BRIDGES IN MATHEMATICS 4

GRADE 4 SUPPLEMENT

Set C3 Geometry: Circles & Angles

Includes

Skills & Concepts

★ develop benchmark angles including 60°, 90°, and 120° to estimate angle measurement

★ identify the angles associated with different fractions of a complete turn

★ identify and describe the center, radius, circumference, and diameter of a circle

★ exemplify points and line segments

★ use appropriate tools to measure and draw line segments to the nearest quarter-inch, eighth-inch, and millimeter

★ recognize angles as geometric shapes that are formed wherever two rays share a common endpoint

★ an angle is measured with reference to a circle with its center at the common endpoint of the rays. An angle that turns through $\frac{1}{360}$ of a circle is called a "one-degree angle," and can be used to measure angles.

★ an angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

★ measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

★ recognize angle measure as additive. Solve addition and subtraction problems to find unknown angles on a diagram using an equation with a symbol for the unknown angle measure.

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Bridges in Mathematics Grade 4 Supplement
Set C3 Geometry: Circles & Angles

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Bridges in Mathematics is a standards-based K–5 curriculum that provides a unique blend of concept development and skills practice in the context of problem solving. It incorporates the Number Corner, a collection of daily skill-building activities for students.

The Math Learning Center is a nonprofit organization serving the education community. Our mission is to inspire and enable individuals to discover and develop their mathematical confidence and ability. We offer innovative and standards-based professional development, curriculum, materials, and resources to support learning and teaching. To find out more, visit us at www.mathlearningcenter.org.

Set C3 ★ Activity 1

ACTIVITY

Pattern Block Angles

Overview

Students review the terms right angle, straight angle, and interior angle. Then they use right and straight angles as benchmarks to determine the interior angles of each pattern block. Finally, they use pattern blocks to measure angles of rotation on a clock face and determine the fraction of a complete turn represented by each angle of rotation.

Skills & Concepts

- ★ measure angles in geometric figures
- ★ develop benchmark angles including 60°, 90°, and 120°
- ★ identify the angles associated with different fractions of a complete turn

You'll need

- ★ Pattern Block Angles (page C3.4, 1 copy for display, plus a class set)
- ★ Using Pattern Blocks to Measure Angles on a Clock Face (pages C3.5 and C3.6, 1 copy for display, plus a class set)
- ★ a set of pattern blocks for each pair of students consisting of at least: 1 hexagon, 2 trapezoids, 2 squares, 3 triangles, 3 blue rhombuses, and 3 white rhombuses
- ★ pattern blocks for display
- ★ Word Resource Cards (right angle and straight angle) optional

Advance Preparation Consider assigning student pairs ahead of time so that students who may struggle with spatial relationships are paired with peers who can help them.

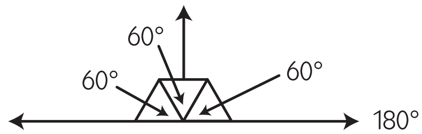
Instructions for Pattern Block Angles

1. Begin by displaying a copy of Pattern Block Angles. Explain that today students are going to work in pairs to determine what the measurements of the *interior angles* (angle on the inside of a shape) of each pattern block are. Use your Word Resource Cards to review the terms *right angle* (exactly 90 degrees), and *straight angle* (exactly 180 degrees).

2. Now give each pair of students a small set of pattern blocks and 2 copies of Pattern Block Angles. Ask them to talk to each other about how they could use the right and straight angles at the bottom of the page to determine the interior angle measurements of the triangle. Give them a few moments to work, and then invite a few pairs to share their ideas with the whole group. It's fine if they have not yet calculated the exact angle measurements: the point is to make sure that everyone has some ideas about how to start this activity.

> ***Students*** *We put triangles on the straight angle until they filled it up. These three angles are each a third of a straight angle.*
> *A straight angle is 180°, so each of the triangles must be 60°, right?*
> *That's what we think because 3 × 60 is 180.*

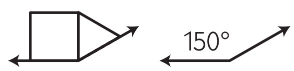
Activity 1 Pattern Block Angles (cont.)



3 × 60 = 180

3. Give students time to work in pairs to determine all the interior angles of the pattern blocks and label them on their sheets. Circulate around the room to see how students are working and to listen to their conversations. If you like, you might ask students to pause so you can share some helpful strategies you've observed. For example, "I saw some people finding angles on some of the pattern blocks that are equal to the angles they'd already figured out on another block. That helped them figure out those angles quickly."

4. If some students finish quite a bit earlier than the rest of the group, you can invite them to tour the room and help others. Ask them to suggest strategies for finding the angle measurements rather than simply sharing the answers with their classmates. You might also ask them to turn over their papers and trace the sides of the pattern blocks to show how they can make other angles by adding the interior angles of different blocks. Here is an example:



We can make a 150-degree angle by putting these two angles together. 90 + 60 = 150

5. When students have completed their sheets, reconvene the group and review the page to make sure everyone has their pattern block angles labeled correctly. Explain that they will use the blocks to measure other angles in this activity and the next, so it's important that they are working with the correct angle measures.

6. Now explain that angles measure the amount of rotation. For example, in the course of an hour, the minute hand on the clock rotates all the way around the clock face. Ask students to follow along using a pencil as you model this on the projector. Start with the pencil at the 12 position and rotate it a full 360 degrees, keeping the eraser anchored in one place so that you're not sliding the pencil, just rotating it about the eraser. Ask students how many degrees they rotated the pencil when they turned it one full turn. (360 degrees) Many students may connect this to a full turn on a skateboard or snowboard, referred to as a 360.

7. Now ask students to rotate their pencils one-fourth of a full turn. If they start at the 12 position, where does the pencil end up pointing after one-fourth of a turn? (the 3 position)

8. Post the display copy of Using Pattern Blocks to Measure Angles on a Clock Face and read the directions out loud. Explain that the students are now going to use the interior angles of the pattern blocks to measure different angles of rotation on the clock face. Use the square pattern block, and then three white rhombuses, to show how they can measure the example angle with different combinations of pattern blocks.

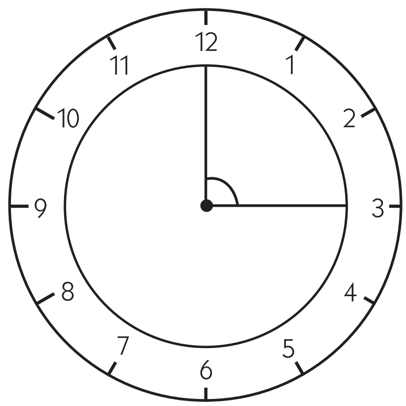
Activity 1 Pattern Block Angles (cont.)

9. Invite students to ask any questions they might have, and then give them the rest of the period to complete the two pages. If students finish before the period is over, review the sheets as a whole group. If not, review the sheets at the beginning of the next activity in this set.

Note *Save, or have students save, their completed sheets for use in Activity 2.*

Extension

- When reviewing the answers, invite students to express the rotations not only in terms of the fraction of a whole turn, but also as equivalent fractions based on the numbers on the clock face and on the number of degrees. You'll need to gauge your students' comfort level with fractions to determine whether this would be a worthwhile exercise for the group, or for just a few students.



$\frac{1}{4} = \frac{90}{360}$ and $\frac{1}{4} = \frac{3}{12}$

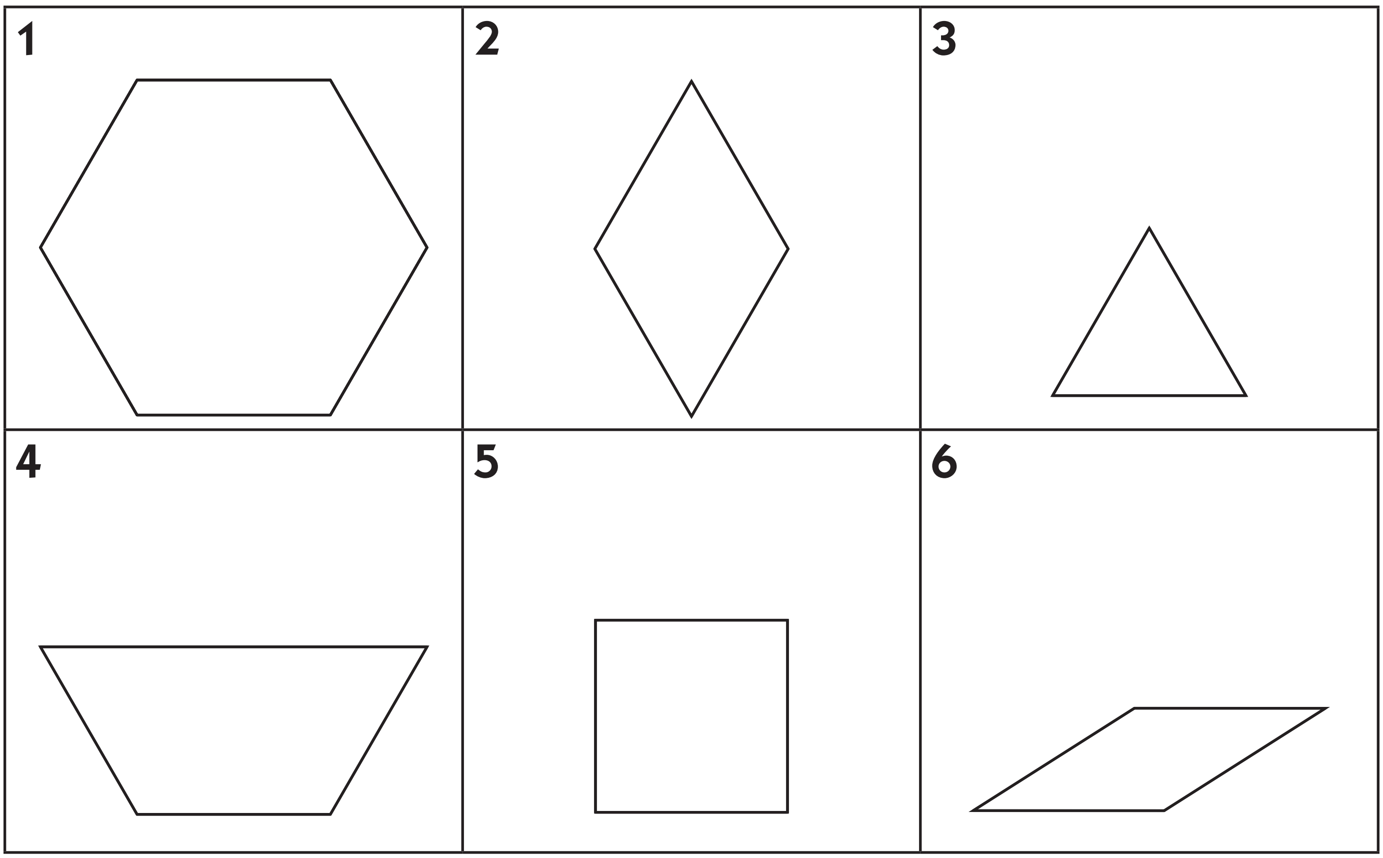
INDEPENDENT WORKSHEET

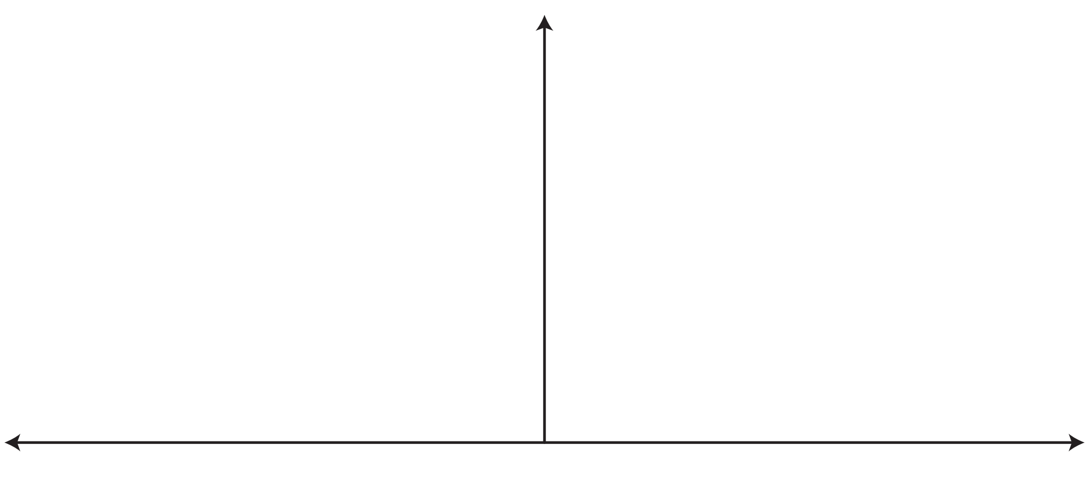
Use Set C3 Independent Worksheet 1 (page C3.21) to provide students with practice measuring the interior angles of polygons using pattern blocks. Use Independent Worksheet 2 (page C3.23) to provide students with practice estimating angle measures against benchmarks of 90˚, 60˚, and 180˚. Use Independent Worksheet 3 (pages C3.25 and C3.26) to provide students with practice drawing polygons with specified angles and side lengths.

NAME ____________________________________ **DATE** ________________

Pattern Block Angles

Label the interior angles of each pattern block shown below. Use the straight and right angles below to help determine what the angles are.





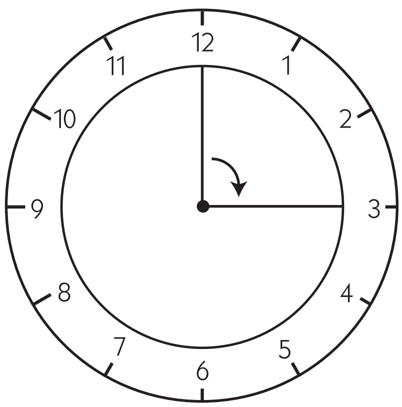
NAME ______________________________ **DATE** ______________

Using Pattern Blocks to Measure Angles on a Clock Face

Use your pattern blocks to measure each angle on the clock faces below. Then write the fraction of a whole turn each angle represents.

