quadrilaterals (i.e., quadrilaterals with "dents" in them), because it can be very difficult to tear off the vertices. Ask them instead to make convex quadrilaterals (i.e., ones without "dents").



Concave Quadrilaterals

Convex Quadrilaterals

Before they move on to steps b–e, ask students to check each other's quadrilaterals to make sure they are, in fact, quadrilaterals and that they are convex.

Sketching and Describing the Results of the Exploration

As they finish, ask students to tape their assembled angles for the triangle and quadrilateral they made in their student journals. Then ask them to respond to the prompt below, which you can write on the whiteboard if you like.

Activity 2 Angle Measures in Triangles & Quadrilaterals (cont.)

> Explain the results of today's work with triangles and quadrilaterals. Include any observations or generalizations you've made.

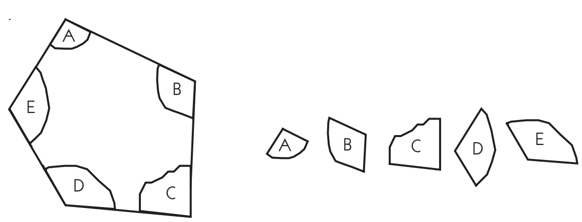
You can also ask the following questions if students need a little more direction.

- What happened when you taped together the 3 vertices from the triangle? What about the 4 vertices from the quadrilateral?
- Did it work that way for others around you?
- How would you describe the sum of the 3 angles you taped together? What about the sum of the 4 angles you taped together?
- Why do you think it works that way?
- Do you think it would work this way for all quadrilaterals, including those that have equal sides (squares or rhombuses)?

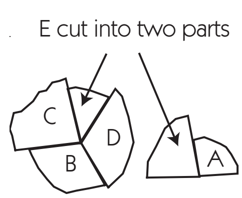
CHALLENGE

Consider creating a bulletin board display of some of the angles students have taped together, along with written explanations drawn from their journals, revised and edited as necessary. This demonstration is more powerful when students see that the angles from a wide variety of triangles all combine to make a straight angle, and that the sum of the angles in a wide variety of quadrilaterals is always 360°. Other polygons could be included on the bulletin board if students choose to experiment with other polygons at home or in spare moments in class.

> ***Jade*** *I wonder why you get twice the number of degrees when all you're adding is one more angle. I wonder what would happen if you tried a pentagon.*
>
> ***Blanca*** *Maybe it would double the amount again, and you'd get 720°!*
>
> ***Morgan*** *Let's try it!*



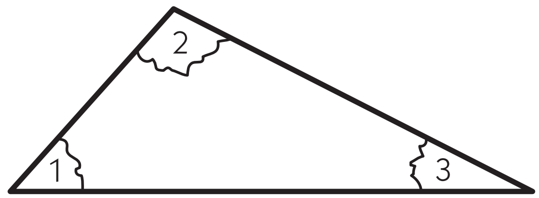
This is an irregular pentagon with its vertices marked and then ripped off.



The vertices are assembled to show that together they add up to 360 + 180 or 540 degrees.

Angle Measures in Figures

1a On a piece of construction paper, use your ruler to draw a large triangle that has 3 different side lengths (a scalene triangle) and mark the angles 1, 2, and 3.



b Cut out the triangle you just drew.

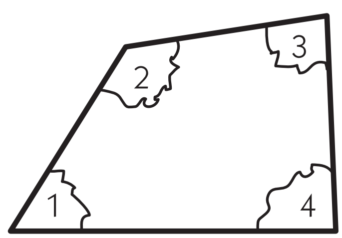
c Tear off the 3 vertices. Leave plenty of space so you can see the numbers clearly and fit the 3 torn pieces together easily.

d Line up the edges of the vertices so they all come together and tape them together.

e Determine (in degrees) the sum of the 3 angle measures in a triangle.

f The sum of the 3 angle measures in a triangle is _______ degrees.

2a On a piece of construction paper, use your ruler to draw a quadrilateral with 4 different side lengths and mark the angles 1, 2, 3, and 4.



b Cut out the quadrilateral you just drew.

c Tear off the 4 vertices. Leave plenty of space so you can see the numbers clearly and fit the 4 torn pieces together easily.

d Line up the edges of the vertices so they come together and tape them together.

e Determine (in degrees) the sum of the 4 angle measures in a quadrilateral.

f The sum of the 4 angle measures in a quadrilateral is _______ degrees.

Activity 3

PROBLEMS & INVESTIGATIONS

Angle Measure From Pattern Blocks to Protractors

WORK SAMPLE

Overview

Students first review some terms related to angles and then use what they know about the angle measures of some pattern blocks to investigate the protractor. The sheets students complete today may be saved as work samples in their math portfolios.

Actions

1. The class reviews some terms related to angles.
2. Students estimate and measure an angle using pattern blocks and a protractor.
3. The class reconvenes to share their discoveries about how to use the protractor.
4. Students practice measuring and constructing more angles.

Skills & Concepts

Please see Activity 1 for additional skills and concepts addressed in this lesson.

★ Use a protractor to measure angles in whole degrees (4.MD.6)

You'll need

★ Experimenting with Angle Measurement (Teacher Master 7 run 1 copy for display)

★ Experimenting with Angle Measurement, pages 1 and 2 (Teacher Masters 8 and 9, run a class set, plus 1 for display)

★ Word Resource Cards (acute angle, obtuse angle, ray, right angle, straight angle, vertex, zero angle) See note.

