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BREAKOUT

The Number Rack

Trends in Student Thinking Data Tool

The MATH LEARNING CENTER

Using the Trends in Student Thinking Data Tool

Eliciting and using evidence of student mathematical thinking is a critical practice for effective mathematics teaching (NCTM, 2014). Drawing out evidence of what students know and can do shouldn't wait until the end of a unit. Rather, watching students work and listening to them explain their ideas provides rich opportunities for supporting both student and teacher learning on a daily basis (NCTM, 2020). When viewed in this way, assessment becomes a process where every interaction is an opportunity to make sense of how students are constructing their understanding and making sense of the mathematics. It is an ongoing learning conversation between teacher and student, rather than a singular assessment event.

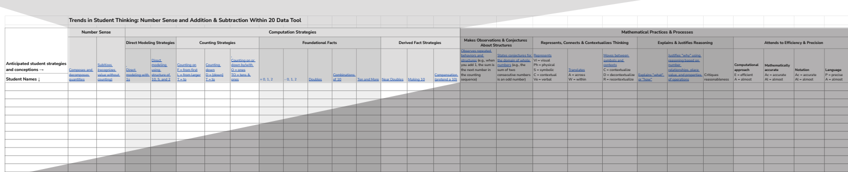
The *Trends in Student Thinking: Number Sense and Addition & Subtraction Within 20 Data Tool* is designed to support the systematic documentation and analysis of the thinking that students demonstrate. The tool can be used to identify, record, and analyze a range of concepts, practices, and processes central to computing within 20. It helps teachers identify, for a particular fact, where students are along a developmental continuum from direct modeling, to counting, to using known facts to derive other facts. Gathering and documenting evidence of student mathematical thinking in this way can inform decisions about next instructional steps in terms of interactions with the mathematics and with others (i.e., teacher, students, or both).

Background

Embedded in the *Trends in Student Thinking: Number Sense and Addition & Subtraction Within 20 Data Tool* is a continuum of mathematical knowledge and skills needed to develop fact fluency with meaning. Also included is the set of mathematical practices and processes that convey how students engage as doers, knowers, and sensemakers in mathematics. The first section of the tool builds from foundational number sense (i.e., composing, decomposing, subitizing) to direct modeling strategies and from counting strategies to foundational facts, all in support of students working from known facts to efficiently and confidently derive facts. The next section of the tool then identifies the practices and processes in which students will engage, beginning from students observing and conjecturing to representing and making connections across and within various representations, including story contexts. This section also includes levels of students' mathematical discourse, as well as attention to efficiency and precision with both solutions and language. Specific explanations and samples of student thinking are included on the "Articulation and Examples" sheet (second tab in the Trends Tool) along with strategy-specific assessing and advancing questions that may be useful when interacting with students.

Capturing results of the analysis of student thinking on the tool can be accomplished with a combination of tallies and codes. At times it is also helpful to include additional details about a students' thinking; for example, it may be helpful when a student uses a near doubles strategy to solve 6 + 7 to notate which double they used, 6 + 6 or 7 + 7. Trends can be examined for an individual student or across a group of students over time to determine the extent to which students are able to flexibly and appropriately select a computational strategy.

Number Sense		Computation Strategies												
		Direct Modeling Strategies		Counting Strategies			Foundational Facts					Derived Fact Strategies		
Composes and decomposes quantities	Subitizes (recognizes value without counting)	Direct modeling with 1s	Direct modeling using structure of 10, 5, and 2	Counting on F = from first L = from larger T = to	Counting down D = [down] T = to	Counting on or down by/with O = ones TO = tens & ones	+ 0, 1, 2	- 0, 1, 2	Doubles	Combinations of 10	Ten and More	Near Doubles	Making 10	Compensation (pretend a 10)



Mathematical Practices & Processes											
Makes Observations & Conjectures About Structures		Represents, Connects & Contextualizes Thinking			Explains & Justifies Reasoning			Attends to Efficiency & Precision			
Observes repeated behaviors and structures (e.g., when you add 1, the sum is the next number in the counting sequence)	States conjectures for the domain of whole numbers (e.g., the sum of two consecutive numbers is an odd number)	Represents Vi = visual Ph = physical S = symbolic C = contextual Ve = verbal	Translates A = across W = within	Moves between symbols and contexts C = contextualize D = decontextualize R = recontextualize	Explains "what" or "how"	Justifies "why" using reasoning based on number relationships, place value, and properties of operations	Critiques reasonableness	**Computational approach** E = efficient A = almost	**Mathematically accurate** Ac = accurate Al = almost	**Notation** Ac = accurate Al = almost	**Language** P = precise A = almost

Try it

Take a moment to watch the following short video clips of students solving the problem 6 + 7. As you watch and listen, make note on a copy of the Trends Tool of where you see the student thinking falling along the continuum of Number Sense and Computation Strategies.

Student 1

Student 2

Student 3

Student 4

Check Your Thinking

Below is a summary of what each student says and does, as well as comments about student mathematical understanding and next steps. These data are also recorded on an excerpt of the tool provided below. Before we turn to how these students' thinking can be recorded on the tool, compare your thinking to this summary.

