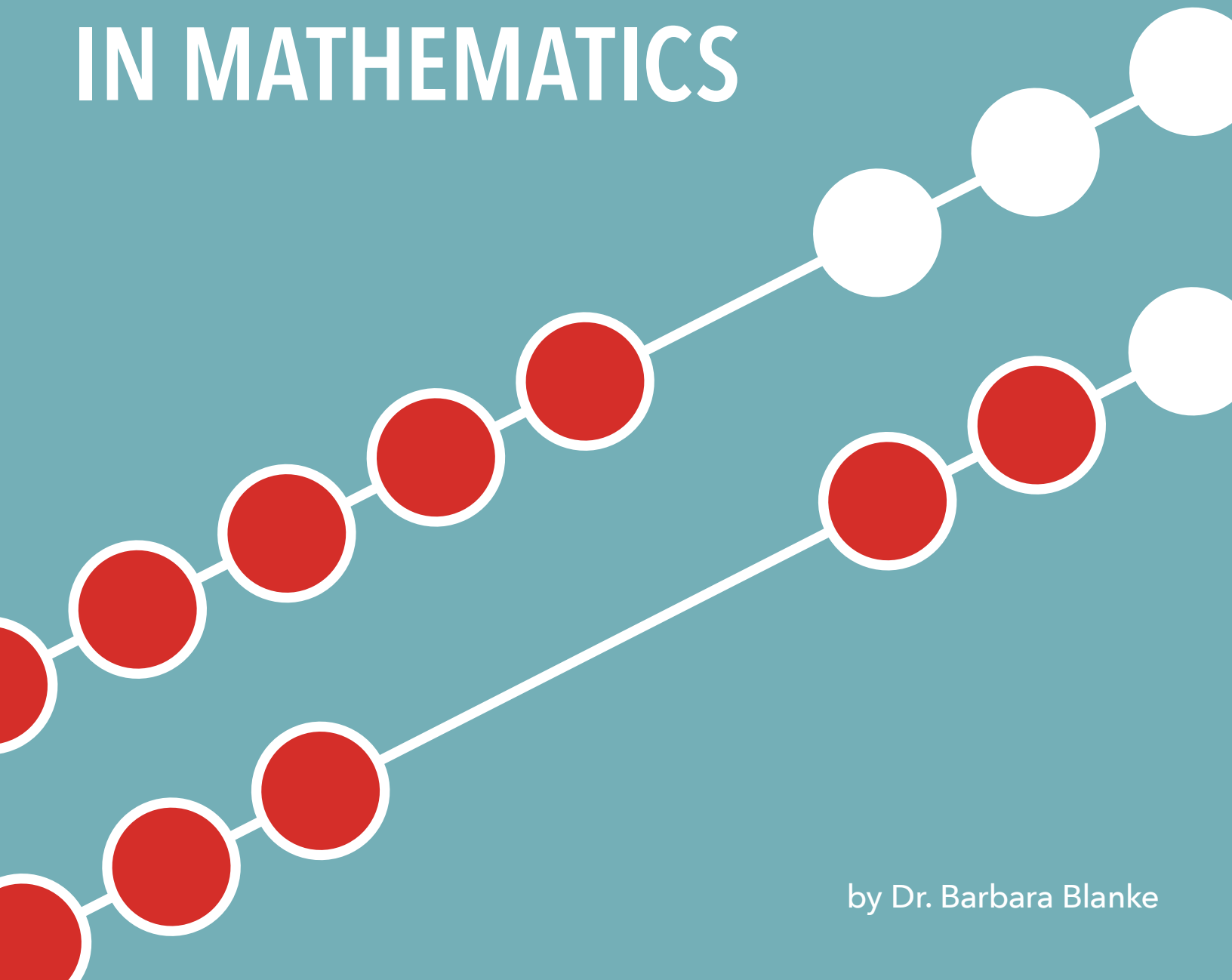


USING THE REKENREK AS A VISUAL MODEL FOR STRATEGIC REASONING IN MATHEMATICS



by Dr. Barbara Blanke

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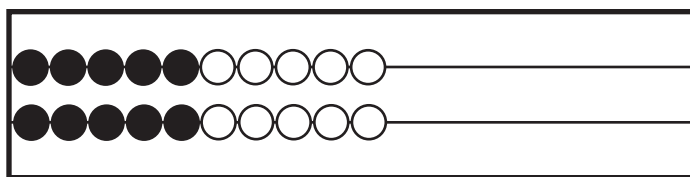
Using the Rekenrek as a Visual Model for Strategic Reasoning in Mathematics

"...materials cannot transmit knowledge: the learner must construct the relationships" Gravemeijer, 1991

What is a Rekenrek?

