



GRADE 5 SUPPLEMENT

Set E2 Data Analysis: Fundamental Counting Principle

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Skills & Concepts

- ★ use the fundamental counting principle on sets with up to 4 items to determine the number of possible combinations
- ★ determine the probability of events occurring in familiar contexts or experiments, and express probability as fractions from zero to one

Bridges in Mathematics Grade 5 Supplement

Set E2 Data Analysis: Fundamental Counting Principle

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Prepared for publication on Macintosh Desktop Publishing system.

Printed in the United States of America.

P201304

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Bridges in Mathematics is a standards-based K–5 curriculum that provides a unique blend of concept development and skills practice in the context of problem solving. It incorporates the Number Corner, a collection of daily skill-building activities for students.

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Set E2 ★ Activity 1



ACTIVITY

Counting the Possible Outcomes

Overview

In this activity, students use charts and tree diagrams to show the possible outcomes of probability experiments. The teacher then guides the class to understand and apply the fundamental counting principle, which states that the total number of outcomes is equal to the number of possibilities in a set of choices multiplied by the number of possibilities in each other set of choices.

Skills & Concepts

- ★ use the fundamental counting principle on sets with up to 4 items to determine the number of possible combinations
- ★ determine the probability of events occurring in familiar contexts or experiments, and express probability as fractions from zero to one

You'll need

- ★ Counting the Possible Outcomes (pages E2.6–E2.7, run 1 copy of each sheet on a transparency)
- ★ Amber's Experiment (page E2.8, run a class set)
- ★ Student Math Journals
- ★ a piece of paper to mask portions of the overhead
- ★ a paper lunch sack
- ★ 4 tile, one each in green, red, yellow, and blue
- ★ a penny

Instructions for Counting the Possible Outcomes

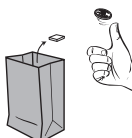
1. Ask students to get out their math journals and let them know that you are going to investigate some more probability experiments today. Then show just the top section of Counting the Possible Outcomes, page 1, at the overhead.

Set E2 Data Analysis: Fundamental Counting Principle Blackline Run 1 copy on a transparency

Counting the Possible Outcomes page 1 of 2

Rafael put 4 tile in a bag, one green, one red, one yellow, and one blue. Then he shook the bag to mix the tile.

If he flips a penny and pulls 1 tile out of the bag without looking, what is the probability that the penny will land on heads and the tile he pulls out will be green?



2. Read the text with the class. Model the experimental set-up by placing 4 tile, one in each color, into a paper lunch sack. Close the sack and shake it to mix the tile. Show students the penny and repeat the question: what is the probability that if you flip the penny and pull 1 tile from the sack without looking, the penny will come up heads and you will get a green tile? Ask them to think about this question privately and write a response in their journals that includes both an answer and an explanation.

3. After a few minutes, have students pair-share their responses. Then ask volunteers to share their thinking with the class.

Activity 1 Counting the Possible Outcomes (cont.)

Students *I said the probability of getting heads and green is unlikely because it seems like there are a lot of things that can happen, like you could get heads and red, or tails and blue.*

I said he has a 1 out of 6 chance of getting heads and green because the penny has 2 sides and there are 4 different colors of tile in the bag. Two plus four is 6.

I said maybe 8 because you can get heads with 4 different colors or tails with 4 different colors.

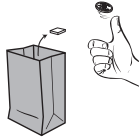
4. Reveal the next question on the overhead and discuss it with the class. What would they have to do to determine the probability of getting heads and a green tile on one try? Chances are, some students will suggest that you try it and find out. Challenge them to think of something besides actually conducting the experiment themselves. After some discussion, reveal the lower half of the overhead. Read it with the class, and work with their input to fill in the chart at the bottom of the sheet.

Set E2 Data Analysis: Fundamental Counting Principle Blackline Run 1 copy on a transparency

Counting the Possible Outcomes page 1 of 2

Rafael put 4 tile in a bag, one green, one red, one yellow, and one blue. Then he shook the bag to mix the tile.

