



GRADE 5 SUPPLEMENT

Set A12 Number & Operations: Dividing Fractions & Whole Numbers

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Skills & Concepts

- ★ Interpret quotients of whole numbers
- ★ Write story problems or describe problem situations to match a division expression or equation
- ★ Multiply a whole number by a fraction
- ★ Solve story problems involving multiplying a whole number or a fraction by a fraction
- ★ Solve story problems involving multiplication of fractions and mixed numbers
- ★ Divide a unit fraction by a whole number
- ★ Divide a whole number by a unit fraction
- ★ Write story problems involving division of a unit fraction by a whole number
- ★ Solve story problems involving division of a unit fraction by a whole number
- ★ Solve story problems involving division of a whole number by a unit fraction

Bridges in Mathematics Grade 5 Supplement
Set A12 Number and Operations: Dividing Fractions & Whole Numbers

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Bridges in Mathematics is a standards-based K–5 curriculum that provides a unique blend of concept development and skills practice in the context of problem solving. It incorporates the Number Corner, a collection of daily skill-building activities for students.

The Math Learning Center is a nonprofit organization serving the education community. Our mission is to inspire and enable individuals to discover and develop their mathematical confidence and ability. We offer innovative and standards-based professional development, curriculum, materials, and resources to support learning and teaching. To find out more, visit us at www.mathlearningcenter.org.

Background for the Teacher

Making Sense of Division with Fractions

"Division by fractions, the most complicated operation with the most complex numbers, can be considered as a topic at the summit of arithmetic."

Liping Ma (1999)

"Division of fractions is often considered the most mechanical and least understood topic in elementary school."

Dina Tirosh (2000)

Division of fractions often evokes anxiety in adults. Many recall a process of inverting and multiplying but very few understand why that procedure works. By providing a three-year period—Grades 5, 6, and 7—for students to learn to multiply and divide with fractions, the authors of the Common Core State Standards aim to help generations of learners understand these operations. Their goals for fifth graders are limited and reasonable. Specifically, Common Core requires fifth grade students to:

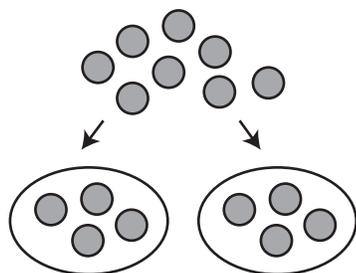
- Interpret division of a fraction by a whole number and division of a whole number by a fraction by, for instance, writing story problems to match expressions such as $6 \div \frac{1}{4}$ and $\frac{1}{2} \div 5$.
- Compute such quotients using visual models to represent and solve the problems. (Other than the expectation that students be able to write equations to represent story problems involving division of fractions, there is no call for specific numeric methods or algorithms.)
- Explain or confirm their answers by using the inverse relationship between multiplication and division (e.g., I know that $4 \div \frac{1}{3} = 12$ is correct because $12 \times \frac{1}{3} = 4$).

In order to comprehend and solve problems such as $\frac{1}{3} \div 4$ and $4 \div \frac{1}{3}$, we have to understand that there are two different interpretations of division: sharing and grouping. When we interpret division as sharing (sometimes called equal sharing, fair sharing, or partitive division), we share out a quantity equally, as shown below at left. We know how many groups we have to make; we have to find out what the size of each group is. When we interpret division as grouping (sometimes called measurement or quotative division), we know what the size of each group is; we have to find out how many groups we can make given the dividend with which we're working, as shown below at right.

$$8 \div 2 = 4$$

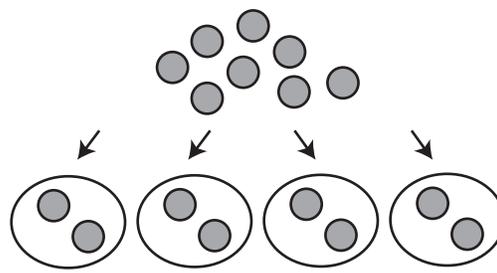
Sharing Interpretation

Here we interpret $8 \div 2$ to mean 8 divided or shared evenly, as between 2 people.



Grouping Interpretation

In this interpretation of $8 \div 2$, we determine how many groups of 2 we can make with 8.



Notice that the answer is the same in both interpretations, but it means something different in each case. In the sharing interpretation of division the result of dividing 8 by 2 tells us the size of each group;

Background for the Teacher Making Sense of Division with Fractions (cont.)

each person getting 4. In the grouping interpretation, we already know the size of the group—2. The result of dividing 8 by 2 tells us how many groups of 2 are in 8. (There are 4.)

The importance of knowing and understanding both interpretations of division cannot be overstated because both are required to make sense of division with fractions. Consider the following: $4 \div \frac{1}{3}$. If you read this expression and try to grapple with it in any kind of sensible way, the sharing interpretation of division seems unreasonable. How do you equally share 4 things with a third of a person? On the other hand, the grouping interpretation makes better sense. How many groups of one-third can you get from 4? In other words, how many thirds are there in 4? We can reason that—there are 3 thirds in 1, so there must be 4×3 or 12 thirds in 4. We can solve the problem sensibly without resorting to inverting and multiplying. In fact, there are a couple of visual models that make it possible for fifth graders to picture and solve the problem, as shown below.

$$4 \div \frac{1}{3}$$

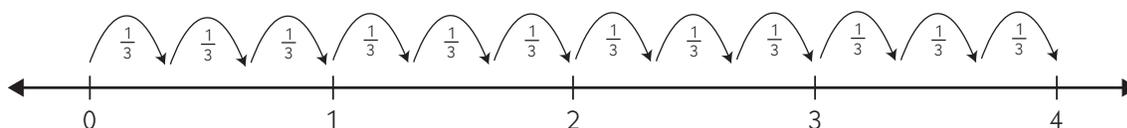
Grouping Interpretation of Division (Measurement or Quotative Division)

I have 4 cups of trail mix. How many $\frac{1}{3}$ cup sacks can I make with this amount of trail mix?

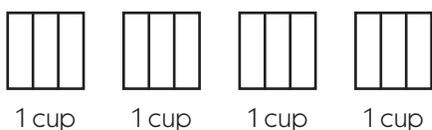
Basic Question: I know **what size** my groups (servings) are.

How many groups (servings) can I make?

Suggested Models: Number Line or Discrete Objects



I can make twelve one-third cup sacks with 4 cups of trail mix.



There are 3 thirds in each cup, so I can see there are 12 thirds in 4 cups. That means I can make Twelve one-third cup servings with 4 cups of trail mix.

I can also see that $4 \div \frac{1}{3} = 12$ because 12 thirds add up to 4, or $12 \times \frac{1}{3} = 4$

What about $\frac{1}{3} \div 4$? Can we use the grouping interpretation of division to help evaluate this expression? How many groups of 4 can you take out of $\frac{1}{3}$? Since that makes little sense, what about the sharing interpretation? Is it possible to divide $\frac{1}{3}$ into 4 equal shares? If you divide $\frac{1}{3}$ into 4 equal shares, each share is $\frac{1}{12}$. This may seem more difficult than figuring out how many thirds there are in 4, but a visual similar to the geoboard model students encountered in Supplement Set A9 for multiplying fractions enables fifth graders to represent and solve situations that involve dividing a fraction by a whole number, as shown next.

Background for the Teacher Making Sense of Division with Fractions (cont.)

$$\frac{1}{3} \div 4$$

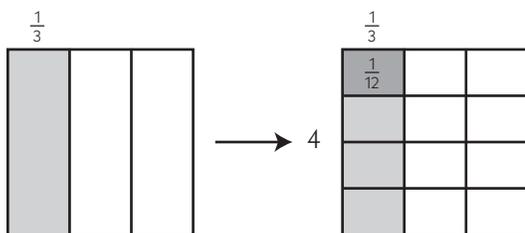
Sharing Interpretation of Division (Fair Sharing or Partitive Division)

4 people are going to share $\frac{1}{3}$ a pan of brownies. What fraction of the pan will each person get?

Basic Question: I know **how many** groups (servings) are going to be formed.

What size will each group (serving) be?

Suggested Model: Geoboard, Sketches of Open Arrays (see below)



Each person gets $\frac{1}{12}$ of a pan of brownies.

I can also see that $\frac{1}{3} \div 4 = \frac{1}{12}$ because 4 one twelfths add up to $\frac{1}{3}$, or $4 \times \frac{1}{12} = \frac{1}{3}$

The pre-assessment in Activity 1 addresses the competencies Common Core expects from fifth graders in relation to dividing fractions by whole numbers and vice versa, and will give you an opportunity to see how your students do with the following skills and concepts prior to instruction:

- Solving story problems that involve dividing a fraction by a whole number
- Solving story problems that involve dividing a whole number by a fraction
- Choosing the correct operation when presented with a story problem that requires multiplying rather than dividing a whole number by a fraction
- Interpreting division of whole numbers by fractions and fractions by whole numbers

Note If you have students who solve the problems on the assessment using an invert and multiply strategy, be aware that these children may benefit at least as much from the instruction in Activities 2–7 as those who have no way to tackle such problems yet, because the activities will give them an opportunity to make sense of an algorithm they may not really understand.

The models and instructional strategies you use during this supplement set will lead nicely into the work students do with multiplying and dividing fractions in Grades 6 and 7. Math educators Suzanne Chapin and Art Johnson caution us, however, that some of the division situations students will encounter in sixth and seventh grade include fractions that cannot be easily modeled using pictures or materials (e.g., $\frac{3}{4} \div \frac{2}{3}$). Chapin and Johnson go on to explain that,

It is important to realize that not all division situations are represented by actions based on partitive division or repeated subtraction (grouping division). For example, if the area of a rectangle is 10 square centimeters and the width is $\frac{1}{2}$ centimeter, the length of the rectangle can be found by calculating $10 \div \frac{1}{2}$ Area is a multidimensional quantity that is the product of length and width. The “invert and multiply” algorithm, which relies on the inverse relationships between

Background for the Teacher Making Sense of Division with Fractions (cont.)

multiplication and division, and between reciprocals, enables us not only to make sense of other situations but also to divide ‘messy’ fractions.

Math Matters: Understanding the Math You Teach

So, have no doubt that there is still a place for invert and multiply, but not in fifth grade. What you do with the students this year to meet the Common Core expectations will lay solid foundations on which middle school teachers can build so their students are able to use the algorithm with good understanding.

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Set A12 ★ Activity 1



ACTIVITY

Dividing Fractions & Whole Numbers Pre-Assessment

Overview

This pre-assessment launches a set of activities that deal with division of fractions by whole numbers and whole numbers by fractions. It is strongly recommended that teachers read the background information below and conduct the pre-assessment before teaching Activities 2–7 in this set. During Activity 8, students will take an expanded version of today’s assessment.

Skills & Concepts

- ★ Multiply a whole number by a fraction (CCSS 5.NF.4a)
- ★ Solve story problems involving multiplying a whole number by a fraction (CCSS 5.NF.4a)
- ★ Divide a unit fraction by a whole number (CCSS 5.NF.7a)
- ★ Divide a whole number by a unit fraction (CCSS 5.NF.7b)
- ★ Write story problems involving division of a unit fraction by a whole number (CCSS 5.NF.7b)
- ★ Solve story problems involving division of a unit fraction by a whole number (CCSS 5.NF.7c)
- ★ Solve story problems involving division of a whole number by a unit fraction (CCSS 5.NF.7c)

You’ll need

- ★ Operating with Fractions & Whole Numbers Pre-Assessment (pages A12.3–A12.5, run a class set plus a copy for display)
- ★ Operating with Fractions & Whole Numbers Pre-Assessment Class Checklist (optional, page A12.6, run 1 or 2 copies)

Instructions for Dividing Fractions & Whole Numbers Pre-Assessment

1. Explain to students that over the next couple of weeks, the class will study division of fractions and whole numbers. Today they’ll take a pre-assessment that will provide you information about what they already know and what they still need to learn regarding the skills and concepts involved. Explain that in about two weeks, they will take a similar assessment, at which time they will have additional ways to handle problems that may seem challenging today.
2. Give students each a copy of the pre-assessment. Ask them to write their name and the date at the top of each page. Read and review the problems together and have students circle the “doing” words as you go.

Activity 1 Dividing Fractions & Whole Numbers Pre-Assessment (cont.)

Set A12 Number & Operations: Dividing Fractions & Whole Numbers Blackline Run a class set plus 1 copy for display

NAME _____ DATE _____

Operating with Fractions & Whole Numbers
Pre-Assessment page 1 of 3

1 Solve each of the three story problems below. For each problem:

- Write an expression to represent the problem.
- Use numbers, visual models, labels, and/or words to solve the problem.
- Complete the sentence below with your solution to the problem.

a Shelly made cupcakes and now she wants to frost them. She has 4 cups of frosting. It takes $\frac{1}{3}$ a cup of frosting for each cupcake. How many cupcakes can she frost?

Expression: _____

3. Before students start to work, be sure they understand they have to show their work and/or explain their thinking for problems 1 and 2; the answers alone will not be adequate. Remind them to write a story problem to match the expression in problem 3, to solve the problem and write the answer. Remind students of the difference between an expression ($12 \div 2$) and an equation ($12 \div 2 = 6$). An expression is a mathematical phrase without an equal sign. An equation completes the expression with a solution after an equal sign. You might list examples for the students on the board (e.g. expressions: $3 + 4$, $27 + 9$, equations $3 + 4 = 7$, $27 + 9 = 36$.) Also, alert them to the fact that this assessment includes multiplication and division situations because it's important to determine which operation is called for in a given problem.

Let students know that you can't explain the tasks to them, but you will reread any of the problems to them if needed during the assessment period. Although they may not be sure how to solve some of the problems, encourage them to attempt each one. Partial solutions are fine, and if they are unable to answer a particular question or solve a particular problem they can write, "I don't know yet." You might also have them underline any words they don't understand.

4. Students will complete a similar assessment in Set A12, Activity 8, at which time a scoring guide will be included for your use. We recommend that you use the results of today's pre-assessment to help guide your instruction as you teach this set of activities. To help, you can use the Dividing Fractions & Whole Numbers Pre-Assessment Class Checklist on page A12.6 if you like. By compiling results for your entire class, you can get a sense of the areas in which the class as a whole may need extra support.

NAME _____

DATE _____

Operating with Fractions & Whole Numbers

Pre-Assessment page 1 of 3

- 1** Solve each of the three story problems below. For each problem:
- Write an expression to represent the problem.
 - Use numbers, visual models, labels, and/or words to solve the problem.
 - Complete the sentence below with your solution to the problem.

a Shelly made cupcakes and now she wants to frost them. She has 4 cups of frosting. It takes $\frac{1}{3}$ a cup of frosting for each cupcake. How many cupcakes can she frost?

Expression: _____

Shelly can frost _____ cupcakes.



(Continued on next page.)

NAME _____

DATE _____

Operating with Fractions & Whole Numbers Pre-Assessment page 2 of 3

b Jake and his dad are making flags for a scouting project. They are going to make 6 flags and need $\frac{2}{3}$ a yard of cloth for each flag. How many yards of cloth will they need in all?

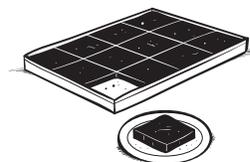
Expression: _____

Jake and his dad will need _____ yards of cloth in all.

c There is $\frac{1}{2}$ a pan of brownies left. Four children are going to share it equally. What fraction of the whole pan of brownies will each child get?

Expression: _____

Each child will get _____ of the whole pan of brownies.



(Continued on next page.)

NAME _____

DATE _____

Operating with Fractions & Whole Numbers Pre-Assessment page 3 of 3

2 Cory says that $6 \div \frac{1}{2}$ means the same thing as $\frac{1}{2}$ of 6, so the answer is 3. Do you agree with him? Why or why not? Use numbers, labeled models, and/or words to explain your thinking.

3 Write a story problem to represent the expression in the box below. Then solve your own problem. Show your work with labeled models, numbers, and/or words, and write the answer on the line provided.

$\frac{1}{2} \div 3$

My story problem:

My work:

The answer to my problem is _____.

Operating with Fractions & Whole Numbers Pre-Assessment Class Checklist

<p>Item and Correct Answer</p> <p>1a Write an expression to represent a story problem that involves dividing a whole number by a unit fraction. Solve the problem; show work. Expression: $4 \div \frac{1}{3}$ Answer: 12 cupcakes; student work will vary</p> <p>1b Write an expression to represent a story problem that involves multiplying a whole number by a fraction. Solve the problem; show work. Expression: $6 \times \frac{2}{3}$ OR $\frac{2}{3} \times 6$ Answer: 4 yards of cloth; student work will vary</p> <p>1c Write an expression to represent a story problem that involves dividing a unit fraction by a whole number. Solve the problem; show work. Expression: $\frac{1}{2} \div 4$ Answer: $\frac{1}{8}$ the pan of brownies; student work will vary</p> <p>2 Interpret division of a whole number by a unit fraction. Answer: No; students' explanations will vary. Example: I don't agree because $6 \div \frac{1}{2}$ is not the same thing as half of 6. It means how many halves in 6. Since there are 2 halves in 1, there are 12 halves in 6.</p> <p>3 Write and solve a story problem to represent the expression $\frac{1}{2} \div 3$; show work Answer: $\frac{1}{6}$; Students' story problems and work will vary. Example: I had half a sandwich. I gave it to my 3 friends to share equally. How much of the sandwich did each friend get?</p>	<p>CCSS</p> <p>5.NF.7b 5.NF.7c</p>	<p>Student Name</p>											

Set A12 ★ Activity 2



ACTIVITY

Reviewing the Sharing & Grouping Interpretations of Division

Overview

During this activity, students and teacher review two different interpretations of division—grouping and sharing. The computation and the story problems throughout the activity are deliberately simple, involving only whole numbers, to allow students to investigate and understand both interpretations of division.