★ class set of protractors

★ class set of pattern blocks

★ erasable marker (e.g., Vis-à-Vis)

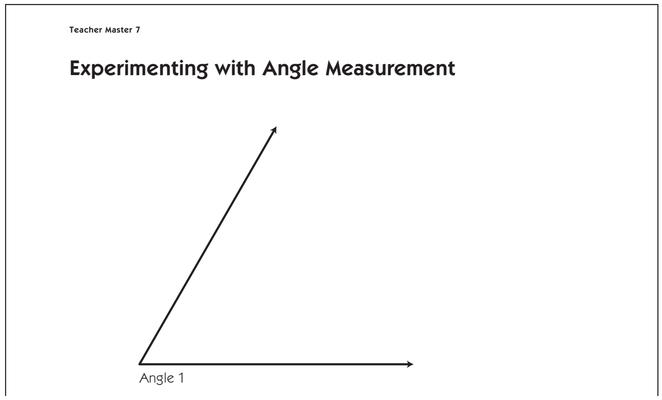
★ piece of paper to mask portions of the Teacher Master on display

Note Teacher Masters to make Word Resource Cards are located at the end of this packet. Locate the necessary term. Run on cardstock and fold in half, tape edges. Place the Word Resource Cards listed above on display before the session starts.

Reviewing Angle Vocabulary

Display the top portion of the Measuring Angles Teacher Master and ask students to describe what they notice. While mathematicians commonly define an angle as the union of two rays (the sides of the angle) that have the same endpoint (the vertex), students will describe the angle in less formal terms.

Activity 3 Angle Measure: From Pattern Blocks to Protractors (cont.)



> ***Students*** *It's an angle.*
> *The angle is acute because it's smaller than 90º.*
> *The angle has 2 arrows that go in different directions.*
> *The lines start in the same place and then go different directions.*
> *The place where they both start is the vertex, I think. It's kind of like a corner for the angle.*
> *And the arrow lines are like the sides of the angle.*

After a bit of discussion, draw students' attention to the Word Resource Cards on display. Have them consider how the terms relate to the angle on the overhead by asking how the angle on the overhead compares to the angles pictured on the Word Resource Cards. Also ask them to locate the vertex and the rays on the overhead.

Estimating and Measuring an Angle with the Pattern Blocks

WORK SAMPLE

Distribute the Experimenting with Angle Measure Teacher Masters 8 and 9. Ask students to get out pencils, protractors, and pattern blocks. Review the instructions on the sheet and explain that you'll do the first angle together. Ask everyone to record an estimate of the measure of Angle 1 in the appropriate box at the top of their record sheet. Then ask volunteers to share and explain their estimates as you record them on the projector beside Angle 1.

> ***Students*** *It's less than 90º, that's for sure, because it's smaller than a right angle. I said it's 70º.*
> *I said it was 65º. It looks bigger than half a right angle, somewhere between 45º and 90º.*
> *My estimate is 60º. It looks like it's about 2/3 of the way to a 90º angle.*
> *Mine was close to that. I said 55º. Half of 90 is 45, and it looks bigger than half of a 90º angle, but not all that much bigger.*

Activity 3 Angle Measure: From Pattern Blocks to Protractors (cont.)

Teacher Master 7 Run a class set.

NAME ____________________ DATE __________

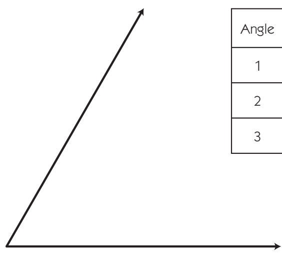
Experimenting with Angle Measurement page 1 of 2

1 For each angle below:

a Estimate how many degrees you think it measures.

b Use a pattern block to check the measure. (Each angle below matches one or more of the angles in your pattern blocks.)

c Measure it with your protractor.



Angle 1

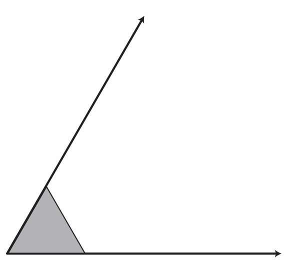
Angle	How many degrees? (estimate)	How many degrees? (actual measure)
1	70° 65° 60° 55°	
2		
3		

Now ask students to work in pairs to find at least 1 pattern block in the set that fits into the angle exactly. After a bit of experimentation, they'll discover that any of the three angles on the green triangle, as well as the acute angles on the trapezoid and the blue rhombus, fit. What does that tell them about the measure of Angle 1? Allow students a few minutes to reconstruct and pair-share some of their findings concerning the measure of the various pattern block angles, and then call two or three volunteers to the projector to share their conclusions.