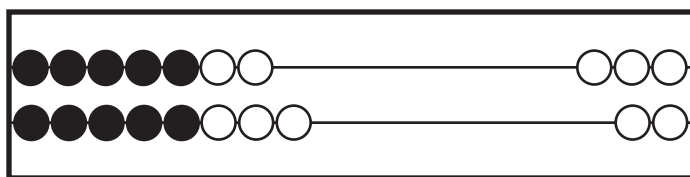
Directly translated, rekenrek means calculating frame, or arithmetic rack. Adrian Treffers, a mathematics curriculum researcher at the Freudenthal Institute in Holland, designed it to support the natural mathematical development of children and to help them generate a variety of addition and subtraction strategies, including doubles plus or minus 1, making 10's, and compensation. Students can use the rekenrek to develop computation skills or solve contextual problems. Once children understand the operations of addition and subtraction, and can model various situations, it is important that they automatize the basic facts by finding and using patterns and relationships. Unlike drill and practice worksheets and flashcards, the rekenrek supports even the youngest learners with the visual models they need to discover number relationships and develop automaticity.

The rekenrek looks like an abacus, but it is not based on place value columns or used like an abacus. Instead, it features two rows of 10 beads, each broken into two sets of five, much like the ten frames used in Bridges in Mathematics.



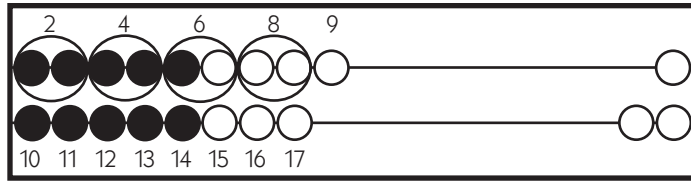
The rekenrek

Like ten frames, this tool helps students see the quantity of five as a whole and develop strategies to solve equations like $5 + 2$, $5 + 3$, etc. The figure below shows how $7 + 8$ would be set up on the rekenrek. In working with this model, children might find the total by adding $7 + 7 + 1$, or $5 + 5 + 2 + 3$, or $10 + 5$. They could also choose to count on, but the rekenrek is likely to stretch children to see groups of five.

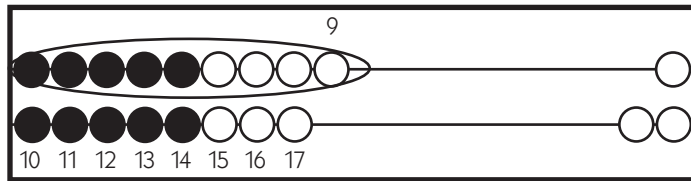


$$7 + 8$$

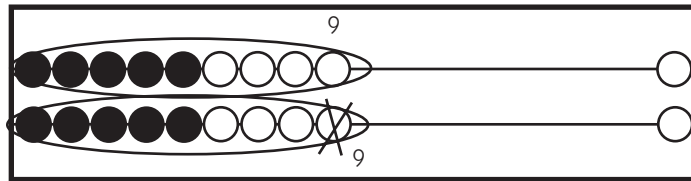
Here are some strategies we've seen students use for solving $9 + 8$ on the rekenrek.



