BRIDGES IN MATHEMATICS 5

GRADE 5 SUPPLEMENT

Set D2 Measurement: Volume

Includes

Skills & Concepts

- ★ Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
- ★ Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
- ★ Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and by multiplying the height by the area of the base.
- ★ Represent threefold whole-number products as volumes, the associative property of multiplication.
- ★ Apply the formulas V=l x w x h and V=b x h to find volumes of right rectangular prisms with whole number edge lengths in the context of solving mathematical problems.
- ★ Solve real world problems involving multiplication of fractions and mixed numbers, using visual fraction models or equations to represent the problem.
- ★ Convert among different-sized standard measurement units within a given measurement system
- ★ Make a line plot to display a data set of measurements in fractions of a unit (½, ¼, ⅛).
- ★ Use operations on fractions for this grade to solve problems involving information presented in line plots

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Bridges in Mathematics Grade 5 Supplement
Set D2 Measurement: Volume

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Bridges in Mathematics is a standards-based K–5 curriculum that provides a unique blend of concept development and skills practice in the context of problem solving. It incorporates the Number Corner, a collection of daily skill-building activities for students.

The Math Learning Center is a nonprofit organization serving the education community. Our mission is to inspire and enable individuals to discover and develop their mathematical confidence and ability. We offer innovative and standards-based professional development, curriculum, materials, and resources to support learning and teaching. To find out more, visit us at www.mathlearningcenter.org.

Set D2 ★ Activity 1

ACTIVITY

Introducing Volume

Overview

In this activity, students move toward increasingly efficient methods of finding the volume of cubes and rectangular solids.

Skills & Concepts

- ★ determine volume by finding the total number of same-sized units of volume that fill a three-dimensional shape without gaps or overlaps
- ★ understand a cube that is one unit on an edge is the standard unit for measuring volume
- ★ select appropriate units, strategies, and tools for solving problems that involve estimating or measuring volume
- ★ measure necessary attributes of shapes to use volume formulas to solve problems

You'll need

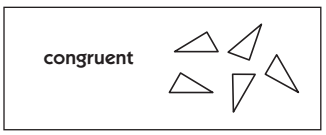
- ★ Cubes & Rectangular Solids (page D2.4, run 1 for display)
- ★ Paper Box Pattern (page D2.5, run a class set)
- ★ centimeter cubes (class set)
- ★ scissors
- ★ scotch tape
- ★ rulers (class set)
- ★ Student Math Journals
- ★ Word Resource Cards (congruent, edge, face, parallel, perpendicular, vertex) optional

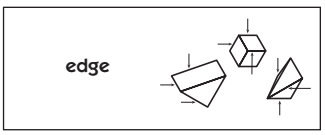
Advance Preparation Display the Word Resource Cards where students can see them before conducting the activity.

Instructions for Introducing Volume

1. Give students each a centimeter cube and allow several minutes for them to record as many observations as they can about the cube in their math journals. Call their attention to the Word Resource Cards before they start writing and challenge them to include at least 3 of the words in their observations.

2. Have them pair-share their observations, and then call for whole-group sharing. Record some of their observations at the top of the Cubes and Rectangular Solids, keeping the rest of the page covered for now. If it doesn't come up in the discussion, ask students to find examples of parallel, perpendicular, and congruent edges and faces as they examine their cubes.





3. Ask students to estimate the length of one of the edges of their cube. Then have a volunteer measure to confirm that each edge is 1 centimeter. Next, ask students to determine the area of one of the cube's faces. Finally, explain that because their cube is 1 centimeter long, wide, and high, it is called a *cubic centimeter.* Just as centimeters are used to measure length and square centimeters are used to measure area, *cubic centimeters* are used to measure *volume.* Add this information to the display, along with the abbreviations for each measure.

Activity 1 Introducing Volume (cont.)

4. Next, reveal the picture of the rectangular solid on the display. Have students write at least 3 observations about this figure in their journals. Then invite volunteers to share their observations with the class as you record at the display. After you've recorded 8–10 observations, work with input from the students to label all 3 dimensions of the solid: length, width, and height.

Set D2 Measurement: Volume Blackline Run 1 copy on a transparency.

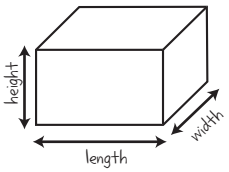
Cubes and Rectagular Solids

Cube

length of edge = 1 cm
area of face = 1 sq cm (cm^2)
volume of cube = 1 cubic cm (cm^3)

- square on every side
- 8 vertices
- has parallel and perpendicular sides
- all faces are congruent
- all edges are congruent
- has 3 pairs of parallel faces
- sides that touch are perpendicular
- 6 faces
- 12 edges

Rectangular Solid



- faces are rectangles
- 6 faces, 8 vertices, 12 edges
- all right angles
- sides that touch are perpendicular
- 3 pairs of parallel sides
- opposite sides are congrunent

5. Give each table a good supply of centimeter cubes. Ask each student to build several different rectangular solids that have a volume of exactly 12 cubic centimeters. Be sure they understand that their constructions have to be solidly filled in, without gaps or holes between cubes. Ask them to share and compare their constructions as they're working.

6. After a few minutes, call a halt to the construction process. Ask several volunteers to describe their constructions by length, width, and height. Record each description at the board, along with an equation to confirm that the total is 12 cubic centimeters.

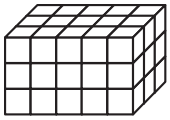
Rectangular Solids with Volume = 12 cm^3

Length = 2 cm Width = 2 cm Height = 3 cm	Length = 6 cm Width = 2cm Height = 1cm	Length = 12 cm Width = 1 cm Height = 1 cm	Length = 3 cm Width = 4 cm Height = 1 cm
$2 \times 2 \times 3 = 12\ cm^3$	$6 \times 2 \times 1 = 12\ cm^3$	$12 \times 1 \times 1 = 12\ cm^3$	$3 \times 4 \times 1 = 12\ cm^3$

7. Now reveal the rectangular solid at the bottom of the display. Ask students to replicate it with their cubes and determine its volume without counting every cube one by one. As they finish, invite volunteers to share their strategies with the class, as you record at the display. If it doesn't come from one of the students, ask them what would happen if you multiplied *length* × *width* × *height*. Would it result in the same answer they've shared? Why or why not? Press them to explain their thinking and then work with their input to write the equation and solve the multiplication problem.

Activity 1 Introducing Volume (cont.)

Build this rectangular solid with your centimeter cubes. Find the volume *without* counting each cube 1 by 1.



$15 + 15 + 15 = 45\ cm^3$

$9 \times 5 = 45\ cm^3$

counted by rows of 5 ($5 \times 9 = 45\ cm^3$)

$18 + 18 = 36\ \ 36 + 9 = 45\ cm^3$

length x width x height

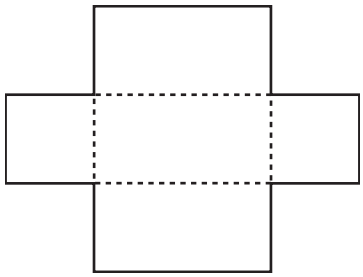
$5 \times 3 \times 3 = 45\ cm^3$

8. Ask students to clear their cubes to the side for now and get out their scissors. Give each student a copy of the Paper Box Pattern and supply each table with some scotch tape. Have them cut, fold, and tape their paper patterns to make a box. Ask early finishers to help others near them.

Set D2 Measurement: Volume Blackline Run a class set.

Paper Box Pattern

Cut out this pattern. Fold along the dashed lines and tape to make a box.



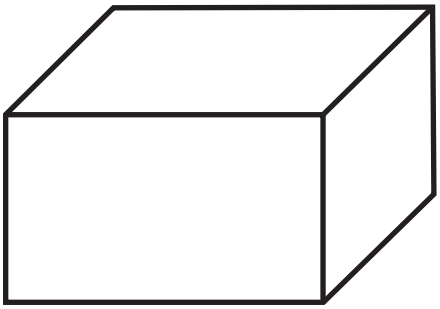
9. When everyone has finished, ask students to estimate the volume of the box. How many centimeter cubes do they think it will take to fill the box completely? Record some of their estimates on the board. Then challenge them to work in pairs to determine the actual volume of the box *without* filling it to the top with cubes, dumping them out, and counting them one by one. As they finish, have them record their solution in their journal, along with a detailed description of their strategy.

10. Toward the end of the period, reconvene the class. Ask volunteers to share their strategies and solutions with the class. If the idea of measuring the dimensions of the box and multiplying them doesn't come from one of the students, ask them to get out their rulers and try it. Does it result in the same solution they got using other methods? Why? (Students should find that the taped box holds 54 centimeter cubes. It is 6 centimeters long, 3 centimeters wide, and 3 centimeters high. $6 \times 3 \times 3 = 54\ cm^3$.)