> ***Blanca*** *We found out that the green triangle fits into Angle 1 exactly. But we couldn't remember how big that angle was.*

Teacher Master 10

Experimenting with Angle Measurement



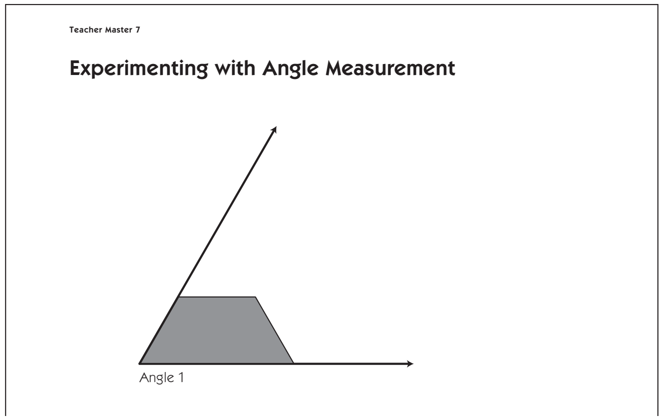
Angle 1

> ***Yolanda*** *Then we remembered from the other day that if you add all the angles on a triangle together, you get 180º. Since all the angles on this triangle are the same, we figured each one must be 60º because 180 ÷ 3 = 60.*

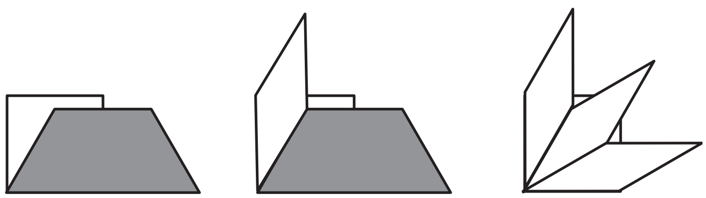
> ***Darius*** *That's pretty close to what we estimated. Lots of people thought Angle 1 was going to be more than 50º but less than 90º for sure.*

Activity 3 Angle Measure: From Pattern Blocks to Protractors (cont.)

> ***Nick*** *We found out that the trapezoid fits into Angle 1 like this. But we couldn't remember how big that angle was on the trapezoid.*



> ***Armin*** *So here's what we did. We put the trapezoid on top of the square to compare it to 90º, like this. Then we saw there was still room for one of those skinny rhombuses. But that still didn't help because we didn't know how big that one was either. Finally, we saw that we could fit exactly 3 of those skinny rhombuses into the square corner. Then we knew that each one of those was 30º because 90 ÷ 3 = 30. That meant that the angle on the trapezoid that fit into Angle 1 had to be 60º.*

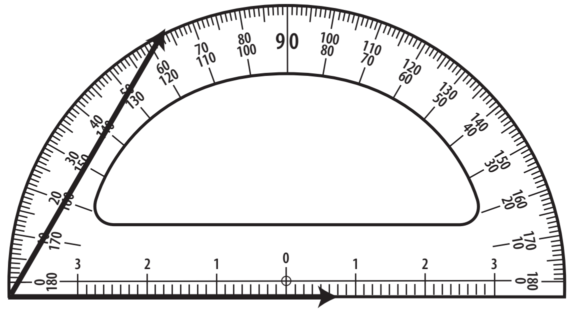


Measuring Angle 1 with a Protractor

Once the class has reached consensus that Angle 1 is 60º, ask students to use the protractors to show that the measure is 60º. You'll want to give them some time to discover for themselves how they can position and read the protractor to get the same result. Encourage them to work in pairs and table groups to share their discoveries and help one another. After a few minutes of experimentation, ask one or more pairs to share their strategies, using a protractor on Angle 1 at the overhead.

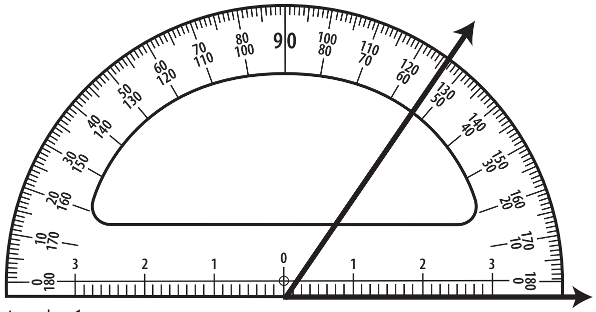
> ***Kamela*** *First we tried just lining up the protractor on the angle like this. The top of the angle kind of crossed over where it says 60 and 120 on the protractor, but it didn't really seem to land right on the 60.*

Activity 3 Angle Measure: From Pattern Blocks to Protractors (cont.)



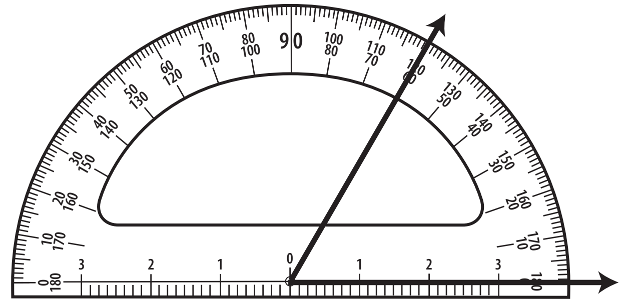
Angle 1

Jade *Then we thought maybe if we put the middle of the protractor right on the corner of the angle it would work, like this, but it didn't. We tried some other stuff and after we moved the protractor around for awhile, we saw that if you put the little hole right over the vertex and make sure the lines on both sides of the hole line up with the ray on the bottom, it comes out right.*



Angle 1

This didn't work.



Angle 1

This did! The other ray landed exactly on the 60°.

Measuring and Constructing More Angles

Give students the rest of the session to work with a partner to complete Teacher Masters 8 and 9. Reconvene the group as needed to talk about how the protractor can be used to confirm the pattern block measures. You might ask students who are comfortable using the protractor to help others who are experiencing difficulty. You might also work with a small group of students who are having difficulty.

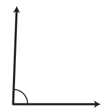
Activity 3 Angle Measure: From Pattern Blocks to Protractors (cont.)