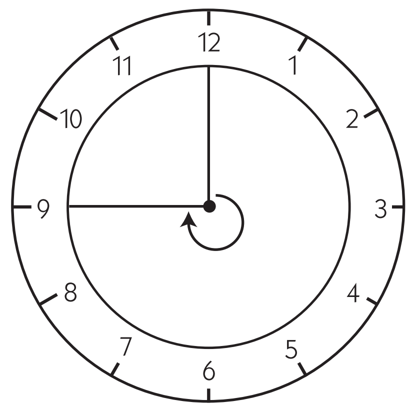
ex a angle measure __________

b fraction of a whole turn __________



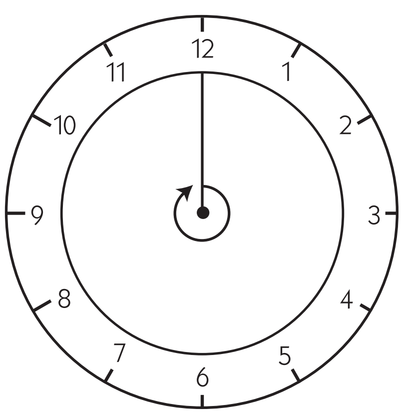
1a angle measure __________

b fraction of a whole turn __________



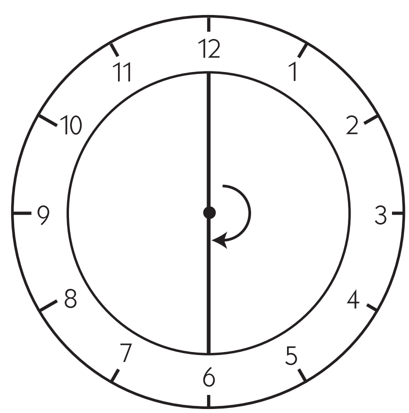
2a angle measure __________

b fraction of a whole turn __________



3a angle measure __________

b fraction of a whole turn __________



(continued on back.)

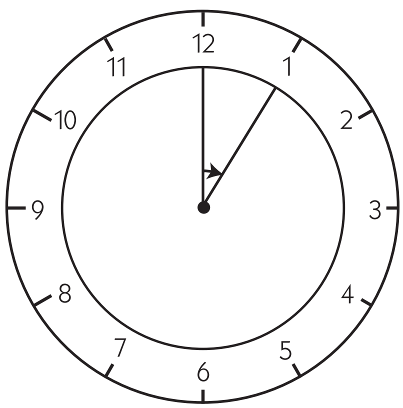
NAME ______________________________ **DATE** ________________

Using Pattern Blocks to Measure Angles on a Clock Face (cont.)

Use your pattern blocks to measure each angle on the clock faces below. Then write the fraction of a whole turn each angle represents.

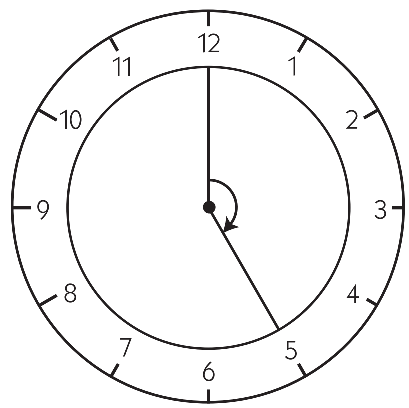
4a angle measure __________

b fraction of a whole turn __________



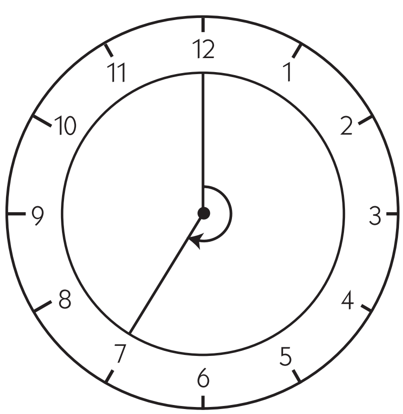
5a angle measure __________

b fraction of a whole turn __________



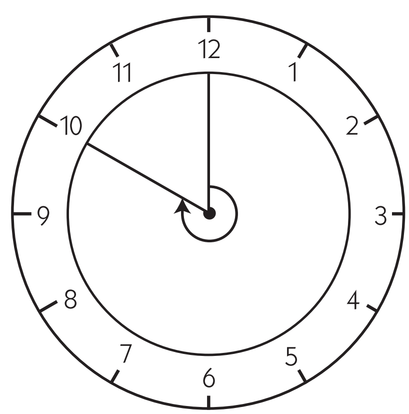
6a angle measure __________

b fraction of a whole turn __________



7a angle measure __________

b fraction of a whole turn __________



Set C3 ★ Activity 2

ACTIVITY 2

Human Angles

Overview
Students work in pairs to sketch and then estimate the angle of rotation for a number of different joints in their bodies, again using right angles, straight angles, and the interior angles of the pattern blocks as benchmarks. They also estimate the approximate fraction of a complete turn represented by each angle.

Skills & Concepts
★ measure angles in geometric figures

★ use benchmark angles including 60°, 90°, and 120° to estimate angle measurements

★ identify the fraction of a complete turn represented by different angles

You'll need
★ Using Pattern Blocks to Measure Angles on a Clock Face (pages C3.5 and C3.6, completed copies from Activity 1)

★ Range of Motion in Human Joints (page C3.10, run 1 copy for display)

★ Measuring the Range of Motion of Your Joints, pages 1 and 2 (pages C3.11 and C3.12, 1 copy for display, plus a class set)

★ a set of pattern blocks for each pair of students consisting of at least: 1 hexagon, 2 trapezoids, 2 squares, 3 triangles, 3 blue rhombuses, and 3 white rhombuses

★ pattern blocks for display

Note Since today's activity involves students' bodies, it is probably best to let students choose their own partners to ensure they are working with a classmate they like and feel comfortable with.

Instructions for Human Angles
1. If you did not have time in Activity 1 to review students' completed sheets, Using Pattern Blocks to Measure Angles on a Clock Face, do so now. Before working on today's activity, students will need to make sure that they have identified the angles and fractions of a turn correctly.

2. Now explain that in today's activity, students will be estimating the range of motion in some different joints in their bodies, including their wrists, elbows, knees, and shoulders. Explain that different kinds of joints can rotate different amounts. Sometimes, when people are injured, they go to physical therapy to help regain the full range of motion in the injured joint. For example, if a soccer player hurts her knee, she might need physical therapy to regain the full range of motion in her knee, and if a quarterback injures his shoulder, he also might need physical therapy to move his shoulder the way he needs to in order to make a long pass.

Physical therapists use a device called double-armed goniometer to measure the exact range of motion of their patients' joints. A goniometer consists of a stationary arm holding a protractor that is placed parallel with a stationary body segment, a pin (the axis of the goniometer) that is placed over the joint, and a movable arm that moves along a moveable body segment. (If you or the students do an Internet search for *goniometer*, you can find a variety of illustrations and photos of this measuring device.)

3. Place the Range of Motion in Human Joints sheet on display and explain that students will work in pairs to test the range of motion of different joints in their own bodies. Each student will move his or her joints as shown on this display, and his or her partner will sketch where the motion ended. Then they will use the pattern blocks and their sheets from yesterday to estimate the angle of rotation and fraction of a full turn for each joint movement.

4. Now invite a volunteer to show how to perform each movement shown on the display and help correct them if they don't do it quite right. Emphasize that they are to rotate their joints only as far as it is comfortable. If they start to feel like they are straining, they must stop. Explain that they can injure themselves if they try to push their joints farther than they should go.

5. Now show the display copy of Measuring the Range of Motion of Your Joints. Have a volunteer show the knee motion again, and model how to sketch the ending point of that rotation. Then model how to use the interior angles of the pattern blocks to estimate the angle of rotation to the nearest ten degrees. While the volunteer is doing the motion and while you are sketching it, model how students can use 90 degrees as a benchmark in their sketch and in making their estimates. Ask students to refer to their sheets from yesterday to help estimate the fraction of a complete turn this angle represents.

6. Give each student a copy of the sheets and answer questions they have about the activity. Then give them most of the rest of the period to work in pairs. Circulate around the room to answer questions and to make sure students are conducting the activity safely and respectfully.

7. When you have five or ten minutes left in the period, reconvene the class and ask them to share their work by asking questions like the following:

- Which joint had the greatest range of motion?
- Which joint had the smallest range of motion?
- Which fractions of a full turn were most difficult to estimate? How did you handle it?
- Did any of your estimates surprise you?

You might also ask students to share and compare the estimates for the joints of their choice.

Extensions

- Have students research the different kinds of joints in the body. Each kind of joint performs a different kind of motion and, as a result, has a different possible range of motion.
- Invite students to research different birds, who have an impressive range of motion in their necks. Ask them to draw sketches of each bird's range of neck motion.
- Invite students to make sketches showing different angles of rotation in skateboarding, ice skating, and snowboarding tricks. You might consider having them make posters and present the information to the class.

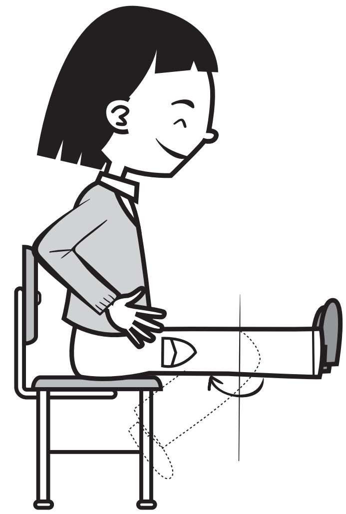
INDEPENDENT WORKSHEET

Use Independent Worksheet 4 (pages C3.27 and C3.28) to provide students with more practice identifying and drawing different angles of rotation and relating them to fractions of a complete turn about a circle.

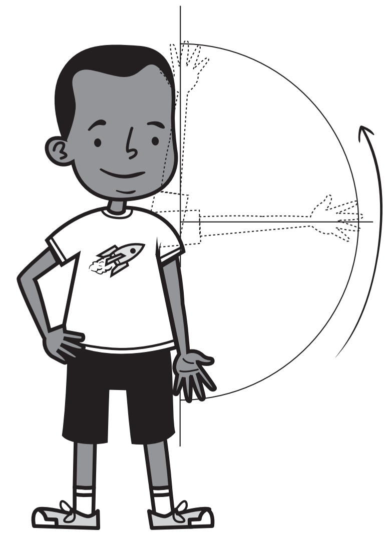
NAME ______________________ **DATE** ______________

Range of Motion in Human Joints

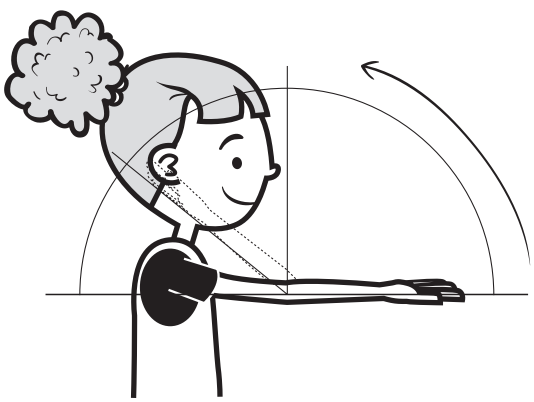
1 Knee



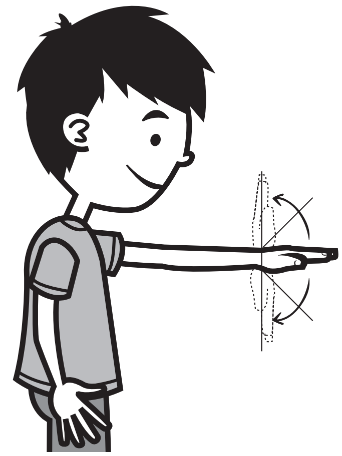
2 Shoulder: To the Side



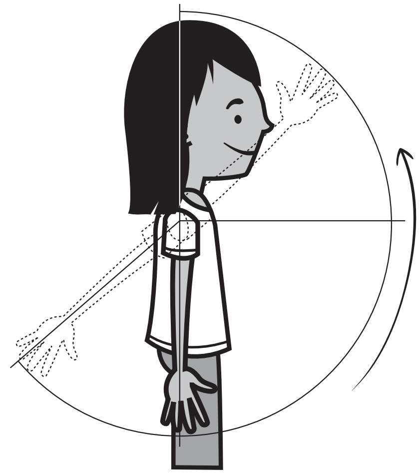
3 Elbow



4 Wrist



5 Shoulder: Back and Front



NAME ______________________________ **DATE** ______________

Measuring the Range of Motion of Your Joints

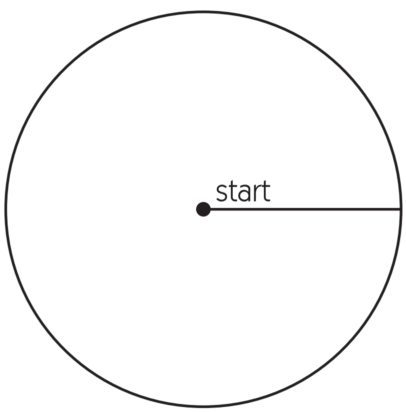
Work with a partner to test how much you can rotate each of the joints shown below. Each of you will sketch. First sketch the ending points of the joint's rotation. Then use your pattern blocks to estimate the degree of rotation to the nearest 10 degrees. When testing your joints, only bend as far as is comfortable: don't rotate your joints until it feels difficult or painful!