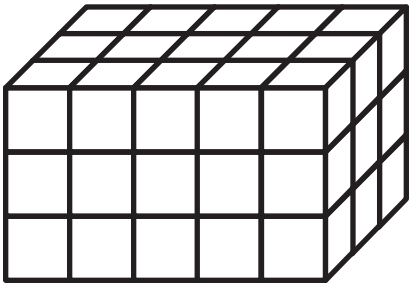
Cubes & Rectangular Solids

Cube

Rectangular Solid

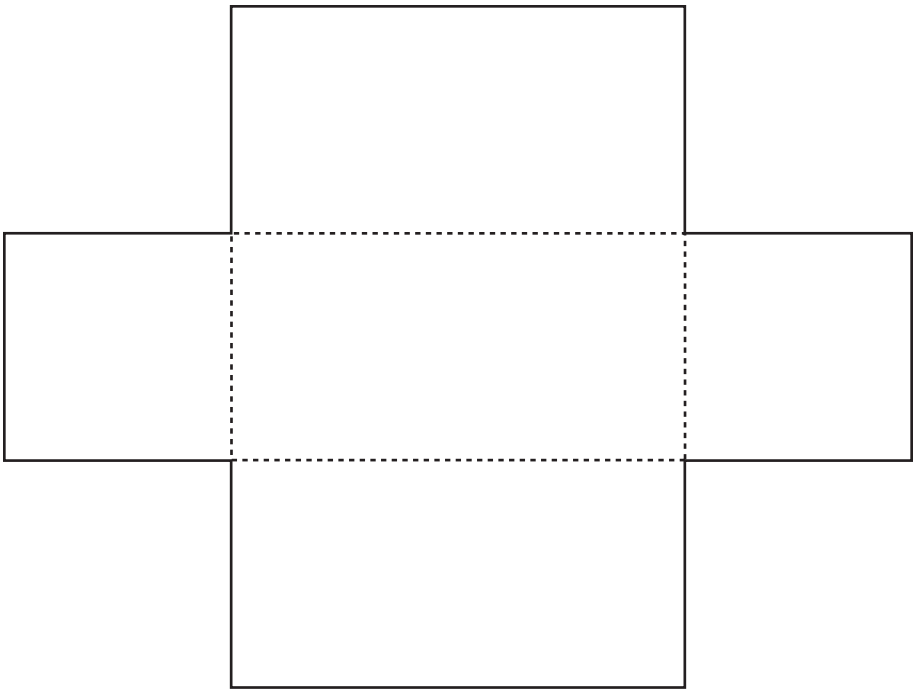


Build this rectangular solid with your centimeter cubes. Find the volume *without* counting each cube 1 by 1.



Paper Box Pattern

Cut out this pattern. Fold along the dashed lines and tape to make a box.



Set D2 ★ Activity 2

ACTIVITY

More Paper Boxes

Overview

Using paper boxes and centimeter cubes, students work together to generate efficient methods, including the standard formulas, for finding the volume of cubes and rectangular solids.

Skills & Concepts

- ★ determine volume by finding the total number of same-sized units of volume that fill a three-dimensional shape without gaps or overlaps
- ★ select appropriate units, strategies, and tools for solving problems that involve estimating or measuring volume
- ★ measure necessary attributes of shapes to use volume formulas to solve problems

You'll need

- ★ More Paper Boxes (page D2.10, run a half-class set, plus a few extra)
- ★ Student Math Journals or Journal Grid Page (page D2.11, run as needed, optional)
- ★ centimeter cubes (class set)
- ★ scissors
- ★ scotch tape
- ★ rulers (class set)
- ★ *Counting on Frank* by Rod Clement (optional)

Instructions for More Paper Boxes

1. Ask students to pair up, or assign partners. Students will need their rulers, scissors, and journals for this activity. Give each pair a copy of More Paper Boxes, along with some scotch tape. Have them cut out and tape together Box A, leaving Boxes B and C uncut for now. Ask early finishers to help others nearby.

2. When most students have finished constructing Box A, ask them to estimate how many centimeter cubes it will take to fill the box completely. Have them each record an estimate in their journals. Then ask volunteers to share and explain their estimates as you record at the board.

> ***Lauren*** *It looks like it's going to take about 10 to fill the bottom, and it's about 3 cubes high, so I think 30 cubes will do it.*
>
> ***Tonio*** *I say 40 because it's maybe 10 on the bottom and 4 up. That would be 4 × 10, so that's 40.*
>
> ***Marisa*** *I said 54 cubes because it looks like it's 3 across and maybe 6 long. That's 18. I think it's going to be 3 layers high, so I multiplied 3 × 18 to get 54.*

3. Distribute centimeter cubes and ask student pairs to find the actual volume of Box A. Encourage students to use any efficient method they choose, but moving away from filling the box completely, dumping out the cubes and counting by ones. As they finish, have them record their answer, along with a description of their strategy in their journal.

4. After they've had a few minutes to work, ask volunteers to share their solutions and strategies with the class.

Activity 2 More Paper Boxes (cont.)

Carter *It took 21 cubes to cover the bottom of the box. Then we stacked cubes in one corner to find out how high the box was. It was 4 cubes up, so we said 4 × 21 is 84 cubes.*

Abby *We just used the cubes to make kind of an outline inside the box. It was 7 on the long side and 3 on the short side, so we knew the first layer would be 21. Then we went up one corner like Carter and Xavier, and it was 4. Then we knew it was 84 cubic centimeters because 4 × 21 is 84.*

5. If the idea of measuring the dimensions of the box and multiplying them doesn't come from the students, ask them to get out their rulers and try it. Does this strategy result in the same solution they got using other methods? Why? Work with class input to record an equation that matches what they just did: $7 \times 3 \times 4 = 84$ cm^3.

6. Ask students to cut out and tape together Box B and record an estimate of the volume in their journals. As they're working, collect the centimeter cubes. When most have finished, ask volunteers to share their estimates as you record at the board. Then challenge students to find the actual volume of the box using their rulers instead of cubes. Have them record the answer, along with any computations they made, in their journal.

7. After they've had some time to work, ask volunteers to share their solutions and strategies with the class. Then work with input from the class to write a general formula for finding the volume of a rectangular solid (length × width × height = volume), along with an equation for Box B ($6 \times 4 \times 2 = 48$ cm^3). Have students record this information in their journals.

8. Now tell them that some fifth graders in another class said they thought they could find the volume of Box C without cutting and taping it together. Do your students agree with these fifth graders? Why or why not? Have them pair-share their responses and then ask volunteers to share their thinking with the class.

Students *We said you could do it by just using a ruler, but you should cut out the box and put it together first.*
We think they're right. It looks like it's going to be a cube, so if you just measured one edge, you could figure it out.

9. Ask students to measure one or more edges of the uncut box to help make as accurate an estimate as possible. Have them record their estimate, along with an explanation in their journal. (If they're sure their estimate matches the actual volume, that's fine.)

10. After a few volunteers have shared and explained their estimates, ask students to cut out and tape together Box C. Have them measure it to determine the actual volume, and record the answer, along with any calculations they made, in their journals.

11. Have volunteers share and explain their solutions and strategies. Was it possible to determine the volume of the figure by measuring only 1 edge? Why or why not? Would it have been possible to find the answer without cutting and taping the cube? Why or why not? Then have students write an equation for the volume of Box C ($4 \times 4 \times 4 = 64$ cm^3) in their journals.

Activity 2 More Paper Boxes (cont.)

Extensions

- Explain that because the length, width, and height of a cube are all equal, mathematicians generally represent $s \times s \times s$ by s^3. Using this notation, the volume of a cube is s^3 where s is the length of one edge of the cube.