Skills & Concepts

- ★ Find whole-number quotients of whole numbers using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. (CCSS 5.NBT.6)
- ★ Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (CCSS 5.NBT.6)

You'll need

- ★ Poster Problems (page A12.12, run 1 copy on paper, see Advance Preparation)
- ★ Two Interpretations of $15 \div 3$ (page A12.13, run 1 copy for display)
- ★ Grouping or Sharing? (pages A12.14 and A12.15, run 1 copy for display and a class set on paper)
- ★ Students' math journals or 1 piece of scratch paper per student
- ★ colored tile (15 per student)
- ★ 15 tile for display
- ★ 2 sheets chart paper (see Advance Preparation)
- ★ marking pens

Advance Preparation Run a copy of the Poster Problems Blackline. Trim the problems and glue one to the top of each piece of unlined chart paper. Display the charts where all the students can see them.

Jeff made 12 cookies, and divided them equally among his 4 cousins. How many cookies did each cousin get?

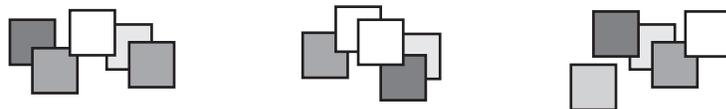
Jeff made 12 cookies and packed them into sacks of 4. How many sacks of cookies was he able to make?

Instructions for Reviewing the Sharing & Grouping Interpretations of Division

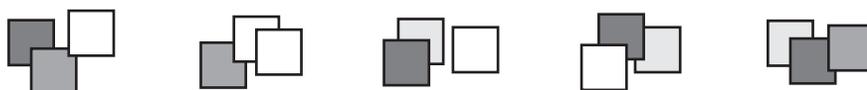
1. Open today's instruction by letting students know that you're going to take a day to investigate two different ways to think about division before starting to work on problems that involve dividing with fractions.
2. Have helpers place a container of tile at each table or cluster of desks as you write the expression $15 \div 3$ on the board. Ask students to each model the expression with tile. After they've had a minute or two to work, have them share their work with the people sitting next to them. Then call on a volunteer to build his model at the display and explain his thinking to the class.

Activity 2 Reviewing the Sharing & Grouping Interpretations of Division (cont.)

Craig I got 15 tiles and divided them into 3 groups because it says to divide by 3. There were 5 in each group when I got finished.



3. Ask the students if anyone got a different answer or had a different way of thinking about the expression. If no one responds, explain that there are two different ways to think about division. The first is called a sharing interpretation because the total number of objects is shared out evenly among a certain number of groups, 3 in this case. The second is called a grouping interpretation. Model this interpretation yourself if it hasn't come from any of the students by counting out 15 tile as the students watch and then pulling off groups of 3 until none of the tile remain in the original group.



4. Discuss the grouping interpretation with the students. In what way does this interpretation show $15 \div 3$? Is the answer still 5? If so, 5 what?

Alicia I get the way Craig showed us. With that way, you just take 15 and split it into 3 groups. You get 5 in each group. That's the answer. I don't get that other way you showed.

Xavier I think I get it. With the other way, you start with 15, right? Then you see how many groups of 3 you can make. It's 5. You can make 5 groups of 3. The answer is still 5, but it means how many groups instead of how many tile in each group.

5. Follow the discussion by placing the Two Interpretations of $15 \div 3$ master. Give students a few moments to examine the master quietly. Then work with the students to determine and record the meaning of each number in the equation for each model.

Teacher Let's take a look at the sharing interpretation of division first. In this interpretation, what does the number 15 mean?

Students It's the number you start with.
15 tells the number of tile you have to start with.

Teacher And what does the number 3 mean in this interpretation of division? Talk to the person next to you about this, and show thumbs up when you know. Cory and Mariah, what did you decide?

Mariah We said the 3 tells you how many groups to split the tiles into.

Cory It's kind of like it tells you how many people are going to share 15 tiles or something.

Teacher And what about the 5? What does that mean in the sharing interpretation?

Activity 2 Reviewing the Sharing & Grouping Interpretations of Division (cont.)

Students *That's the answer.*

It tells how many each group gets.

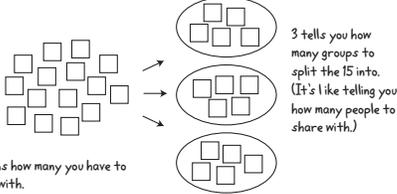
It's like if you had 15 of something and shared it with 3 kids, each kid would get 5.

Here's how the master might look when you and the class have completed step 5.

Set A12 Number & Operations: Dividing Fractions & Whole Numbers Run 1 copy or display

Two Interpretations of $15 \div 3$

Sharing



15 means how many you have to start with.

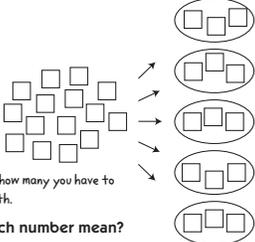
3 tells you how many groups to split the 15 into. (It's like telling you how many people to share with.)

5 is the answer. It tells how many tiles each group gets.

What does each number mean?

$15 \div 3 = 5$

Grouping



15 means how many you have to start with.

5 is the answer. It tells how many groups you were able to make.

3 tells you how many to put in each group.

What does each number mean?

$15 \div 3 = 5$

6. Then draw students' attention to the two charts you posted. Read the story problem at the top of each sheet with the class, and solicit the answer to both—3. Pose the following question before you move along to the next step: Even though the answer to both problems is 3, does the 3 mean the same thing in each case?

7. Rather than taking time to discuss the question with the class right away, ask students to get out their math journals, or give them each a piece of scratch paper to work on. Have them:

- Draw a line down the middle of the next available journal page or sheet of paper, and label the top of one side *sharing* and the other side *grouping*.
- Have them model each situation using simple dots and loops.
- Ask them to label each visual model with an equation to match, and explain, in writing, the meaning of each number in the equation.

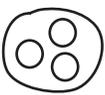
8. When students have had 7–10 minutes to work on the task, reconvene the class. Ask one volunteer to describe her model for the first problem. Then have the other students explain the meaning of each number in the matching equation. Repeat this with the second problem. As students are explaining, record their thinking on the two pieces of chart paper.

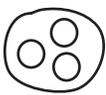
Activity 2 Reviewing the Sharing & Grouping Interpretations of Division (cont.)

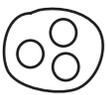
9. As you discuss the two problems with the class, pose the following questions and prompts, and record students' responses on the charts:

- What information is provided in each problem, and what do you have to find out?
- What is the key question in each problem?
- What interpretation of division does each story problem illustrate? (If your students want to devise their own names for the sharing and grouping strategies, that's fine.)

Jeff made 12 cookies, and divided them equally among his 4 cousins. How many cookies did each cousin get?

Kid 1


Kid 2


Kid 3


Kid 4


$12 \div 4 = 3 \text{ cookies}$

How many he started with

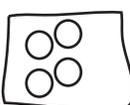
How many kids he shared with or
How many groups he divided the cookies into

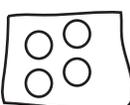
How many each kid got or
How many in each group

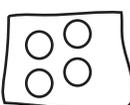
Key Question:
How many cookies does each kid get? (SIZE of groups)

Sharing Division

Jeff made 12 cookies and packed them into sacks of 4. How many sacks of cookies was he able to make?

Sack 1


Sack 2


Sack 3


$12 \div 4 = 3 \text{ sacks}$

How many he started with

How many he's going to put in each sack or
How many he put in each group

How many sacks he was able to make with 12 cookies

Key Question:
How many groups can you make if you put 4 in each group (NUMBER of groups)

Grouping Division

10. Give students each a copy of Grouping or Sharing? and display the masters for everyone to see. Review and discuss the tasks with the class. Give students the option of working on these sheets independently, in pairs, or with you.

11. As students finish, have them share and compare their work with a classmate. If there are differences in some of their responses, have them work together to resolve those differences or check with a third classmate before consulting you. Have them check each others' story problems to be sure each correctly illustrates the designated interpretation of division.

If students need more time to complete the assignment, ask them to finish it as homework or as part of their morning seatwork the following day before you conduct Activity 3.

***Note** Leave the story problem posters you made with the class today on display for students' reference throughout Activities 3–7 in this supplement set.*

Activity 2 Reviewing the Sharing & Grouping Interpretations of Division (cont.)



INDEPENDENT WORKSHEET

See Set A12 Independent Worksheet 1 on page A12.75 for more practice with identifying sharing and grouping situations, identifying the operation needed to solve a story problem, and multiplying fractions and whole numbers.

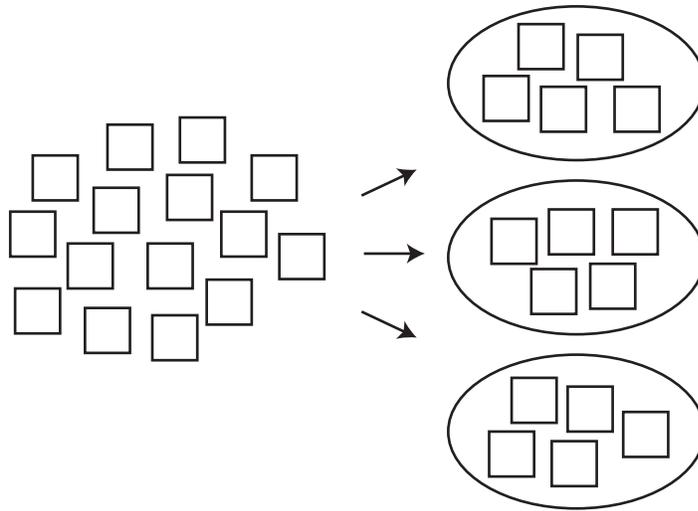
Poster Problems

Jeff made 12 cookies, and divided them equally among his 4 cousins. How many cookies did each cousin get?

Jeff made 12 cookies and packed them into sacks of 4. How many sacks of cookies was he able to make?

Two Interpretations of $15 \div 3$

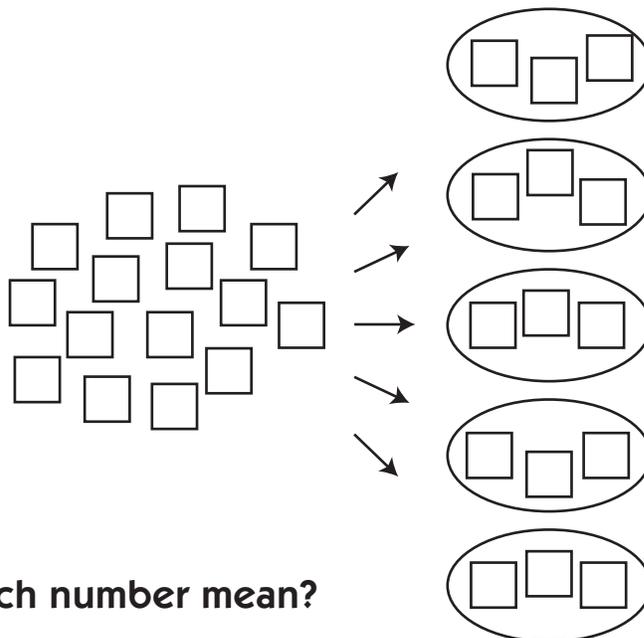
Sharing



What does each number mean?

$$15 \div 3 = 5$$

Grouping



What does each number mean?

$$15 \div 3 = 5$$

NAME _____

DATE _____

Grouping or Sharing? Page 1 of 2

1 Decide whether each of the story problems below involves the sharing or the grouping interpretation of division. Circle your choice. You don't need to solve the problems, but it may help to think about whether the answer will mean how many items are in each group or how many groups can be made.

a Frank picked 12 flowers. He divided the flowers evenly between 3 vases. How many flowers did he put in each vase?

Sharing

Grouping



b Erica had 20 baseball cards. She put them up on her bulletin board in rows containing 5 cards. How many rows was she able to make?

Sharing

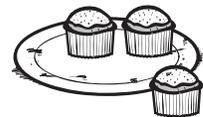
Grouping



c Darius and his dad made 28 cupcakes for the bake sale. They put 7 cupcakes on each plate. How many plates of cupcakes were they able to make?

Sharing

Grouping



d Kiara and her sister are collecting pennies. They have 120 pennies so far. They put their pennies into stacks of 10. How many stacks were they able to make?

Sharing

Grouping



e Carlos has 15 matchbox cars. He wants to share them with 2 of his friends so all three boys have the same number. How many cars will each boy get?

Sharing

Grouping



f Jade and her sister made \$5.00 doing chores for their mom. They split the money evenly between themselves. How much money did each girl get?

Sharing

Grouping



(Continued on next page.)

NAME _____

DATE _____

Grouping or Sharing? Page 2 of 2

2 Read each story problem. Then:

- write an equation (including the answer) for the problem.
- fill in the bubble to show whether the answer means the size of each group or the number of groups

a Mai and her mother made 24 invitations to Mai’s birthday party. They put the invitations into stacks of 4. How many stacks did they make?

Equation: _____

The answer means:

- the size of each group (for example, the number of items each person got)
- the number of groups

b Troy and his mom got a case of bottled water for the soccer game. They divided the 24 bottles evenly among the 12 boys on the team. How many bottles of water did each boy get?



Equation: _____

The answer means:

- the size of each group (for example, the number of items each person got)
- the number of groups

3 Choose one of the expressions below and circle your choice. Write a sharing story problem and a grouping story problem about the same expression.

$18 \div 6$

$60 \div 12$

$108 \div 4$

$400 \div 25$

Sharing Story Problem	Grouping Story Problem

Set A12 ★ Activity 3



ACTIVITY

Grouping Stories

Overview

Today's activity opens with a short string of division problems. The last problem in the set involves division of a whole number by a fraction. Students review the sharing and grouping interpretations of division, decide which makes more sense in this context, and test their decision by solving and discussing two related story problems. The activity concludes with a story designed to help students make generalizations about the effect of dividing a whole number by a fraction.

You'll need

- ★ Division String (page A12.25, run 1 copy for display)
- ★ Two Problems (page A12.26, run 1 copy for display)
- ★ Division Posters from Activity 2 (displayed where all the students can see them)
- ★ Students' math journals or 1 piece of scratch paper per student
- ★ class set of whiteboards, pens, and erasers or a second piece of scratch paper per student
- ★ marking pens
- ★ a piece of paper to mask portions of the display
- ★ a clean, empty half-gallon milk carton (optional, see Advance Preparation)
- ★ 2 one-quart bottles from the Grade 5 Number Corner Kit (optional, see Advance Preparation)
- ★ 1-cup and 2-cup liquid measuring cups from the Grade 5 Number Corner Kit (optional, see Advance Preparation)
- ★ small bottle of food coloring (optional, see Advance Preparation)
- ★ empty clean plastic bowls or containers in the following numbers: 4 two-cup containers, 8 one-cup containers, 16 $\frac{1}{2}$ -cup containers (optional, see Advance Preparation)

Skills & Concepts

- ★ Divide a whole number by a unit fraction using visual models (CCSS 5.NF.7b)
- ★ Solve story problems involving division of a whole number by a unit fraction (CCSS 5.NF.7c)

.....

Advance Preparation The half-gallon milk carton, quart bottles, measuring cups, food coloring, and bowls or other containers are optional, and designed to provide support to students who are struggling with the concepts presented in this activity. We suggest you read through the activity, including the support suggestion on the last page, and decide whether or not your students are likely to need a concrete model to help them understand that dividing a whole number by a fraction results in a quotient that's larger than the dividend. If so, fill the half-gallon milk carton with 8 cups of water and add about 10

Activity 3 Grouping Stories (cont.)

drops of food coloring. Arrange the carton of water, the 2 quart bottles and the liquid measuring cups, and the plastic bowls or containers on a tray prior to conducting this activity.

.....

Instructions for Grouping Stories

1. Open today's instruction by letting students know that you're going to start with a short review of the two different interpretations of division the class investigated last session—sharing and grouping.
2. Display just the top row of the Division String master, keeping the rest of the sheet covered for now. Read the combination with the class, and have the students supply the answer as you write it in. Then work with input from the class to write a sharing and a grouping story problem to match, using the theme of children and food (or any other theme you choose). Take this opportunity to review the two interpretations of division, using the posters from the previous session for reference. Reflect, also, with the students that the quotients mean two different things—in the first case, the size of each group; in the second, the number of groups.

Set A12 Number & Operations: Dividing Fractions & Whole Numbers Blackline Run 1 copy or display		
NAME _____		DATE _____
Division String		
Equation	Sharing Problem	Grouping Problem
1 $6 \div 3 = \underline{2}$	Max made 6 cookies. He gave them to his 3 sisters to share among themselves. How many cookies did each sister get?	Brianna made 6 cookies. She put them in sacks of 3 to take to the neighbors. How many sacks was she able to make?

3. Reveal the next two equations on the master, one by one, and repeat step 2. Then reveal the final expression— $6 \div \frac{1}{2}$. Read it with the class, and ask students to talk with the people sitting next to them about the answer. After a few moments, invite several volunteers to share their answers with the class. Record all responses without comment, even if one or more of them are incorrect.
4. Since at least some of your students will likely give 3 as an answer, press them to explain their thinking. How can $6 \div 2$ and $6 \div \frac{1}{2}$ yield the same result?