1 Knee

a approximate degrees of rotation

____

b approximate fraction of a complete turn ________________

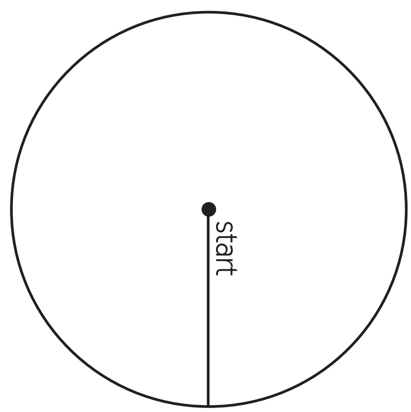


2 Shoulder: To the Side

a approximate degrees of rotation

____

b approximate fraction of a complete turn________________



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NAME ____________________________ **DATE** ______________

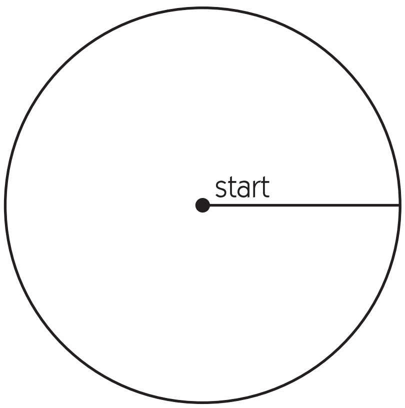
Measuring the Range of Motion of Your Joints (cont.)

3 Elbow

a approximate degrees of rotation

____

b approximate fraction of a complete turn _______________

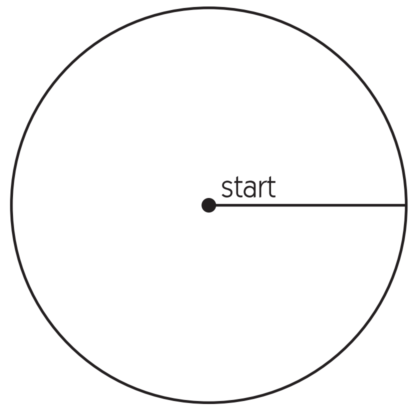


4 Wrist

a approximate degrees of rotation

____

b approximate fraction of a complete turn _______________

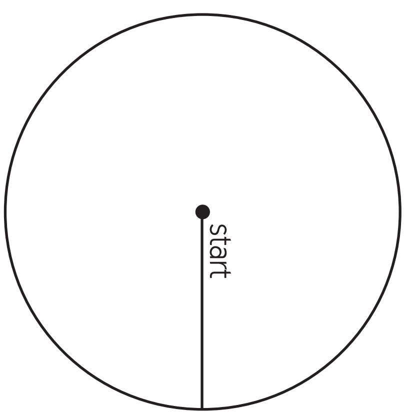


5 Shoulder: Back and Front

a approximate degrees of rotation

____

b approximate fraction of a complete turn _______________

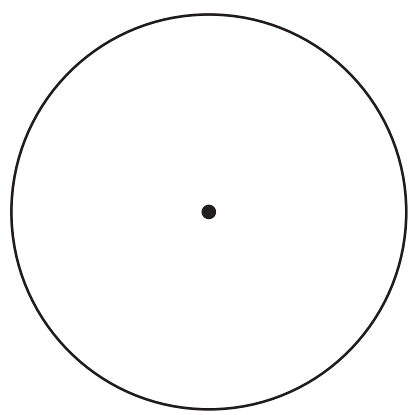


6 Your Choice _______________

a approximate degrees of rotation

____

b approximate fraction of a complete turn _______________



Set C3 ★ Activity 3

ACTIVITY

Measuring Circles

Overview

Students share what they already know about circles. The teacher introduces a set of circle words and works with input from the class to label a circle at the display. Students then measure the radius and diameter of several circles and share ideas about how these dimensions might relate to each other.

Skills & Concepts

★ identify and describe the center, radius, circumference, and diameter of a circle

★ exemplify points and line segments

★ use appropriate tools to measure to the nearest quarter-inch

You'll need

★ A Circle (page C3.16, run 1 copy for display)

★ Circles (page C3.17, run enough copies so that each student gets 1 circle. Cut apart.)

★ Circles to Measure & Label (pages C3.18 and C3.19, run a class set plus 1 for display)

★ class set of rulers

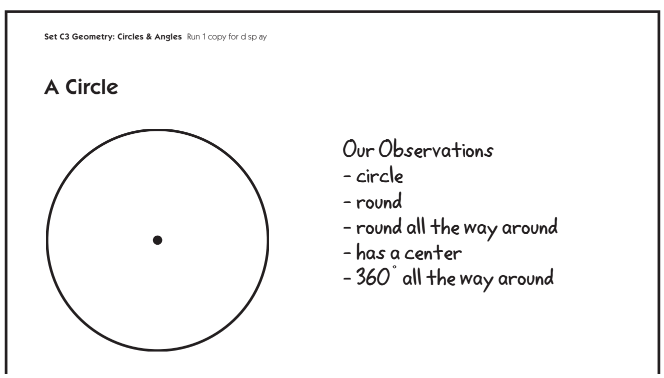
★ scissors

★ a piece of paper to mask parts of the display

★ 2 feet of string for each pair of students (optional)

Instructions for Measuring Circles

1. Display only the top portion of A Circle. Ask students to pair-share what they already know and what they notice about the shape. Then call on volunteers to share their ideas with the class as you record on the display.


Set C3 Geometry: Circles & Angles Run 1 copy for display

A Circle

Our Observations
- circle
- round
- round all the way around
- has a center
- 360° all the way around

2. Reveal the first word on the vocabulary list at the bottom of the master: *center.* Ask students to explain where the center of the circle is and how they know. Is there any way they can *prove* that it is the center? After a bit of discussion, give each student a copy of the circle and explain that it is an exact copy of the one on the display. Ask them to work in pairs to see if the point that appears to be in the middle of this circle is actually in the middle.

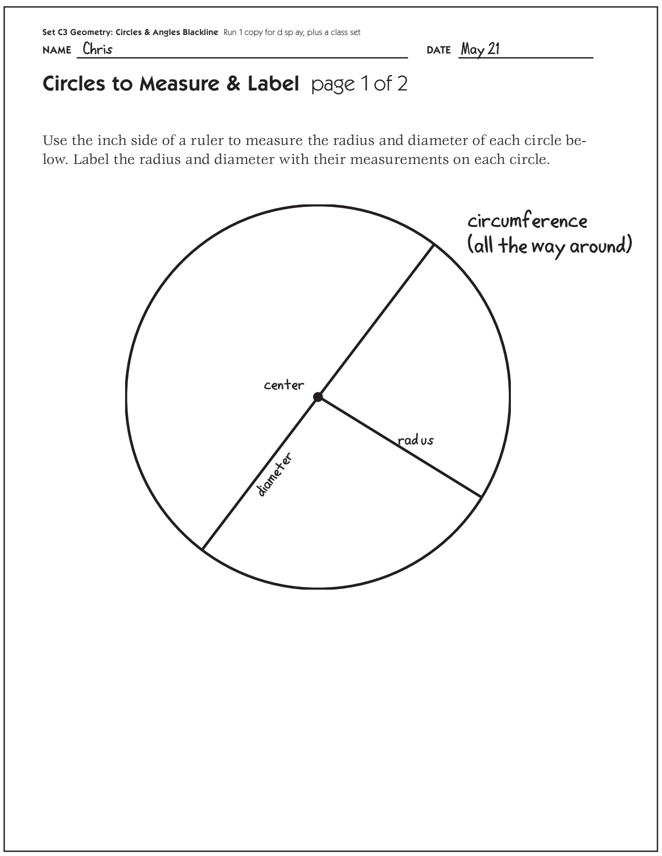
3. When they have had a few minutes to work, ask students to share their conclusions and strategies. Is the point actually at the center of the circle? What did they do to find out?

Activity 3 Measuring Circles (cont.)

> ***Students*** *We cut out the circle and folded it in half. The point landed right on the fold, so we said it must be the center.*
> *We measured from the point to the edge of the circle. It was exactly an inch and a half on one side and then on the other side. That means the dot is right in the middle.*
> *We kept measuring from the point to different places on the circle. It was an inch and a half every time. It must be in the center.*

4. Affirm students' experiments and explanations by explaining that a circle is a set of points that are all the same distance from the center. Give students each a copy of Circles to Label & Measure. Label the center of your circle as students label the center of the top circle on their sheets.

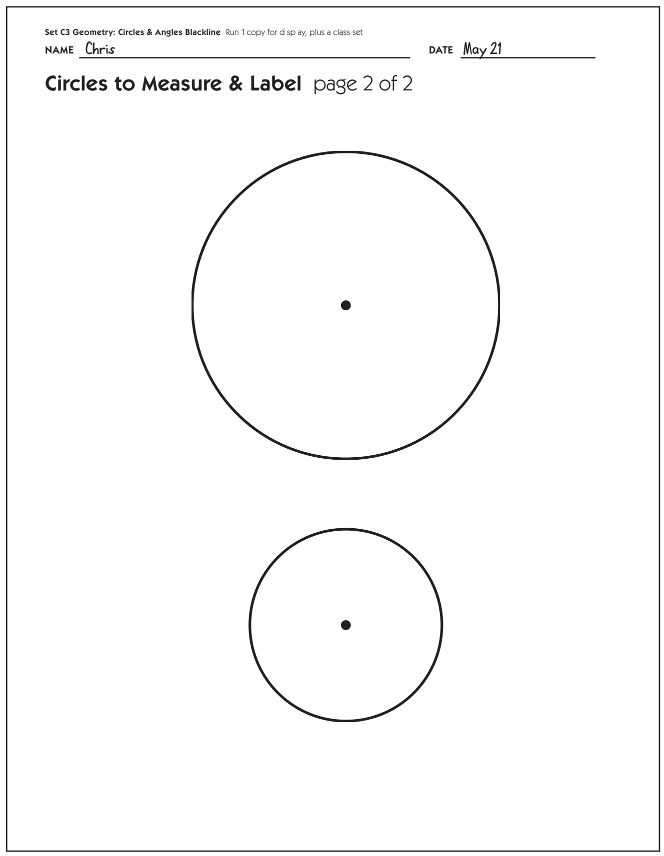
5. Reveal the other vocabulary words at the bottom of the Circle display copy one by one. As you show each one, read and discuss the word and its definition with the class. Challenge students to use what they already know about points and line segments to figure out how to construct and label these dimensions on the first circle. Provide as much support as needed, but encourage them to think for themselves.


Set C3 Geometry: Circles & Angles Blackline Run 1 copy for display, plus a class set

NAME Chris DATE May 21

Circles to Measure & Label page 1 of 2

Use the inch side of a ruler to measure the radius and diameter of each circle below. Label the radius and diameter with their measurements on each circle.


Set C3 Geometry: Circles & Angles Blackline Run 1 copy for display, plus a class set

NAME Chris DATE May 21

Circles to Measure & Label page 2 of 2

6. Have students use the inch side of their ruler to measure the radius and the diameter of the circle on the first page. Ask them to measure carefully to the nearest half-inch and record the measurements on their sheets. How do the two measurements compare? Why? Students will notice that the radius is exactly half the diameter (the diameter is exactly twice the length of the radius).

7. Ask students to test any theories they develop about the relationship between the radius and diameter of the circle by marking and measuring both dimensions on the other two circles on their sheets. Is the length of the radius always exactly half the length of the diameter?

Activity 3 Measuring Circles (cont.)

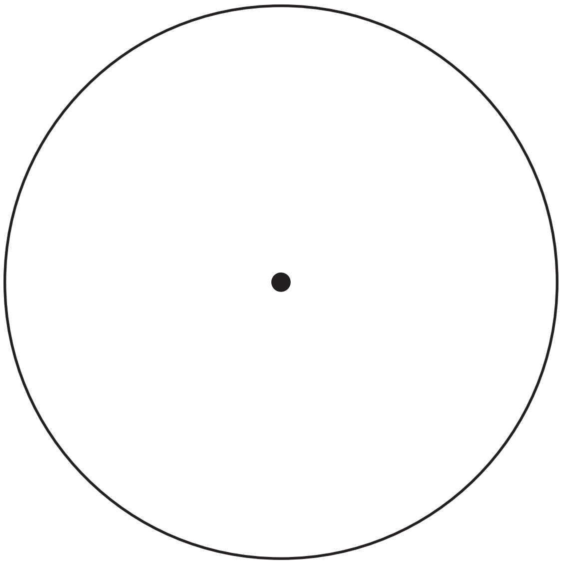
Extensions

Give each pair of students a 2-foot length of string. Have them use it to measure the circumference of each circle. Advise them to start with the largest circle on their sheets and work their way down to the smallest. That way, they can keep cutting the string and won't need more than the original length. Ask them to record the circumference of each circle on the sheet, and then reflect on their results. What do they notice? (If you try this for yourself, you will see that the circumference is just a little more than 3 times the diameter of every circle. This is a good, hands-on demonstration of the formula for finding the circumference of a circle, which can be expressed as either diameter $\times$ π or $2\pi r$.)

INDEPENDENT WORKSHEET

Use Set C3 Independent Worksheet 5 (page C3.29) to provide students with more practice identifying and drawing diameters and radii of circles, as well as drawing specific angles of rotation and measuring to the nearest millimeter.