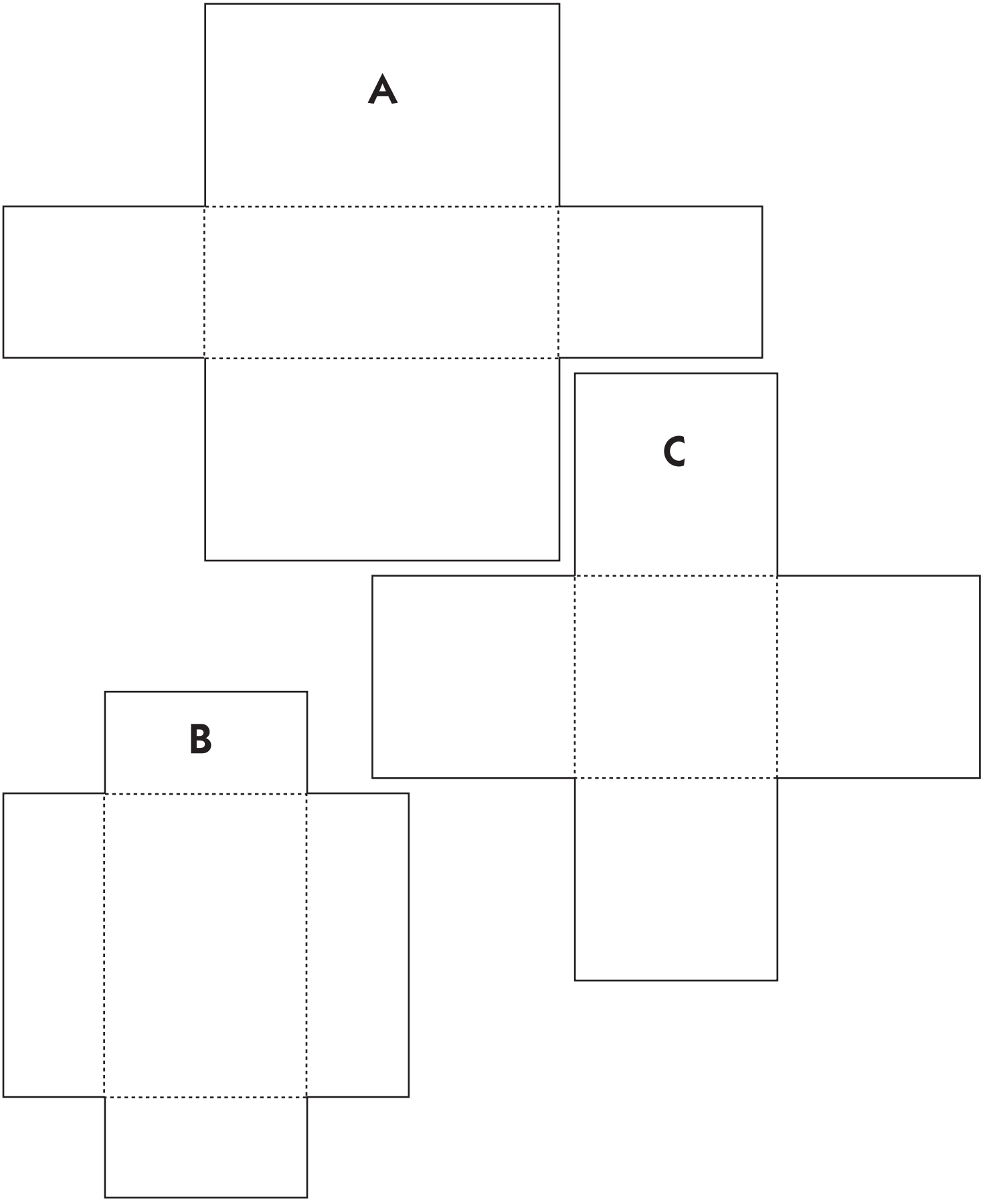
  $$s \times s \times s = s^3$$

  Record this at the board, and ask students to compare it to the formula for finding the volume of a rectangular prism. How are the two alike? How are they different? Ask them to record the general formula for finding the volume of a cube in their journals.
- Have volunteers use lightweight cardboard and tape to construct a cubic inch and a cubic foot, and share them with the class. Ask students to list in their journals some of the things they'd measure in cubic inches and some of the things they'd measure in cubic feet.
- Read *Counting on Frank* by Rod Clement before or after this session.

INDEPENDENT WORKSHEET

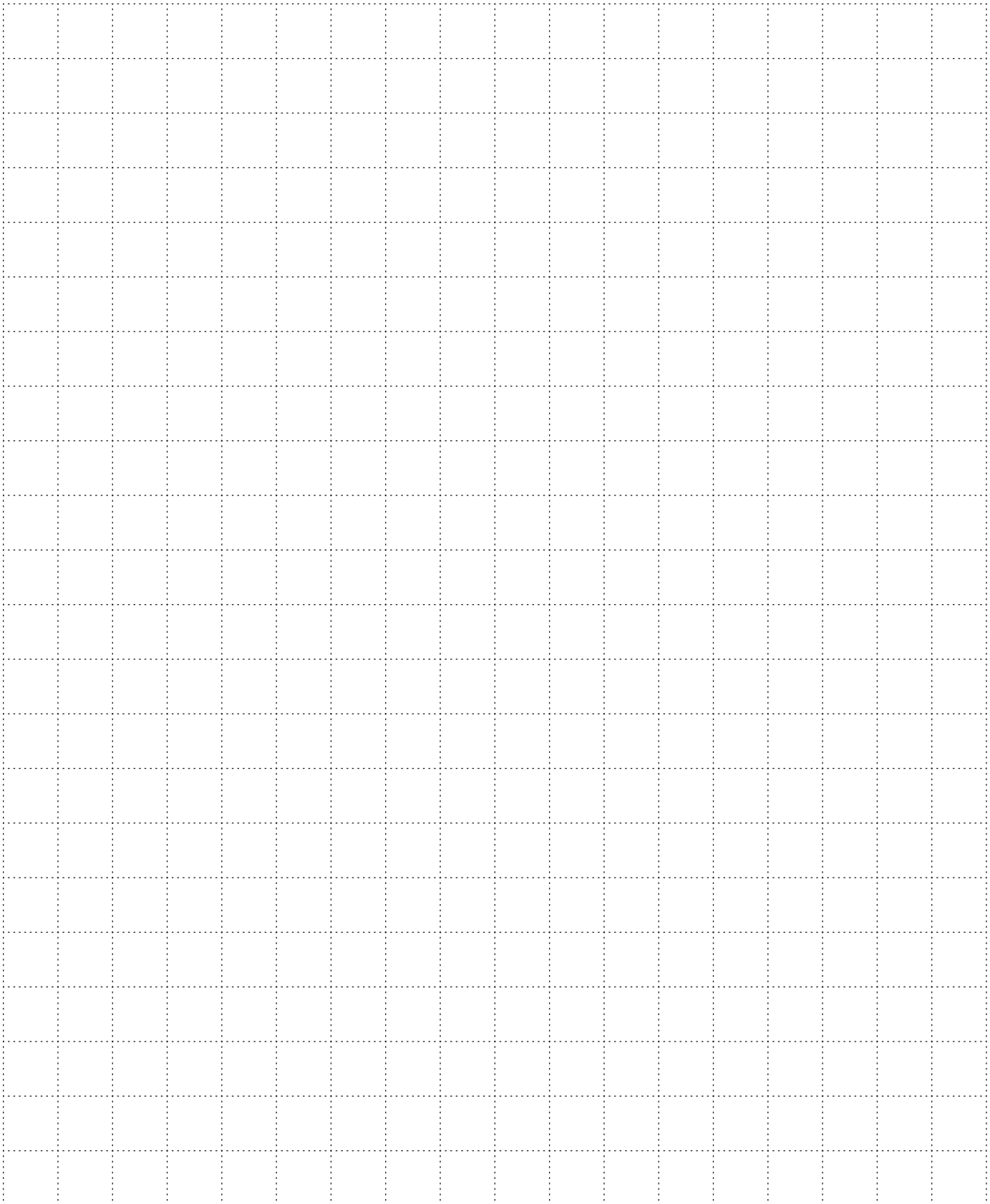
See Set D2 Independent Worksheet 1, Volume Review (page D2.35) and Independent Worksheet 2, The Camping Trip (page D2.39) for more practice selecting and using appropriate units and formulas to determine length, area, and volume.

More Paper Boxes



NAME _____ **DATE** _____

Journal Page Grid



Set D2 ★ Activity 3

ACTIVITY

Fish Tank Volume

Overview

During this activity, students will demonstrate their understanding of the properties of solid figures and determine the volume of rectangular prisms. They convert measurement units within a given measurement system, and use these conversions in solving multi-step, real world problems with volume.

Skills & Concepts

- ★ Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
- ★ Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
- ★ Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base.
- ★ Represent threefold whole-number products as volumes, the associative property of multiplication.
- ★ Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems.

You'll need

- ★ Design a Fish Tank (page D2.16 run a calss set plus 1 for display)
- ★ Fish Tank Models (page D2.17 run a calss set plus 1 for display)
- ★ Student math journals or Journal Grid Page (page D2.118, run as needed, optional)
- ★ Centimeter cubes (class set)
- ★ Calculators (optional)
- ★ Masking tape (optional)

Instructions for Fish Tank Volume

1. Begin by telling students that over the next few days they will build several models of fish tanks that meet specific volume requirements. Activate their prior knowledge about fish tanks by asking the following questions: How many students have a fish tank at home? What do you know about fish tanks? Where could a fish tank go in the office area?

2. Read the Design a Fish Tank scenario. Discuss the properties of a cube and invite one student to draw a cube in the display area, while others do the same in their journals. (Some students may need additional support when drawing a cube but this is not a critical skill and a quick sketch will do). Restate the total fish tank volume would be 8 cubic feet of water.