Hannah's Strategy: counts by 2's to 8 then counts on by 1's

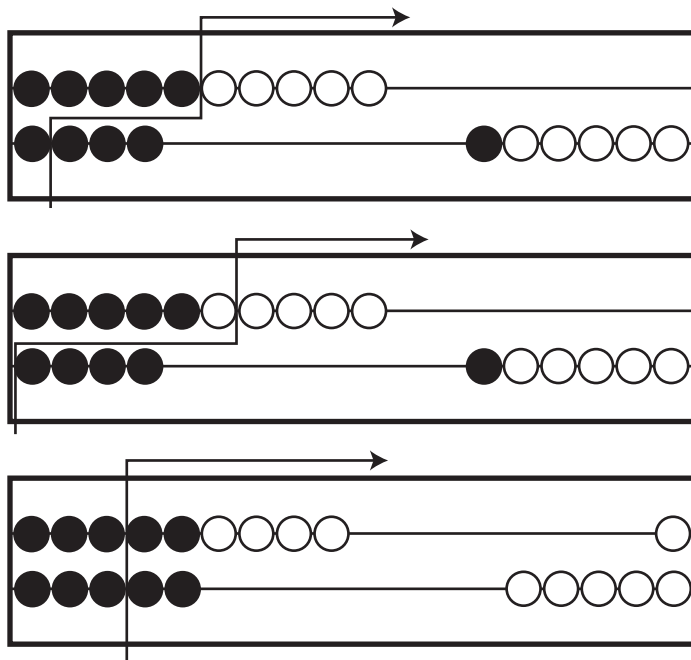


Ryan's Strategy: sees 9 and counts on by 1's



Ellie's Strategy: doubles minus 1. Shows $9 + 9$ and removes 1.

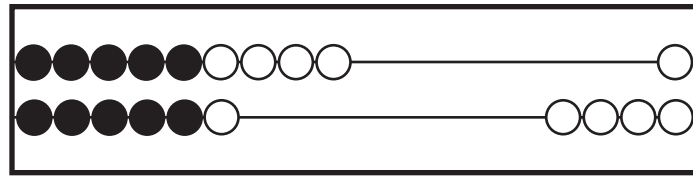
The rekenrek also allows students to develop a variety of subtraction strategies. Stanford professor Robert McKim (1980) states that, "the thinker who fluently produces a number of possible solutions to a problem is usually more successful than the thinker who settles for one solution". The figure below shows three different ways that children might solve $14 - 8$ using the rekenrek.



Various solutions for $14 - 8$

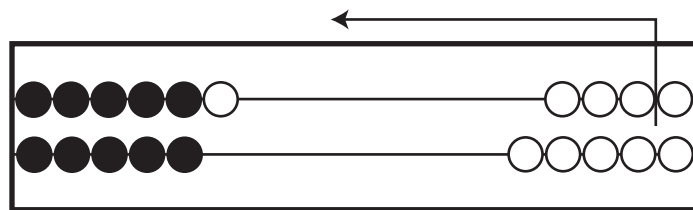
While students can use the rekenrek to generate different strategies for solving basic facts, they can also use it to solve story problems such as the ones below.

***There** were 15 kids in the house. 9 were playing video games upstairs. How many were in the kitchen having snacks?*



A student might show the 9 children upstairs with 9 beads in the top row, and then count on additional beads in the bottom row until reaching 15. One can then see there were 6 children in the kitchen.

***There** were 6 kids doing math home connections upstairs. 5 kids were downstairs reading books. 3 more kids came home to study. How many were in the house?*



Various way students solved this problem:

$$5 + 5 + 1 + 3 =$$

$$10 + 4 =$$

$$9 + 5 =$$

$$5 + 5 + 4 =$$

How to Get Started Using the Rekenrek

Using 5 and 10 as anchors for counting, adding, and subtracting is far more efficient than one-by-one counting. With the help of the rekenrek, kindergartners can start making the transition about the middle of the school year. The activities on the following pages show how. Each activity assumes familiarity with rekenrek operations in a previous activity.

Common Core State Standards

The table below reflects the Common Core State Standards addressed in Activities 1-10.