Teacher Many of you are saying that the answer to $6 \div \frac{1}{2}$ is 3. Would someone be willing to explain why?

Marcus Simple— $\frac{1}{2}$ of 6 is 3.

Teacher So, you're reading the expression 6 divided by $\frac{1}{2}$ to mean $\frac{1}{2}$ of 6? Thumbs up if you agree with Marcus. Kiara, would you like to comment on this?

Kiara I agree with Marcus. I don't know what else it could mean. I think $6 \div \frac{1}{2}$ means that you should cut 6 in $\frac{1}{2}$, and that's 3.

Teacher Okay, I'm confused. Earlier, we all decided that six divided by two is three. How can six divided by $\frac{1}{2}$ give us the same answer?

Activity 3 Grouping Stories (cont.)

Raven *I think it's just a different way of writing the same problem. If you divide something by 2, it's like cutting it in $\frac{1}{2}$, and when it says to divide by $\frac{1}{2}$ I think it's another way of saying to cut the number in $\frac{1}{2}$.*

5. Resist the temptation to correct student thinking at this point. Instead, move along to posing story problems that represent $6 \div \frac{1}{2}$. Using the first three sharing problems on the master for reference (as well as the division posters from the previous session if they seem helpful), solicit agreement from the students that a sharing problem for $6 \div \frac{1}{2}$ would be like sharing 6 with half a person. Maybe grouping makes more sense for $6 \div \frac{1}{2}$.

6. After a minute or two of discussion and debate, table the attempt to write a sharing problem, and explore the possibility of writing a grouping problem to match the combination. This time, solicit agreement from the students that a grouping problem for $6 \div \frac{1}{2}$ would mean dividing 6 into groups of $\frac{1}{2}$; in other words, determining how many halves there are in 6. Is this possible?

If the students are able to come to some sort of resolution about the question and generate a valid grouping problem for $6 \div \frac{1}{2}$, record it in the correct box on the master. If not, leave the question unresolved for now, and move on to step 7. (While some students may quickly see that $6 \div \frac{1}{2}$ is 12, and come up with a story problem to match, others will likely be quite convinced that $6 \div \frac{1}{2}$ is 3, and not easily dissuaded.)

7. Whether or not the class reached resolution about the meaning of $6 \div \frac{1}{2}$, explain that you have a couple of story problems to pose that might further their thinking on the issues raised. Display the Two Problems master as students get out their math journals or helpers pass out scratch paper.

8. Give students a few moments to examine the sheet quietly. Then read and review the instructions and the problems, providing clarification as needed without telling the students how to solve either of the problems. Be sure to emphasize the requirement that they make a visual fraction model to show and solve the problem they choose.

Activity 3 Grouping Stories (cont.)

Set A12 Number & Operations: Dividing Fractions & Whole Numbers Run 1 copy for display

Two Problems

Choose *one* of the two story problems below and copy it into your journal (or onto your piece of paper) Then:

- Use numbers, labeled visual models, and/or words to solve the problem and explain your strategy.
- Write an equation to match.

You will have about 10 minutes to work. If you finish one problem, do the other, or solve the challenge version of the problem you started with.

1a Little Snail is going to visit his grandma, who lives 6 miles away. He can crawl $\frac{1}{2}$ mile a day. How many days will it take him to get to his grandma's house?

Equation: _____ 

1b Challenge What if Little Snail crawled $\frac{2}{3}$ of a mile a day? Then how many days would it take him to get to his grandma's house?

Equation: _____

2a How many $\frac{1}{2}$ cup servings are there in 6 cups of ice cream?

Equation: _____ 

2b Challenge How many $\frac{3}{4}$ cup servings are there in 6 cups of ice cream?

Equation: _____

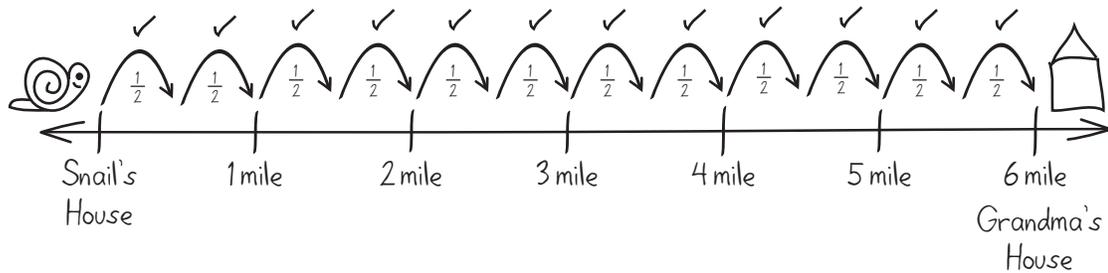
9. When students understand what to do, have them go to work. Circulate to observe and provide clarification if necessary, but avoid the temptation to tell students how to model or solve the problems. (Encourage students who may be getting frustrated at this point to see if they can draw a picture that might help them understand the situation. These students might also be encouraged to see how others around them are handling the problems.)

10. As they finish one of the problems, ask the students to share and compare their work with a classmate, and then either solve the other problem or work on the challenge associated with the problem they chose first.

11. After about 10 minutes, reconvene the class, even if some of the students haven't finished. Solicit and record all answers to the first problem. Then invite several volunteers to share their work with the class, either at the display, or by replicating their visual model on the board and explaining it to the class.

Carlos I just drew a path from the snail's house to his grandma's, like this. I put him here, and his grandma 6 miles away. Then I marked off half-miles to see how many days it would take him, like this. After that, I counted up the number of halves—that's what those checkmarks are for, and it came out to 12.

Activity 3 Grouping Stories (cont.)



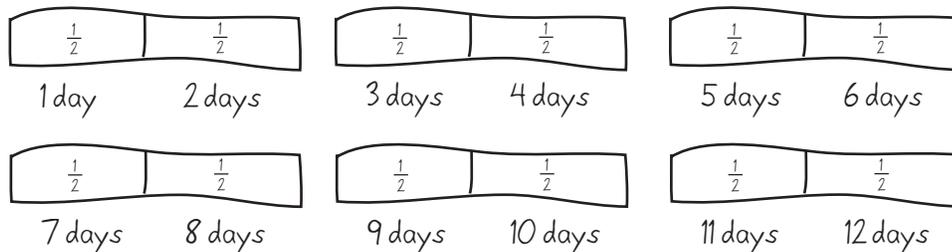
Teacher Does anyone have any questions for Carlos? No? I do. Carlos, how many days did you decide it would take the snail to get to his grandma's?

Carlos I think it must be 12.

Amber I agree with Carlos. I drew a picture kind of like his. Then I saw that there were 2 half-miles in every mile, so I just said, okay, if the snail can go one half mile every day, then it would be 2, 4, 6, 8, 10, 12 days for him to get there.

Teacher Did anyone have a different visual model to represent and solve this problem?

DeAndre My idea is kind of the same, but I did it one mile at a time, like this.



Teacher I have another question for those of you who are thinking it took the snail 12 days to get to his grandma's. What equation did you write to represent the problem?

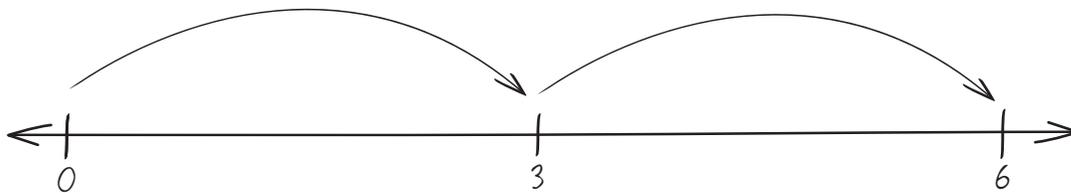
Carly I got 12, and I wrote $6 \div \frac{1}{2} = 12$.

Teacher Can someone repeat Carly's equation for me? I want to record it on the board.

Sam She said $6 \div \frac{1}{2} = 12$, but I respectfully disagree with the answer. I said $6 \div \frac{1}{2}$, but I think the answer is 3.

Teacher Did you make a visual model to show and solve the problem, Sam?

Sam Yes. First I wrote the equation, and I know the answer, so then I made the picture. See? Half-way to 6 is 3, so it took the snail 2 days.

Activity 3 Grouping Stories (cont.)

Teacher Comments or questions for Sam, anyone?

Jasmine I get what you did, Sam, but if it only took 2 days, that means the snail had to go 3 miles a day, and the problem says he could only go half a mile a day.

12. Once several students have shared their work, move along to the second problem, even if there wasn't consensus on the answer to the first. Again, solicit and record answers, and invite a few volunteers to share their models and strategies with the class. (Despite the fact that students who are able to model the situations accurately will arrive at an answer of 12, there may still be a few who are confused or convinced that the answer is 3 because 3 is half of 6.)

13. When you have discussed both problems with the class, question them closely about the answer most of them may agree upon at this point. When we divide, we usually get an answer that's smaller than the dividend. How can one possibly start with 6 and get an answer of 12?

Students Think about it with cookies. If you have 6 cookies and break each one of them in $\frac{1}{2}$, you get 12 halves, right?

Or if you have 6 sandwiches, and you cut each one in $\frac{1}{2}$, you get 12.

I think if you break something into groups that are smaller than 1, you get more of them.

I don't get it—I just don't.

14. Explain that you are going to finish up today's lesson by telling a story that may give students another way to think about dividing by whole numbers and fractions. (Even if many of your students grasp the situation clearly at this point, their understandings are likely fragile, and it's quite possible that a few are still confused by the idea of dividing a whole number by a fraction.)

15. Give each student a whiteboard, pen, and eraser, or a piece of scratch paper, and explain that as you tell the story, they'll write a set of equations to match. Ask them to start at the top of their board or paper and write large enough for you to see, but leave enough room to record several equations in a list. Then start the story, using or adapting the text below as you see fit.

One day, Anthony decided to make a pot of soup. First, he set a nice batch of beans on the stove to cook. Then he started chopping up onions, carrots, celery, and potatoes to add to the beans. As he was working, he started thinking about inviting some friends over to share the soup with him. By the end of the afternoon, he had 8 cups of bean soup. He was very hungry by then, so he got out his biggest soup bowls, and thought, "This soup is so good, and I'm so hungry. I think I'll give everyone a 4-cup serving. How many people could Anthony serve"

16. Stop at this point and ask students to record a division equation to match the situation. Have them hold up their boards or papers, and then call on a volunteer who has correctly recorded $8 \div 4 = 2$ to share and explain her equation. As she does so, record it on the board, and review the meaning of each

Activity 3 Grouping Stories (cont.)

of the numbers with the class (8 is the number of cups of soup Anthony made. Four is the serving size; he's planning to give each person a 4-cup serving. Two is the number of people he can serve.) Ask her if it's a sharing or grouping problem and why. Then continue the story.

When Anthony realized that he could only make two 4-cup servings, he said, "That means I can only invite 1 other person besides myself. That's not much of a party. Maybe I'll give each person a 2-cup serving instead." How many people could Anthony serve now?

17. Pause again and have students write a second equation under the first to represent the situation. Have a different volunteer share and explain his equation. Record it on the board, and review the meaning of each of the numbers with the class. Then continue the story.

When Anthony realized that he could only make four 2-cup servings, he said, "That means I can only invite 3 other people besides myself. I have more friends than that. Maybe I'll give each person a 1-cup serving instead."

18. Pause again and have students write a third equation to represent the situation. Have a different volunteer share and explain her equation. Record it on the board, and review the meaning of each of the numbers with the class. Then discuss the set of equations so far:

- What patterns, if any, can students find in the numbers?
- What is the effect of decreasing the serving size?

$$8 \div 4 = 2 \text{ people}$$

$$8 \div 2 = 4 \text{ people}$$

$$8 \div 1 = 8 \text{ people}$$

Students *He has 8 cups of soup every time.*

The serving sizes are getting smaller. First he was going to give everyone 4 cups, then 2, and now just 1 cup of soup.

The numbers you're dividing by are getting cut in half—4, then 2, now 1.

And the answers are doubling! Look—2, 4, and now 8.

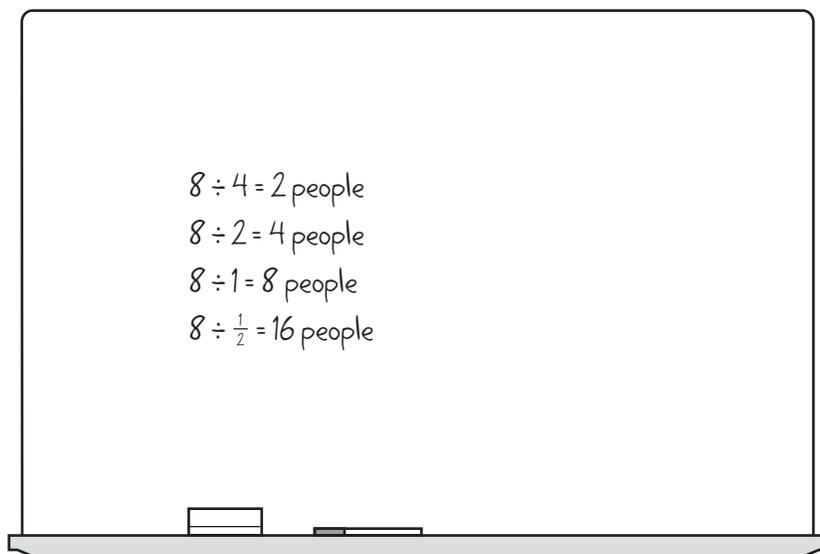
When you make smaller servings, you can invite more people.

19. Continue the story:

Anthony was in a dither. He didn't have time to make any more soup, but 7 people besides himself didn't come close to the number of friends he had in the village. He walked around and around in circles trying to figure out what to do. Suddenly he stopped, smacked his forehead and said, "I've got it! I'll give everyone just half a cup of soup. That will mean I can have 15 people and myself—16 servings!"

Activity 3 Grouping Stories (cont.)

20. Have students record and discuss the matching equation. Does it work? Is it true that if Anthony reduces the serving size to half a cup each, he can have a total of 16 people (including himself) at his party? Have students share thoughts about the situation, first in pairs and then as a whole group.



Students *It fits with the pattern, because you keep cutting the amount of soup in half, and you can have twice as many people.*

There's 2 halves in one cup, right? So there must be 16 halves in 8.

All you have to do is multiply by 2 to get the answer!

Weird—I don't get it.

I think I do now. If you go $8 \div 2$, and think about how many 2s there are in 8, it's 4. If you go $8 \div \frac{1}{2}$, and think about how many $\frac{1}{2}$ -cups there are in 8, it's 16.

You have to use division the grouping way! If you think about the sharing way, it doesn't even make sense. You can't share 8 with half of a person!

Support If more than a very few students seem confused at this point, do a live demonstration with the half-gallon of colored water, liquid measuring tools, and bowls or plastic containers, as described here:

- Review the fact that there are 8 cups in a half gallon. Explain that the water in the half-gallon milk carton you've prepared represents Anthony's batch of soup.
- Go back to the first part of the story and the first equation, in which Anthony decides to give each person a 4-cup serving of soup. Pour 4 cups of colored water from the milk carton into each of two quart bottles. Solicit agreement from the students that each serving is quite large, and only 2 people can be served.
- Go back to the second part of the story and the second equation, in which he decides to give each person a 2-cup serving of soup. Pour all the water back into the $\frac{1}{2}$ -gallon milk carton and then measure 2 cups into each of four smaller containers. Confirm with the students that this results in 4 servings.
- Pour the water back into the milk carton. Repeat the pouring and measuring process twice more, first creating eight 1-cup servings, and finally creating sixteen $\frac{1}{2}$ -cup servings.

Challenge If students are making sense of the situation, and understand that 8 cups of soup divided into $\frac{1}{2}$ -cup servings makes it possible to serve 16 people, continue the story. Anthony is so big-hearted

NAME _____

DATE _____

Division String

Equation	Sharing Problem	Grouping Problem
1 $6 \div 3 = \underline{\quad}$		
2 $6 \div 2 = \underline{\quad}$		
3 $6 \div 1 = \underline{\quad}$		
4 $6 \div \frac{1}{2} = \underline{\quad}$		

Two Problems

Choose *one* of the two story problems below and copy it into your journal (or onto your piece of paper) Then:

- Use numbers, labeled visual models, and/or words to solve the problem and explain your strategy.
- Write an equation to match.

You will have about 10 minutes to work. If you finish one problem, do the other, or solve the challenge version of the problem you started with.

1a Little Snail is going to visit his grandma, who lives 6 miles away. He can crawl $\frac{1}{2}$ mile a day. How many days will it take him to get to his grandma's house?



Equation: _____

1b Challenge What if Little Snail crawled $\frac{2}{3}$ of a mile a day? Then how many days would it take him to get to his grandma's house?

Equation: _____

2a How many $\frac{1}{2}$ cup servings are there in 6 cups of ice cream?



Equation: _____

2b Challenge How many $\frac{3}{4}$ cup servings are there in 6 cups of ice cream?

Equation: _____

Set A12 ★ Activity 4



ACTIVITY

Dividing a Whole Number by a Fraction

Overview

After a short review of the sharing and grouping models of division, students work independently to solve two story problems. The teacher reconvenes the class and works with the students to record several strategies for dividing whole numbers by unit fractions. Students then complete a related assignment.