If he flips a penny and pulls 1 tile out of the bag without looking, what is the probability that the penny will land on heads and the tile he pulls out will be green?





Unlikely because there are lots of possibilities.
 1 out of 6 chance because $2 + 4 = 6$
 $1/8$ because $2 \times 4 = 8$ possibilities.

What do you have to do to find out?
 Try it ourselves.
 Figure out what all the possibilities are.
 Make a list to see all the things that could happen.

To determine probability, you need to know all the different things that can happen. A list of all the possible outcomes is called a *sample space*.

You can make a sample space for a probability experiment by thinking of all the possibilities and writing them down. Here are two other methods:

1 Make a chart.

	Green	Red	Yellow	Blue
Heads 	HG	HR	HY	HB
Tails 	TG	TR	TY	TB

5. When the chart is complete, ask students to describe what it tells them about Rafael's experiment.

Students *It shows all the things that can happen.*

There are 8 different possibilities.

It says he has a 1 in 8 chance of getting green and heads on one try because there are 8 different combinations.

Students *If you know how many different things can happen, you can tell what the probability is of getting something. I was right. There are 8 possibilities because you can get a head with 4 different colors, or a tail with 4 different colors.*

Activity 1 Counting the Possible Outcomes (cont.)

Teacher How would you express the probability as a fraction?



Ramani There are 8 possibilities, so getting heads and green is 1 out of 8. That's $\frac{1}{8}$.

6. Now show the top section of the second overhead. Explain that people sometimes use a tree diagram to create a sample space, or a list of all the possibilities. Give students a few moments to examine the diagram quietly. Work with their input to complete the diagram. Then reveal and discuss the question at the bottom of the sheet.


Set E2 Data Analysis: Fundamental Counting Principle Blackline Run 1 copy on a transparency

Counting the Possible Outcomes page 2 of 2

2 Make a tree diagram.

Coin Face	Tile Color	Outcomes
 heads	Green	Heads & Green
	Blue	Heads & Blue
	Yellow	Heads & Yellow
	Red	Heads & Red
 tails	Green	Tails & Green
	Blue	Tails & Blue
	Yellow	Tails & Yellow
	Red	Tails & Red

3 If Rafael flips a penny and pulls 1 tile out of the bag without looking, what is the probability that the penny will land on tails and the tile he pulls out will be blue? How do you know?



7. Give students each a copy of Amber's Experiment. Read the first question together and give students a few minutes to record their responses privately. Then ask them to pair-share, and have a few volunteers share and explain their predictions to the class.



Activity 1 Counting the Possible Outcomes (cont.)

Set E2 Data Analysis: Fundamental Counting Principle Blackline (Run a class set)

NAME _____ DATE _____

Amber's Experiment

1 Amber is going to flip a penny and roll a die at the same time. The die has the numbers 1, 2, 3, 4, 5, and 6 on it. What is the probability that the penny will land on heads and the die will land on 4? Write your prediction here and explain your thinking.






2 You are going to make 2 kinds of sample spaces for this experiment. Remember, a sample space is a list of all the possible outcomes.

a Think before you start. What are the possible outcomes for the penny flip?

b What are the possible outcomes for the die roll?

c Complete the chart below to show all the possible outcomes of Amber's experiment.

Number Rolled	2			5	
Heads 	H2				
Tails 				T5	

d On the back of this sheet, make a tree diagram to show all the possible outcomes of Amber's experiment.

3 How many possible outcomes are there in this experiment? _____

4 What is the probability that the penny will land on heads and the die will land on 4? Express your answer as a fraction.

8. Review the rest of the tasks on the sheet together. When students understand what to do, give them most of the remaining time to complete the work. Leave Counting the Possible Outcomes, page 2 on display at the overhead so students can see an example of a tree diagram. Circulate as they are working and give assistance as needed.