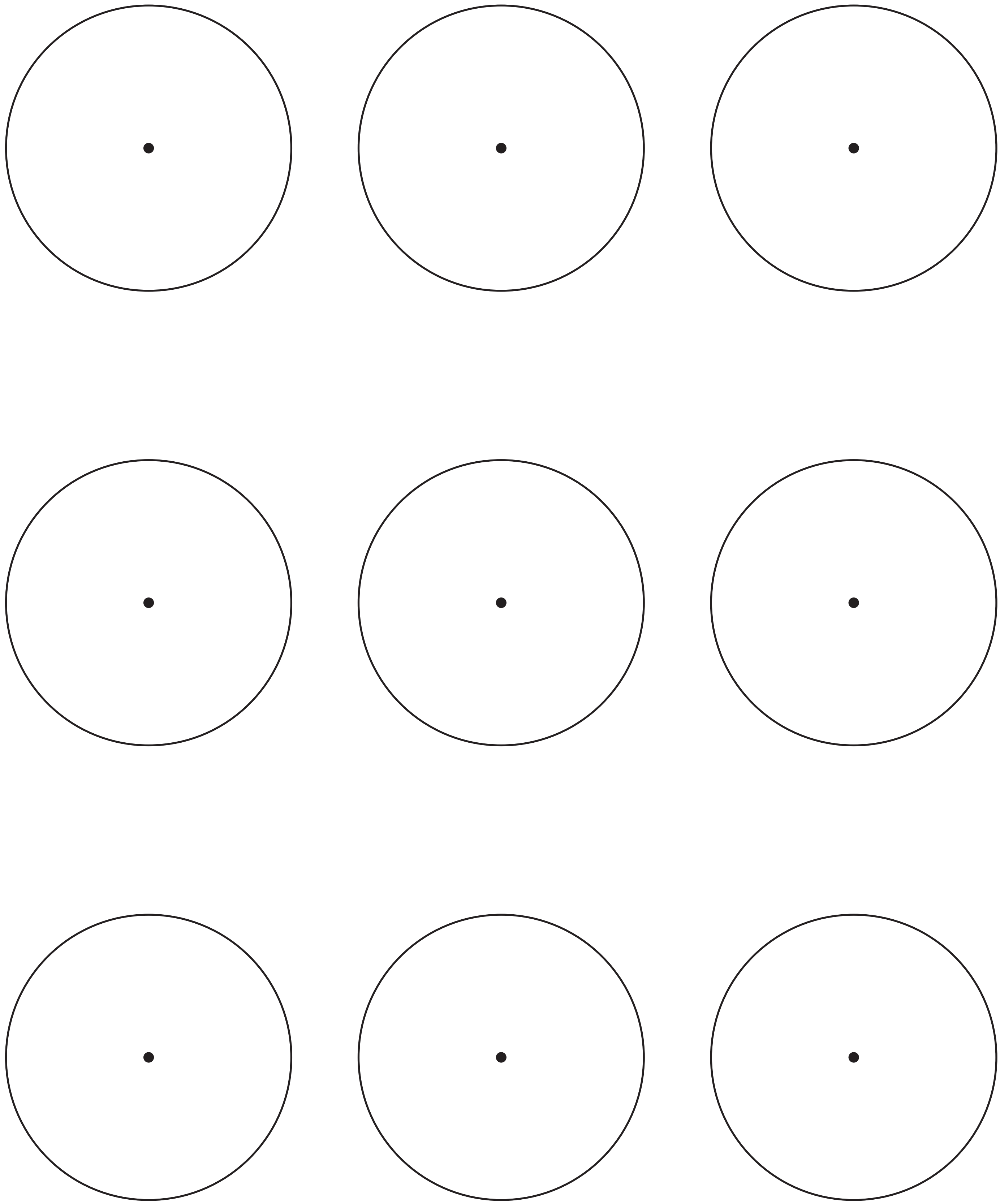
A Circle



Circle Words

- center
- radius—a line segment from the center of the circle to any point on the circle
- diameter—a line segment that passes through the center of the circle and has endpoints on the circle
- circumference—the distance around a circle

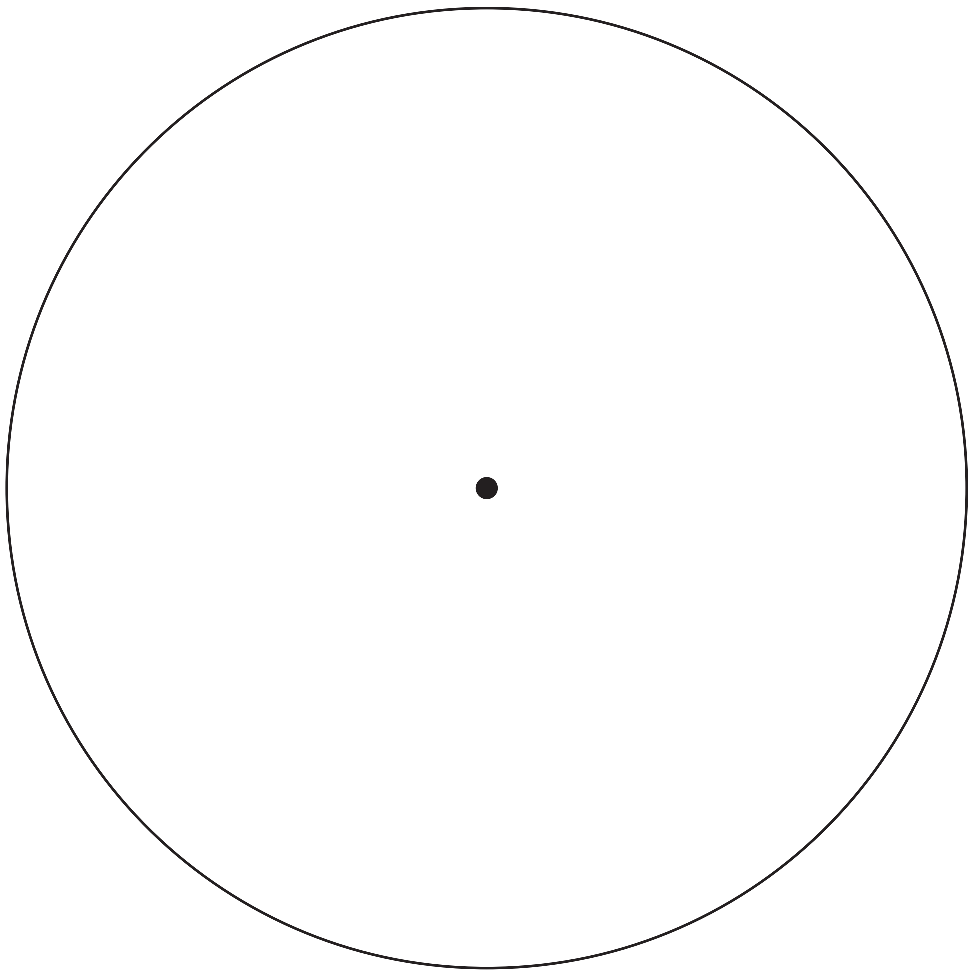
Circles



NAME ______________________ **DATE** ______________

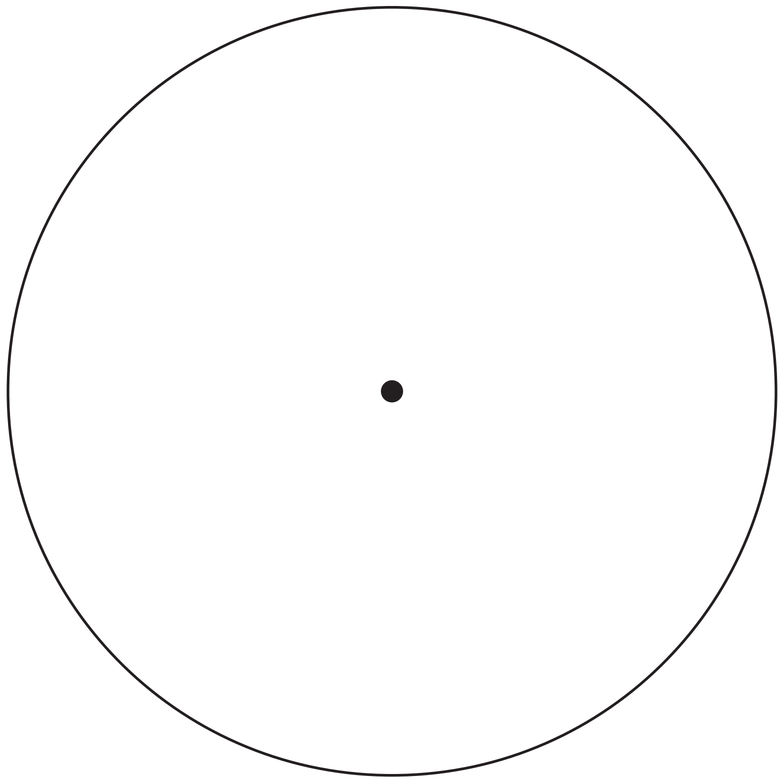
Circles to Measure & Label page 1 of 2

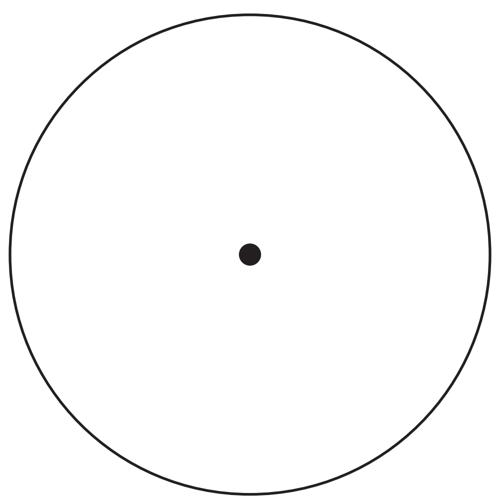
Use the inch side of a ruler to measure the radius and diameter of each circle. Label the radius and diameter with their measurements on each circle.



NAME ______________________ **DATE** ______________

Circles to Measure & Label page 2 of 2





Set C3 ★ Activity 4

ACTIVITY

From Pattern Blocks to Protractors

Overview

Students first review some terms related to angles and then use what they know about the angle measures of some pattern blocks to investigate the protractor. The sheets students complete today may be saved as work samples in their math portfolios.

Skills & Concepts

- ★ using a protractor to measure angles
- ★ selecting tools to construct angles
- ★ recognizing acute, right, and obtuse angles

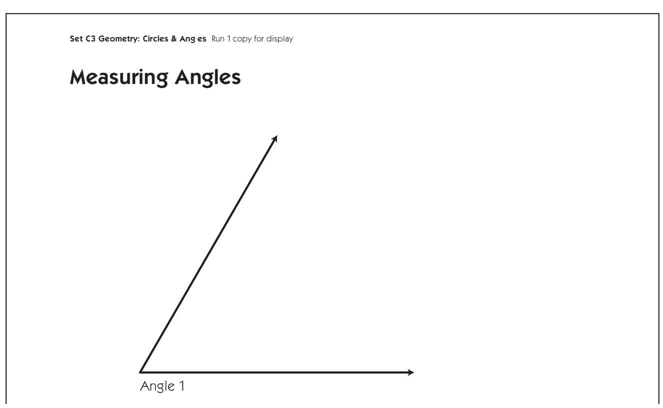
You'll need

- ★ Measuring Angles (page C3.28, run a class set plus 1 copy for display)
- ★ Experimenting with Angle Measurement (pages C3.26 and C3.27, run a class set)
- ★ Word Resource Cards (acute angle, obtuse angle, ray, right angle, straight angle, vertex, zero angle), optional
- ★ 180° protractor class set plus 1 for display
- ★ pattern blocks class set plus 1 for display
- ★ pens
- ★ piece of paper to mask portions of the display

Advance Preparation Display the Word Resource Cards listed above before the session starts.

Instructions for From Pattern Blocks to Protractors

1. Display the top portion of Measuring Angles and ask students to describe what they notice. While mathematicians commonly define an angle as the union of two rays (the sides of the angle) that have the same endpoint (the vertex), students will describe the angle in less formal terms.



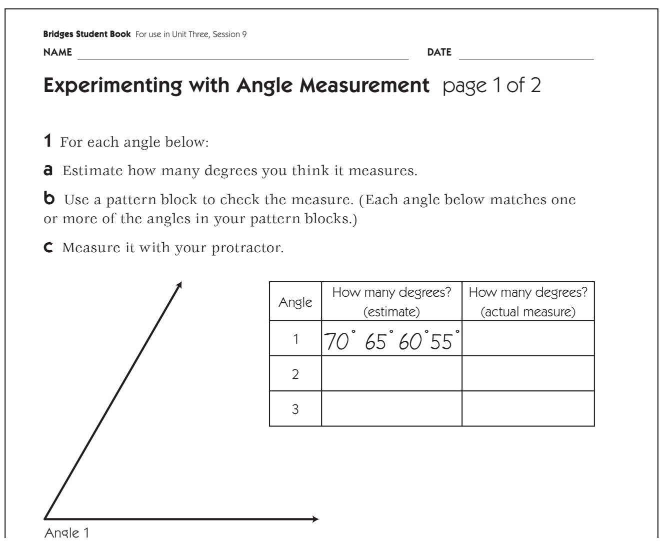
Students *It's an angle.*
The angle is acute because it's smaller than 90º.
The angle has 2 arrows that go in different directions.
The lines start in the same place and then go different directions.

The place where they both start is the vertex, I think. It's kind of like a corner for the angle. And the arrow lines are like the sides of the angle.

After a bit of discussion, draw students' attention to the Word Resource Cards on display. Have them consider how the terms relate to the angle on the display by asking how the angle on the display compares to the angles pictured on the Word Resource Cards. Also ask them to locate the vertex and the rays on the display.

2. Pass out copies of Experimenting with Angle Measure. Ask students to get out pencils, protractors, and pattern blocks. Review the instructions on the sheet and explain that you'll do the first angle together. Ask everyone to record an estimate of the measure of Angle 1 in the appropriate box at the top of their record sheet. Then ask volunteers to share and explain their estimates as you record them on the display beside Angle 1.

Students *It's less than 90º, that's for sure, because it's smaller than a right angle. I said it's 70º.*
I said it was 65º. It looks bigger than half a right angle, somewhere between 45º and 90º.
My estimate is 60º. It looks like it's about 2/3 of the way to a 90º angle.
Mine was close to that. I said 55º. Half of 90 is 45, and it looks bigger than half of a 90º angle, but not all that much bigger.



Bridges Student Book For use in Unit Three, Session 9
NAME ______________________ DATE ____________

Experimenting with Angle Measurement page 1 of 2

1 For each angle below:

a Estimate how many degrees you think it measures.

b Use a pattern block to check the measure. (Each angle below matches one or more of the angles in your pattern blocks.)

c Measure it with your protractor.