Skills & Concepts

- ★ Divide a whole number by a unit fraction using visual models (CCSS 5.NF.7b)
- ★ Create a story context for dividing a whole number by a unit fraction (CCSS 5.NF.7b)
- ★ Solve story problems involving division of a whole number by a unit fraction (CCSS 5.NF.7c)

You'll need

- ★ Two More Problems (page A12.33, run 1 copy for display)
- ★ Do-It-Yourself Story Problems (pages A.12.34–36, run 1 copy for display and a class set on paper)
- ★ Division Posters from Activity 2 (displayed where all the students can see them)
- ★ Students' math journals or 1 piece of scratch paper per student
- ★ 2 pieces of chart paper
- ★ marking pens

Instructions for Dividing a Whole Number by a Fraction

1. Open today's instruction by letting students know that you're going to start with a short review of the two different interpretations of division the class investigated last session and the one previous—sharing and grouping.
2. Draw students' attention to the two posters created by the class during Activity 2. Give students a few moments to study the posters quietly. Then write $10 \div 2$ on the board and ask them to brainstorm, first in pairs and then as a whole group, a sharing and a grouping story problem to match. Record an example of both types of story problems on the board, along with the solution to each. Note with students that the quotient means two different things: the size of the group in the case of the sharing problem, and the number of groups in the grouping problem.

$10 \div 2$	
<p>Sharing Story Problem</p> <p>I had 10 marbles. I split them evenly with a friend. How many marbles did we each get?</p> <p>Answer: 5 marble each</p>	<p>Grouping Story Problem</p> <p>I had 10 socks. I put them into pairs. How many pairs was I able to make?</p> <p>Answer: 5 pairs of socks</p>

Activity 4 Dividing a Whole Number by a Fraction (cont.)

3. Next, write $10 \div \frac{1}{2}$ on the board and read it with the class. Then go back and read it to the class as a sharing and then a grouping situation. Which interpretation makes better sense?

Teacher Okay, so we agree that this expression says ten divided by $\frac{1}{2}$. I'm going to phrase it in two different ways now—sharing first, and then grouping. Think privately about which version makes more sense, and then we'll talk.

Ten shared evenly with half a person.

Ten split into groups of a half.

Students You can't share something with half a person.

I think you have to use the grouping way.

It's 5! Just split 10 in half, and it's 5!

But this is different. If you think about grouping, it's like asking how many groups of a half are in 10.

Right! Half of 10 is 5!

That's right, but I'm talking about halves. Little halves, smaller than 1. There are two halves in 1 thing, right? So this is like how many halves in 10 things?

4. Even if a few students are still somewhat confused by the situation, confirm what many probably understand by now—when you divide a whole number by a unit fraction, the grouping interpretation better sense than the sharing interpretation. $10 \div \frac{1}{2}$ requires the problem-solver to figure out how many halves, as in a half of 1, are in 10.

5. Have students brainstorm, first in pairs and then as a whole group, a grouping story problem to match $10 \div \frac{1}{2}$. Record one of their ideas on the board, and work with the class to solve the problem.

$10 \div 2$	
<p>Sharing Story Problem I had 10 marbles. I split them evenly with a friend. How many marbles did we each get? Answer: 5 marble each</p>	<p>Grouping Story Problem I had 10 socks. I put them into pairs. How many pairs was I able to make? Answer: 5 pairs of socks</p>
<p>Sharing Story Problem</p>	<p style="text-align: center;">$10 \div \frac{1}{2}$</p> <p>Grouping Story Problem I had 10 cookies I cut each of them in $\frac{1}{2}$. How many halves was I able to make? Answer: 20 half cookies</p>

6. Next, display the Two More Problems master as students get out their math journals or scratch paper.

7. Give students a few moments to examine the sheet quietly. Then read and review the instructions and the problems, providing clarification as needed without telling the students how to solve either of the problems. Be sure to emphasize the requirement that they make a labeled model to show and solve the problem they choose.

Activity 4 Dividing a Whole Number by a Fraction (cont.)

Set A12 Number & Operations: Dividing Fractions & Whole Numbers Blackline Run 1 copy or display

Two More Problems

Choose *one* of the two story problems below and copy it into your journal (or onto your piece of paper). Then:

- create a labeled visual model to represent and solve the problem.
- write an equation to match.

You will have about 10 minutes to work. If you finish one problem, do the other, or solve the challenge version of the problem you started with.

1a It takes $\frac{1}{3}$ of a yard of ribbon to make a hair bow. Maya has 5 yards of ribbon. How many hair bows can she make?

Equation: _____

1b Challenge Fancy hair bows take $\frac{3}{4}$ of a yard of ribbon each. How many fancy hair bows can Maya make with 5 yards of ribbon?

Equation: _____

2a Our aquarium holds 5 liters of water. If we use a scoop that holds $\frac{1}{6}$ of a liter of water, how many scoops will be needed to fill the aquarium?

Equation: _____

2b Challenge If we use a larger scoop that holds $\frac{2}{3}$ of a liter of water, how many scoops will be needed to fill the aquarium?

Equation: _____



Note The challenge problems are optional, and push beyond the expectation that fifth graders will learn to divide whole numbers by unit fractions, as both involve proper fractions and remainders. You likely have a few students who understand fractions well enough to devise visual strategies that make sense and result in the correct answers. If they get stuck, the attempt is still valuable. Rather than telling them how to solve these problems, encourage them to use visual models in order to see and make sense of the situations involved. See the second note below step 10 for examples of such student-generated strategies.

8. When students understand what to do, have them go to work. Circulate to observe and provide clarification if necessary, but avoid the temptation to tell students how to model or solve the problems. (Encourage students who may be getting frustrated at this point to see if they can draw a picture that might help them understand the situation. These students might also be encouraged to see how others around them are handling the problems.)

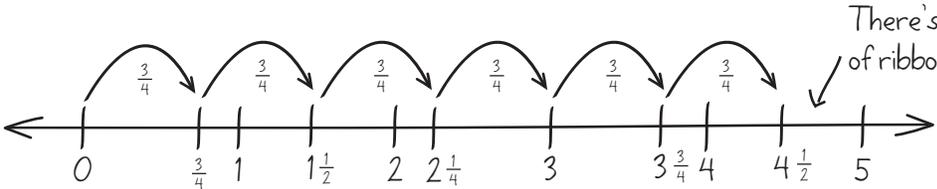
9. As they finish one of the problems, ask the students to share and compare their work with a classmate, and then either solve the other problem or work on the challenge associated with the problem they chose first.

10. After about 10 minutes, reconvene the class, even if some of the students haven't finished. Solicit and record all answers to the first problem. Then invite several volunteers to share their work with the class, either at the display, or by replicating their visual model on the board and explaining it to the class. Repeat this process with the second problem.

Activity 4 Dividing a Whole Number by a Fraction (cont.)

Note The illustration below shows a strategy for each of the challenge problems, along with the correct answer. These, again, are representative of the types of strategies capable students may use."

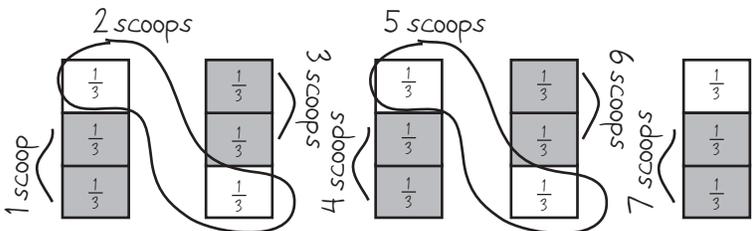
1b Challenge: Fancy hair bows take $\frac{3}{4}$ of a yard of ribbon each. How many fancy hair bows can Maya make with 5 yards of ribbon?



$5 \div \frac{3}{4} = 6 \text{ r } \frac{1}{2}$

She can make 6 hair bows, and there's $\frac{1}{2}$ a yard of ribbon left over.

2b Challenge: If we use a larger scoop that holds $\frac{2}{3}$ of a liter of water, how many scoops will be needed to fill the aquarium?



$5 \div \frac{2}{3} = 7 \frac{1}{2}$ scoops

It's 7 scoops that are each $\frac{2}{3}$ of a cup. There's still a $\frac{1}{3}$ of a cup left to fill at the end. Since $\frac{1}{3}$ is half of $\frac{2}{3}$, the answer is $7 \frac{1}{2}$ scoops.

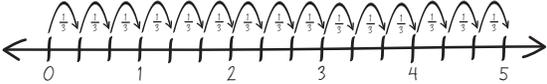
After each volunteer has shared, work with input from the class to replicate the model and summarize the strategy used to solve the problem on a piece of chart paper.

Activity 4 Dividing a Whole Number by a Fraction (cont.)

Models and Strategies for Dividing
a Whole Number by a Unit Fraction

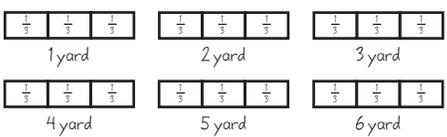
$5 \div \frac{1}{3}$

- Draw a number line as long as you need. Then mark it off in thirds, and count how many there are.



$5 \div \frac{1}{3} = 15$ She can make 15 hair bows.

- Draw each yard of ribbon and divide it into thirds. Then count how many one-thirds there are. 15

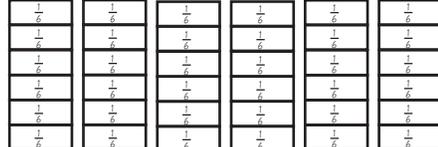


- Think about how many one-thirds there are in one yard. It's 3. Then multiply by 5 because she has 5 yards of ribbon, not 1. $3 \times 5 = 15$ one-thirds, so she can make 15 bows.

Models and Strategies for Dividing
a Whole Number by a Unit Fraction

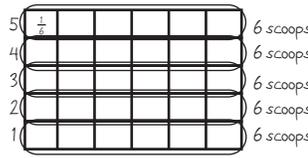
$5 \div \frac{1}{6}$

- Draw a rectangle for each liter. Then divide that into sixths because the scoop holds $\frac{1}{6}$ liter. Add up to find out how many one-sixths you get.



$6 + 6 + 6 + 6 + 6 = 30$ scoops

- Draw the aquarium. Divide it into 5 layers, one for each liter. Then divide it into $\frac{1}{6}$ s the other way because it takes 6 scoops to make a liter.



$5 \div \frac{1}{6} = 30$ one-sixths,
so 30 scoops

- If each scoop is $\frac{1}{6}$ of a liter, it takes 6 scoops to make a liter, so if you multiply 6×5 because there are 5 liters in the aquarium, you see it's 30 scoops.

Note The strategies shown on the charts above represent some of the ways fifth graders are likely to deal with dividing whole numbers by fractions in the context of selected story problems. Some of these strategies are more efficient than others, and it's entirely possible that your students will devise methods that are not shown above. It should be stressed that you are not responsible for teaching any of these strategies directly. The goal of your instruction at this point should be to provide students with opportunities to make sense of situations that involve division of whole numbers by fractions; to ask that they bring perseverance, a willingness to take risks, and the knowledge they already possess about division and fractions to bear on problems that require them to stretch a little (or a lot, in some cases).

11. Give students each of copy of the Do-It-Yourself Story Problems and display the masters for everyone to see. Review and discuss the tasks with the class. Give students the option of working on these sheets independently, in pairs, or with you. Encourage students to make use of the strategies just shared by their classmates.

12. As students finish, have them share and compare their work with a classmate. If they have selected different divisors or dividends for each problem, challenge them to be sure each other's answers are correct. Have them check each other's story problems to be sure they make sense and have been solved correctly.

If students need more time to complete the assignment, ask them to finish it as homework or as part of their morning seatwork the following day before you conduct Activity 5.

.....
Note Save the strategy posters from today's activity for use in Activity 7.
.....



INDEPENDENT WORKSHEET

See Set A12 Independent Worksheet 2 on page A12.77 for more practice with dividing a whole number by a fraction, identifying the operation needed to solve a story problem, and multiplying fractions and whole numbers.

Two More Problems

Choose *one* of the two story problems below and copy it into your journal (or onto your piece of paper). Then:

- create a labeled visual model to represent and solve the problem.
- write an equation to match.

You will have about 10 minutes to work. If you finish one problem, do the other, or solve the challenge version of the problem you started with.

1a It takes $\frac{1}{3}$ of a yard of ribbon to make a hair bow. Maya has 5 yards of ribbon. How many hair bows can she make?

Equation: _____

1b Challenge Fancy hair bows take $\frac{3}{4}$ of a yard of ribbon each. How many fancy hair bows can Maya make with 5 yards of ribbon?

Equation: _____

2a Our aquarium holds 5 liters of water. If we use a scoop that holds $\frac{1}{6}$ of a liter of water, how many scoops will be needed to fill the aquarium?

Equation: _____

2b Challenge If we use a larger scoop that holds $\frac{2}{3}$ of a liter of water, how many scoops will be needed to fill the aquarium?

Equation: _____



NAME _____

DATE _____

Do-It-Yourself Story Problems Page 1 of 3

1 Solve each of the story problems below. For each problem:

- Choose and circle one of the numbers in parentheses, depending on how challenging you want the problem to be.
- Write an expression to represent your problem.
- Use numbers, labeled visual models, and/or words to solve the problem and explain your strategy.
- Complete the sentence below with your solution to the problem

a It takes $(\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{3}{8})$ of a cup of sugar to make a batch of cookies. I have 5 cups of sugar. How many batches of cookies can I make?

Expression: _____

I can make _____ batches of cookies.



(Continued on next page.)

NAME _____

DATE _____

Do-It-Yourself Story Problems Page 2 of 3

b The road-repair crew can fix $\frac{1}{2}$ a mile of road per day. How many days will it take them to fix a stretch of road that is (4 miles, 5 miles, $6\frac{1}{2}$ miles, $8\frac{3}{4}$ miles)?

Expression: _____

It will take _____ days to fix the road.



c How many ($\frac{1}{2}$ cup, $\frac{1}{3}$ cup, $\frac{1}{4}$ cup, $\frac{2}{3}$ cup) servings are there in a quart of ice-cream? There are 4 cups in a quart.

Expression _____

There are _____ servings in a quart of ice-cream.



(Continued on next page.)

NAME _____

DATE _____

Do-It-Yourself Story Problems Page 3 of 3

d Little Snail is going to visit his friend over at the next pond, 3 miles away. He can crawl ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{2}{3}$) of a mile per day. How many days will it take him to get there?

Expression: _____

It will take _____ days to get there.



2 Choose *one* of the expressions below and circle your choice. Write a story problem about the expression you circled. Then solve your own problem using a fractional visual model.

$$10 \div \frac{1}{2}$$

$$12 \div \frac{1}{3}$$

$$20 \div \frac{1}{4}$$

$$24 \div \frac{2}{3}$$

My story problem:

My work:

The answer to my problem is _____.

Set A12 ★ Activity 5



ACTIVITY

Sharing Stories

Overview

Today's activity opens with a short string of division problems. The last problem in the set involves division of a fraction by a whole number. Students review the sharing and grouping interpretations of division, decide which makes more sense in this context, and test their decision by solving and discussing several story problems. The activity concludes with a story designed to help students make generalizations about the effect of dividing a fraction by a whole number.

Skills & Concepts

- ★ Divide a unit fraction by a whole number using visual models (CCSS 5.NF.7a)
- ★ Solve story problems involving division of a unit fraction by a whole number (CCSS 5.NF.7c)

You'll need

- ★ Another Division String (page A12.45, run 1 copy for display)
- ★ Fractions on a Geoboard (page A12.46, run 1 copy for display)
- ★ Brownies & Bars (page A12.47, run 1 copy for display)
- ★ Division Posters from Activity 2 (displayed where all the students can see them)
- ★ geoboards and rubber bands (class set, plus 1 board and some bands for display)
- ★ Students' math journals or 2 pieces of scratch paper per student
- ★ marking pens in black, blue, and yellow
- ★ 1 blue and 1 yellow crayon or colored pencil, per student
- ★ a piece of paper to mask portions of the display

Instructions for Sharing Stories

1. Let students know you're going to start today's session with another look at the two different interpretations of division the class has been working with lately—sharing and grouping.
2. Display just the top row of the Another Division String master, keeping the rest of the sheet covered for now. Read the combination with the class, and have the students supply the answer as you write it in. Then work with input from the class to write a sharing and a grouping story problem to match, using the theme of dogs and dog biscuits (or any other theme you choose). Review the fact that the quotients mean two different things—in the first case, the size of each group; in the second, the number of groups.

Set A12 Number & Operations: Dividing Fractions & Whole Numbers Blackline Run 1 copy for display

Equation	Sharing Problem	Grouping Problem
$1 \frac{12}{4} \div 4 = \underline{\quad 3 \quad}$	Brandon had 12 small dog biscuits. He divided them evenly among his 4 dogs. How many biscuits did each dog get?	Tifani made 12 dog biscuits. She put them in bags of 4 to give to her grandma's dogs. How many bags of 4 did Tifani make?

Activity 5 Sharing Stories (cont.)

3. Reveal the next two equations on the master, one by one, and repeat step 2. Then reveal the final expression— $\frac{1}{2} \div 4$. Read it with the class, and ask students to talk with the people sitting next to them about the answer. After a few moments, invite several volunteers to share their answers with the class. Record all responses without comment, even if some of them are incorrect. (While a few of your students may reason that $\frac{1}{4}$ of $\frac{1}{2}$ is $\frac{1}{8}$ and $\frac{1}{8}$ of $\frac{1}{2}$ is $\frac{1}{16}$, many will likely be confused when presented with the idea of dividing a fraction by a whole number. After all, how can you possibly divide a number less than one by 4?)