Teacher Master 9 Run a class set.

NAME ______________________ DATE __________

Experimenting with Angle Measurement page 2 of 2

2 Lan says the angle below measures about 120°. Do you agree or disagree with her? Explain your answer.



3 Using a protractor, construct a 60° angle below or on a separate piece of paper. (If you use another sheet of paper, attach it to this assignment.) Check your work with a pattern block, and include the pattern block in your angle sketch.

CHALLENGE

4 Look around your classroom for acute angles. Choose several. For each angle you choose:

a Estimate how many degrees you think it measures.

b Measure it with your protractor.

c Record your work on the chart below.

Acute Angles in the Classroom	How many degrees?	How many degrees?

CHALLENGE

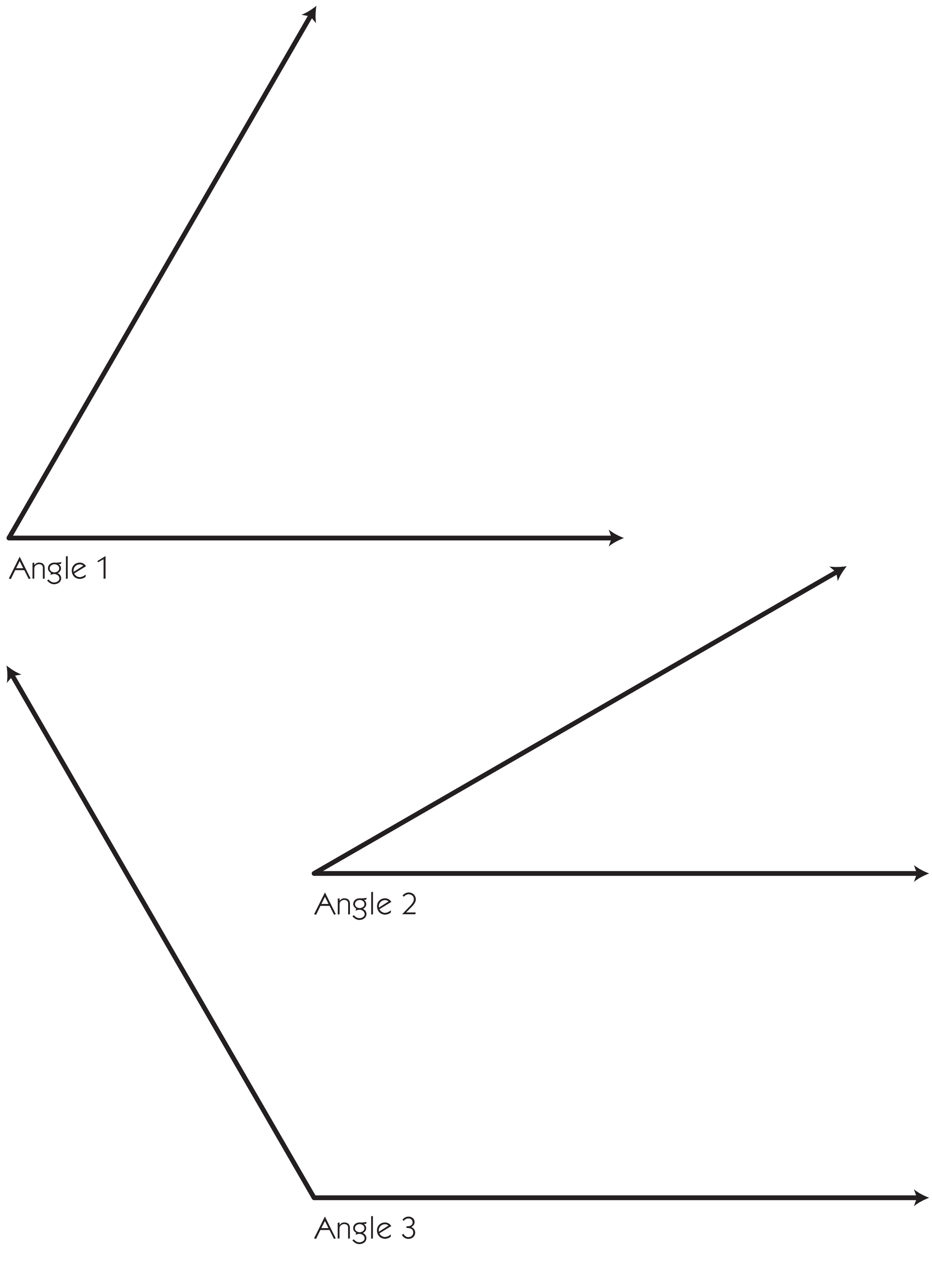
Measuring More Angles Around the Classroom

Some students may have time to work on problem 4, which challenges them to estimate and measure acute angles they find around the classroom. If student interest in this problem is high, you may want to devote a section of your whiteboard to angle measurement, setting up a chart similar to the one shown below, which students can add to over the next few days.