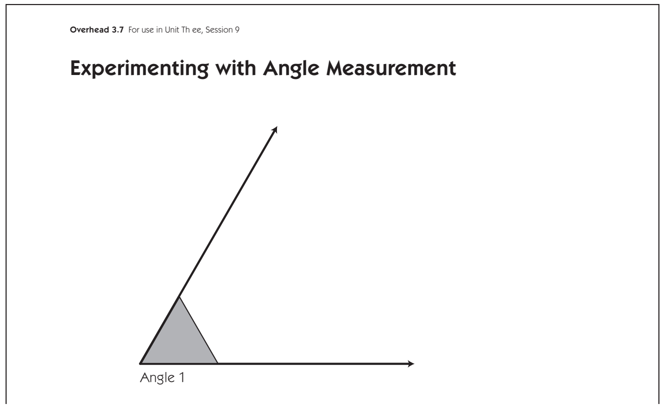
Angle	How many degrees? (estimate)	How many degrees? (actual measure)
1	70° 65° 60° 55°	
2		
3		

Angle 1

3. Now ask students to work in pairs to find at least 1 pattern block in the set that fits into the angle exactly. After a bit of experimentation, they'll discover that any of the three angles on the green triangle, as well as the acute angles on the trapezoid and the blue rhombus, fit. What does that tell them about the measure of Angle 1? Allow students a few minutes to reconstruct and pair-share some of their findings concerning the measure of the various pattern block angles, and then call two or three volunteers to the display to share their conclusions.

Blanca *We found out that the green triangle fits into Angle 1 exactly. But we couldn't remember how big that angle was.*

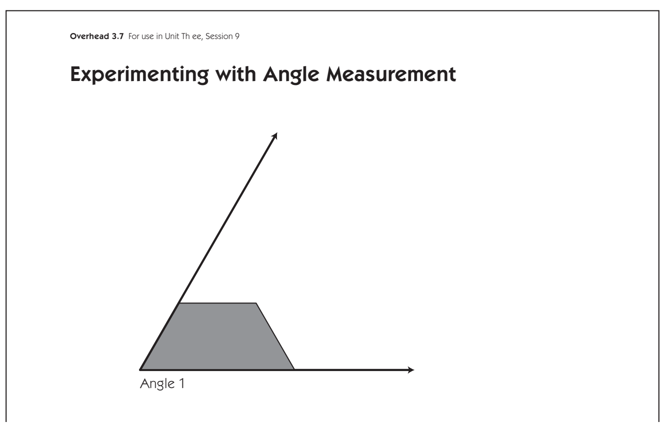
Activity 4 From Pattern Blocks to Protractors (cont.)



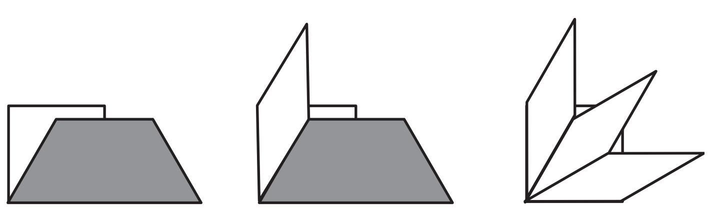
Yolanda *Then we remembered from the other day that if you add all the angles on a triangle together, you get 180º. Since all the angles on this triangle are the same, we figured each one must be 60º because 180 ÷ 3 = 60.*

Darius *That's pretty close to what we estimated. Lots of people thought Angle 1 was going to be more than 50º but less than 90º for sure.*

Nick *We found out that the trapezoid fits into Angle 1 like this. But we couldn't remember how big that angle was on the trapezoid.*



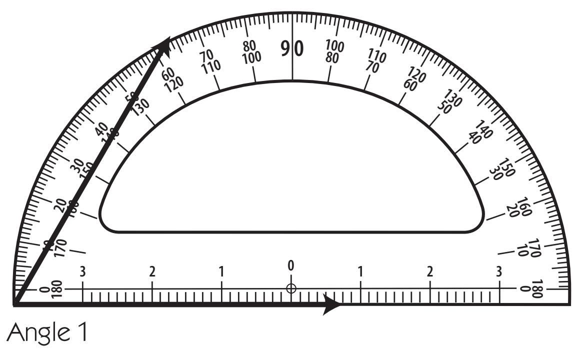
Armin *So here's what we did. We put the trapezoid on top of the square to compare it to 90º, like this. Then we saw there was still room for one of those skinny rhombuses. But that still didn't help because we didn't know how big that one was either. Finally, we saw that we could fit exactly 3 of those skinny rhombuses into the square corner. Then we knew that each one of those was 30º because 90 ÷ 3 = 30. That meant that the angle on the trapezoid that fit into Angle 1 had to be 60º.*



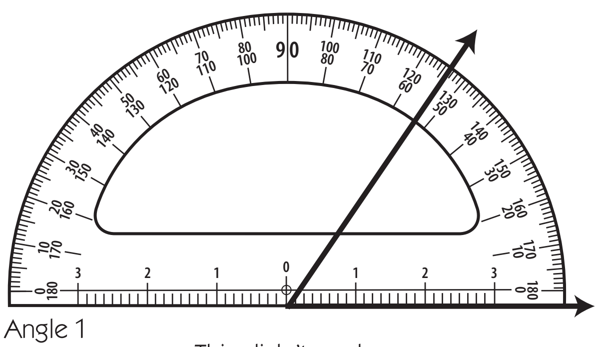
Activity 4 From Pattern Blocks to Protractors (cont.)

4. Once the class has reached consensus that Angle 1 is 60º, ask students to use the protractors to show that the measure is 60º. Because most fourth graders are new to using protractors, you'll want to give them some time to discover for themselves how they can position and read the protractor to get the same result. Encourage them to work in pairs and table groups to share their discoveries and help one another. After a few minutes of experimentation, ask one or more pairs to share their strategies, using a protractor on Angle 1 at the display.

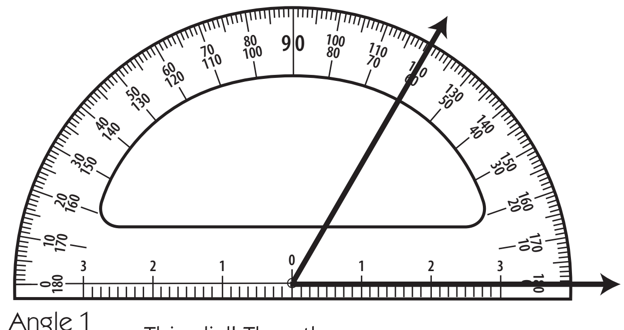
> ***Kamela*** *First we tried just lining up the protractor on the angle like this. The top of the angle kind of crossed over where it says 60° and 120° on the protractor, but it didn't really seem to land right on the 60°.*



> ***Jade*** *Then we thought maybe if we put the middle of the protractor right on the corner of the angle it would work, like this, but it didn't. We tried some other stuff and after we moved the protractor around for awhile, we saw that if you put the little hole right over the vertex and make sure the lines on both sides of the hole line up with the ray on the bottom, it comes out right.*



This didn't work.



This did! The other ray landed exactly on the 60º.

5. Give students the rest of the session to work with a partner to complete Experimenting with Angle Measurement. Reconvene the group as needed to talk about how the protractor can be used to confirm the pattern block measures. You might ask students who are comfortable using the protractor to help others who are experiencing difficulty. You might also work with a small group of students who are having difficulty.

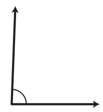
Activity 4 From Pattern Blocks to Protractors (cont.)

Bridges Student Book For use n Un t Three, Session 9

NAME ______________________ DATE ____________

Experimenting with Angle Measurement page 2 of 2

2 Lan says the angle below measures about 120°. Do you agree or disagree with her? Explain your answer.



3 Using a protractor, construct a 60° angle below or on a separate piece of paper. (If you use another sheet of paper, attach it to this assignment.) Check your work with a pattern block, and include the pattern block in your angle sketch.

CHALLENGE

4 Look around your classroom for acute angles. Choose several. For each angle you choose:

a Estimate how many degrees you think it measures.

b Measure it with your protractor.

c Record your work on the chart below.

Acute Angles in the Classroom	How many degrees?	How many degrees?

Extension

Some students may have time to work on problem 4, which challenges them to estimate and measure acute angles they find around the classroom. If student interest in problem 4 is high, you may want to devote a section of your whiteboard to angle measurement, setting up a chart similar to the one shown below, which students can add to over the next few days.

Measuring Angles in Our Classroom

Less than 90°	Exactly 90°	More than 90° but less than 180°
Point on my collar = 75°	Corner of a piece of paper = 90°	Hexagon Pattern Block = 120° Bench leg = 107° Trapezoid table corner = 120°

INDEPENDENT WORKSHEET

Use Set C3 Independent Worksheet 6 (on page C3.49) for additional practice with angle measurement.

NAME ______________________________ **DATE** ______________

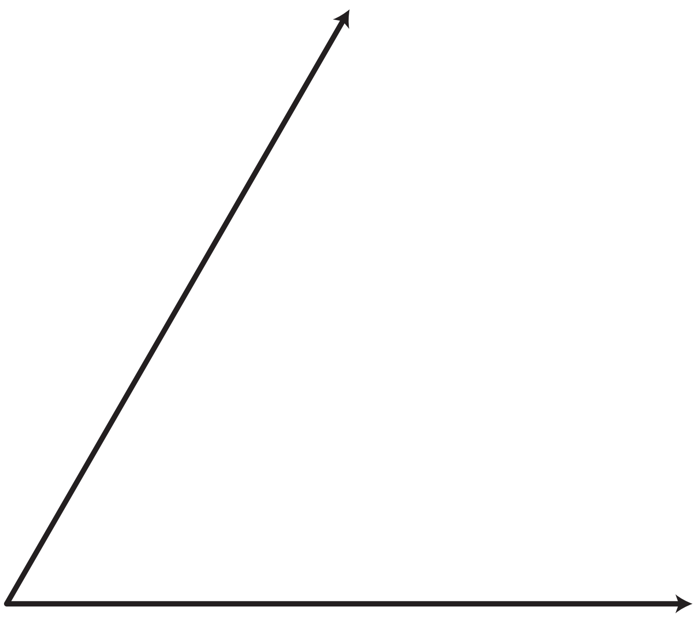
Experimenting with Angle Measurement page 1 of 2

1 For each angle below:

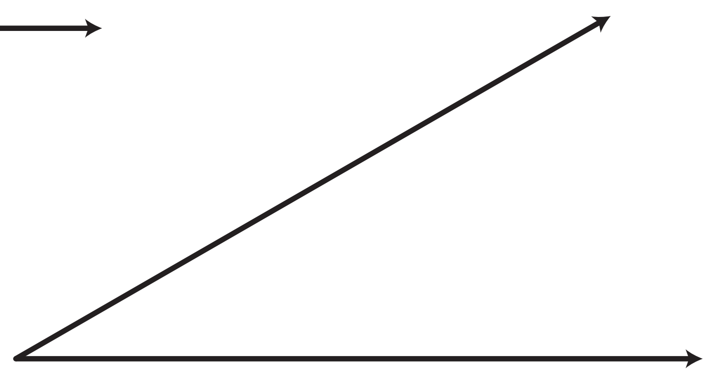
a Estimate how many degrees you think it measures.

b Use a pattern block to check the measure. (Each angle below matches one or more of the angles in your pattern blocks.)

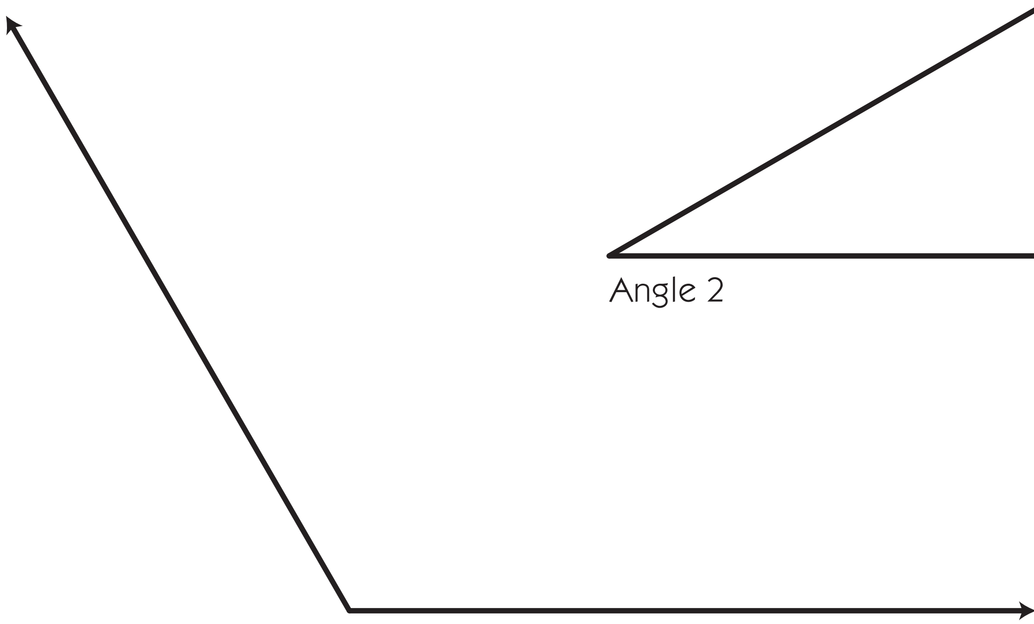
c Measure it with your protractor and label it.



Angle 1



Angle 2



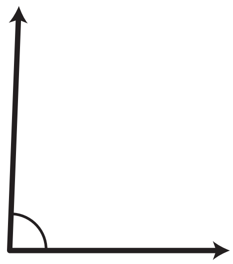
Angle 3

(Continued on back.)

NAME ______________________ DATE ______________

Experimenting with Angle Measurement page 2 of 2

2 Lan says the angle below measures about 120º. Do you agree or disagree with her? Explain your answer.



3 Using a protractor, construct a 60º angle below. Check your work with a pattern block, and include the pattern block in your angle sketch.

CHALLENGE

4 Look around your classroom for acute angles. Choose several. For each angle you choose:

a Estimate how many degrees you think it measures.