Teacher *What does the word "cubic" mean?*

Activity 3 Fish Tank Volume (cont.)

Sarah *A cube is a three-dimensional figure where all sides are equal.*

Jacob *You show the total volume by adding the little 3 behind whatever unit you are measuring in. The little 3 means that the shape is 3-D.*

Teacher *How do we find the area of a cube?*

Students *Length times width times height.*

Teacher *So if the tank has a total volume of 8 cubic feet how can we find the length, width, and height of the tank?*

Kristen *I think we should take the 8 and divide it by 3 because there are three types of sides.*

Betsy *Wait! If we divided 8 by 3 we would get an answer of* $2\ ^{2}/_{3}$ *Now if I use the formula of length x width x height and plug in* $2\ ^{2}/_{3}$ *that number would be greater than 8 cubic feet.*

Teacher *We know that we need all the sides to be the same in length when our tank is in the shape of a cube. What is one way to represent the quantity, 8?*

Betsy *I know if the cube had the dimension of* $2 \times 2 \times 2 = 8$ *ft.*^{3}

3. Distribute handfuls of centimeter cubes to each group and ask each student to count out 8 cubes. Have them build the cube model they have drawn and then label the dimensions in their journals. Now ask the students to manipulate the same 8 cubes in a variety of ways to create different rectangular prisms. Ask students to demonstrate their ideas as a class, while you record a quick sketch and the dimensions on the teacher master. Have students record the same information in their journals.

4. Give students a few moments to consider the next question: How many cubic inches are in *8 cubic feet*? Have students turn and share their thinking and then record a solution in their journal. Calculators might be helpful. Solicit strategies from the class and discuss how the students computed the answer 13,824 in.^{3}

5. Set the stage with some background information. People often buy a fish tank that is too small for even one fish. Encourage students to estimate about how much space a fish would need to survive. A general ratio is one fish per one gallon of water. One gallon of water is approximately cubic feet of water.

6. Display question 2 of the Design a Fish Tank master. As a class, determine how much water is needed for a fish to survive in feet and then in inches, and record the solutions.

7. Ask students to discuss with a partner how many fish could live comfortably in 1 cubic foot of water.

8. Move on to Fish Tank Models. Display a copy, read the scenario and specifications at the top. Have students brainstorm, first in pairs and then as a whole group, what are some possible dimensions of fish tanks that will work with these specifications. Using cubes, grid paper or quick sketches, have the students begin their inquiry.

9. Encourage students to record the models in their journals, including labels for the dimensions. Once 3 different models have been built, select three pairs of students to share their thinking with the class.

10. As you discuss the different tank volumes, pose the following questions:

- What might be a more efficient modeling strategy than using centimeter cubes?

Activity 3 Fish Tank Volume (cont.)

- What formula might apply in this situation?
- What is an equation or expression that matches these models?
- What would happen if we changed the order of the numbers?

Teacher *What might be a more efficient modeling strategy than using centimeter cubes?*

Sammy *We could use boxes that are in the shapes of cubes or draw a picture to represent the units- a 3-D box with labels.*

Teacher *Good, pictures could be a definite solution.*

Reece *Well we have a length, a width, and a height. Could we use those to make the formula of Length x Width x Height?*

Abby *Yes Reece, we could even shorten it to L x W x H.*

Teacher *Which letter is which on the model?*

Abby *The Length is the bottom measurement facing us. The width is how deep the object is. That's the measurement going away from us. And the height is what measurement is pointing towards the top of the paper.*

Teacher *What is the equation or expression that matches these models?*

Sarah *I think we just need to plug in the numbers we are using into the formula to create an equation that represents our model.*

Teacher *Great work everyone! Now what would happen if we changed the position and/or order of the numbers? Would we get the same answer?*

Reece *This is multiplication formula so the order doesn't matter. We would get the same number any way we change the order of the numbers around.*

Abby *I agree with Reece. I think it's the commutative property.*

11. Continue to discuss solution strategies for the three questions below the table as a class.

Extension

Have students measure out the length and width of their model in feet with masking tape on the floor to build a correlation between the model and the actual size of the tank.

INDEPENDENT WORKSHEET

Use Independent Worksheet 3, Tank Volumes (page D2.41) for additional practice with volume.

Design a Fish Tank

Your school wants to install several fish tanks in the office area. Your class has been asked to design the fish tanks and choose which fish to buy.

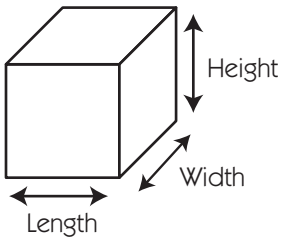
1 One tank the office considered purchasing held a total of 8 cubic feet of water.

a What would be the dimensions of the tank be if the shape was a cube?

b What other dimensions could the tank be if it didn't have to be a cube?

c How many cubic inches would that tank hold?

2 The table that follows represents the dimensions and area of a fish tank for just one fish (without rocks and plants). If each fish needs 1 cubic unit with the dimensions of $\frac{1}{2}$ of a foot for the length, width and height of water to live in, what is the total volume in feet? Inches?



Number of Fish	Volume of Water	Total Volume
1	$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$	_____ ft.^{3}
1	____ × ____ × ____ ×	_____ in.^{3}

3 If one small fish needs $\frac{1}{8}$ cubic feet of water to live in, what is the maximum number of small fish that would fit in 1 cubic foot of water? How do you know?

Fish Tank Models

1 Your classmates will be presenting several fish tank models to the office staff. The staff will vote on which tank sizes would work the best. Your job is to create 3 models that meet these specifications:

- The volume of the actual fish tank can be no larger than 16 cubic feet.
- The volume of the actual fish tank can be no smaller than 4 cubic feet.
- The minimum base size of the tank can be no smaller than 2 square feet.

For your design, each cm cube is equal to 1 foot cubed ($1\ cm^3 = 1\ ft.^3$)

Tank	Length (in cm)	Width (in cm)	Height (in cm)	Volume of the model (in cm^3)	Actual volume of the tank (in $ft.^3$)
1					
2					
3					

2 Answer the following questions based on the information above.

a Which tank model has the largest base? _____
Explain.

b Which tank model has the smallest base? _____
Explain.

c If the office selects the largest tank volume, how many fish can that tank hold? (Remember each fish needs $\frac{1}{8}$ cubic feet of water to live in).

NAME _____ **DATE** _____

Journal Page Grid

Set D2 ★ Activity 4

ACTIVITY

Number of Fish

Overview

The saga of the fish tanks in the office continues today, as students determine the number of fish that will live comfortably in a variety of different fish tanks. Students will select fish of different lengths and create a line plot to show their data. They will use this data to determine the mean, median, range and mode of their data set in fractional units. The second half of the session provides additional practices with a second scenario.

You'll need

- ★ Number of Fish (page D2.21, run 1 for display)
- ★ Length of Fish, Line Plot Data Analysis (page D2.22, run a class set, plus 1 for display)
- ★ Second Tank Needs Fish (page D2.23 and D2.24, run a class set, plus 1 for display)
- ★ Word Resource cards for range, median, mode, mean, line plot and x-axis (optional)
- ★ Colored Tiles (class set)

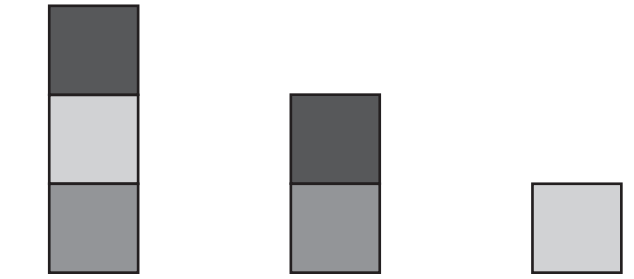
Skills & Concepts

- ★ Solve word problems involving addition and subtraction of fractions using visual fraction models or equations to represent the problem.
- ★ Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.
- ★ Solve word problems involving division of whole numbers using visual fraction models or equations to represent the problem
- ★ Solve real world problems involving multiplication of fractions and mixed numbers, using visual fraction models or equations to represent the problem.
- ★ Convert among different-sized standard measurement units within a given measurement system
- ★ Make a line plot to display a data set of measurements in fractions of a unit (½, ¼, ⅛).
- ★ Use operations on fractions for this grade to solve problems involving information presented in line plots

Instructions for Number of Fish

1. Open today's session by displaying the Number of Fish master. Read the top portion together and invite the students to work with a partner to select 8 fish from the collection. Select 8 children to share their choices, ensuring that there are at least 3 different types of fish represented in the collection, then record the final choices on the teacher master.

2. Today, each of the colored tiles will represent 1/4 of a foot. Students will work in teams to build a model for all 8 of the fish lengths chosen by the class, representing the actual fish length in tile.