Measuring Angles in Our Classroom

Less than 90°	Exactly 90°	More than 90° but less than 180°
Point on my collar = 75°	Corner of a piece of paper = 90°	Hexagon Pattern Block = 120° Bench leg = 107° Trapezoid table corner = 120°

Experimenting with Angle Measurement



NAME ______________________________ **DATE** ______________

Experimenting with Angle Measurement page 1 of 2

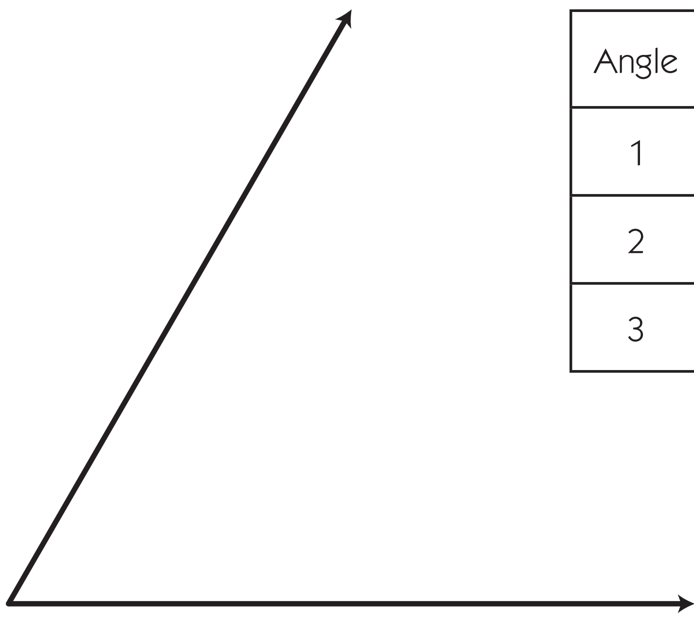
1 For each angle below:

a Estimate how many degrees you think it measures.

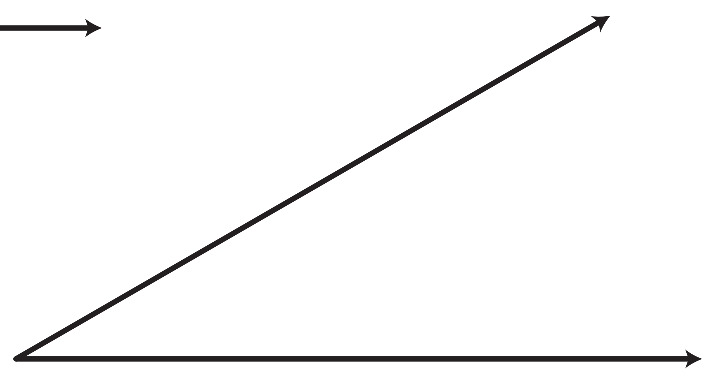
b Use a pattern block to check the measure. (Each angle below matches one or more of the angles in your pattern blocks.)

c Measure it with your protractor.

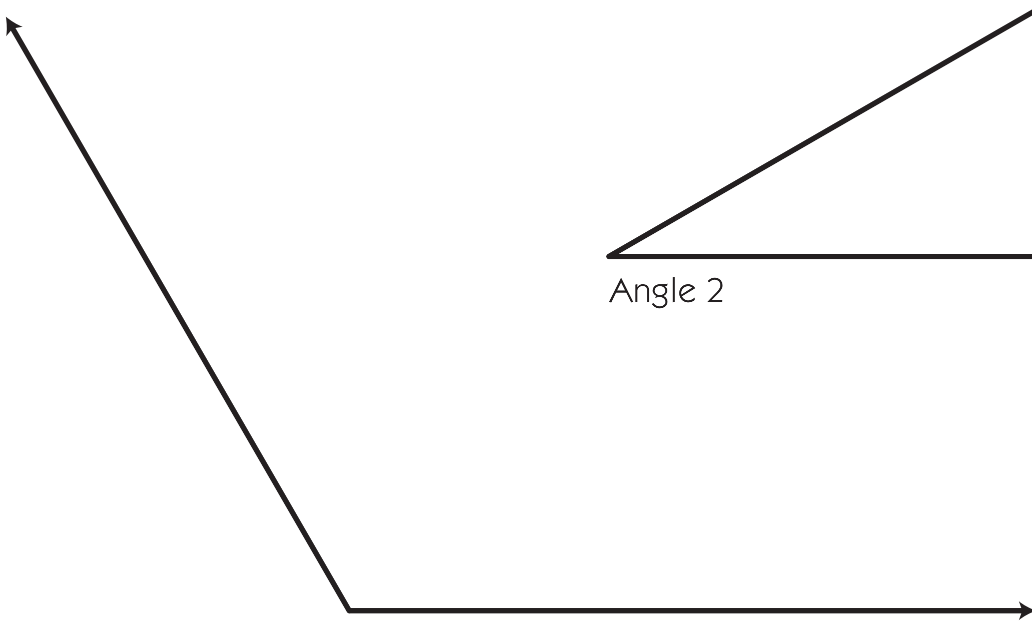
Angle	How many degrees? (estimate)	How many degrees? (actual measure)
1		
2		
3		



Angle 1



Angle 2



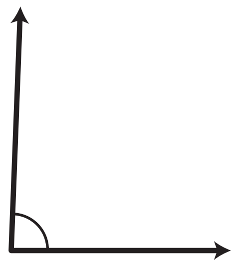
Angle 3

(Continued on back.)

NAME ______________________ **DATE** ______________

Experimenting with Angle Measurement page 2 of 2

2 Lan says the angle below measures about 120º. Do you agree or disagree with her? Explain your answer.



3 Using a protractor, construct a 60º angle below or on a separate piece of paper. (If you use another sheet of paper, attach it to this assignment.) Check your work with a pattern block, and include the pattern block in your angle sketch.