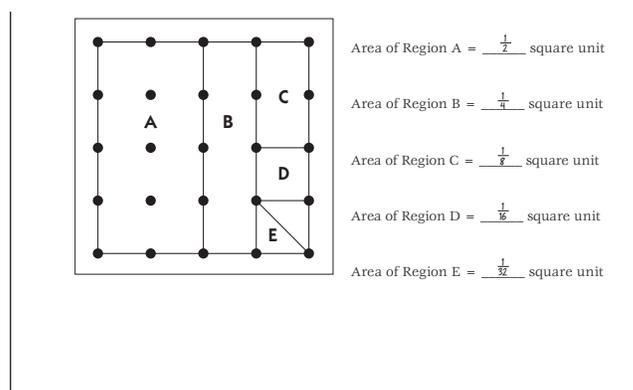
4. Resist the temptation to correct student thinking at this point. Instead, move along to posing story problems that represent $\frac{1}{2} \div 4$. Using the first three grouping problems on the master for reference (as well as the division posters from Session 2 if they seem helpful), solicit agreement from the students that a grouping problem for $\frac{1}{2} \div 4$ would mean pulling as many groups of 4 as possible out of $\frac{1}{2}$. Does that make sense?

5. After a minute or two of discussion and debate, table the attempt to write a grouping problem, and explore the possibility of writing a sharing problem to match the combination. This time, solicit agreement from the students that a sharing problem for $\frac{1}{2} \div 4$ would mean splitting a half of something 4 ways. Is this possible?

While some students may suggest that it would be possible to share half a dog biscuit among 4 dogs, others may argue that the portions would be too small and/or it would be too difficult to split half a biscuit into 4 equal shares. Too, there may be some doubt about the fraction of the biscuit each dog would receive. So, for now, leave the last row on the master blank, promising to come back to it in a few minutes.

6. Acknowledge that it might be difficult to split a half a dog biscuit four ways, and propose instead to model a situation where 4 dogs share a half pan of cornbread. Explain that the geoboard, which was useful in modeling and solving multiplication with fractions, will prove useful here as well.

7. Display a copy of the Fractions on a Geoboard master while helpers pass out geoboards and bands. Use the master to quickly review with the class some of the fractions that can be formed on the geoboard if the entire board is assigned an area of 1 unit. Have students build each of the regions shown on the master, and report the area as a fraction of the whole. Ask a volunteer or two to explain their thinking before you record the area of each region on the master.

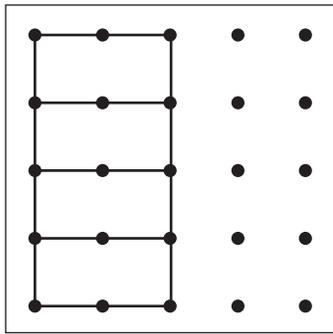


8. When you've completed this brief review, have students clear their geoboards of rubber bands and build Region A again as you do so on your display board. Explain that you'll use this to represent the

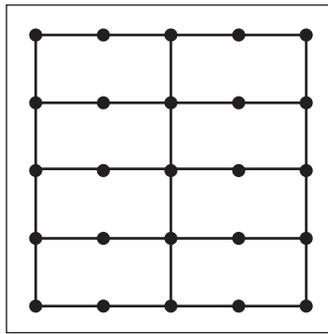
Activity 5 Sharing Stories (cont.)

half pan of cornbread the 4 dogs are going to share. Then ask the students to use their rubber bands to divide Region A into 4 equal parts and determine what the size of each part is. As they finish, have them share and compare their results with the people sitting nearest them.

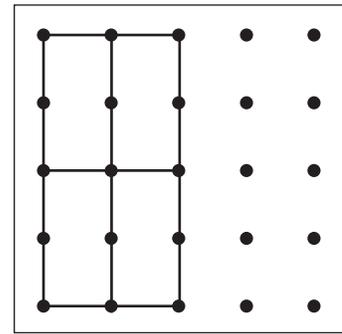
9. When most students have finished, ask the class to report the size of each part. Record all responses on the board, and then invite 2 or 3 volunteers to display their geoboards (one at a time) and explain their thinking to the class.



Max



Emma



Demetrius

Max I just divided the half into 4 parts. I can see that each one of them is the same size as Region C, so they're $\frac{1}{8}$ each.

Emma I did kind of the same thing, but I divided up the whole pan. I could see there were 8 parts, so each one must be an $\frac{1}{8}$.

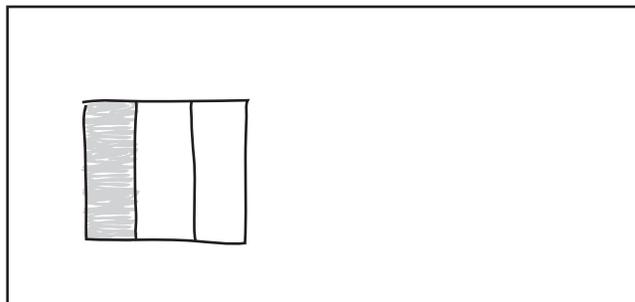
Demetrius I split the half into 4 parts, but I put my rubber bands going the other way. Each piece is still an eighth; it's just going up and down instead of sideways. Also, I know we're right because 4 times $\frac{1}{8}$ is $\frac{1}{2}$. If you add them up it's $\frac{4}{8}$, and that's the same as $\frac{1}{2}$.

10. Return to the Division String master, and take a minute to confirm that the model students just built and explained represents $\frac{1}{2} \div 4$. Work with students to fill in the answer for the last equation on the sheet, and record a story problem in the sharing column.

$4 \frac{1}{2} \div 4 = \frac{1}{8}$	There was a half a pan of cornbread left over from supper. Talia's mom said she could divide it evenly among the 4 dogs. What fraction of the pan did each dog get?	
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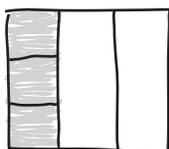
11. Now explain that the geoboard will be a useful way to model and solve some of today's problems, but there will be other problems that will require a visual model instead. Have students find the next available page in their journals, or give them each a piece of scratch paper. Let them know they'll need a pencil, and crayons or colored pencils in yellow and blue as well.

Have the students each draw a square on their sheet while you do so at the board. Then divide the square into thirds and shade in the third farthest to the left yellow, and have students do the same.

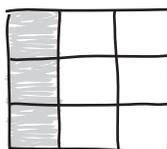
Activity 5 Sharing Stories (cont.)

12. Explain that there was a third of a pan of cornbread leftover from supper the other night, and Alonzo's mother gave him permission to divide it evenly among their three dogs. Then work with input from the students to write an equation to represent the situation: $\frac{1}{3} \div 3$. Give them a few moments to pair-share ideas about what fraction of the pan of cornbread each dog will receive, and then ask students to work with the visual model they've made on their paper to determine the answer. (Reassure them that it's fine to make additional copies of the model if they need to.)

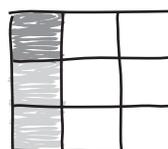
13. Reconvene the class after a minute or two, even if some of the students are still working, or have become stuck. Invite several volunteers to share their thinking with the class by placing their papers on display at the document camera, or drawing on the board.



Josh



Vincent



Maya

Josh I split the third into 3 parts, but I don't know how much each of those is worth.

Vincent I did kind of the same thing, but I split the whole pan into 3 parts. I can see there are 9 in the whole pan, so each part of the cornbread must be $\frac{1}{9}$.

Maya I agree with you, Vincent. I did the same thing as you, and then I used my blue crayon to color in one of the parts. It's one out of 9, so that's a ninth.

14. If no one in your class suggests a strategy similar to Maya's, model it yourself. Explain or confirm for students that dividing the entire pan, rather than just the fraction of cornbread remaining, makes it possible to determine the size of each portion accurately, as Vincent and Maya have explained above. Then work with students to complete the equation, and reflect on it for a few moments. Does it make logical sense that $\frac{1}{3}$ divided into 3 equal shares would result in $\frac{1}{9}$? Why or why not?

Activity 5 Sharing Stories (cont.)

$\frac{1}{3}$ of the pan of cornbread is left

1 out of 9 pieces, $\frac{1}{9}$

If we split it into 3 equal parts, we can see that each is $\frac{1}{9}$ of the pan.

$$\frac{1}{3} \div 3 = \frac{1}{9}$$

15. Now explain that you have a couple of story problems to pose that might further students' thinking about dividing a fraction by a whole number. Display the Brownies and Bars master as students find the next page in their journals or flip their scratch paper over.

16. Give students a few moments to examine the sheet quietly. Then read and review the instructions and the problems, providing clarification as needed. Be sure to emphasize the requirement that they create a labeled visual model to show and solve the problem they choose.

Set A12 Number & Operations: Dividing Fractions & Whole Numbers Run 1 copy or display

Brownies & Bars

Choose one of the two story problems below and copy it into your journal (or onto your piece of paper). Then:

- create a labeled visual model to represent and solve the problem.
- write an equation to match.

You will have about 10 minutes to work. If you finish one problem, do the other, or solve the challenge version of the problem you started with.

1a There is $\frac{1}{4}$ of a pan of brownies left. Jake and his three brothers are going to share it equally. What fraction of the pan of brownies will each of the 4 boys get?

Equation: _____

1b Challenge What if there was $\frac{2}{8}$ of a pan of brownies left? Then what fraction of the pan of brownies would each boy get if they divided it equally among themselves?

Equation: _____

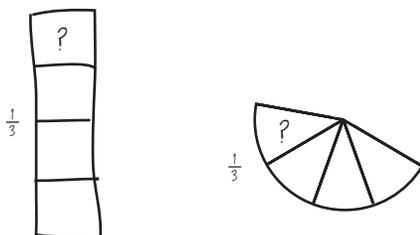
17. When students understand what to do, have them go to work. Circulate to observe and provide clarification as needed. Encourage children who are struggling to use a geoboard to model and solve the first problem, and then record a visual model and an equation to show their work; the other problem and the two optional challenges require models similar to the one you created for the class.

18. As they finish one of the problems, ask the students to share and compare their work with a classmate, and then solve the other problem. If they finish both, invite them to work on one or both of the challenges.

Activity 5 Sharing Stories (cont.)

19. After about 10 minutes, reconvene the class, even if some of the students haven't finished. Solicit and record all answers to the first problem. Then invite several volunteers to share their work with the class, either at the display, or by replicating their visual model on the board and explaining it to the class. Repeat with the second problem.

Note Both of the story problems feature fractions of baked goods remaining in pans. They were written this way to encourage students to use geoboards or visual models of squares, both of which can be subdivided easily. This model works well for representing and dividing unit or proper fractions by whole numbers. There are other models that can be used, but none that make it quite so easy to determine the fractions that result. For example, some students might try to solve the second problem by modeling the third of the pan of lemon bars that remains, and then dividing that third into 4 equal parts. Problems may emerge when these students try to determine the fractional size of the resulting parts.



Josie I drew the third of a pan and split it up into 4 equal parts, but I have no idea what fraction they are. I guess you could say each of them is a fourth of a third, but that's kind of weird.

Matt I did the same thing, but I used a circle. I know how to make a third of a circle, but when I divide it into 4 parts, I don't know what those fractions are.

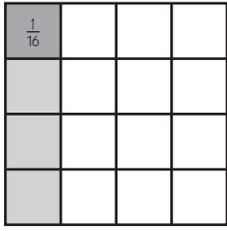
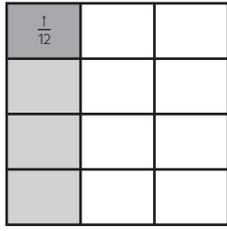
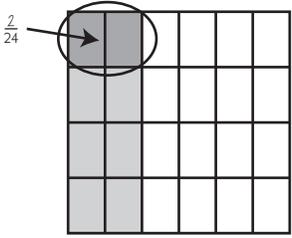
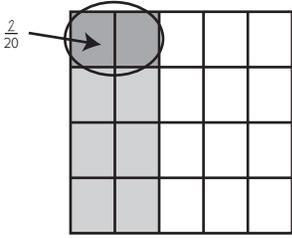
A model less likely to emerge given the context of today's problems would be the number line. It's certainly possible to draw a line, label it 0 at one end and 1 at the other, divide it into thirds, and then subdivide one of the thirds into 4 parts, but unless the student already knows that a fourth of a third is a twelfth, or thinks to subdivide each third into 4 parts, it can be hard to determine the fractional size of each division.



Gregg I thought a number line would work for solving the lemon bar problem, but I didn't know what size each piece was after I finished making my model.

It's likely that at least some of your students will use the geoboard or models of squares successfully to find the correct answer to each of the problems. The visual models and answers below are included for your reference. (The challenge problems are optional, but the square models work equally well when dividing unit or proper fractions by whole numbers.)

Activity 5 Sharing Stories (cont.)

<p>Problem 1a</p>  <p>$\frac{1}{4} \div 4 = \frac{1}{16}$</p>	<p>Problem 2a</p>  <p>$\frac{1}{3} \div 4 = \frac{1}{12}$</p>
<p>Problem 1b Challenge</p>  <p>$\frac{2}{6} \div 4 = \frac{2}{24}$ or $\frac{1}{12}$</p>	<p>Problem 2b Challenge</p>  <p>$\frac{2}{5} \div 4 = \frac{2}{20}$ or $\frac{1}{10}$</p>

Explain that you are going to finish up today's lesson by telling a story that may give students another way to think about dividing fractions and whole numbers. (Even if students grasp the situation clearly at this point, their understandings are likely fragile, and it's quite possible that a few are still confused by the idea of dividing a fraction by a whole number.)

20. Have students find the next page in their journals or give each of them another a piece of scratch paper, and have them keep their yellow and blue crayons or colored pencils handy. Explain that as you tell the story, they'll write equations to match, and make visual models to solve the problems that come up. Then start the story, using or adapting the text below as you see fit.

One day, Grandma was baking brownies. Her two grandchildren arrived just as she was cutting up the last pan and packing the brownies into boxes for the bake sale. Grandma said, "I've got to run to the store for a few minutes. There's half a pan of brownies left. Get out the milk, and you can share the half pan between you." The children's eyes got wide at the thought of milk and half a pan of Grandma's best brownies!

21. Stop at this point and ask students to record a division expression to match the situation. Have them hold up their boards or papers, and then call on a volunteer who has correctly recorded $\frac{1}{2} \div 2$ to share and explain her thinking. As she does so, record it on the board, and model the situation as the students do so on their boards. When there is general agreement that $\frac{1}{2}$ divided by 2 is $\frac{1}{4}$, record the answer on the board. Then continue the story.

Just as the children sat down to enjoy the brownies, there was a knock at the door. It was their cousin, Junior, from next door. They couldn't leave him out, so they poured a glass of milk for him and re-divided the the half-pan of brownies fair and square.

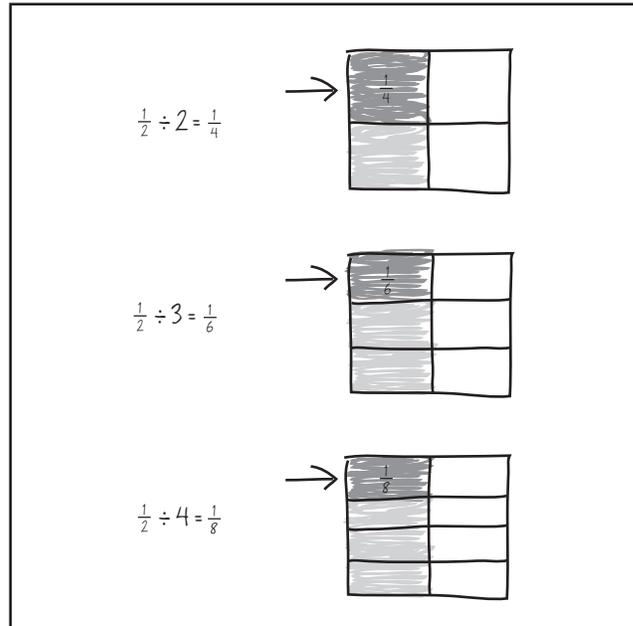
Activity 5 Sharing Stories (cont.)

22. Pause again and have students write a second expression under the first to represent the situation. Have them hold up their boards or papers, and then call on a volunteer who has correctly recorded $\frac{1}{2} \div 3$ to share and explain her thinking. As she does so, record it on the board, and model the situation as the students do so on their boards. When there is general agreement that $\frac{1}{2}$ divided by 3 is $\frac{1}{6}$, record the answer on the board. Then continue the story.

Just as the children sat down to enjoy the brownies, there was a knock at the door. It was Junior's little brother. They couldn't leave him out, so they poured a glass of milk for him and re-divided the half-pan of brownies fair and square.

23. Pause again and take time with the students to model and solve the problem, and record an equation to match. Review the meaning of each of the numbers in the equation with the class. Then discuss the list of equations so far:

- What patterns, if any, can students find in the numbers?
- What is the effect of increasing the number of people who are sharing the brownies?



24. If time allows, ask students to predict what fraction of the pan each child would get if Junior's little sister arrived and there were 5 children. And what if Junior's friend, Maria, came over from across the street and there were 6 children? Press students to explain their predictions as they share.

Students *I'm pretty sure it would be $\frac{1}{10}$ with 5 kids and $\frac{1}{12}$ with 6 kids.*

I agree, Sergio. If you multiply the bottom part of the fraction and the whole number, you get the answer, so 2×5 is 10 and 2×6 is 12.

The parts keep getting smaller and smaller.

This is starting to remind me of when we were multiplying fractions. It's kind of like we're finding out $\frac{1}{2}$ of $\frac{1}{2}$, then $\frac{1}{3}$ of $\frac{1}{2}$, then $\frac{1}{4}$ of $\frac{1}{2}$.