CCSS	ACT 1	ACT 2	ACT 3	ACT 4	ACT 5	ACT 6	ACT 7	ACT 8	ACT 9	ACT 10
K.CC.4	X	X								
K.CC.5	X	X								
K.OA.1			X	X	X	X				
K.OA.2			X	X	X	X				
K.OA.3		X	X	X	X	X				
K.OA.4			X	X	X	X				
K.OA.5			X	X	X	X				
K.NBT.1		X								
1.OA.2							X		X	
1.OA.3			X	X	X	X	X	X	X	X
1.OA.4				X	X	X	X	X	X	X
1.OA.5			X	X	X	X	X	X	X	X
1.OA.6			X	X	X	X	X	X	X	X
1.OA.8				X	X	X	X	X	X	X
1.NBT.1		X								
1.NBT.2		X							X	
2.OA.2			X	X	X	X	X	X	X	X
2.OA.3					X	X				

Activity 1

What the Heck is a Rekenrek?

Grade Levels

K–2

Common Core Standards

K.CC.4, K.CC.5

You Will Need

- ★ One rekenrek for teacher demonstration
- ★ Something to “hide” the rekenrek’s bottom row of beads (a folded 8” × 5” index card or piece of construction paper, or a small piece of fabric)

Skills

- ★ Demonstrate familiarization with the calculating frame.
- ★ Subitize 5 and 10.
- ★ Develop vocabulary for rekenrek use.

Instructions for What the Heck is a Rekenrek

As students watch, cover the bottom row of beads on your rekenrek with a folded index card or construction paper or fabric. Then slide all the beads in the visible row over to the far right. (All beads over to the right is the “start” position for this activity and others in this collection.)

Ask your students what they notice about today’s mathematical tool.

Teacher *What do you notice about the rekenrek?*

Jose *I see red and white beads.*

Ellie *I think there are ten.*

Teacher *How can we prove or disprove Ellie’s thought?*

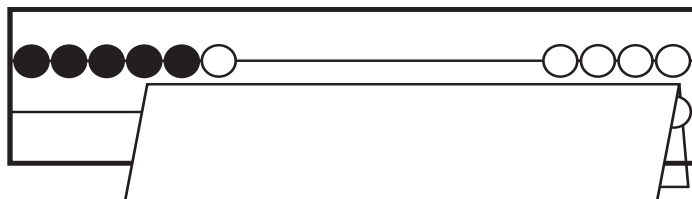
Hannah *I counted five red and five white.*

Daniel *That is right, because I saw that half were red and half were white.*

Marshall *5 plus 5 makes 10 total!*

Explain that you’re going to slide some beads on your rekenrek from one side of the wire to the other. Ask students to watch carefully and be ready to share how many you move.

- Slide 3 red beads to the left. Ask, “How many red beads do you see?”
- Return the beads to the right side of the rekenrek.
- Next slide 5 red beads to the left. Say, “Think privately. How many beads did I move? Quietly share with a friend why you think your method is right.”
- Return the beads to the right side of the rekenrek.
- Slide over 6 beads. Ask “What do you notice?”

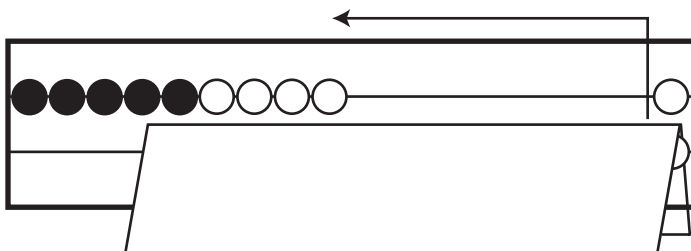


Activity 1 What the Heck is a Rekenrek? (cont.)

At this point let several children share how they can prove there are 6 beads... 3 and 3, counting by ones, five and 1 more, 1 and five more, 2 and 2 and 2 etc... Celebrate the multiple ways of “seeing” 6, especially the ones that did not involve one-by-one counting.

- Now slide 3 more beads over to the left for a total of 9. Ask, “How many do you see now?”

Keep soliciting different ways to prove there are 9 beads. Hopefully, someone will use the anchor of 10 saying something like, “I know there are 10 beads, so one less than 10 is 9” or, “I know there are 5, and 5 more would be 10, minus one is 9.” Your goal is to look for non-counting strategies that assist students in subitizing 5 and 10. We want these numbers to become anchors for the students that they quickly visualize.