CHALLENGE

4 Look around your classroom for acute angles. Choose several. For each angle you choose:

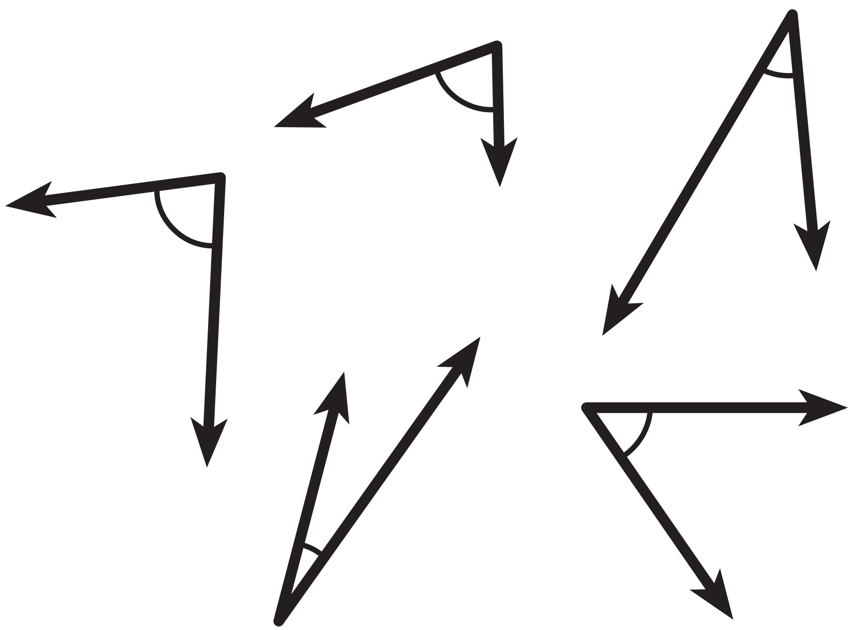
a Estimate how many degrees you think it measures.

b Measure it with your protractor.

c Record your work on the chart below.

Acute Angles in the Classroom	How many degrees?	How many degrees?

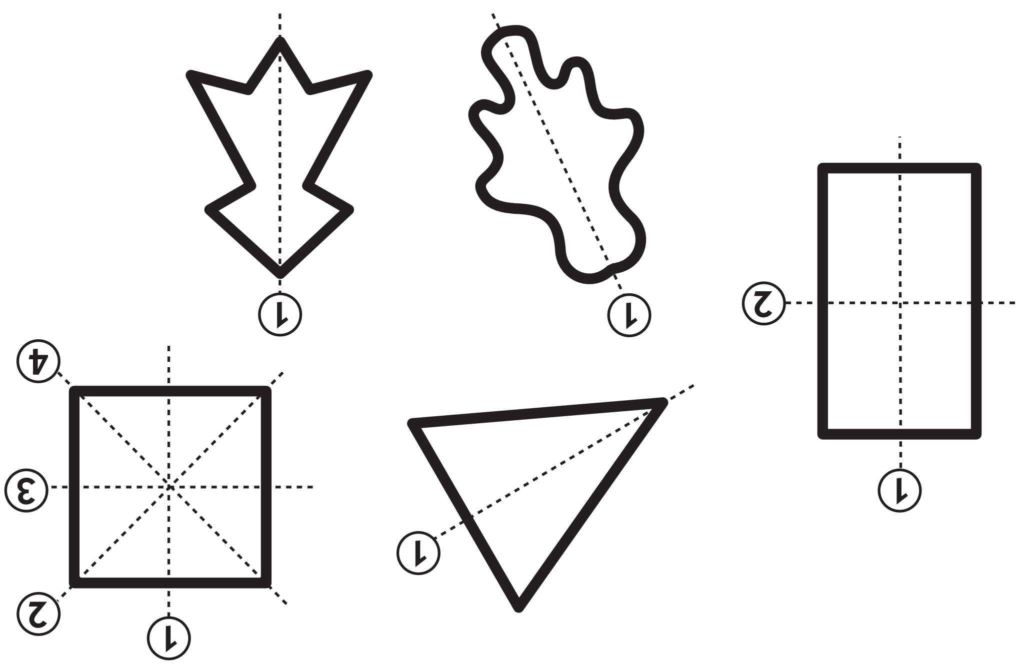
acute angle: an angle that has a measure less than 90°



acute angle

measures less than 90°

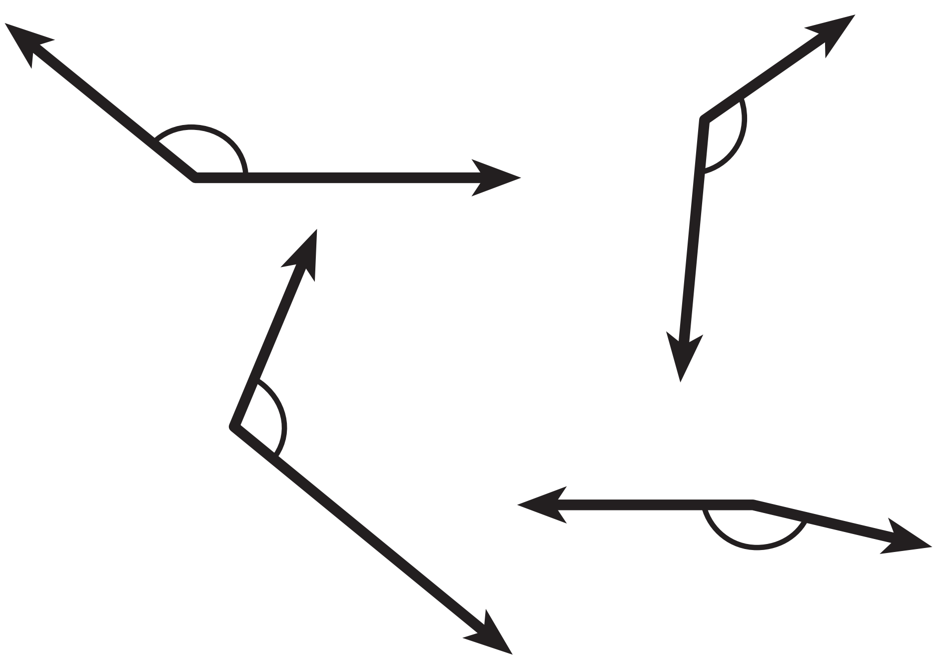
line of symmetry: a line that divides a figure into two mirror images