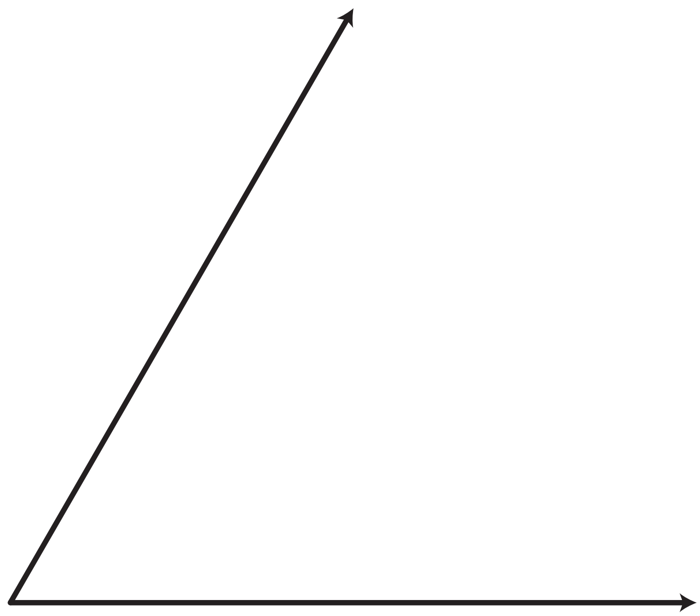
b Measure it with your protractor.

c Record your work on the chart below.

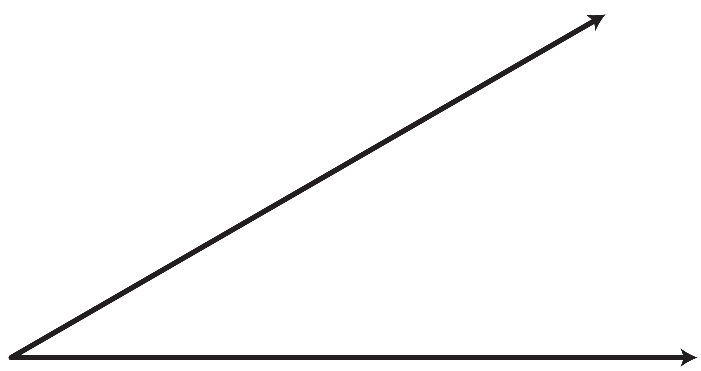
Acute Angles in the Classroom	How many degrees? (estimate)	How many degrees? (actual measure)

NAME ______________________________ **DATE** ______________

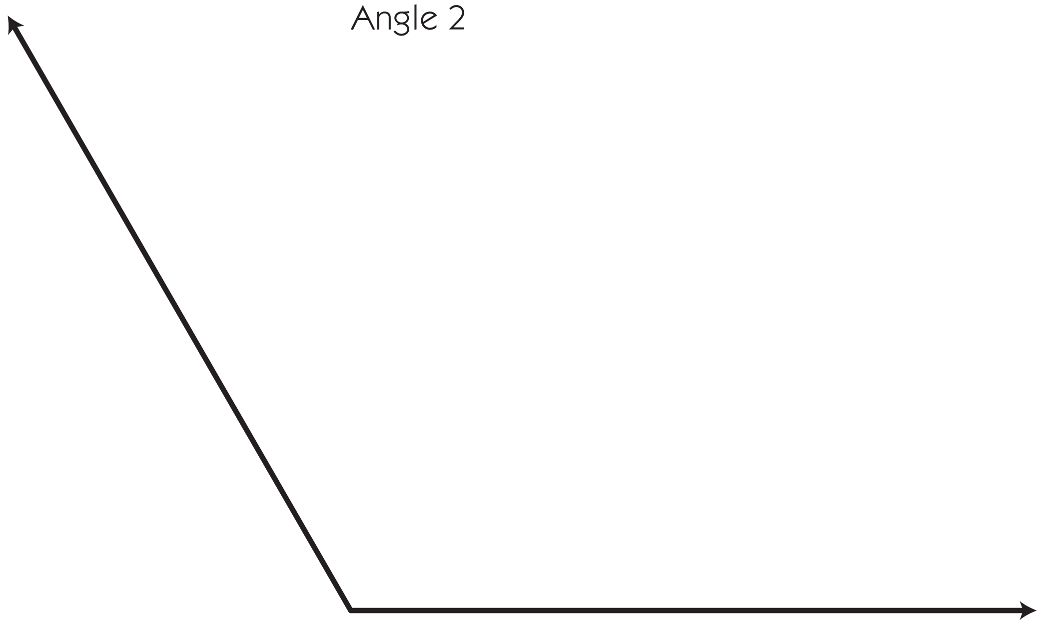
Measuring Angles



Angle 1



Angle 2



Angle 3

Set C3 ★ Activity 5

ACTIVITY

Drawing Stars

Overview

Students discuss how to use the protractor to draw angles. Then they use the protractors and their understanding of rotations to create star designs.

Skills & Concepts

- ★ constructing figures with tools
- ★ using a protractor to measure angles
- ★ measuring angles
- ★ using measurement tools to accurately construct angles
- ★ describing rotational symmetry

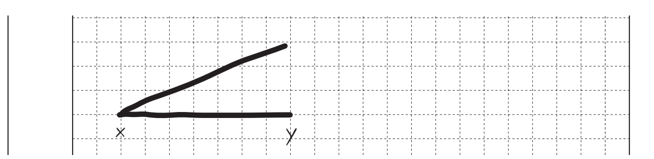
You'll need

- ★ 5-Point Star Template (page C3.34, run a class set plus 1 copy for display.)
- ★ Instructions for Drawing 5-Point Stars (page C3.35, run a class set)
- ★ Challenge 9-Point Star Template (page C3.36, run as needed. Optional.)
- ★ Student Journals or Journal Page Grid (page C3.37 run a class set, optional)
- ★ rulers, class set
- ★ protractors, class set
- ★ crayons, markers, or other art supplies for decorating designs

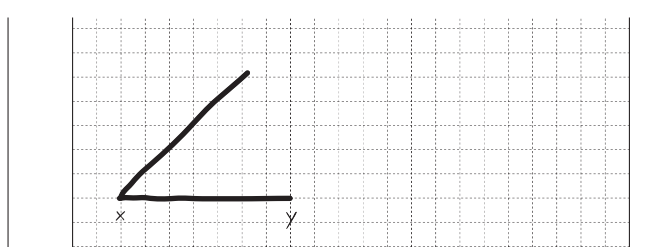
Instructions for Drawing Stars

1. Start the session by having students open their journals to the next available page. Then ask them to use their ruler to draw and label a 3 ½″ line segment, XY, as you do so at the display. Next, ask them to draw a second line segment that meets the first at a 45º angle, using a ruler to make sure the segment is straight, but *estimating* the size of angle itself. Encourage them to talk to their neighbors about how they made their estimates.

> ***Nick*** *Here's my angle. It looks like about 45º to me. I know it's less than a right angle.*



> ***Rian*** *I know that 90º is a square corner and 45 is half of 90, so I made a diagonal right through the squares.*



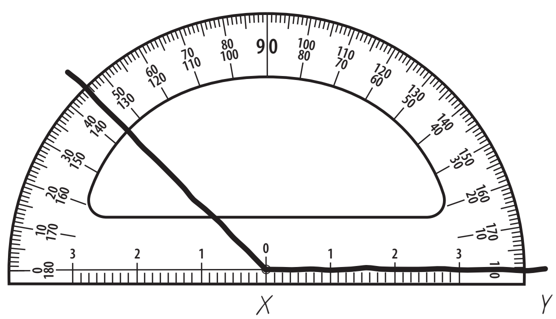
Activity 5 Drawing Stars (cont.)

2. Ask how they could check the angles they have drawn and adjust them if they don't measure 45º. Give them a moment to share ideas in pairs and then discuss their ideas as a whole class.

> ***Students*** *We said you could make a 90˚ angle and then cut it in half.*
> *That's what I did in my journal! I drew the first line across the page and drew the other one so it cut across the squares diagonally. It was like cutting 90º in half.*
> *You could take a paper square, fold it on the diagonal, and use it to check your angle.*
> *You could use a geoboard the same way. Just make a square and one of its diagonals and then set it on top of the angle you made in your journal to check it.*
> *We said you could use a protractor to check your angle too.*

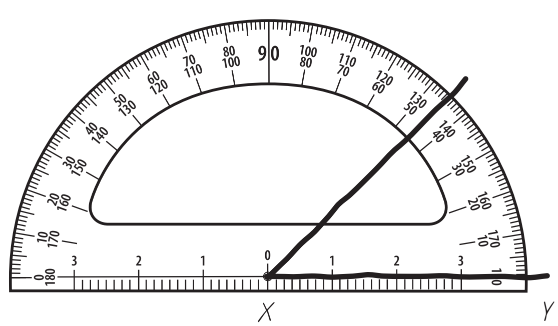
3. After students have shared their ideas, ask them to measure their angles with a protractor. If their angle doesn't measure 45º, have them leave their original angle but draw in a second line segment that does meet XY at a 45º angle. When they are finished, have them compare their results in pairs, and then invite 2 or 3 pairs of volunteers to share their work with the class, starting with the line segment XY you drew on the display.

> ***Jaime*** *The first angle I drew was way too skinny. To make a new one, I put that little hole at the bottom of the protractor at the end of the line where the X is. Then I went to the top point at 45˚, made a mark, and drew my angle in.*



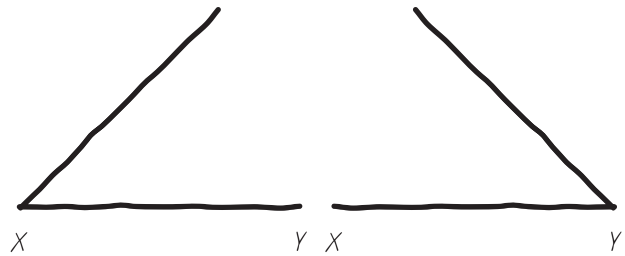
> ***Josie*** *When Jaime showed me, I noticed that angle he got was obtuse, and a 45º has to be smaller than a right angle.*

> ***Jaime*** *So then I fixed it by erasing my line and making my mark at the other 45º place on the protractor. When I drew the line and looked at it with the protractor on top again, I could see it had to be right because it's half as big as a right angle.*



> ***Justin*** *When Jon and I talked, we saw that we made the same angles, and they were reflections!*

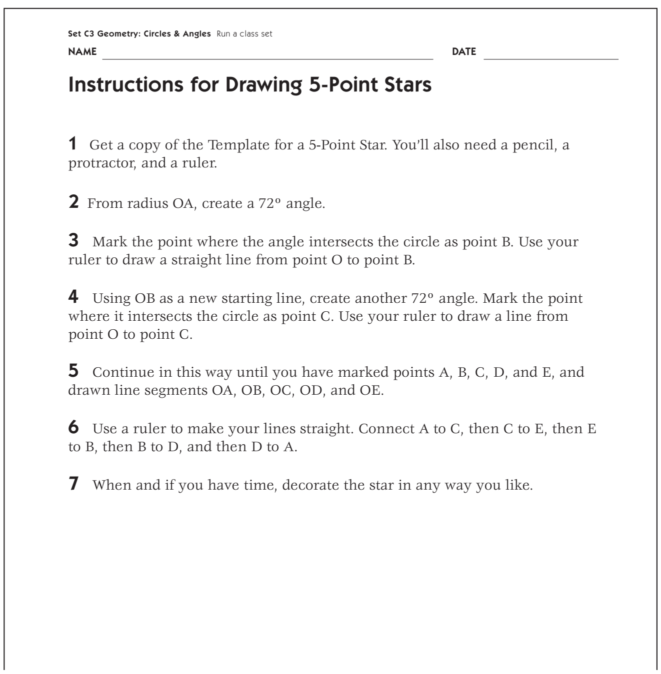
Activity 5 Drawing Stars (cont.)



> ***Kamela*** *When I checked my first angle, the line was at about the 50˚ mark. All I had to do was make a little dot by the 45º mark and draw in my new angle. It was almost the same.*

In the following activity, students will have more practice using the protractor as a drawing tool. If some students already find it easy to construct angles with a protractor, you might station at least one of them at each table to help classmates who find the task more challenging.

2. Distribute a copy of Instructions for Drawing Stars and of the 5-Point Star Template to each student.



Set C3 Geometry: Circles & Angles Run a class set

NAME ______ DATE ______

Instructions for Drawing 5-Point Stars

1 Get a copy of the Template for a 5-Point Star. You'll also need a pencil, a protractor, and a ruler.

2 From radius OA, create a 72º angle.

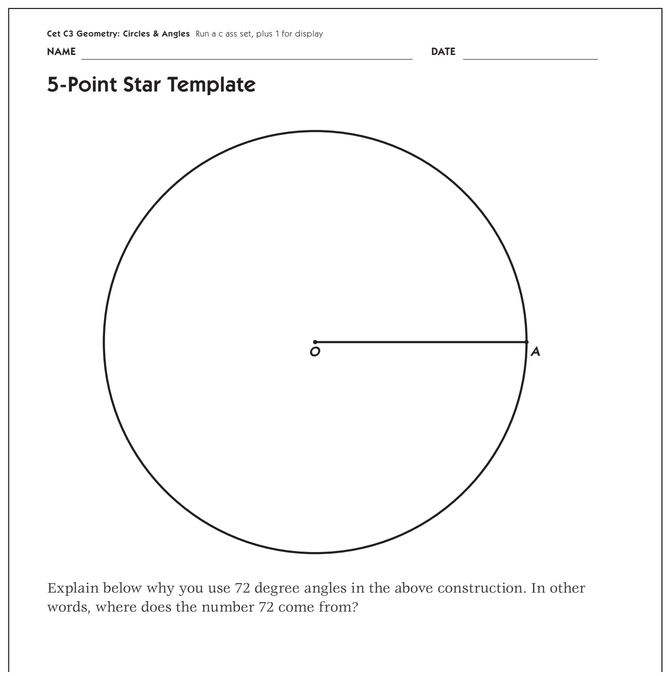
3 Mark the point where the angle intersects the circle as point B. Use your ruler to draw a straight line from point O to point B.

4 Using OB as a new starting line, create another 72º angle. Mark the point where it intersects the circle as point C. Use your ruler to draw a line from point O to point C.