9. Toward the end of the period, or at the start of the next, reconvene the class to discuss the work they have completed. Here are some questions to pose:

- How many possible outcomes did you find for Amber's experiment? (12)
- What is the probability that she will get heads on the penny and a 4 on the die in one try? ($1/12$)
- What is the probability that she will get tails on the penny and a 5 on the die in one try? ($1/12$)
- What is the probability that she will get heads on the penny and an even number on the die? ($3/12$)
- Which do you think is easier and more effective as a sample space, the chart or the tree diagram? Why?
- Is there an easier way to determine the number of possible outcomes than making a list, a chart, or a tree diagram? (Yes. You can multiply the number of possible outcomes for the penny by the number of possible outcomes for the die. That is 2×6 , or 12 possible outcomes. Chances are, some of your students will have noticed this on their own. If not, call it to their attention.)

10. After you have discussed the last question with the class, explain that mathematicians have developed a generalization called the fundamental counting principle. This principle tells us that the total number of outcomes is equal to the number of possibilities in a set of choices multiplied by the number of possibilities in each other set of choices. Then place the first overhead back on display. Does the principle work to tell how many outcomes there are for Rafael's experiment?

Activity 1 Counting the Possible Outcomes (cont.)

Students *Yep! There are 2 ways the penny can land and 4 different colors. $2 \times 4 = 8$. You can see it on the chart, but just multiplying is way easier than making a chart.*

11. Ask students to think about the fundamental counting principle in relation to the Odd Coin Game. Can they use the counting principle to predict how many different outcomes there are for flipping 3 coins at the same time?

Students *There are 2 possibilities for each penny. You can get heads or tails. So that's 2 outcomes for the first penny, 2 for the second, and 2 for the third. It would be $2 \times 2 \times 2 = 8$. That's right! There were 8 different combinations, remember?*



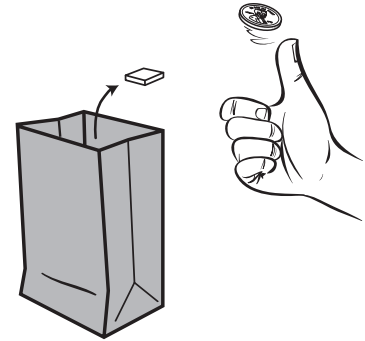
INDEPENDENT WORKSHEET

Use Set E2 Independent Worksheets 1 and 2 to provide students with more practice creating sample spaces and applying the fundamental counting principle.

Counting the Possible Outcomes page 1 of 2

Rafael put 4 tile in a bag, one green, one red, one yellow, and one blue. Then he shook the bag to mix the tile.

If he flips a penny and pulls 1 tile out of the bag without looking, what is the probability that the penny will land on heads and the tile he pulls out will be green?



What do you have to do to find out?

To determine probability, you need to know all the different things that can happen. A list of all the possible outcomes is called a *sample space*.

You can make a sample space for a probability experiment by thinking of all the possibilities and writing them down. Here are two other methods:

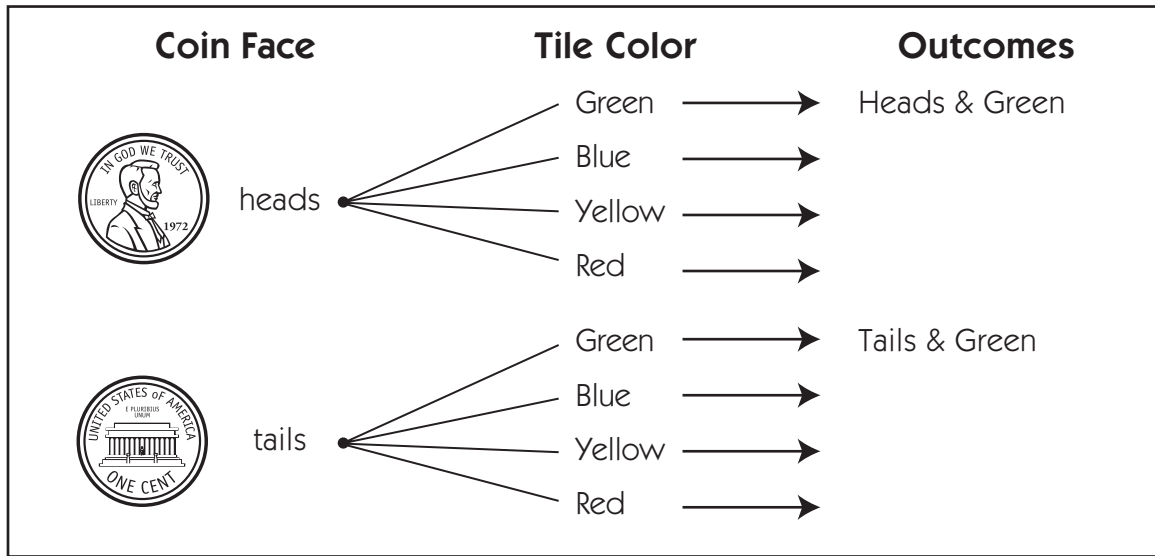
1 Make a chart.