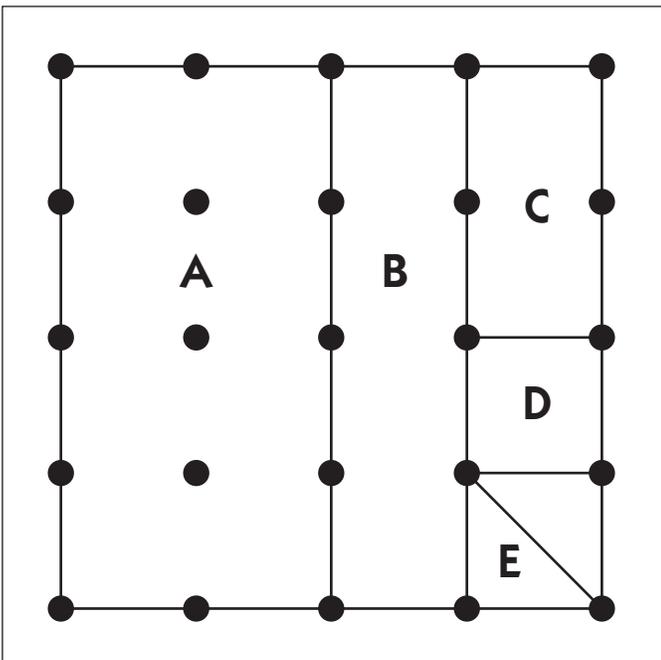
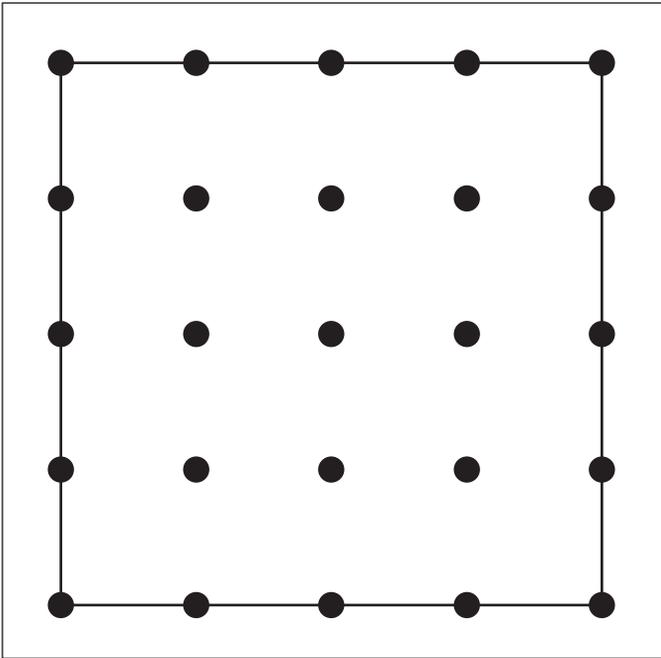
25. Let students know you'll return to the problem of dividing fractions by whole numbers in the next activity.

Another Division String

Equation	Sharing Problem	Grouping Problem
1 $12 \div 4 = \underline{\quad}$		
2 $8 \div 4 = \underline{\quad}$		
3 $4 \div 4 = \underline{\quad}$		
4 $\frac{1}{2} \div 4 = \underline{\quad}$		

Fractions on a Geoboard

If the biggest square on the geoboard has an area of 1 unit, what is the area of each lettered region? Label the area of each region below.



Area of Region A = _____ square unit

Area of Region B = _____ square unit

Area of Region C = _____ square unit

Area of Region D = _____ square unit

Area of Region E = _____ square unit

Brownies & Bars

Choose one of the two story problems below and copy it into your journal (or onto your piece of paper). Then:

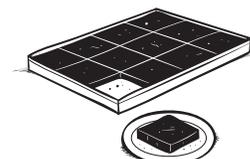
- create a labeled visual model to represent and solve the problem.
- write an equation to match.

You will have about 10 minutes to work. If you finish one problem, do the other, or solve the challenge version of the problem you started with.

1a There is $\frac{1}{4}$ of a pan of brownies left. Jake and his three brothers are going to share it equally. What fraction of the pan of brownies will each of the 4 boys get?

Equation: _____

1b Challenge What if there was $\frac{2}{6}$ of a pan of brownies left? Then what fraction of the pan of brownies would each boy get if they divided it equally among themselves?

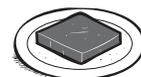


Equation: _____

2a There is $\frac{1}{3}$ of a pan of lemon bars left. Sara and her three cousins are going to share it equally. What fraction of the pan of lemon bars will each of the 4 girls get?

Equation: _____

2b Challenge What if there was $\frac{2}{5}$ of a pan of lemon bars left? Then what fraction of the pan of lemon bars would each girl get if they divided it equally among themselves?



Equation: _____

Set A12 ★ Activity 6



ACTIVITY

Dividing a Fraction by a Whole Number

Overview

After a short review of the sharing and grouping models of division, students work independently to solve two story problems. The teacher reconvenes the class and works with the students to record several strategies for dividing unit fractions by whole numbers. Students then complete a related assignment.

Skills & Concepts

- ★ Divide a unit fraction by a whole number using visual models (CCSS 5.NF.7b)
- ★ Create a story context for dividing a unit fraction by a whole number (CCSS 5.NF.7b)
- ★ Solve story problems involving division of a unit fraction by a whole number (CCSS 5.NF.7c)

You'll need

- ★ Rope & Candy Problems (page A12.54, run 1 copy for display)
- ★ More Do-It-Yourself Story Problems (pages A.12.55–A12.57, run 1 copy for display and a class set on paper)
- ★ Division Posters from Activity 2 (displayed where all the students can see them)
- ★ Students' math journals or 1 piece of scratch paper per student
- ★ 2 pieces of chart paper
- ★ marking pens

Instructions for Dividing a Fraction by a Whole Number

1. Open today's instruction by letting students know that you're going to start with a short review of the two different interpretations of division the class has been using lately—sharing and grouping.
2. Draw students' attention to the two posters created by the class during Activity 2. Give students a few moments to study the posters quietly. Then write $4 \div 2$ on the board and ask them to brainstorm, first in pairs and then as a whole group, a sharing and a grouping story problem to match. Record an example of both types of story problems on the board, along with the solution to each. Note with students that the quotient means two different things: the size of the group in the case of the sharing problem, and the number of groups in the grouping problem.

$4 \div 2$	
<p>Sharing Story Problem</p> <p>I had 4 horse posters. I split them evenly with my best friend. How many posters did we each get?</p> <p>Answer: 2 posters</p>	<p>Grouping Story Problem</p> <p>I had 4 sports posters. I pinned up 2 of them on each wall in my room until I ran out. How many walls got posters?</p> <p>Answer: 2 walls</p>

Activity 6 Dividing a Fraction by a Whole Number (cont.)

3. Next, write $\frac{1}{4} \div 2$ on the board and read it with the class. Then go back and read it to the class as a sharing and then a grouping situation. Which interpretation makes sense?

Teacher *Okay, I think we all agree that this expression says $\frac{1}{4}$ divided by 2. I'm going to phrase it in two different ways now—sharing first, and then grouping. Think privately about which version makes more sense, and then we'll talk.*

One-fourth of something shared evenly between 2 people.

The number of groups of 2 you can get out of $\frac{1}{4}$.

Students *You can't get any groups of 2 out of $\frac{1}{4}$. Two is way bigger than $\frac{1}{4}$!*

I think you have to use the sharing way.

But if you had $\frac{1}{4}$ of something really big, you could give it to lots of pairs of people.

I respectfully disagree, Jason. You can't say that 2 is smaller than $\frac{1}{4}$ all of a sudden. Two is, like... 8 times bigger than one-fourth because there are $\frac{1}{4}$ in 1.

Sharing works, though. You can share a fourth of a cookie or a sandwich with someone. You don't get very much, but you can do it.

If you cut a $\frac{1}{4}$ in $\frac{1}{2}$ to share it with someone else, it's $\frac{1}{8}$, because there are $\frac{2}{8}$ in a $\frac{1}{4}$.

4. Even if a few students are still somewhat confused by the situation, confirm what many probably understand by now—when you divide a unit fraction by a whole number, the sharing interpretation makes better sense, than the grouping interpretation. In fact, $\frac{1}{4} \div 2$ requires the problem-solver to divide one-fourth into two equal parts and then determine the size of each fractional part that results.

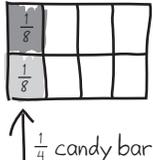
5. Have students brainstorm, first in pairs and then as a whole group, a sharing story problem to match $\frac{1}{4} \div 2$. Record one of their ideas on the board, and work with the class to solve the problem, using a visual model similar to the ones you introduced last session. Encourage students who are still struggling with the concept of dividing a fraction by a whole number to model the situation on a geoboard.

$\frac{1}{4} \div 2$

Sharing Story Problem

I had $\frac{1}{4}$ of a candy bar left.
I divided it equally with my brother.
What fraction of the candy bar did we each get?
Answer: $\frac{1}{8}$ of a candy bar

~~Grouping Story Problem~~



6. Next, display the Rope & Candy Problems master as students get out their math journals or scratch paper.

7. Give students a few moments to examine the sheet quietly. Then read and review the instructions and the problems, providing clarification as needed without telling the students how to solve either of

Activity 6 Dividing a Fraction by a Whole Number (cont.)

the problems. Be sure to emphasize the requirement that they create a labeled visual model to show and solve the problem they choose.

Set A12 Number and Operations: Dividing Fractions & Whole Numbers Run 1 copy or display

Rope & Candy Problems

Choose *one* of the two story problems below and copy it into your journal or paper). Then:

- create a labeled visual model to represent and solve the problem.
- write an equation to match.

You will have about 10 minutes to work. If you finish one problem, do the other, or solve the challenge version of the problem you started with.

1a John has $\frac{1}{2}$ of a yard of rope. He wants to cut the rope into 3 equal pieces. What fraction of a yard will each piece of rope be?

Equation: _____

1b Challenge How many inches long will each piece of rope be?

Equation: _____ 

2a Lori had $\frac{1}{2}$ of a bag of candy and shared it with 3 friends. What fraction of the bag of candy did each of the 4 children get?

Equation: _____

2b Challenge There were 64 pieces of candy in the whole bag. How many pieces of candy did each friend get?

Equation: _____ 

8. When students understand what to do, have them go to work. Circulate to observe and provide clarification if necessary, but avoid the temptation to tell students how to model or solve the problems. (Encourage students who may be getting frustrated at this point to see if they can model and solve the second problem on a geoboard, and then create a labeled visual model to show their work.)

9. As they finish one of the problems, ask the students to share and compare their work with a classmate, and then either solve the other problem or work on the challenge associated with the problem they chose first.

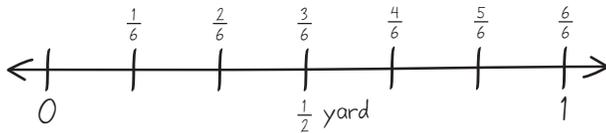
10. After about 10 minutes, reconvene the class, even if some of the students haven't finished. Solicit and record all answers to the first problem. Then invite several volunteers to share their work with the class, either at the display, or by replicating their model on the board and explaining it to the class. Repeat this process with the second problem.

After each volunteer has shared, work with input from the class to replicate the model and summarize the strategy used to solve the problem on a piece of chart paper.

Activity 6 Dividing a Fraction by a Whole Number (cont.)Models and Strategies for Dividing
a Unit Fraction by a Whole Number

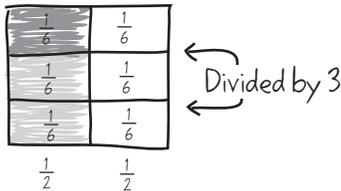
$$\frac{1}{2} \div 3$$

- Draw a number line up to 1. Mark it at $\frac{1}{2}$, and then divide each half into 3 parts. That way, you can tell how many there are in 1, and what size they each are.



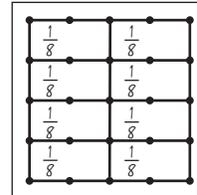
If you split each half into 3 parts, there are 6 in all, so each part must be $\frac{1}{6}$ of a yard.

- Draw a square to stand for the whole yard. Divide it in half one way, and in 3 parts the other way. Then you can see how big each part is when you divide $\frac{1}{2}$ into 3 parts. It's $\frac{1}{6}$, so each part of the rope must be $\frac{1}{6}$ of a yard.

Models and Strategies for Dividing
a Unit Fraction by a Whole Number

$$\frac{1}{2} \div 4$$

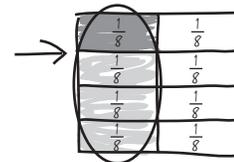
- You can do this one on the geoboard. First divide the board in half one way. Then divide it into 4 equal parts the other way. You can see that each part is $\frac{1}{8}$ of the board.



- Draw a square to stand for the whole box of candy. She only has $\frac{1}{2}$ the box left, so divide it in $\frac{1}{2}$ and color in the $\frac{1}{2}$ to show what she has left. Then divide the square into 4 equal parts the other way. That way, you can see how much of the box each part is.

Here's the $\frac{1}{2}$ she has left.

If you split the whole box into 4 equal parts the other way, you can see that there are 8 parts, so each one must be $\frac{1}{8}$ of the whole box.



- I think dividing $\frac{1}{2}$ by 4 is like finding $\frac{1}{4}$ of $\frac{1}{2}$. That's multiplication, and the answer is $\frac{1}{8}$.

$$\frac{1}{2} \div 4 = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

Note The strategies shown on the charts above represent some of the ways fifth graders are likely to deal with dividing fractions by whole numbers in the context of selected story problems. Some of these strategies are more efficient than others, and it's entirely possible that your students will devise methods that are not shown above. It should be stressed that you are not responsible for teaching any of these strategies directly. The goal of your instruction at this point should be to provide students with opportunities to make sense of situations that involve division of unit fractions by whole numbers; to ask that they bring perseverance, a willingness to take risks, and the knowledge they already possess about division and fractions to bear on problems that require them to stretch a little (or a lot, in some cases).

11. Give students each a copy of the More Do-It-Yourself Story Problems and display the masters for everyone to see. Review and discuss the tasks with the class. Give students the option of working on these sheets independently, in pairs, or with you. Encourage students to make use of the strategies just shared by their classmates.

Activity 6 Dividing a Fraction by a Whole Number (cont.)

12. As students finish, have them share and compare their work with a classmate. If they have selected different divisors or dividends for each problem, challenge them to be sure each other's answers are correct. Have them check each other's story problems to be sure they make sense and have been solved correctly.

If students need more time to complete the assignment, ask them to finish it as homework or as part of their morning seatwork the following day before you conduct Activity 7.

Note Save the strategy posters from today's activity for use in Activity 7.

**INDEPENDENT WORKSHEET**

See Set A12 Independent Worksheet 3 on page A12.79 for more practice with dividing a fraction by a whole number, using the relationship between multiplication and division to check answers, and multiplying fractions and whole numbers.

Rope & Candy Problems

Choose *one* of the two story problems below and copy it into your journal or paper). Then:

- create a labeled visual model to represent and solve the problem.
- write an equation to match.

You will have about 10 minutes to work. If you finish one problem, do the other, or solve the challenge version of the problem you started with.

1a John has $\frac{1}{2}$ of a yard of rope. He wants to cut the rope into 3 equal pieces. What fraction of a yard will each piece of rope be?

Equation: _____

1b Challenge How many inches long will each piece of rope be?

Equation: _____



2a Lori had $\frac{1}{2}$ of a bag of candy and shared it with 3 friends. What fraction of the bag of candy did each of the 4 children get?

Equation: _____

2b Challenge There were 64 pieces of candy in the whole bag. How many pieces of candy did each friend get?

Equation: _____



NAME _____

DATE _____

More Do-It-Yourself Story Problems Page 1 of 3

- 1** Solve each of the story problems below. For each problem:
- Choose and circle one of the numbers in parentheses.
 - Write an expression to represent your problem.
 - Use numbers, labeled visual model, and/or words to solve the problem.
 - Complete the sentence below with your solution to the problem

a (Two, three, four) dinner guests shared $(\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{3}{4})$ of a pan of cornbread. What fraction of the cornbread did each guest get?

Expression: _____

Each guest got _____ pan of cornbread.



b Five cousins shared $(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{3}{4})$ of a birthday cake. What fraction of the cake did each cousin get?

Expression: _____

Each cousin got _____ of the birthday cake.



(Continued on next page.)

NAME _____

DATE _____

More Do-It-Yourself Story Problems Page 2 of 3

C Sara had $(\frac{1}{2}, \frac{1}{3}, \frac{2}{3})$ of a cup of grated cheese. She divided it equally between 4 salads. What fraction of a cup of cheese did each salad get?

Expression: _____



There was _____ a cup on each salad.

d Mr. Brown had $\frac{1}{8}$ of a pack of paper. He divided the pack equally among equally among 3 students. What fraction of the pack of paper did each student get?

Expression: _____

Each student got _____ a pack of paper.

(Continued on next page.)

NAME _____

DATE _____

More Do-It-Yourself Story Problems Page 2 of 3



CHALLENGE

e There were 504 sheets of paper in the whole pack. How many sheets of paper did each of the three students get?

Expression: _____

Each student got _____ sheets of paper.