Activity 4 Number of Fish (cont.)

3. Reveal the Length of Fish, Line Plot Data Analysis page and work together to represent the length of 8 fish on a line plot. Title the line plot and label the x-axis the appropriate half, quarter or whole unit. You may need to review the definition of an x-axis.

4. Determine the median, range and mode of the fish lengths. To efficiently find a solution, the fish lengths should be displayed from least to greatest. Use the Word Resource cards to review the terms as needed.

5. Challenge students to find the total length of all 8 fish combined. Solicit several possible strategies and encourage each team to write an equation to match their computation.

6. Ask student pairs to find the average or mean length of the fish using the tile or a computation strategy. Invite several volunteers with varying ideas to share their work with the class.

> ***Kale*** *We used the tile to make equal columns.*
>
> ***Sandra*** *I found the average by adding up all of the fourths and dividing by 4. My answer is* $^{16}/_{4}$ *which equals 4 and then I divided by 4 to get 1 foot.*
>
> ***Jessy*** *You are partially correct Sandra. You do need to add all of the fraction but then you need to divide by how many fish we had, not the four lengths. I would have used the 4 and divided by 8 to get* $^{1}/_{2}$ *of a foot. We had 8 fish.*

7. Introduce the second scenario, Second Tank Needs Fish and follow the same teaching sequence. Allow students to choose to work alone, in pairs or with you. You may choose to reteach in a small group with the tile models while other students work more independently.

Extension

Challenge problems are provided.

INDEPENDENT WORKSHEET

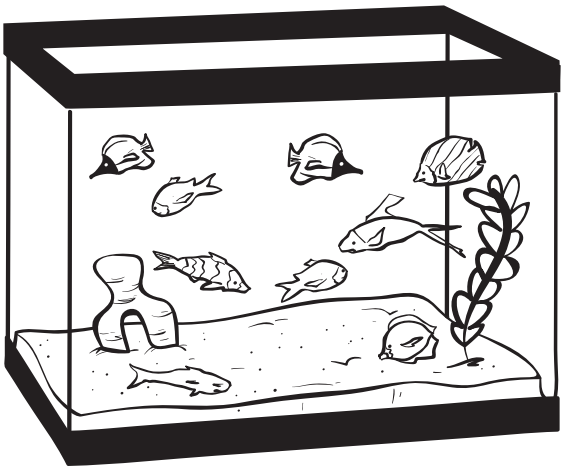
Use Independent Worksheet 4, Third Tank Needs Fish (page D2.43) for additional practice with line plots.

Number of Fish

The school secretary decided that she wants *8 fish* in the largest fish tank model you built yesterday. These fish are going to need more water than a normal fish due to their sizes. The secretary would like to see a minimum of *3 different* types of fish in this tank. Work with a partner to select 8 fish. Be ready to defend your choices.

Tank Number One

Fish Type	Length	Number of Fish
Suckermouth Catfish	$\frac{4}{4}$ ft.	
Red Cap Oranda Goldfish	$\frac{3}{4}$ ft.	
Silver Dollar	$\frac{1}{2}$ ft.	
Angelfish	$\frac{1}{2}$ ft.	
Tiger Barb	$\frac{1}{4}$ ft.	
Congo Tetra	$\frac{1}{4}$ ft.	



NAME _____ **DATE** _____

Length of Fish, Line Plot Data Analysis

1 Create a line plot to represent the length of the fish chosen by the class. Be sure to label the x-axis and title the line plot.

Title _____

_____

_____ _____ _____ _____ _____

_____

(label unit)

2 Find the median fish length.

3 Find the range of the fish lengths.

4 What is the mode of the fish lengths?

5 What is the total length of all of the fish combined?

6 What is the average (mean) length of these fish?

NAME _____ **DATE** _____

Second Tank Needs Fish

The school secretary has chosen the following combination of 8 fish for the second fish tank in the office.

Fish Type	Length	Number of Fish
Suckermouth Catfish	$\frac{4}{4}$ ft.	2
Red Cap Oranda Goldfish	$\frac{3}{4}$ ft.	1
Silver Dollar	$\frac{1}{2}$ ft.	2
Angelfish	$\frac{1}{2}$ ft.	2
Tiger Barb	$\frac{1}{4}$ ft.	0
Congo Tetra	$\frac{1}{4}$ ft.	1

1 With a partner, create a line plot showing the fish lengths. Be sure to title the line plot and label the appropriate units.

Title _____

0 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$ $\frac{4}{4}$

_____

(label unit)

(Continued on next page.)

NAME _____ **DATE** _____

Second Tank Needs Fish (cont.)

2 Find the median fish length.

3 Find the range of the fish lengths.

4 Find the mode of the fish lengths.

5 What is the average (mean) length of the fish in feet?

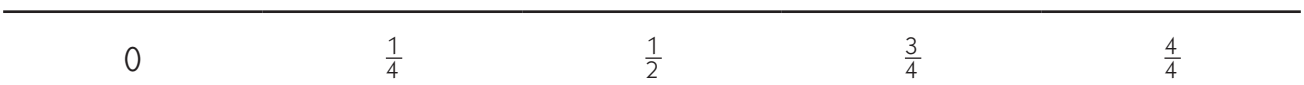
CHALLENGE

6 What is the difference in the fish lengths? Compare the median, range, mode and mean.

7 What is the total length of all of the fish combined in feet?

8 Create a new set of data, different from the second fish tank, which has the same mean and median as the second fish tank. Record the data below on the line plot and give this data a title and appropriate unit label.

Title _____



_____ (label unit)

Set D2 ★ Activity 5

ACTIVITY

Fish Tank Plants with a Line Plot

Overview

Students interpret and analyze line plot data to determine the median, range, mode, and mean, in fractional units, with unlike denominators. They also discover what happens with a "0" value. Students also create a visual fraction model from paper strips to display a set of measurements.

You'll need

- ★ Fish Tank Plants (pages D2.28 and D2.29, run a class set, plus 1 for display)
- ★ Plants for the First Fish Tank (page D2.30, run 1 copy for display)
- ★ Student journals or Journal Grid Page (page D2.31, run as needed, optional)
- ★ Class set of 1 ½" × 12" construction paper strips in any color

Advance Preparations Cut a class set of 1 ½" × 12" long strips of paper (any color)

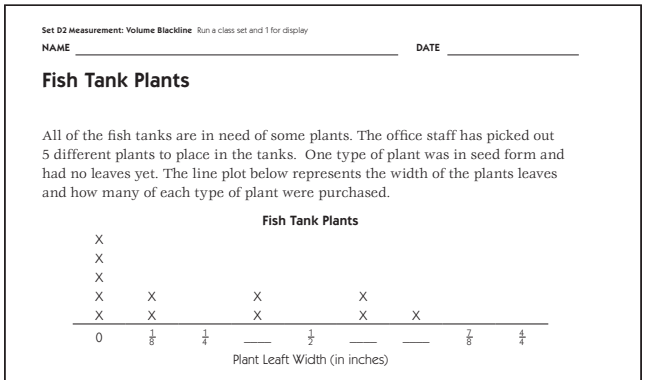
Skills & Concepts

- ★ Solve word problems involving addition and subtraction of fractions using visual fraction models or equations to represent the problem.
- ★ Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.
- ★ Solve word problems involving division of whole numbers using visual fraction models or equations to represent the problem
- ★ Solve real world problems involving multiplication of fractions and mixed numbers, using visual fraction models or equations to represent the problem.
- ★ Convert among different-sized standard measurement units within a given measurement system
- ★ Make a line plot to display a data set of measurements in fractions of a unit (½, ¼, ⅛).
- ★ Use operations on fractions for this grade to solve problems involving information presented in line plots

Instructions for Fish Tank Plants

1. Display the Fish Tank Plants master and have students read the scenario. Ask the class to tell a partner what information is given in the problem. What information is missing?

2. Discuss with the class what each symbol represents within the line plot display. Students might notice the "X" symbol represents the seeds that have not grown yet. If they don't make this connection yet, don't be concerned. Question 3 will ask the class to clarify the value of 0.



Set D2 Measurement: Volume Blackline Run a class set and 1 for display

NAME _____ DATE _____

Fish Tank Plants

All of the fish tanks are in need of some plants. The office staff has picked out 5 different plants to place in the tanks. One type of plant was in seed form and had no leaves yet. The line plot below represents the width of the plants leaves and how many of each type of plant were purchased.

Fish Tank Plants

Activity 5 Fish Tank Plants with a Line Plot (cont.)

3. Students might notice that the fractions have unlike denominators. Invite them to consider how this is different than the line plots they have previously seen. Have students write an equivalent fraction for the denominators below the fourths and halves, and then fill in the missing values on their record sheet.

4. Work together as a class to solve questions 1–6 with the Plant Leaf Width line plot. For question 7, provide some additional time for students to record an answer. Then invite volunteers to come up and share their work.