	Green	Red	Yellow	Blue
Heads				
Tails				

Counting the Possible Outcomes page 2 of 2

2 Make a tree diagram.



3 If Rafael flips a penny and pulls 1 tile out of the bag without looking, what is the probability that the penny will land on tails and the tile he pulls out will be blue? How do you know?



NAME _____

DATE _____

Amber's Experiment

1 Amber is going to flip a penny and roll a die at the same time. The die has the numbers 1, 2, 3, 4, 5, and 6 on it. What is the probability that the penny will land on heads and the die will land on 4? Write your prediction here and explain your thinking.



2 You are going to make 2 kinds of sample spaces for this experiment. Remember, a sample space is a list of all the possible outcomes.

a Think before you start. What are the possible outcomes for the penny flip?

b What are the possible outcomes for the die roll?

c Complete the chart below to show all the possible outcomes of Amber's experiment.

Number Rolled

Heads



Tails



	2			5	
	H2				
				T5	

d On the back of this sheet, make a tree diagram to show all the possible outcomes of Amber's experiment.

3 How many possible outcomes are there in this experiment? _____

4 What is the probability that the penny will land on heads and the die will land on 4? Express your answer as a fraction.

NAME _____

DATE _____

Set E2 ★ Independent Worksheet 1



INDEPENDENT WORKSHEET

Charlie's Marbles

Charlie put 4 marbles in a bag. One of the marbles was green, one was yellow, one was blue, and one was red. He shook the bag to mix up the marbles.

Then he put 4 more marbles in another bag. One of the marbles was green, one was yellow, one was blue, and one was red. He shook the bag to mix up the marbles.

When the bags were ready, Charlie said to his friend, Sara, "I am going to pull a marble out of both bags at the same time without looking. What are my chances of getting a red marble out of the first bag and a blue marble out of the second bag?"

Sara said, "I think your chances of getting a red marble out of the first bag and a blue marble out of the second bag are 1 in 16."