2 Choose one of the expressions below and circle your choice. Write a story problem about the expression you circled. Then solve your own problem.

$$\frac{1}{2} \div 2$$

$$\frac{1}{2} \div 6$$

$$\frac{1}{3} \div 4$$

$$\frac{1}{8} \div 4$$

My story problem is:

My work:

The answer to my problem is: _____

Set A12 ★ Activity 7



ACTIVITY

The Division Poster Project

Overview

Today, students are asked to deal with situations that involve dividing whole numbers by unit fractions and unit fractions by whole numbers within the same instructional period. To start, the class examines $4 \div \frac{1}{3}$ and $\frac{1}{3} \div 4$, thinking about which of the two interpretations of division makes more sense with regard to solving each problem. Students then generate and solve story problems to match each expression. Finally, students work in pairs to create posters representing their understandings of dividing whole numbers by unit fractions and unit fractions by whole numbers.

Note Plan to take two math periods for this project.

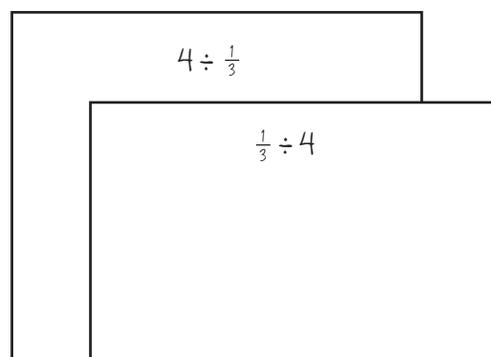
Skills & Concepts

- ★ Divide a whole number by a unit fraction using visual models (CCSS 5.NF.7a)
- ★ Create a story context for dividing a whole number by a unit fraction (CCSS 5.NF.7a)
- ★ Divide a unit fraction by a whole number using visual models (CCSS 5.NF.7b)
- ★ Create a story context for dividing a unit fraction by a whole number (CCSS 5.NF.7b)
- ★ Solve story problems involving division of a whole number by a unit fraction, and a unit fraction by a whole number (CCSS 5.NF.7c)

You'll need

- ★ Division Poster Project (page A12.64, run 1 copy for display)
- ★ Division Posters from Activity 2 (displayed where all the students can see them)
- ★ Strategy Posters from Activities 4 and 6 (displayed where all the students can see them)
- ★ 2 pieces of unlined chart paper (see Advance Preparation)
- ★ marking pens
- ★ 1 piece of scratch paper per student pair
- ★ 2" × 18" drawing paper, 1 sheet per student pair
- ★ additional scratch paper (optional, see note at the end of the activity)
- ★ crayons, colored pencils, and/or markers for student use
- ★ students' name tags or some other way to draw children's names at random

Advance Preparation Write $4 \div \frac{1}{3}$ at the top of one of the pieces of unlined chart paper. Write $\frac{1}{3} \div 4$ at the top of the other piece.



Instructions for The Division Poster Project

1. Open today's instruction by letting students know that they're going to work in pairs to create posters to show what they've learned about dividing whole numbers and fractions a little later in the session, but first you're going to do some review work together.

Activity 7 The Division Poster Project (cont.)

2. Draw students' attention to the two pieces of chart paper you've prepared for today's activity. Read the expressions at the top of each sheet with the class, and ask students to share, first in pairs and then as a whole group, which interpretation of division makes better sense for each. (Encourage students to refer to the Division Posters the class made during Session 2 and/or the Strategy Posters from Sessions 4 and 6 if those seem helpful.)

Natasha Okay, on that first one, it says 4 divided by a third. Sharing doesn't make any sense with that one, because how can you share 4 things with a third of a person?

Eric I think sharing makes sense with the other one, though. You can take $\frac{1}{3}$ of something and share it with 4 people, like if you have $\frac{1}{3}$ of a pizza or something like that.

Amber On that first one, I think you have to use grouping. You can take groups of one third out of 4. There are 3 thirds in 1, right? So there must be 12 thirds in 4.

Carlos But grouping doesn't make any sense with $\frac{1}{3} \div 4$, because there aren't any groups of 4 in $\frac{1}{3}$. One-third is smaller than 4 by a long shot.

3. When there is general consensus that $4 \div \frac{1}{3}$ makes the most sense when interpreted as a situation that requires grouping, while $\frac{1}{3} \div 4$ makes more sense in the context of sharing, work with students to record that information below each expression on the chart paper. Have them help you write out a verbal "translation" of each expression as well (see illustration below).

4. Next, work with the class to generate and record a story problem for each expression. (Don't take a lot of time with this step, because students will soon be working in pairs to devise their own story problems for the two expressions.)

$$4 \div \frac{1}{3}$$

Grouping

How many groups of $\frac{1}{3}$ are there in 4?
How many thirds are there in 4?

Mrs. Brown had 4 apples. She cut each apple into thirds. How many thirds did she make in all?

$$\frac{1}{3} \div 4$$

Sharing

If 4 people share $\frac{1}{3}$ of a _____, what fraction of the _____ will they each get?

There was $\frac{1}{3}$ of a pizza leftover from dinner. Mom said the 4 kids could split it among themselves for a snack the next day.

5. Have students pair up, or assign partners. Give each pair a piece of scratch paper, and have them write their own story problems, one for $4 \div \frac{1}{3}$ and one for $\frac{1}{3} \div 4$. (Suggest that they write each expression and a matching story problem at the top of one side of the scratch paper.)

6. As each pair finishes writing their two story problems, have them meet with another pair, trade papers, and solve each other's problems. Let them know that they'll need to show and explain their work in writing, using numbers, labeled models, and words, because each pair should be able to understand how the other pair solved their problems by reading their responses without any additional explanation.

Activity 7 The Division Poster Project (cont.)

Teacher *If you, Marcus and Andrew, solve the problems I wrote with Andrea, she and I should be able to see exactly how you solved them just by reading your work—without having to ask you questions about it. That means you're going to have to show and explain each step you take. The answer by itself doesn't explain how the problem was solved.*

7. When students have had adequate time to complete the assigned tasks, reconvene the class. Call on students at random to ensure maximum engagement, asking each to read one of the story problems he or she wrote. Have students identify the matching expression, and work with their input to record the context or situation on the appropriate chart.

Teacher *I have all your name tags. I am going to mix them up and pull them out at random, one by one. If I pull your name, you get to read one of the story problems you wrote without telling us which expression it was for. We have to figure it out and listen carefully enough to your problem to be able to summarize the context or situation and record it on the correct chart. When we finish, we'll have a whole list of ideas for writing more story problems. Maya, I just pulled your name. What do you have for us?*

Maya *There was a third of a pan of brownies on the counter. My mom said my 3 sisters and I could share it. How much did we each get?*

Teacher *Thumbs up if you know which expression Maya's story problem matches. Okay, you can all tell me at once—ready?*

Students $\frac{1}{3} \div 4!$

Teacher *Is that correct, Maya?*

Maya *Yep!*

Teacher *So, Maya has 4 kids sharing a third of a pan of brownies. I'll record that on the correct piece of chart paper here. Ernest, you're up next.*

Here's how the two charts might look after you have called on 8–10 students, which should be adequate.

Activity 7 The Division Poster Project (cont.)

$$4 \div \frac{1}{3}$$

Grouping

How many groups of $\frac{1}{3}$ are there in 4?
How many thirds are there in 4?

Mrs. Brown had 4 apples. She cut each apple into thirds. How many thirds did she make in all?

Contexts:

- 4 giant cookies, each cut into thirds—how many thirds?
- 4 mini-pizzas, each cut into thirds
- 4 pieces of colored paper, each cut into thirds
- 4 yards of cloth, each cut into thirds to make flags

$$\frac{1}{3} \div 4$$

Sharing

If 4 people share $\frac{1}{3}$ of a _____, what fraction of the _____ will they each get?

There was $\frac{1}{3}$ of a pizza leftover from dinner. Mom said the 4 kids could split it among themselves for a snack the next day.

Contexts:

- 4 kids sharing $\frac{1}{3}$ of a pan of brownies
- 4 dogs sharing $\frac{1}{3}$ of a pan of cornbread
- 4 friends sharing $\frac{1}{3}$ of a giant pizza
- 4 sisters sharing $\frac{1}{3}$ of a box of candy
- $\frac{1}{3}$ of a yard of ribbon cut into 4 equal pieces

8. Finally, display the Division Project Poster master. Give students a minute to read it to themselves. Then read it with the class and clarify the assignment as needed.

Set A12 Number and Operations: Dividing Fractions & Whole Numbers Run 1 copy or display

Division Poster Project

You and your partner are going to work together to make a poster showing what you have learned about dividing whole numbers and fractions. Follow these instructions:

1 Choose *one of the pairs* of expressions listed below.

$4 \div \frac{1}{2}$	$\frac{1}{2} \div 4$	$8 \div \frac{1}{4}$	$\frac{1}{4} \div 8$
$5 \div \frac{1}{4}$	$\frac{1}{4} \div 5$	$6 \div \frac{1}{3}$	$\frac{1}{3} \div 6$

CHALLENGE

$6 \div \frac{2}{3}$	$\frac{2}{3} \div 6$	$6 \div \frac{3}{4}$	$\frac{3}{4} \div 6$
----------------------	----------------------	----------------------	----------------------

2 Fold a piece of drawing paper in half. *On each half:*

- write **one** of the expressions from the pair you picked.
- **tell** whether the expression should be solved using the sharing or the grouping interpretation of division, and explain why.
- write a **story** problem to match the expression.
- **solve** your story problem. Show all your work. Your solution must include a clearly labeled visual model.

$4 \div \frac{1}{2}$	$\frac{1}{2} \div 4$
----------------------	----------------------

Activity 7 The Division Poster Project (cont.)

9. When students understand what to do, give each pair a piece of 12" × 18" drawing paper and let them go to work. Remind them that they can use any of the story problem contexts or situations on the charts you made with the class today, or think of new ones if they prefer.

.....

***Note** You may want to have students draft their story problems and solutions on scratch paper, and then check them with another student pair for accuracy, before they commit their work to drawing paper.*

.....

Extensions

- When student pairs have completed their posters, display them in the classroom or the hallway. If you choose to display them in the hallway where they can be viewed by students, teachers, and parents, you might also post a copy of the Division Poster Project master, or perhaps a shorter explanation of the project you (or you and the students) write, so others can appreciate what went into these posters.
- Once the posters are up, have your students take a gallery walk, moving through the display in pairs to examine and admire their classmates' work.

Division Poster Project

You and your partner are going to work together to make a poster showing what you have learned about dividing whole numbers and fractions. Follow these instructions:

1 Choose *one of the pairs* of expressions listed below.

$4 \div \frac{1}{2}$	$\frac{1}{2} \div 4$	$8 \div \frac{1}{4}$	$\frac{1}{4} \div 8$
$5 \div \frac{1}{4}$	$\frac{1}{4} \div 5$	$6 \div \frac{1}{3}$	$\frac{1}{3} \div 6$



CHALLENGE

$6 \div \frac{2}{3}$	$\frac{2}{3} \div 6$	$6 \div \frac{3}{4}$	$\frac{3}{4} \div 6$
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2 Fold a piece of drawing paper in half. *On each half:*

- write one of the expressions from the pair you picked.
- tell whether the expression should be solved using the sharing or the grouping interpretation of division, and explain why.
- write a story problem to match the expression.
- solve your story problem. Show all your work. Your solution must include a clearly labeled visual model.

$4 \div \frac{1}{3}$	$\frac{1}{3} \div 4$

Set A12 ★ Activity 8



ACTIVITY

Dividing Fractions & Whole Numbers Post-Assessment

Overview

The pre-assessment given in Activity 1 is re-administered in somewhat different form during this activity, and includes skills addressed in Supplement Set A9 as well as Supplement Set A12. Students' work on the post-assessment will provide information about what they have learned, as well as the areas in which they need continued support.

Skills & Concepts

- ★ Multiply a whole number or a fraction by a fraction (CCSS 5.NF.4a)
- ★ Solve story problems involving multiplying a whole number or a fraction by a fraction (CCSS 5.NF.4a)
- ★ Solve story problems involving multiplication of fractions and mixed numbers (CCSS 5.NF.6)
- ★ Divide a whole number by a unit fraction (CCSS 5.NF.7a)
- ★ Create a story context for dividing a whole number by a unit fraction (CCSS 5.NF.7a)
- ★ Divide a unit fraction by a whole number (CCSS 5.NF.7b)
- ★ Create a story context for dividing a unit fraction by a whole number (CCSS 5.NF.7b)
- ★ Solve story problems involving division of a whole number by a unit fraction, and a unit fraction by a whole number (5.NF.7c)

You'll need

- ★ Operating with Fractions & Whole Numbers Post-Assessment (pages A12.67–A12.70, run a class set plus a copy for display)
- ★ Grid Paper (page A12.71, copies as needed)
- ★ Operating with Fractions & Whole Numbers Post-Assessment Class Checklist (optional, pages A12.72 and A12.73, run 1 or 2 copies)
- ★ Geoboards and bands available for students who want to use them

Instructions for Dividing Fractions & Whole Numbers Post-Assessment

1. Give each student a copy of the post-assessment and then read and review the tasks with the class. Have students write their names on their papers and circle each “doing” word as you read through the items together.

Set A12 Number & Operations: Dividing Fractions & Whole Numbers Blackline Run a class set plus a copy for display

NAME _____ DATE _____

Operating with Fractions & Whole Numbers
Post-Assessment page 1 of 4

1 Solve each of the five story problems below. For each problem:

- Write an expression to represent the problem.
- Solve the problem. Show your work with labeled visual models, numbers, and/or words.
- Complete the sentence below with your solution to the problem.

Activity 8 Dividing Fractions & Whole Numbers Post-Assessment (cont.)

2. Before students start to work, be sure they understand that they have to use numbers, labeled models, and/or words to show their work and/or explain their thinking for problems 1–3; the answers alone will not be adequate. Remind them that they need to write a story problem to match the expression in problems 4 and 5, but they don't need to solve the problems. Also, alert them to the fact that this assessment includes multiplication as well as division situations because it's important to be able to tell when each operation is applied.

Let them know that they can use geoboards and bands, and/or grid paper to help solve any of the problems on the assessment, and make sure they understand how to access these materials.

3. Remind students that you are available to re-read any of the directions or problems for them while they work. Advise them to complete the items they find easiest and most familiar first, even if that means skipping around and then returning to the questions they find more challenging.

4. If you plan to score this assessment as suggested on the Operating with Fractions & Whole Numbers Post-Assessment Class Checklist, let students know how you will be scoring their papers. In some of the problems, they will be given a point for the answer and a point for showing their work. Story problems will be scored on a 3-point basis as follows:

- 1 point for writing an expression that accurately represents the story problem
- 1 point for using a strategy that could lead to the correct answer.
- 1 point for the correct answer, clearly stated.

While it may seem to create test anxiety, we find it is helpful to share expectations with students before they begin.

5. Give students the rest of the period to complete the assessment. Make sure your students understand what they are expected to do when they complete the assessment and where you want them to place their finished papers.

Note *In addition to scoring students post-assessments as suggested on the Post-Assessment Class Checklist, you may find it helpful to compare them to students' pre-assessments. Although some students may not score particularly well on the post-assessment, you may find they've made progress since the beginning of this supplement set.*

NAME _____

DATE _____

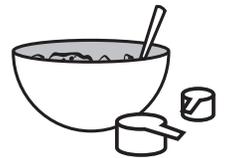
Operating with Fractions & Whole Numbers

Post-Assessment page 1 of 4

- 1** Solve each of the five story problems below. For each problem:
- Write an expression to represent the problem.
 - Solve the problem. Show your work with labeled visual models, numbers, and/or words.
 - Complete the sentence below with your solution to the problem.
- a** It takes $\frac{1}{2}$ of a cup of flour to make a batch of pancakes. Curtis has 4 cups of flour. How many batches of pancakes can he make?

Expression: _____

Curtis can make _____ batches of pancakes.



- b** The fifth graders are painting the bookshelves in their classroom. It takes $\frac{3}{4}$ of a quart of paint to paint each bookshelf. There are 8 bookshelves in the room. How many quarts of paint will the kids need to paint all 8 bookshelves?

Expression: _____

The kids will need _____ quarts of paint to paint all 8 bookshelves.



(Continued on next page.)

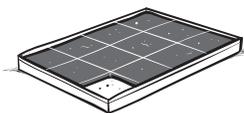
NAME _____

DATE _____

Operating with Fractions & Whole Numbers Post-Assessment page 2 of 4

C There is $\frac{1}{3}$ of a pan of cornbread left. Four children are going to share it equally. What fraction of the whole pan of cornbread will each child get?

Expression: _____



Each child will get _____ of the pan of cornbread.

d There was $\frac{1}{2}$ of a cake left over from Hannah's birthday party. When she and her sister came home from school the next day, they ate $\frac{2}{3}$ of the leftover cake for a snack. How much of the whole cake did the girls have for a snack?

Expression: _____

The girls had _____ of the whole cake for a snack.



(Continued on next page.)

NAME _____

DATE _____

Operating with Fractions & Whole Numbers Post-Assessment page 3 of 4

e The Ruiz family is going to build a raised garden bed for planting flowers in. The bed will be $\frac{3}{4}$ meter wide and $3\frac{1}{2}$ meters long. What will the area of the raised bed be when it is finished?

Expression: _____

The area of the raised flower bed will be _____ square meters.

2 Cory says that $6 \div \frac{1}{2}$ means the same thing as $\frac{1}{2}$ of 6, so the answer is 3. Do you agree with him? Why or why not? Use numbers, labeled models, and/or words to explain your thinking.

3 Jade says she knows that $\frac{1}{4} \div 4 = \frac{1}{16}$ because $\frac{1}{16} \times 4 = \frac{1}{4}$. Is she correct? Explain why or why not.