> ***Jason*** *I found the total leaf widths by adding all of the fractions*
>
> $$\frac{1}{8} + \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{5}{8} + \frac{5}{8} + \frac{3}{4} = \frac{21}{12}$$
>
> ***Bryer*** *I respectfully disagree Jason. The denominators need to be the same before adding all of these fractions. Your problem should have said*
>
> $$\frac{1}{8} + \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{5}{8} + \frac{5}{8} + \frac{6}{8} = \frac{24}{8}$$
>
> ***Sammy*** *I agree with you Bryer, but I knew I could reduce* $\frac{24}{8} = 3$
>
> ***Tracy*** *Bryer, what about adding the 0s to this equation? I thought that you need to show them.*
>
> ***Bryer*** *I didn't add the 0s because 5 zeros added still equals 0 and that would not change the equation.*
>
> ***Teacher*** *What would an equation or expression look like if it matched the line plot? Do the zeros need to be added?*
>
> ***Sammy*** *I think the 5 zeros should be part of the equation. Without them how would we show that there are 5 additional plants on this line plot?*

5. Encourage students to also record an equation to show their thinking.

6. Question 9 asks the student to find the average weight of the plants and explain their thinking. One pound divided by twelve plants would be $\frac{1}{12}$ of a pound.

7. Display the Plants for the First Fish Tank master while students access their journals. Read the top half of the directions as a class. Ask the class to turn to a partner and restate what they are being asked to do. Next, have students get into groups of 4–8 students to share the work for this task. You may want to consider grouping your students strategically.

8. Distribute the 12" long construction paper strips and ask students to construct a model for eighths. Students can achieve this by either folding the strips into eighths (half, half and half again) or by dividing 8 into 12 and measuring out the strip into 1 $\frac{1}{2}$ inch sections. Students will use this visual model to demonstrate an understanding of each question.

9. Circulate while students work in teams, to observe and provide clarification. Let students know they can cut the strips and re-arrange them to find the mean (average).

10. When the majority of students are done, share the models and strategies students used. Invite students to ask questions about this work, agreeing or disagreeing respectfully about the written and oral explanations. A great way to do this is to compare two different explanations. What is the same and

Activity 5 Fish Tank Plants with a Line Plot (cont.)

what is different about the work? What information would we use to prove a correct solution for finding the mean?

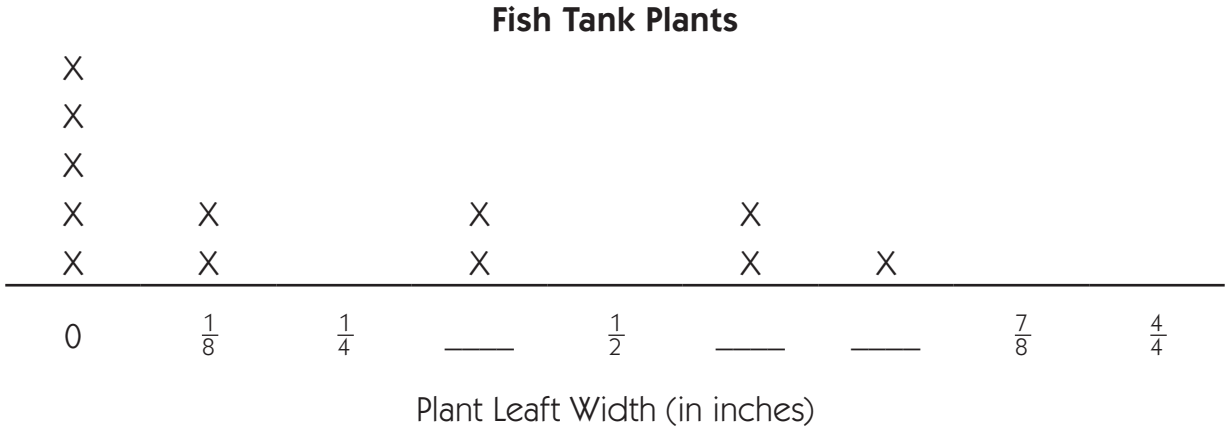
INDEPENDENT WORKSHEET

Assign Independent Worksheet 5, Plants for the Second Fish Tank (page D2.45) for more practice with line plots.

NAME _____ **DATE** _____

Fish Tank Plants

All of the fish tanks are in need of some plants. The office staff has picked out 5 different plants to place in the tanks. One type of plant was in seed form and had no leaves yet. The line plot below represents the width of the plants leaves and how many of each type of plant were purchased.



Use the information in the line plot to answer the questions below.

1 How many plants in all did the office purchase?

2 How many plants were purchased that measured $\frac{3}{8}$ of an inch in width?

3 How many plants were purchased in seed form?

4 What is the median plant leaf width?

5 Find the mode of the plant leaf widths.

(Continued on next page.)

NAME _____ **DATE** _____

Fish Tank Plants (cont.)

6 What is the difference between the widest leaf and the narrowest leaf?

7 What is the total width of the combined leaves?

8 What is the average (mean) width of the plants?

9 The secretary weighed the plants and their total weight was 1 pound.

a What would the average weight of each plant be? (Not including the seeds.)

b Explain your answer.

NAME _____ **DATE** _____

Plants for the First Fish Tank

When the tank needs a cleaning every two weeks, the school secretary would like 8 of the plants to be trimmed to the following lengths (fractions of a foot).

$\frac{4}{8}$	$\frac{7}{8}$	$\frac{6}{8}$	$\frac{5}{8}$	$\frac{8}{8}$	$\frac{6}{8}$	$\frac{4}{8}$	$\frac{8}{8}$

Directions

- Each team member is in charge of cutting 1or 2 paper strips to match the lengths above. Use the strips to create a line plot, including a title, labeled axis and units.
- Then answer the questions in your journal. Use your pieces to prove your thinking.

Questions

1 How many plants needed to be trimmed to $\frac{3}{4}$ of a foot?

2 What is the mode of this data set?

3 What is the median of the plant lengths?

4 Find the range of the data.

5 What is the total length of the plants combined?

6 What is the average (mean) length of the plants?

NAME _____ **DATE** _____

Journal Page Grid

Set D2 ★ Activity 6

ACTIVITY

Water for the Fish Tank Assessment

Overview

Students work on a post-assessment to demonstrate proficiency with fraction computation, line plots and the mean of a data set. This assessment can be given along with the Unit Three Post Assessment.

Skills & Concepts

★ Solve word problems involving addition and subtraction of fractions using visual fraction models or equations to represent the problem.

★ Use operations on fractions for this grade to solve problems involving information presented in line plots

★ Interpret and analyze line plot data to determine the mean, in fractional units

You'll Need

★ Water for the Fish Tank (page D2.34, run a class set)

Instructions for the Water for the Fish Tank Assessment

1. Give each student a copy of the Water for the Fish Tank Assessment and then read and review the tasks with the class.

2. Before students start to work, be sure they understand that they have to use numbers, labeled models, and/or words to show their work and/or explain their thinking; the answers alone will not be adequate.

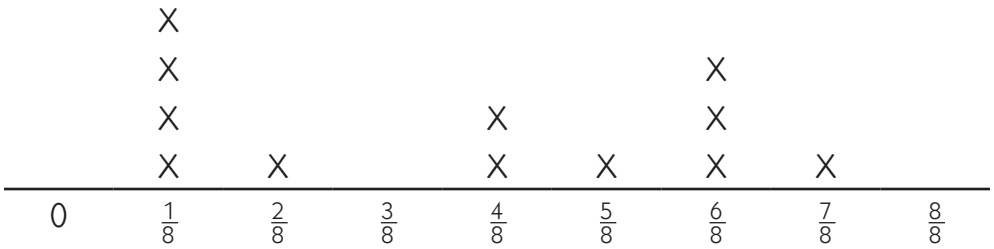
3. Remind students that you are available to re-read any of the directions or problems for them while they work.

NAME _____ **DATE** _____

Water for the Fish Tank

A group of students were asked to fill a 20-gallon classroom fish tank with water. Each student was given a gallon container to fill. The line plot below shows the amount of water each student used to fill their container. Solve the following problems using the data.

Water in the Student Containers



Volume of Water in the Student Containers (in gallons)

1 How much more water would it take to fill the 20 gallon classroom fish tank?

2 If the water in the student gallon containers were redistributed equally, how much water would be in each container?

3 Two students arrive late to school and would like to contribute water to the 20 gallon tank. Each student contributed $\frac{3}{4}$ of a gallon of water to the tank. What would be the new average (mean) contributed by the class? Explain.