1 Do you agree with Sara? Why or why not?

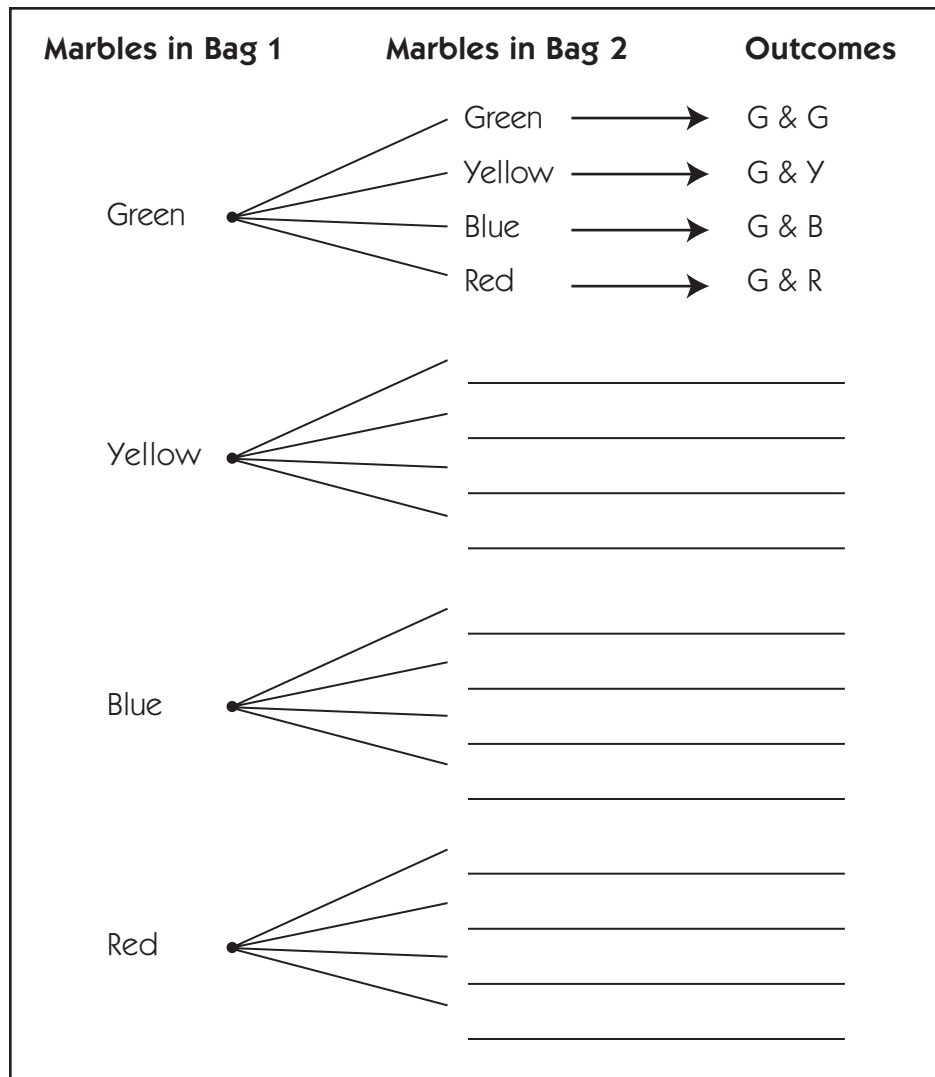
2 Complete the chart below to show all the possible combinations Charlie could get.

		Marbles in Bag 2			
		Green	Yellow	Blue	Red
Marbles in Bag 1	Green	GG			
	Yellow			YB	
	Blue				
	Red		RY		

(Continued on next page.)

Independent Worksheet 1 Charlie's Marbles (cont.)

3 Complete the tree diagram below to show all the possible combinations Charlie could get.



4 What are Charlie's chances of getting a red marble out of the first bag and a blue marble out of the second bag if he pulls one marble out of each bag without looking?



CHALLENGE

5 What are Charlie's chances of getting two marbles the same color if he pulls one marble out of each bag without looking? Explain your answer.

NAME _____

DATE _____

Set E2 ★ Independent Worksheet 2



INDEPENDENT WORKSHEET

Rachel's Outfits

Rachel just got 4 new t-shirts and 3 pairs of pants for summer vacation. One of her new t-shirts is pink. Another is purple. Another is yellow, and the fourth one is turquoise. She got a purple pair of pants, a white pair of pants, and a turquoise pair of pants.

- How many different outfits do you think Rachel will be able to make with her new shirts and pants? Explain your prediction.
- The fundamental counting principle says you can multiply the number of shirts by the number of pants to figure out how many different outfits Rachel can make. Try it here.

$$\underline{\quad} \text{ shirts} \times \underline{\quad} \text{ pairs of pants} = \underline{\quad} \text{ different outfits}$$
- Make a chart or a tree diagram below to show how many outfits Rachel can make. Use a separate piece of paper if you do not have enough room.



- Is there any advantage to making a chart or tree diagram instead of using the fundamental counting principle? Explain your answer.