Continued on next page.

NAME _____

DATE _____

Operating with Fractions & Whole Numbers Post-Assessment page 4 of 4

4 Write a story problem to represent the expression in the box below. Then solve your own problem. Show your work with labeled models, numbers, and/or words, and write the answer on the line provided.

$$\frac{1}{2} \div 3$$

My story problem:

My work:

The answer to my problem is _____.

5 Write a story problem to represent the expression in the box below. Then solve your own problem. Show your work with labeled models, numbers, and/or words, and write the answer on the line provided.

$$6 \div \frac{1}{3}$$

My story problem:

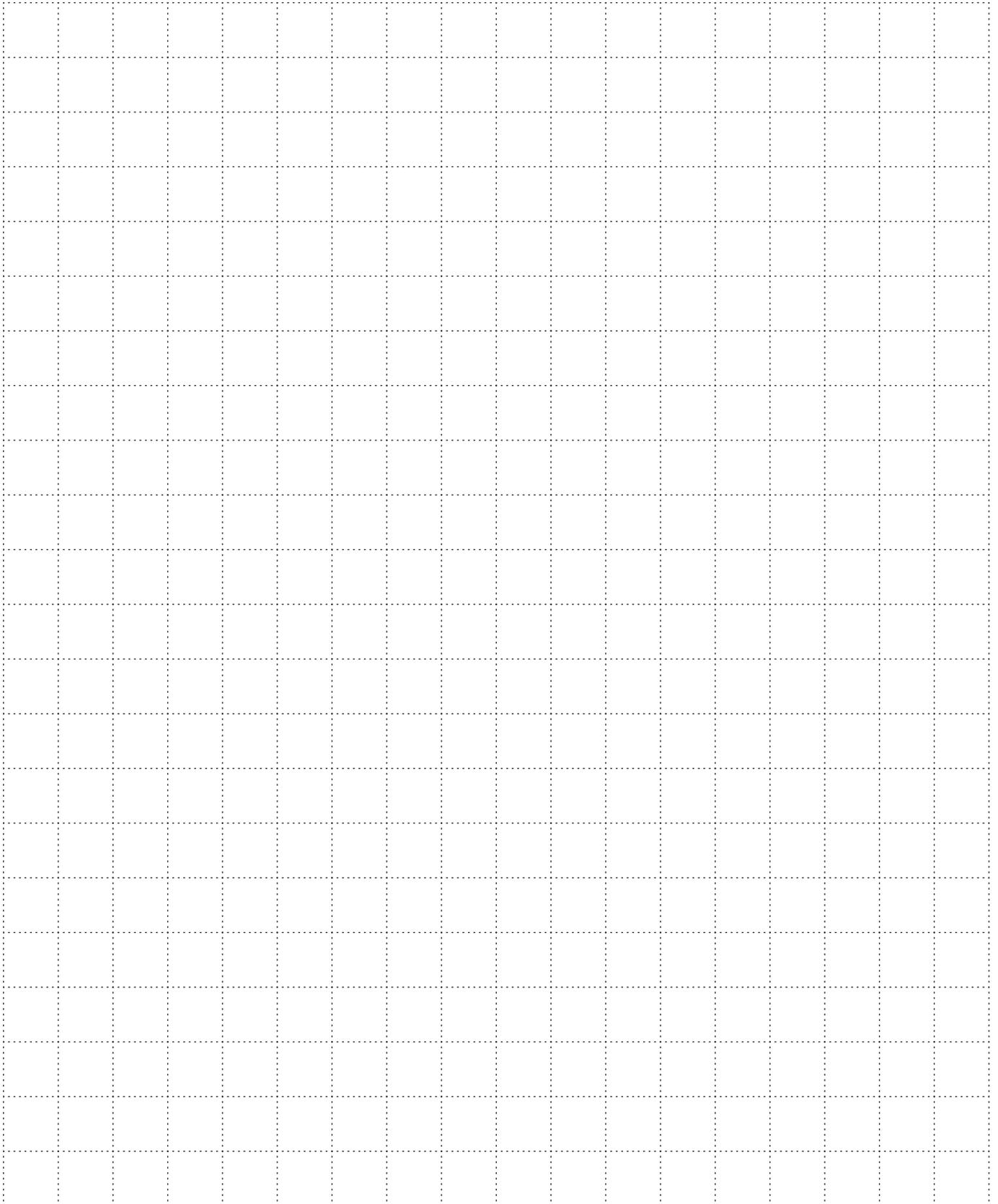
My work:

The answer to my problem is _____.

NAME _____

DATE _____

Grid Paper



Dividing Fractions & Whole Numbers Post-Assessment Class Checklist

Item and Correct Answer	CCSS	Points Possible	Student Names																
<p>1a Write an expression to represent a story problem that involves dividing a whole number by a unit fraction. Solve the problem; show work. Expression: $4 \div \frac{1}{2}$ Answer: 8 batches of pancakes; student work will vary</p>	5.NF.7b 5.NF.7c	3 pts. • 1 pt. for accurate expression • 1 pt for using a strategy that could lead to the right answer • 1 pt for the correct answer																	
<p>1b Write an expression to represent a story problem that involves multiplying a whole number by a fraction. Solve the problem; show work. Expression: $8 \times \frac{3}{4}$ OR $\frac{3}{4} \times 8$ Answer: 6 quarts; student work will vary</p>	5.NF.4a	3 pts. • 1 pt. for accurate expression • 1 pt for using a strategy that could lead to the right answer • 1 pt for the correct answer																	
<p>1c Write an expression to represent a story problem that involves dividing a unit fraction by a whole number. Solve the problem; show work. Expression: $\frac{1}{3} \div 4$ Answer: $\frac{1}{12}$ the pan of cornbread; student work will vary</p>	5.NF.7a 5.NF.7c	3 pts. • 1 pt. for accurate expression • 1 pt for using a strategy that could lead to the right answer • 1 pt for the correct answer																	
<p>1d Write an expression to represent a story problem that involves multiplying a fraction by a fraction. Solve the problem; show work. Expression: $\frac{1}{2} \times \frac{2}{3}$ Answer: $\frac{1}{3}$ of the whole cake; student work will vary</p>	5.NF.4a	3 pts. • 1 pt. for accurate expression • 1 pt for using a strategy that could lead to the right answer • 1 pt for the correct answer																	
<p>1e Write an expression to represent a story problem that involves multiplying a fraction by a mixed number. Solve the problem; show work. Expression: $\frac{3}{4} \times 3 \frac{1}{2}$ Answer: $2 \frac{5}{8}$ square meters; student work will vary</p>	5.NF.6	3 pts. • 1 pt. for accurate expression • 1 pt for using a strategy that could lead to the right answer • 1 pt for the correct answer																	
<p>2 Interpret division of a whole number by a unit fraction. Answer: No; students' explanations will vary. Example: I don't agree because $6 \div \frac{1}{2}$ is not the same thing as half of 6. It means how many halves in 6. Since there are 2 halves in 1, there are 12 halves in 6.</p>	5.NF.7b	2 pts. • 1 pt. for correct answer • 1 pt. for giving a viable explanation																	

Dividing Fractions & Whole Numbers Post-Assessment Class Checklist (cont.)

Item and Correct Answer	CCSS	Points Possible	Student Names															
<p>3 Recognize the relationship between division and multiplication Answer: Yes; students' explanations will vary. Example: I agree because $4 \times \frac{1}{4}$ is a fourth. You can show that by adding $1\frac{1}{4}$ four times. You get $\frac{4}{4}$, which is a fourth, so $\frac{1}{4}$ divided by 4 must be $\frac{1}{16}$.</p>	5.NF.7a	2 pts. • 1 pt. for correct answer • 1 pt. for giving a viable explanation																
<p>4 Write and solve a story problem to represent the expression $\frac{1}{2} \div 3$; show work. Answer: $\frac{1}{6}$; Students' story problems and work will vary. Sample problem: I had half a sandwich. I gave it to my 3 friends to share equally. How much of the sandwich did each friend get?</p>	5.NF.7a	3 pts. • 1 pt. for a story problem that accurately represents the expression • 1 pt for showing work that involves the use of a strategy that could lead to the right answer • 1 pt for the correct answer																
<p>5 Write and solve a story problem to represent the expression $6 \div \frac{1}{3}$; show work. Answer: 18 Students' story problems and work will vary. Sample Problem: I had 6 apples. I cut each apple into thirds. How many thirds did I get in all?</p>	5.NF.7b	3 pts. • 1 pt. for a story problem that accurately represents the expression • 1 pt for showing work that involves the use of a strategy that could lead to the right answer • 1 pt for the correct answer																
Total Score/Level of Proficiency*		25 pts																

* Meeting Standard: 19–25 points (75–100% correct) Approaching Standard: 13–18 points (50–74% correct) Strategic: 7–12 points (25–49% correct) Intensive: 6 points or fewer (24% or less correct)

NAME _____

DATE _____

Set A12 ★ Independent Worksheet 1



INDEPENDENT WORKSHEET

Sharing & Grouping Multiplying & Dividing

1 Read each story problem. Then:

- write an equation (including the answer) for the problem.
- fill in the bubble to show whether the answer means the size of each group or the number of groups

a The swim team is going to a meet across town. There are 35 swimmers on the team, and each van can take 5 of them. How many vans will be needed to take the whole team?

Equation: _____



The answer means:

- the size of each group (for example, the number of items each person got)
- the number of groups

b Jacob picked 28 flowers and divided them equally between 2 vases. How many flowers did he put in each vase?

Equation: _____



The answer means:

- the size of each group (for example, the number of items each person got)
- the number of groups

2 Circle the equation that matches each story problem. Then fill in the correct answer.

a Alexis and her two sisters picked 48 strawberries and shared them equally. How many strawberries did each girl get?

$$48 \div 2 = \underline{\quad} \quad 3 \times 48 = \underline{\quad} \quad 48 \div 3 = \underline{\quad} \quad 48 - 3 = \underline{\quad}$$

(Continued on next page.)

NAME _____

DATE _____

Sharing & Grouping Multiplying & Dividing (cont.)

b Miguel is making valentines. It takes $\frac{1}{2}$ of a sheet of paper for each valentine, and Miguel wants to make 26 valentines. How many sheets of paper will he need?

$$26 \div \frac{1}{2} = \underline{\hspace{2cm}} \quad 26 \times \frac{1}{2} = \underline{\hspace{2cm}} \quad 26 \times 2 = \underline{\hspace{2cm}} \quad 26 - \frac{1}{2} = \underline{\hspace{2cm}}$$

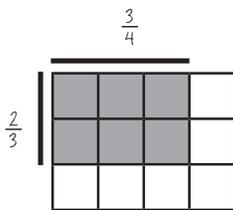
c Ling and her mother are making dumplings. It takes $\frac{3}{4}$ of an ounce of meat for each dumpling, and they are going to make 36 dumplings. How many ounces of meat will they need?

$$36 \times 4 = \underline{\hspace{2cm}} \quad 3 \times 36 = \underline{\hspace{2cm}} \quad \frac{3}{4} \div 36 = \underline{\hspace{2cm}} \quad 36 \times \frac{3}{4} = \underline{\hspace{2cm}}$$

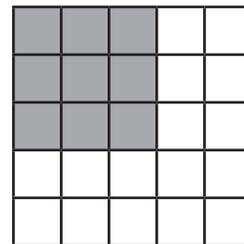
d There was $\frac{1}{2}$ of a pan of cornbread leftover from dinner. Jake and his dad ate half of the leftover cornbread for breakfast. How much of the whole pan did they have at breakfast?

$$\frac{1}{2} \times \frac{1}{2} = \underline{\hspace{2cm}} \quad \frac{1}{4} + \frac{1}{4} = \underline{\hspace{2cm}} \quad \frac{1}{2} \div \frac{1}{2} = \underline{\hspace{2cm}} \quad \frac{1}{2} - \frac{1}{2} = \underline{\hspace{2cm}}$$

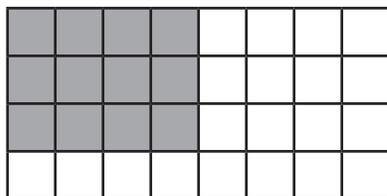
3 Each of the visual models below shows the results of multiplying one fraction by another. Label each of the shaded regions with its dimensions and area. Then write a multiplication equation to match.

ex

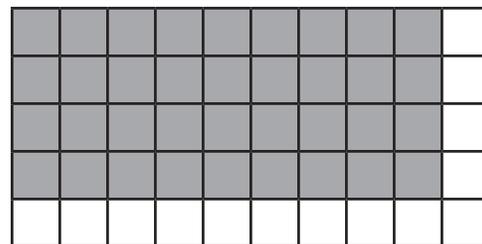
Equation $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$

a

Equation _____

b

Equation _____

c

Equation _____

NAME _____

DATE _____

Set A12 ★ Independent Worksheet 2



INDEPENDENT WORKSHEET

Operating with Fractions & Whole Numbers

- 1** Solve each of the story problems below. For each problem:
- Choose and circle one of the numbers in parentheses, depending on how challenging you want the problem to be.
 - Write an expression to represent your problem.
 - Use numbers, labeled visual models, and/or words to solve the problem and explain your strategy.
 - Complete the sentence below with your solution to the problem.
- a** It takes $(\frac{1}{2}, \frac{1}{3}, \frac{3}{4}, \frac{2}{3})$ of a cup of flour to make a batch of pancakes. I have 4 cups of flour. How many batches of pancakes can I make?

Expression: _____

I can make _____ batches of pancakes.



- b** Little Snail can crawl $(\frac{1}{4}, \frac{1}{3}, \frac{3}{4}, \frac{7}{8})$ of a mile a day. How far can he crawl in 5 days if he crawls the same distance each day?

Expression: _____

Little snail can crawl _____ miles in 5 days.



(Continued on next page.)

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Operating with Fractions & Whole Numbers (cont.)

C Keiko always takes her water bottle with her when she hikes, and she always drinks $2\frac{1}{2}$ cups of water for every mile she hikes. Yesterday, she hiked $\frac{1}{2}$ a mile. How many cups of water did she drink?

Expression: _____



Keiko drank _____ cups of water.

2 Solve each of the multiplication problems below. For each:

- outline a rectangle on the grid that will work for both fractions.
- draw and label the dimensions and area, and write the answer.
- write the problem and answer in words.

<p>ex</p> $\frac{2}{3} \times \frac{4}{8} = \frac{8}{24} = \frac{1}{3}$		<p>Two-thirds of 4-eighths is eight twenty-fourths.</p>
<p>a</p> $\frac{2}{4} \times \frac{3}{5} =$		
<p>b</p> $\frac{2}{3} \times \frac{4}{6} =$		
<p>c</p> $\frac{3}{4} \times \frac{5}{10} =$		

NAME _____

DATE _____

Set A12 ★ Independent Worksheet 3



INDEPENDENT WORKSHEET

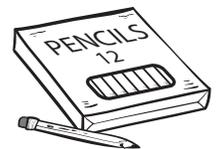
More Fractions & Whole Numbers

- 1** Solve each of the story problems below. For each problem:
- Choose and circle one of the numbers in parentheses, depending on how challenging you want the problem to be.
 - Write an expression to represent your problem.
 - Use numbers, labeled visual models, and/or words to solve the problem and explain your strategy. (Someone should be able to read your paper and tell how you solved each problem without talking to you to find out.)
 - Complete the sentence below with your solution to the problem.

a Mrs. Alvarez had $(\frac{1}{5}, \frac{1}{8}, \frac{3}{8}, \frac{2}{3})$ of a box of pencils. She divided the box equally among (3, 4, 5) students. What fraction of the box of pencils did each student get?

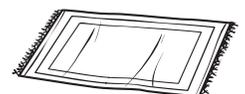
Expression: _____

Each students got _____ of a box of pencils.



b Sara has a rug in her bedroom that is (2, 3, 4 feet) by $(2\frac{1}{2}, 2\frac{3}{4}, 3\frac{3}{8})$ feet. What is the area of Sara's rug?

Expression: _____



The area of Sara's rug is _____ feet.

(Continued on next page.)

NAME _____

DATE _____

More Fractions & Whole Numbers (cont.)

C It takes ($4\frac{1}{2}$, $4\frac{3}{4}$, $4\frac{1}{4}$) feet of craft lace to make a short lanyard for a keychain. John wants to make a lanyard for each of his (5, 6, 7) aunts and uncles. How many feet of craft lace will he need in all?

Expression: _____

John will need _____ feet of craft lace.

2 Use multiplication to check your answer for each of the division problems below.

<p>ex $100 \div 4 = \underline{25}$</p> <p>I know this is correct because</p> $25 \times 4 = 100$	<p>ex $\frac{1}{2} \div 2 = \underline{\frac{1}{4}}$</p> <p>I know this is correct because</p> $\frac{1}{4} \times 2 = \frac{1}{2}$
<p>a $\frac{1}{2} \div 4 = \underline{\hspace{2cm}}$</p>	<p>b $\frac{1}{4} \div 2 = \underline{\hspace{2cm}}$</p>
<p>c $\frac{1}{3} \div 2 = \underline{\hspace{2cm}}$</p>	<p>d $\frac{1}{4} \div 4 = \underline{\hspace{2cm}}$</p>

**CHALLENGE**

3 Maria says that dividing $\frac{1}{2}$ by 3 is the same as multiplying $\frac{1}{2}$ by $\frac{1}{3}$. Do you agree with her? Why or why not? Use numbers, labeled models, and/or words to explain your thinking.