	What the student says and does	**What this might mean**	**What's next**	
			Further assess	**Advance**
Student 1	• Moves 6 beads at once on top row • Moves 7 beads at once on bottom row • Counts 5, 10, 13	• Recognizes 6 and 7 without counting (subitizes) • Direct models using structure of 5 and 10 when forming numbers and finding the sum	Is there another way you could find the total? *Listen for use of known facts, such as 5 + 5.*	How might you use equations to show your thinking?
Student 2	• Moves 6 beads at once on top row • Moves 7 beads one-by-one on bottom row • Counts 6, 7, 8, 9, 10, 11, 12, 13	• Recognizes 6 and likely 7 without counting (subitizes) • Direct models by counting on from 6, the first number, to find the sum	Is there a way you could find the total more efficiently? *Listen for counting on from larger (7) or use of known doubles facts, such as 6 + 6 or 5 + 5.*	What strategy would you use if you were adding 5 and 9? How is your strategy the same or different?
Student 3	• Moves 6 beads (composes with 5 and 1) on top row • Moves 7 beads (composes with 5 and 2) on bottom row • Counts all to find the total	• Sees 6 and 7 as being composed of 5 and some more • Direct models and counts to find the sum	How could you use the number rack to solve 3 + 5? *Watch for counting all or counting on from first (3) or from larger (5).*	Is there a way you could find the total more efficiently? If subitizing 5, ask: Is there a way you could use your thinking from 3 + 5 to solve 6 + 7 differently?
Student 4	• Moves 6 beads at once for both the top and bottom row, then adds 1 bead to the bottom row for 7 • Knows doubles fact (6 + 6) and adds 1 more	• Recognizes 6 without counting (subitizes) • Sees 7 as being composed of 6 and 1 more • Uses known doubles fact, adds 1 more	Can you describe the strategy you used? For which other problems could you use your strategy? *Listen for students' ability to generalize to all near doubles or more generally, adding on or subtracting from any known fact.*	How might you help someone understand why you are doing what you are doing?

Number Sense and Computation Strategies Excerpt

	Number Sense		Computation Strategies												
			Direct Modeling Strategies		Counting Strategies			Foundational Facts					Derived Fact Strategies		
Anticipated student strategies and conceptions → Student Names ↓	Composes and decomposes quantities	Subitizes (recognizes value without counting)	Direct modeling with 1s	Direct modeling using structure of 10, 5, and 2	Counting on F = from first L = from larger T = to	Counting down D = [down] T = to	Counting on or down by/with O = ones TO = tens & ones	+ 0, 1, 2	- 0, 1, 2	Doubles	Combinations of 10	Ten and More	Near Doubles	Making 10	Compensation (pretend a 10)
Student 1		✓ 6, 7		✓											
Student 2		✓ 6, 7?		✓	F										
Student 3	✓			✓											
Student 4	✓	✓								✓ 6 + 6			✓ 6 + 7		

When thinking about the mathematical practices and processes, there is less variance across these students. Because students were prompted to use the number rack to show how they could solve 6 + 7, all students demonstrated the ability to translate the situation from a symbolic to a physical representation [represents—physical (Ph); translates—across (A)]. They also demonstrated the ability to explain how they determined that the sum of 6 and 7 is 13. They all came up with the correct answer [mathematically accurate (Ac)]. Advancing questions such as, "How might you use equations to show your thinking?," "How is your strategy the same or different?," "Does this strategy always work? How do you know?," and "How might you help someone understand why you are doing what you are doing?" would press students to engage in additional mathematical practices and processes.

Mathematical Practices & Processes Excerpt

	Mathematical Practices & Processes									
	Represents, Connects & Contextualizes Thinking			Explains & Justifies Reasoning			Attends to Efficiency & Precision			
Anticipated student strategies and conceptions → Student Names ↓	Represents Vi = visual Ph = physical S = symbolic C = contextual Ve = verbal	Translates A = across W = within	Moves between symbols and contexts C = contextualize D = decontextualize R = recontextualize	Explains "what" or "how"	Justifies "why" using reasoning based on number relationships, place value, and properties of operations	Critiques reasonableness	**Computational approach** E = efficient A = almost	**Mathematically accurate** Ac = accurate Al = almost	**Notation** Ac = accurate Al = almost	**Language** P = precise A = almost
Student 1	Ph	A		✓			E	Ac		
Student 2	Ph	A		✓			A	Ac		
Student 3	Ph	A		✓				Ac		
Student 4	Ph	A		✓			E	Ac		

These brief one-on-one conversations illustrate one way teachers might use the Trends Tool to document assets in students' thinking as opposed to deficits. They can also use the tool as they observe students at work discussing solution strategies or analyze students' written work. Some teachers accomplish this by continuing to add on to the tool, potentially notating in a different color to indicate a different assessment opportunity. Gathering evidence of student thinking in all these ways allows teachers to be more confident in their evaluation of students as they move beyond isolated assessment events toward multiple data sources at multiple points in time. In addition, teachers are also more confident as they take action based on a more complete picture of the complexities of students' mathematical understanding (Rigelman, in press).

Teachers can analyze whole class data using the Trends Tool, answering questions about the range of strategies students are using, the relative sophistication of strategies, the accuracy of solutions, and the extent to which students' explanations include justification of their reasoning or generalization of their thinking across problems. When used regularly, the Trends Tool can also provide information about a student's mathematical journey over time illustrating growth for students and families (e.g., more sophisticated strategies, purposeful selection of strategies, deeper engagement with the practices and processes). Finally, the Trends Tool provides teachers a common language about student thinking and its progression, which is supportive of deeper collaborative analysis and planning and more nuanced differentiation. In each case, the Trends Tool supports improved teaching.

References

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: Author.

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