NAME _____ **DATE** _____

Set D2 ★ Independent Worksheet 1

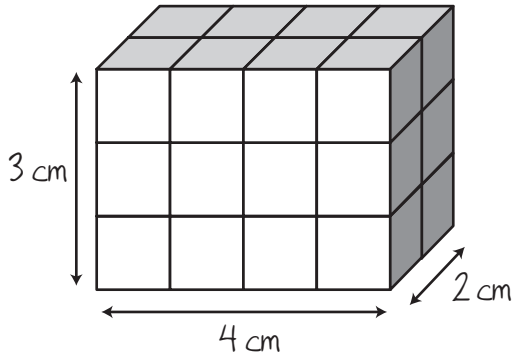
INDEPENDENT WORKSHEET

Volume Review

Volume is the measure of the space occupied by a 3-dimensional object. Volume is measured in cubes of a given size, such as cubic centimeters, cubic inches and cubic feet.

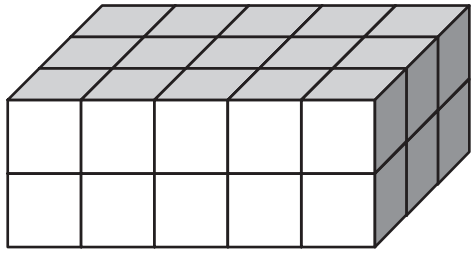
1 Each of the rectangular solids below was built with centimeter cubes. Label each with its dimensions (length, width, and height) and find the volume. Show your work.

example



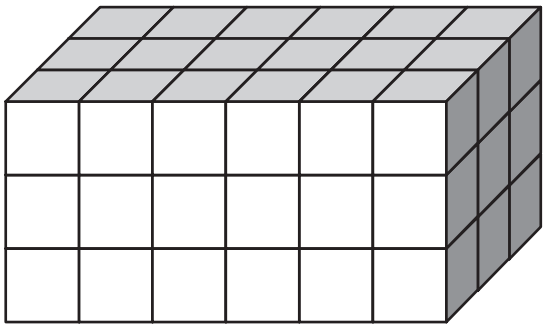
Volume $4 \times 2 \times 3 = 24$ cubic cm (or cm^3)

a



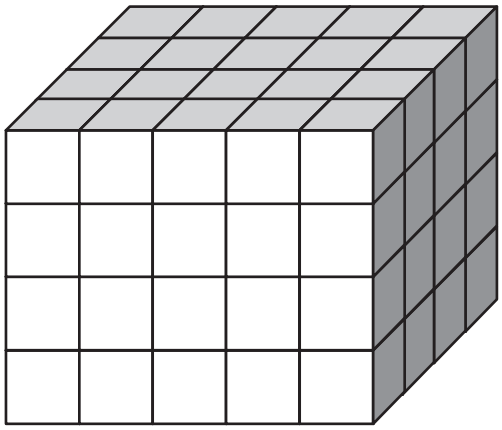
Volume

b



Volume

c



Volume

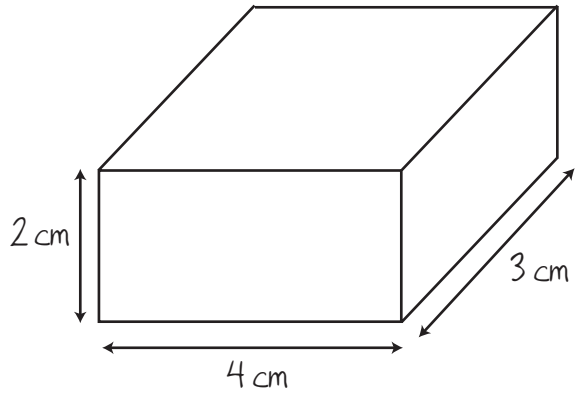
(Continued on next page.)

NAME _____ DATE _____

Independent Worksheet 1 Volume Review (cont.)

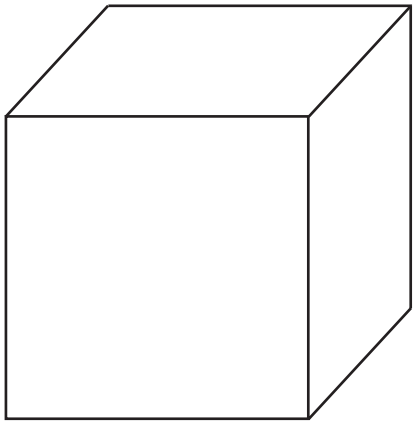
2 Use the centimeter side of your ruler to measure the dimensions of each rectangular solid below. Then find its volume. Show your work.

example



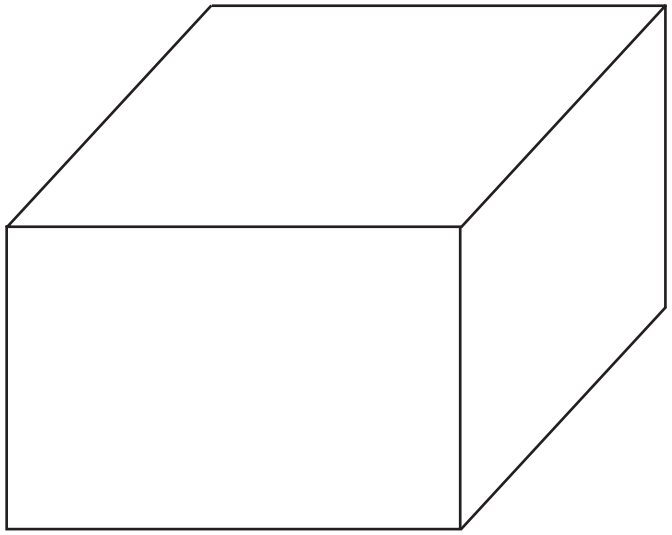
Volume $4 \times 3 \times 2 = 24$ cubic cm (or cm^3)

a



Volume

b



Volume

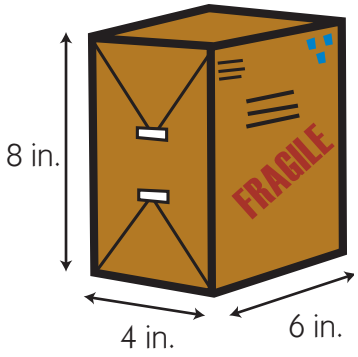
(Continued on next page.)

NAME _____ **DATE** _____

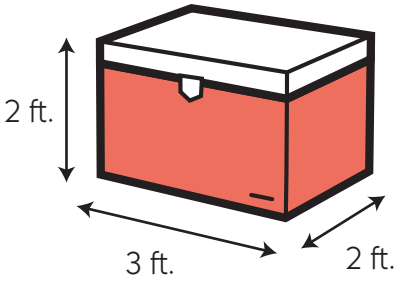
Independent Worksheet 1 Volume Review (cont.)

3 Miguel says you only need to measure one edge of a cube to find its volume. Do you agree with him? Why or why not? Use numbers, labeled sketches, and words to explain your answer.

4 Mia has already measured the dimensions of this packing box. Help her find the volume. Show your work.

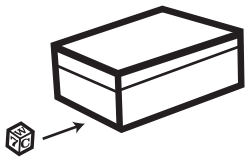


5 Brandon is going on a fishing trip with his family. He wants to find the volume of the family's ice chest. Which expression should he use?



- ○ 2×3
- ○ $3 \times 2 \times 2$
- ○ $3 + 2 + 2$
- ○ $(3 \times 2) - 2$

6 Jeff's little brother is trying to find out how many alphabet blocks will fit into a shoebox. He is measuring:



- ○ the volume of the shoebox
- ○ the area of the shoebox
- ○ the length of the shoebox

(Continued on next page.)

NAME _____ DATE _____

Independent Worksheet 1 Volume Review (cont.)

7 Which of these situations is about volume?

○ determining the amount of fencing it takes to go around a square garden

○ determining how many tiles it will take to cover the kitchen floor

○ determining how many rectangular containers of food will fit into a freezer

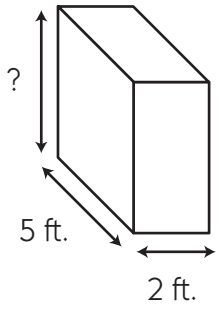
8 Vanesa wants to find the volume of her lunchbox. Which of these units should she use?

○ cubic feet

○ cubic inches

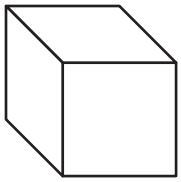
○ cubic yards

9 The volume of this rectangular solid is 40 cubic feet. What is its height? Show your work.



CHALLENGE

10 The volume of this cube is 125 cubic inches. What is the length of each edge? Show your work.



NAME _____ **DATE** _____

Set D2 ★ Independent Worksheet 2

INDEPENDENT WORKSHEET

The Camping Trip

The Gomez family is going on a camping trip next week. There are 4 people in the family: Mr. and Mrs. Gomez and the 11-year-old twins, Ramon and Dora. Help them do some planning for their trip. Circle a correct answer to each question below.