line of symmetry

Working Definition

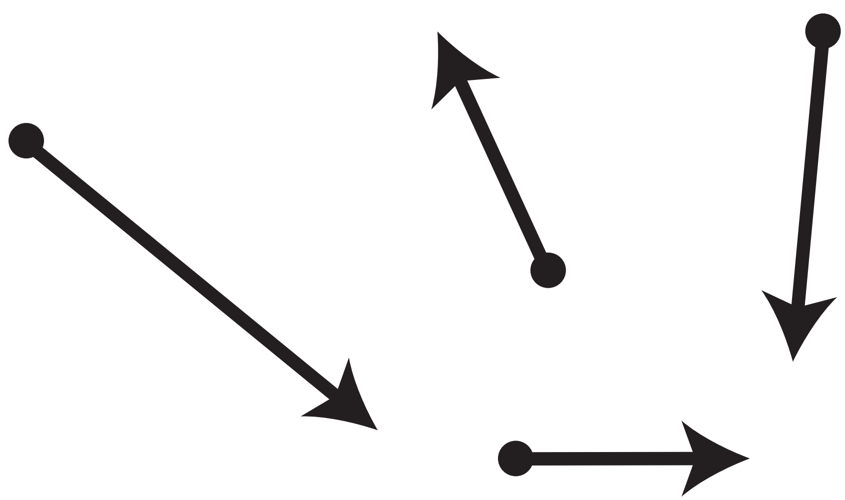
obtuse angle: an angle that has a measure more than 90° and less than 180°



obtuse angle

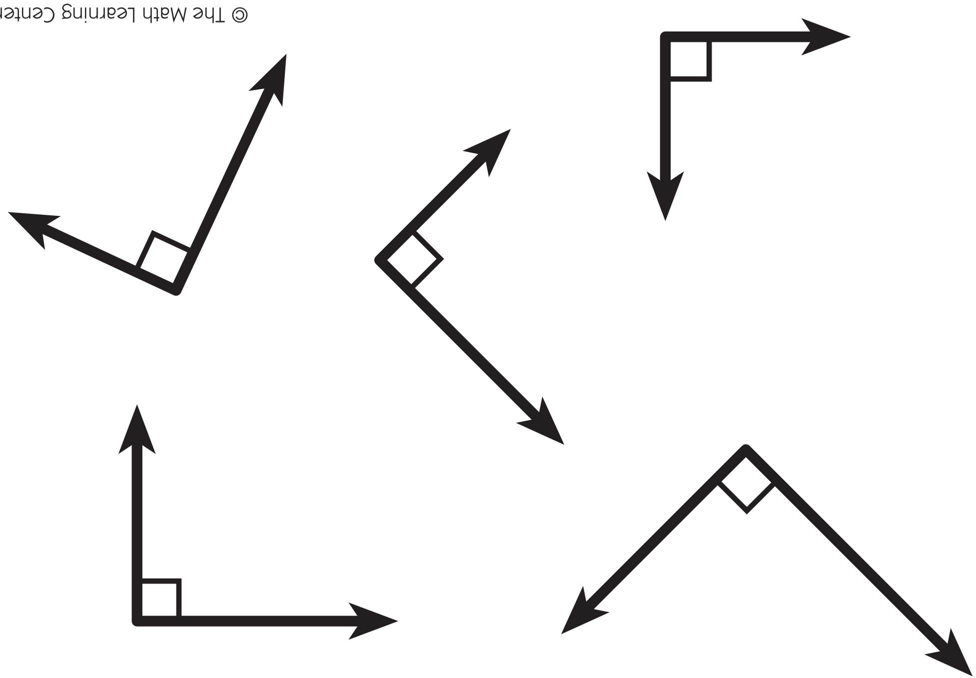
measures more than 90°

ray: a geometric figure that begins at an endpoint and extends forever in one direction



ray

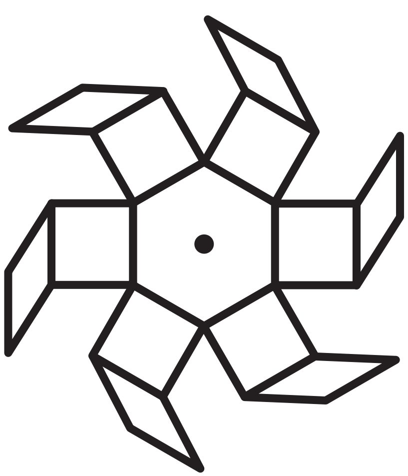
right angle: an angle that has a 90° measure