5 Continue in this way until you have marked points A, B, C, D, and E, and drawn line segments OA, OB, OC, OD, and OE.

6 Use a ruler to make your lines straight. Connect A to C, then C to E, then E to B, then B to D, and then D to A.

7 When and if you have time, decorate the star in any way you like.



Cet C3 Geometry: Circles & Angles Run a c ass set, plus 1 for display

NAME ______ DATE ______

5-Point Star Template

Explain below why you use 72 degree angles in the above construction. In other words, where does the number 72 come from?

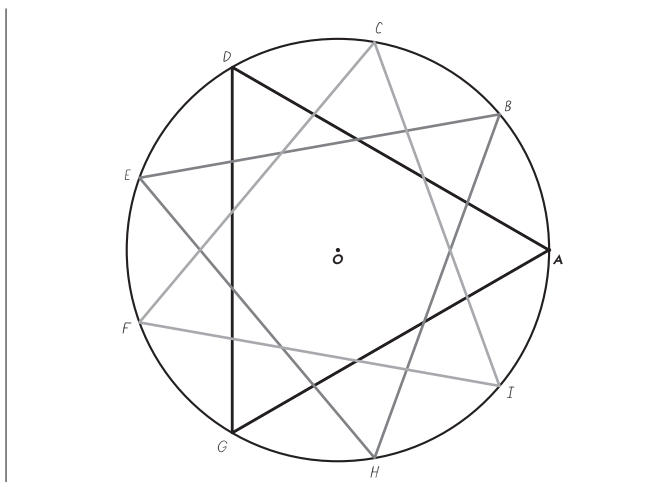
Have students read and follow the first 3 or 4 steps for drawing a 5-point star together as a group. Once each student has drawn at least one 72º angle, ask students to follow the rest of the instructions on their own, consulting with classmates as necessary for assistance. Give students extra copies of the template if they make mistakes that cannot be erased. Give students who finish their 5-point stars well ahead of their classmates the Challange 9-Point Star Template. If you find students need help in getting started or following the directions, you can model how to draw a few points on the 5-Point Star Template.

3. When most or all students have completed the 5-point star, reconvene the class for a group discussion. Some students may need more time to decorate their 5-point stars. You might provide that additional time or ask students to complete and decorate their stars at home. When students' stars are completed, you might display them on the wall or in the hallway.

Activity 5 Drawing Stars (cont.)

4. If you're able to create a wall display of the completed stars, consider gathering students in front of the display to conduct the discussion. Begin by asking students to make some mathematical observations about their stars (both the 5-point stars and the challenge 9-point stars). Many will notice the line symmetry in their stars. During this discussion, you might also ask students why they drew 72° angles to make the 5-point star. Where did the number 72 come from?

> ***Teacher*** *What mathematical observations can you make about your stars?*
>
> ***Yolanda*** *They all have a lot of symmetry to them. And all the 5-pointed stars are congruent.*
>
> ***Raven*** *I noticed that in the 9-pointed stars, they had us make 3 triangles, and each triangle is equilateral.*
>
> ***Teacher*** *How could you show that each triangle was equilateral? And now I'm really curious. Do you think the 9-pointed stars themselves are also equilateral?*
>
> ***Raven*** *Well, I would have to show that all of the sides were the same. I could do that! All the points are equally spaced out along the circle. Look at triangle ADG. To get from A to D you go past 2 points. Same with D to G and then G to A. And if the triangles are equilateral, the star has to be, right?*

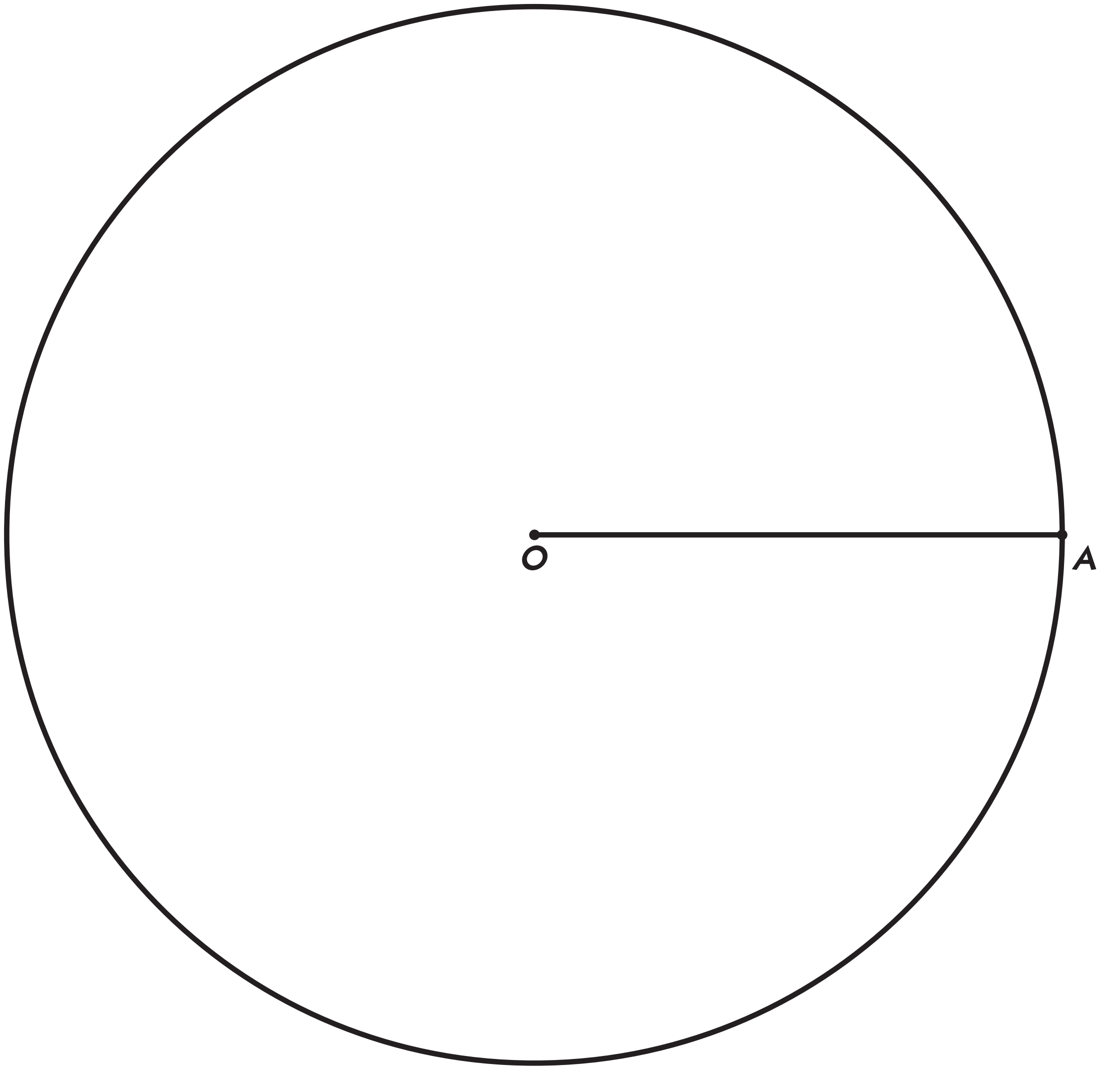


Extension

If students are interested, invite them to use the template to create a 9-Point star. They will use their protractor to identify 9 equally spaced points along the circle: A, B, C, D, E, F, G, H and I. Then use their rulers to connect the points to form triangles, ADG, BEH and CFI and decorate their stars.

NAME ______________________________ **DATE** ______________

5-Point Star Template



Explain below why you use 72 degree angles in the above construction. In other words, where does the number 72 come from?

NAME ______________________ DATE ____________

Instructions for Drawing 5-Point Stars

1 Get a copy of the Template for a 5-Point Star. You'll also need a pencil, a protractor, and a ruler.

2 From radius OA, create a 72º angle.

3 Mark the point where the angle intersects the circle as point B. Use your ruler to draw a straight line from point O to point B.

4 Using OB as a new starting line, create another 72º angle. Mark the point where it intersects the circle as point C. Use your ruler to draw a line from point O to point C.

5 Continue in this way until you have marked points A, B, C, D, and E, and drawn line segments OA, OB, OC, OD, and OE.

6 Use a ruler to make your lines straight. Connect A to C, then C to E, then E to B, then B to D, and then D to A.

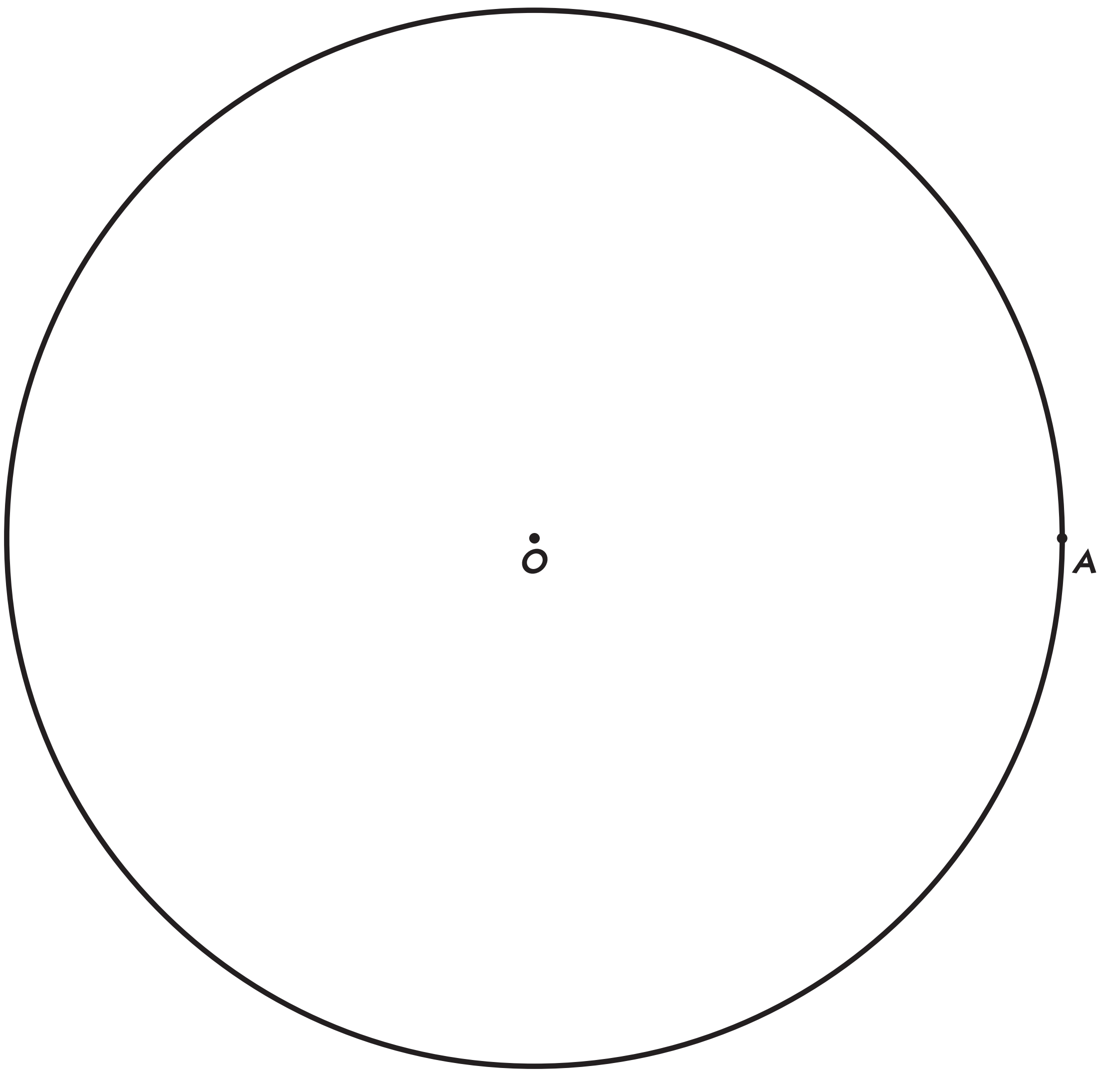
7 When and if you have time, decorate the star in any way you like.

NAME ______________________ **DATE** ______________

Challenge 9-Point Star Template

Instructions for Drawing 9-Point Stars

- Create 9 equally spaced points along the circle: A, B, C, D, E, F, G, H and I. (Hint: Think about the 72 degree angles in your 5-point star and where they came from.)
- Using your ruler, connect the points to form triangles ADG, BEH, and CFI.
- When and if you have time, decorate the star in any way you like.



Explain below how your protractor helped you space the points equally. In other words, where does the angle measurement come from?

NAME ______________________________ **DATE** ______________

Journal Page Grid

NAME ________________________________ **DATE** ________________

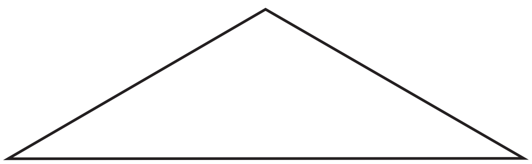
Set C3 ★ Independent Worksheet 1

INDEPENDENT WORKSHEET

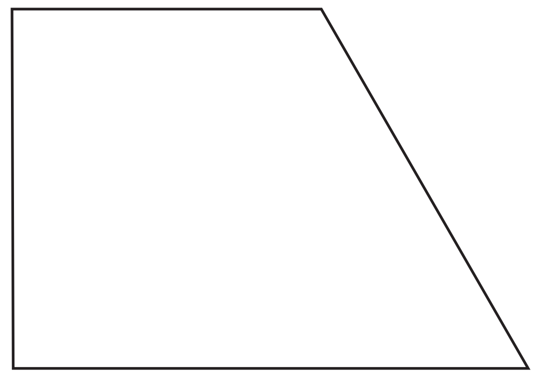
Measuring Interior Angles of Polygons

Use your pattern blocks to measure the interior angles of each polygon below. Label each angle with its measurement. Then write the name of the polygon.