1 Mrs. Gomez wants to cut a piece of rope that's long enough to dry the family's laundry on every day. Which of these units should she use to measure the rope?

inches feet miles

2 Mr. Gomez wants to figure out how far they'll have to drive to get to the campsite. He already knows that it will take about a day to get there. Which of these units should he use?

inches feet yards miles

3 The shoelaces on Ramon's tennis shoes are almost worn out. He has to measure them so he gets the right length at the store. Which of these units should he use?

millimeters centimeters meters kilometers

4 Mrs. Gomez says it's going to be a 3-minute walk from their tent to the lake. Dora wants to measure the distance when they get there. Which of these units should she use?

millimeters centimeters meters kilometers

5 Ramon wants to find the area of his sleeping bag to see how much room he'll have in the family's tent. Which of these units should he use?

square inches square feet square miles

(Continued on next page.)

NAME _____ DATE _____

Independent Worksheet 2 The Camping Trip (cont.)

6 Which formula should Ramon use to find the area of his sleeping bag?

Area = Length + Width Area = Length × Width Area = Length ÷ Width

7 Dora says when they get there, she's going to measure the area of their campsite. Mrs. Gomez says the campsite is big enough for their car, their tent, their picnic table and chairs, and their campfire, with a little room left over. Which of these units should she use?

square inches square yards square miles

8 Which formula should Dora use to find the area of the campsite?

$A = (2 \times l) + (2 \times w)$ $A = (3 \times l) - (2 \times w)$ $A = l \times w$

9 Mr. Gomez wants to find the volume of the family car trunk so he'll know how much luggage will fit back there. Which of these units should he use?

cubic inches cubic feet cubic yards

10 Ramon wants to measure the volume of a shoebox to find out how many CD's he can fit into it for the trip. Which of these units should he use?

cubic inches cubic feet cubic yards

11 Dora is going to collect tiny pebbles at the lake. She wants to measure the volume of a metal band-aid box to keep them in. Which of these units should she use?

cubic centimeters cubic meters cubic kilometers

NAME _____ **DATE** _____

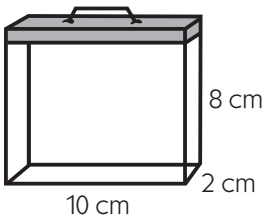
Set D2 ★ Independent Worksheet 3

INDEPENDENT WORKSHEET

Tank Volume

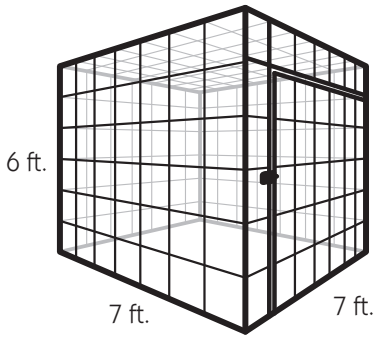
1 At a pet store, the volume of a tank or kennel depends on the size of the pet. Use cubes, sketches and equations to solve for the following questions.

a What is the volume (including units) of the ant farm, if each cubic unit is one centimeter cubed?



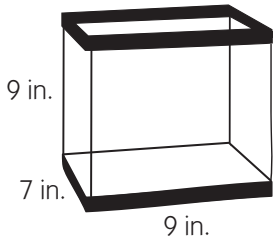
Ant Farm Volume =

b What is the volume (including units) of the dog kennel, if each cubic unit is one foot cubed?



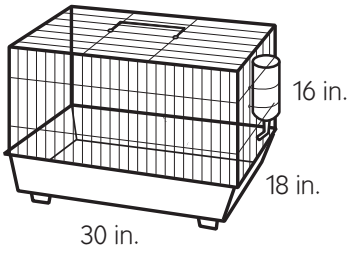
Dog Kennel Volume =

c What is the volume (including units) of the scorpion tank, if each cubic unit is one inch cubed?



Scorpion Tank Volume =

c What is the volume (including units) of the Guinea Pig cage, if each cubic unit is one inch cubed?



Guinea Pig cage Volume =

(Continued on next page.)

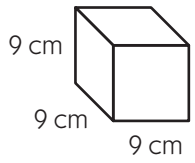
NAME _____ **DATE** _____

Independent Worksheet 3 Tank Volumes (cont.)

Pet Food Volumes

2 The pet store sells pet food and snacks in four sizes. Determine the volume of each container and write an equation to show your thinking.

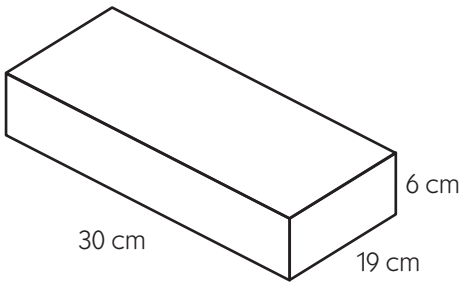
a



Volume _____

Equation

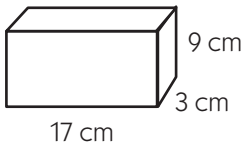
b



Volume _____

Equation

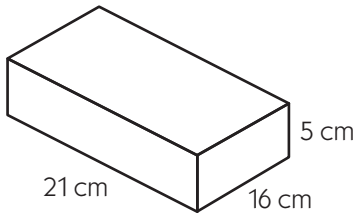
c



Volume _____

Equation

d



Volume _____

Equation

3 Answer the following questions using the infromation above.

a Which figure has a volume less than the volume of figure A?

b What are the volumes of figure A and figure B combined?

e List the figures in order from least to greatest.

_____ _____ _____ _____

NAME _____ **DATE** _____

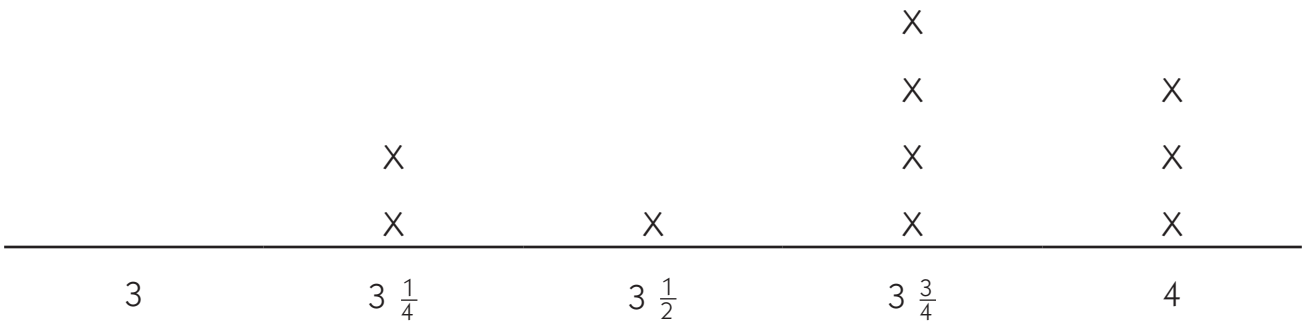
Set D2 ★ Independent Worksheet 4

INDEPENDENT WORKSHEET

Third Tank Needs Fish

Your principal went to the fish store and purchased different fish for the office fish tanks. The prices are shown on the line plot below. Solve the following problems.

Cost of Fish



1 How many fish did the principal purchase? _____

2 What was the difference in cost between the most expensive and least expensive fish?

3 Which fish price cost was the mode?

4 What was the total amount of money spent on the fish? _____

5 What was the average (mean) cost the principal spent on a fish?

NAME _____ **DATE** _____

Set D2 ★ Independent Worksheet 5

INDEPENDENT WORKSHEET

Plants for the Second Fish Tank

The principal went to the pet store and purchased these plants at the prices shown below. These plant prices are displayed in fractions of a dollar. The store owner donated a few extra plants to the school. These donated plants cost the school nothing.

$\frac{1}{2}$	0	$\frac{1}{4}$	$\frac{4}{4}$	0	0	0	$\frac{1}{4}$	0

1 Use the data to create a line plot that has a proper title (including units), the axis labeled correctly, and the data plotted accurately.

Cost of Plants

_____

_____ _____ _____ _____ _____

_____ (in _____)

2 Use information on the line plot to help answer the following questions.

a How many total plants did the principal walk out of the pet store with?

b Not counting the free plants, which plant price was the mode?

(Continued on next page.)

NAME _____ DATE _____

Independent Worksheet 5 Plants for the Second Fish Tank (cont.)

c What was the difference in cost between the most expensive and least expensive plant in the tank, not counting the free plants?

d What was the total amount of money spent on the plants?

e What was the average (mean) cost the principal spent on the plants?

f If the bag of plants the principal purchased weighed 2 pounds, what would the average weight of each plant be? Explain your answer.