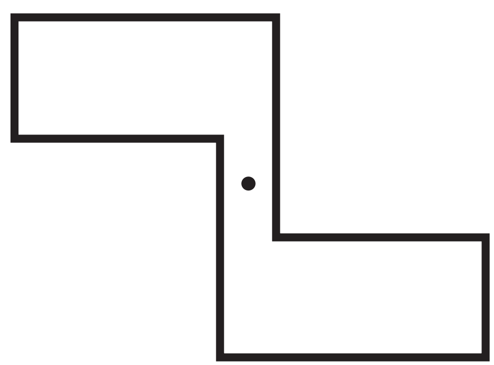


right angle

measures exactly 90°

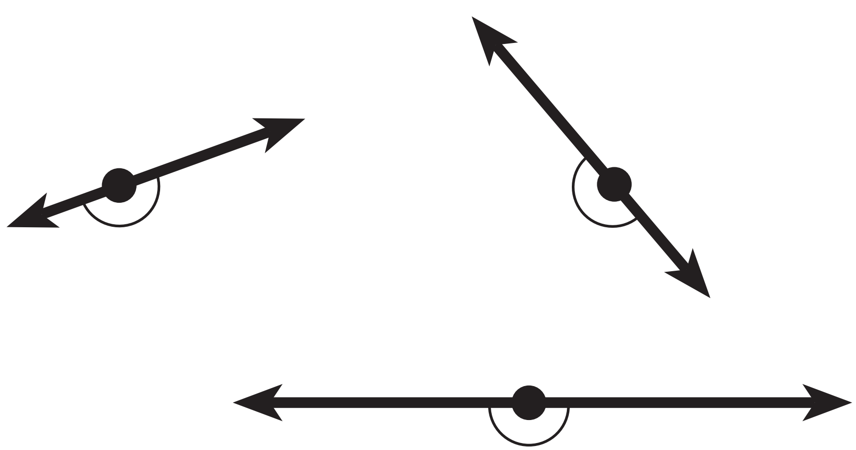
rotational symmetry: the property of a figure that can be turned less than 360 degrees and be identical with itself





rotational symmetry

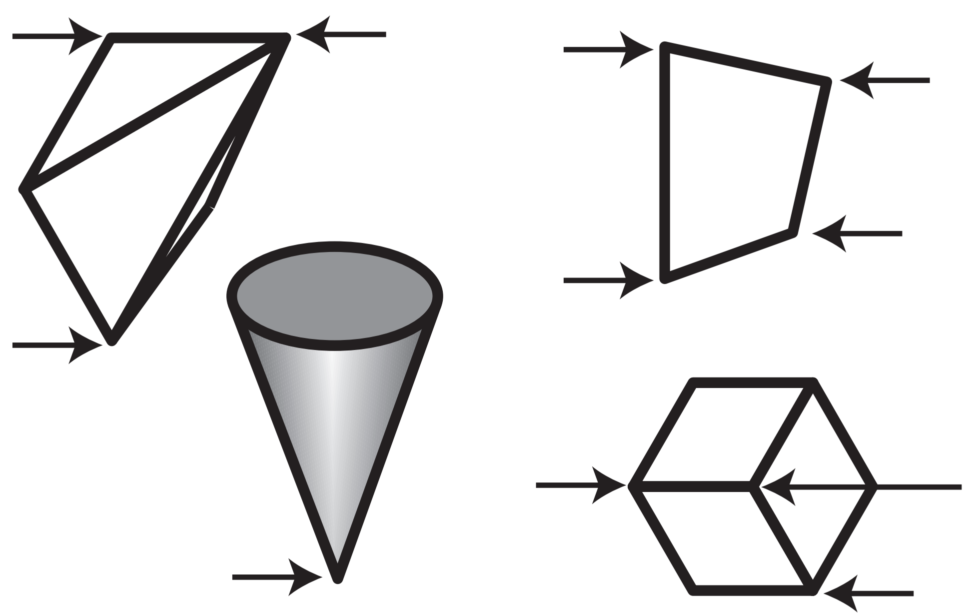
straight angle: an angle whose measure is 180 degrees



straight angle

measures exactly 180°

vertex: the intersection of edges of a polyhedron, the topmost point of a cone, or the point at which the sides of an angle or polygon intersect

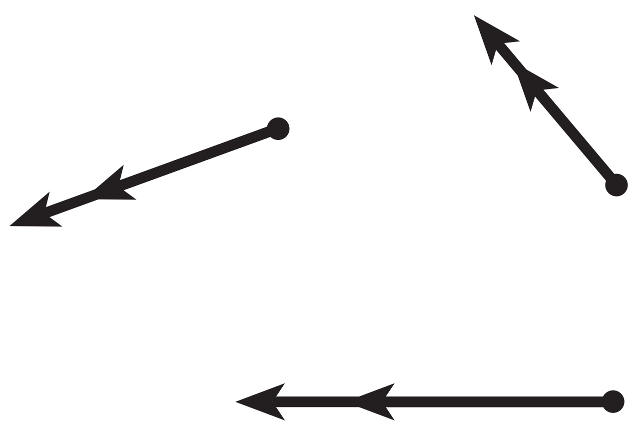


vertex

plural: vertices

Working Definition

zero angle: an angle whose measure is zero degrees



zero angle

measures exactly 0°