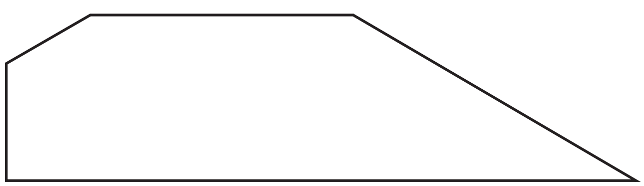
1 ______________________________



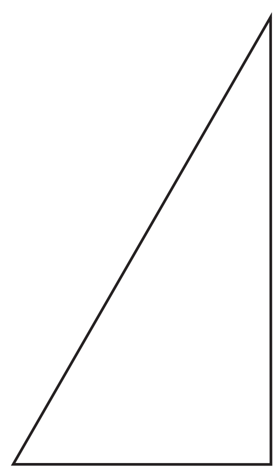
2 ______________________________



3 ______________________________



4 ______________________________



NAME ______________________________ **DATE** ______________

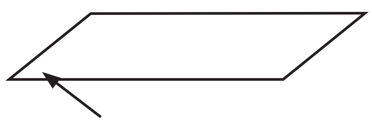
Set C3 ★ Independent Worksheet 2

INDEPENDENT WORKSHEET

Angles in the Classroom

Search your classroom for objects with the angles described below. Complete the chart by sketching each object, labeling it, and drawing an arrow to the angle that is less than, equal to, or greater than the benchmark angle. Use pattern blocks to help.

1 Benchmark Angle: 90°

Less than 90°	Equal to 90°	Greater than 90°
 This angle on the side of my eraser.		

2 Benchmark Angle: 60°

Less than 60°	About Equal to 60°	Greater than 60°

3 Benchmark Angle: 180°

Less than 180°	Equal to 180°	Greater than 180°

NAME ______________________ **DATE** ______________

Set C3 ★ Independent Worksheet 3

INDEPENDENT WORKSHEET

Drawing Polygons

Use a ruler and pattern blocks to draw a polygon that fits each description. Each square on the grid is 1 cm by 1 cm.

1a A rectangle that is 2.5 cm by 5.5 cm

b Label each angle and side length in the rectangle below.

2a An equilateral triangle with sides that are each 3 cm

b Label each angle and side length in the triangle below.

3a A trapezoid with one side exactly 2.5 cm, another side 3 cm, and angles 90°, 90°, 120° and 60°

b Label each angle and side length in the trapezoid below.

4a A rhombus with all sides 3.5 cm, two 60° angles, and two 120° angles

b Label each angle and side length in the rhombus below.

(Continued on next page.)

NAME ______________________________ DATE ______________

Drawing Polygons (cont.)

Use a ruler and pattern blocks to draw a polygon that fits each description. Each square on the grid is a quarter inch by a quarter inch.

5a A rectangle that is $1\frac{1}{2}$ in. by $2\frac{1}{4}$ in.

b Label each angle and side length in the rectangle below.

6a An isosceles triangle with two 30° angles

b Label each angle and side length in the triangle below.

7a A trapezoid with one side exactly 2 in., another side exactly 1 in., and two 60° angles

b Label each angle and side length in the trapezoid below.

8a A rhombus with two 120° angles

b Label each angle and side length in the rhombus below.

NAME ______________________________ **DATE** ______________

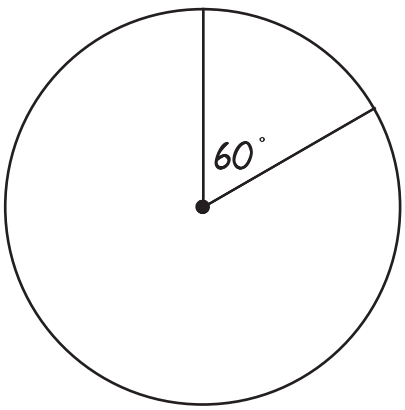
Set C3 ★ Independent Worksheet 4

INDEPENDENT WORKSHEET

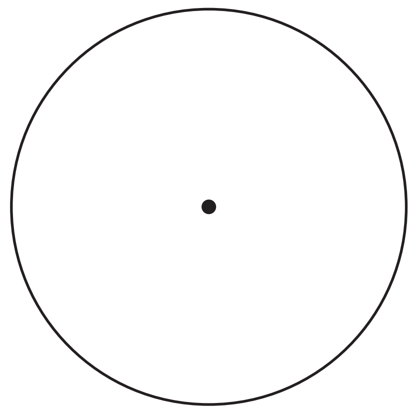
Drawing Angles of Rotation

Draw and label the angles that are equal to each fraction of a whole turn around the circle. Use your pattern blocks to make the angles exact. Remember that there are 360 degrees in a full turn around the circle.

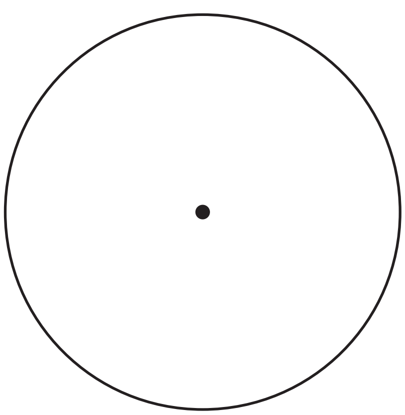
ex Draw and label $\frac{1}{6}$ turn.



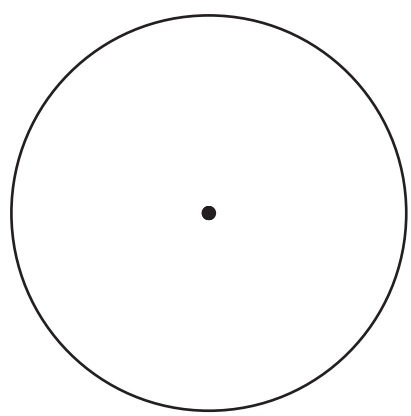
1 Draw and label $\frac{1}{2}$ turn.



2 Draw and label $\frac{1}{4}$ turn.



3 Draw and label $\frac{3}{4}$ turn.



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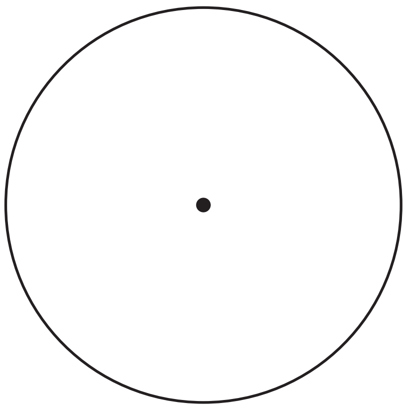
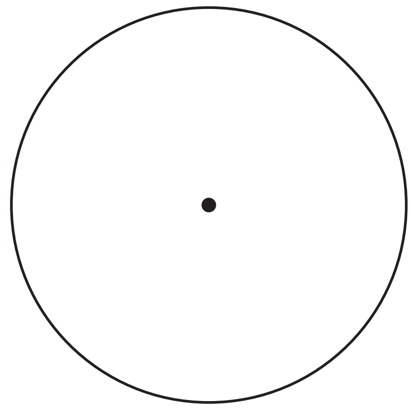
NAME ______________________________ DATE ______________

Drawing Angles of Rotation (cont.)

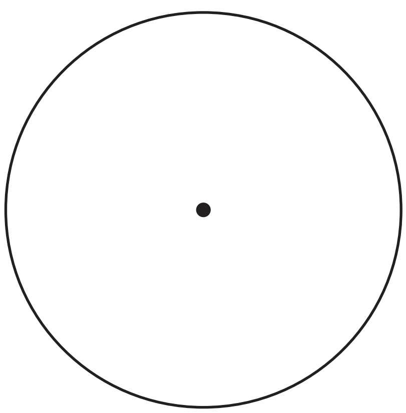
4 Draw and label $\frac{1}{3}$ turn.



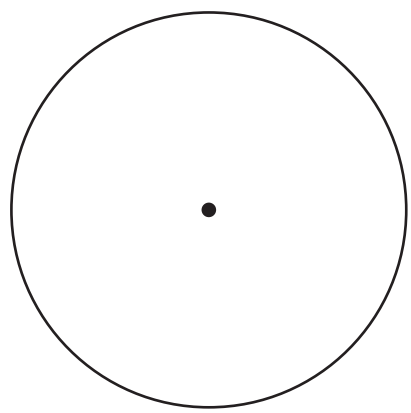
5 Draw and label $\frac{2}{3}$ turn.



6 Draw and label $\frac{5}{6}$ turn.



7 Draw and label $\frac{5}{12}$ turn.



NAME ______________________________ **DATE** ______________

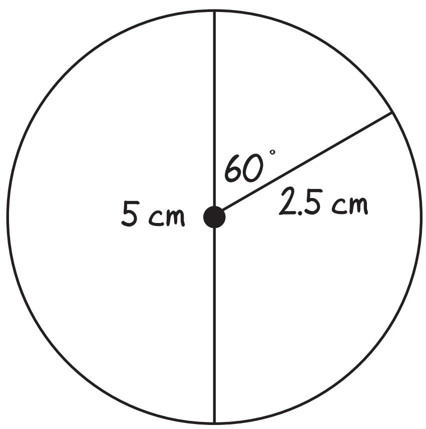
Set C3 ★ Independent Worksheet 5

INDEPENDENT WORKSHEET

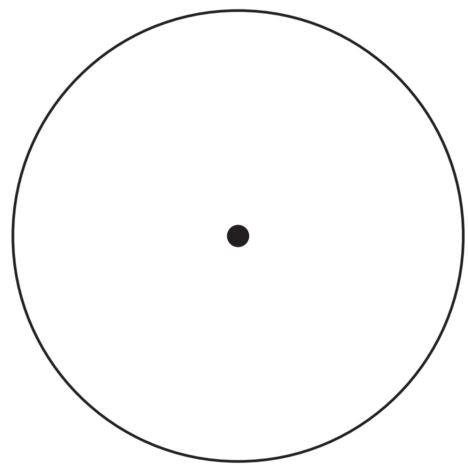
Drawing & Measuring Circles

Draw, measure, and label the radius and diameter of each circle below. Use the centimeter side of your ruler, and measure to the nearest millimeter if necessary. Use your pattern blocks to draw each radius at the given angle from the diameter.

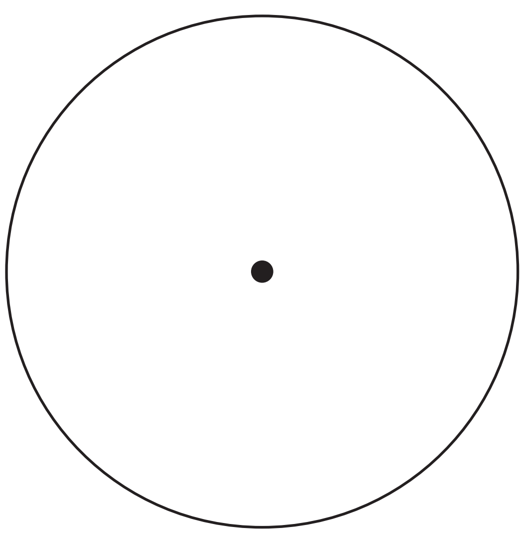
ex Draw the radius at a 60°angle to the diameter.



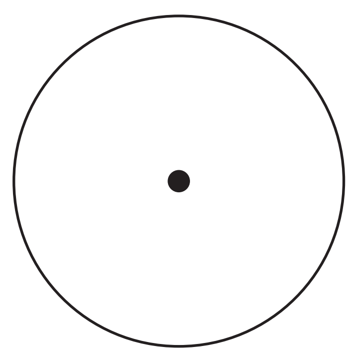
1 Draw the radius at a 30°angle to the diameter.



2 Draw the radius at a 90°angle to the diameter.



3 Draw the radius at a 120°angle to the diameter.



NAME ______________________________ **DATE** ______________

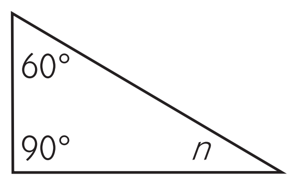
Set C3 ★ Independent Worksheet 6

INDEPENDENT WORKSHEET

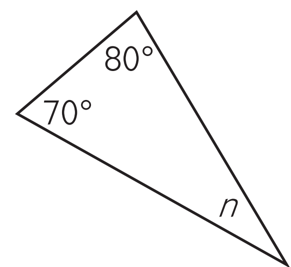
Find the Angle Measure

1 The sum of the angle measures in a triangle is 180 degrees. Below are 4 triangles, each with a missing angle measure labeled *n*. For each one, choose the value of *n*.

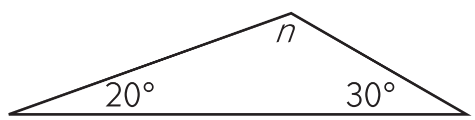
a
- ○ 20 degrees
- ○ 30 degrees
- ○ 50 degrees
- ○ 60 degrees



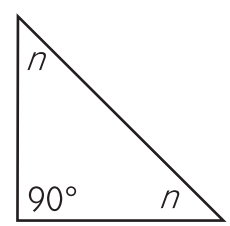
b
- ○ 10 degrees
- ○ 20 degrees
- ○ 30 degrees
- ○ 40 degrees



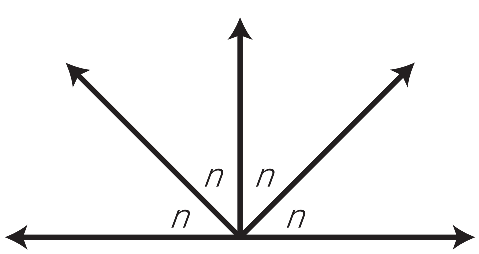
c
- ○ 130 degrees
- ○ 140 degrees
- ○ 150 degrees
- ○ 160 degrees



d
- ○ 30 degrees
- ○ 45 degrees
- ○ 50 degrees
- ○ 60 degrees



2a The 4 angles marked *n* below are congruent and have been put together to form a straight angle. Using sketches, numbers, and words, determine the value of each angle marked *n*. Show your work below.



b The value of each angle marked *n* is ________ degrees.