Activity 2

See and Slide

Grade Levels

K–2

Common Core Standards

K.CC.4, K.CC.5, K.OA.3, K.NBT.1, 1.NBT.1, 1.NBT.2

Skills

- ★ Develop number sense strategies using 5 and 10 as anchor numbers.
- ★ Visualize the numbers 1–20.
- ★ Build numbers efficiently.

You will need

- ★ One rekenrek for each child in the group
- ★ 20 Colored Popsicle sticks (numbers 1–10 in one color, numbers 11–20 in another color)
- ★ 1 container to hold popsicle sticks

Note This activity can be extended on a ten row rekenrek by using 100 colored popsicle sticks numbered from 1–100. Have students represent the numbers using the fewest possible bead moves.

Instructions for See and Slide

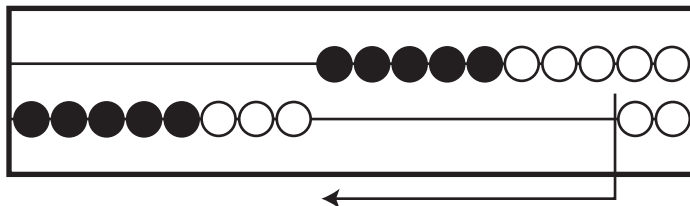
Make sure each child has a rekenrek with all beads moved to the right side. (Do not cover up any beads for this activity)

Tell the students you are going to choose a number (from a container of popsicle sticks numbered 1–10). They are to “slide” that number of beads on the rekenrek, using only one move.

Teacher I chose number 8. Think how you will move eight beads on the rekenrek in only one move. Now slide the beads.

Mike I slid 5 reds and 3 whites on the top row all in one move!

Sandy I did the same thing but on the bottom row.



Pat I moved 5 reds on top and 3 reds on the bottom.

Merrie Is that one move? I thought we had to use only one move.

Pat I used two fingers to make one move!

Teacher All of you “saw” the number eight and made one slide to show that number on your rekenrek.

Activity 2 See and Slide! (cont.)

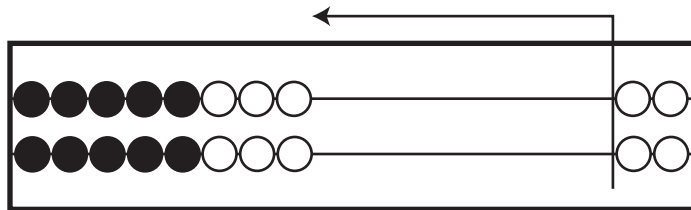
Now include the numbered sticks 11–20 in the container. Tell students they are to use no more than two slides to show numbers larger than 10.

Teacher I chose the number 16. Think about that number. How will it look when we move the beads? Using one or two moves, how can I show 16 on the rekenrek?

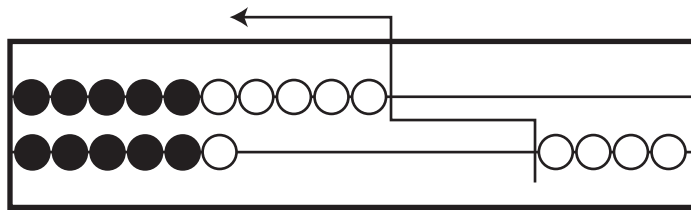
Henry If you move 8 beads on each row, that will make 16.

Marcia Move 10 on the top, 5 on the bottom, and 1 more on the bottom.

Mark But that's three moves. You would need to move 10 on the top and 6 on the bottom.



Doubles



Ten and More

Again, ask students to share their thinking. Continue the activity by asking students to choose and represent additional numbers. Discourage counting one by one by reminding them to use only one slide for numbers 1–10 and at most two slides for numbers 11–20.

Activity 3

Sum it Up

Grade Levels

K–2

Common Core Standards

K.OA.1, K.OA.2, K.OA.3, K.OA.4,
K.OA.5, 1.OA.3, 1.OA.5, 1.OA.6,
2.OA.2

You Will Need

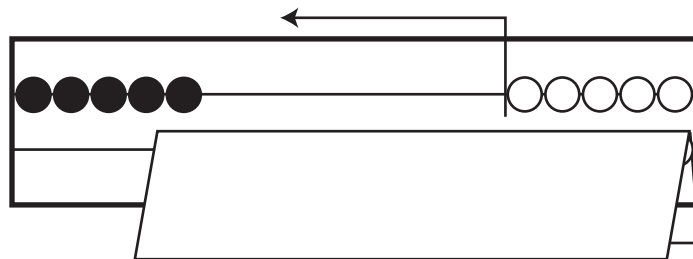
- ★ A rekenrek for each pair of children
- ★ Journal/paper for recording number sentences
- ★ Pencil
- ★ Number cards 1–10 (page 25; Run on cardstock and cut apart.)
- ★ Something to “hide” the rekenrek’s bottom row of beads (a folded 8” × 5” index card or piece of construction paper, or small piece of fabric)

Skills

- ★ Use the anchor numbers 5 and 10 to visualize numbers.
- ★ Use the “counting up” strategy to solve problems with missing addends.
- ★ Build fluency with tens facts.

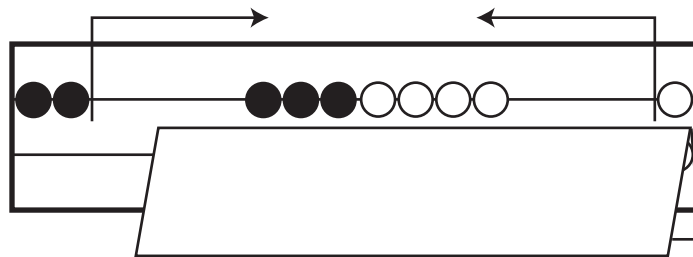
Instructions for Sum it Up

Explain that for this activity you are only going to use the top row of beads. (Cover the bottom row with a folded index card or a piece of fabric.) Begin by sliding the red beads to the left and the white beads to the right on the top row of the rekenrek. Using the number cards, choose a number to build.



How to start the activity

Teacher *I've chosen the number 7. Let's see how many ways we can build it by sliding beads from each side into the middle. I wonder if there is more than one way? What if I slide 3 red beads from the left and 4 white beads from the right. Does that build 7?*

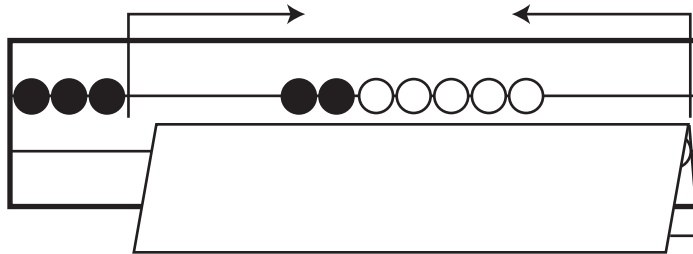


Carrie *Yes, but you can also make 7 by sliding 2 reds and 5 whites.*

Teacher *Let's record all the ways we can build 7.*

Activity 3 Sum it Up! (cont.)

.....
Note record students' combinations as noted below so they might discover the emerging pattern.
.....



$$2 \text{ reds} + 5 \text{ whites} = 7 \text{ beads}$$

$$3 \text{ reds} + 4 \text{ whites} = 7 \text{ beads}$$

$$4 \text{ reds} + 3 \text{ whites} = 7 \text{ beads}$$

$$5 \text{ reds} + 2 \text{ whites} = 7 \text{ beads}$$

Continue the activity by choosing additional numbers from the 1–10 number cards. Invite student pairs to build the chosen number and record the different ways it can be built.

Activity 4

It Takes Two to Build a Number

Grade Levels

K–2

Common Core Standards

K.OA.1, K.OA.2, K.OA.3, K.OA.4,
K.OA.5, 1.OA.3, 1.OA.4, 1.OA.5,
1.OA.6, 1.OA.8, 2.OA.2

You Will Need

- ★ One rekenrek for every two students
- ★ A rekenrek for teacher demonstration
- ★ Student set of number cards 1–10 (page 25)
- ★ Student set of number cards 1–20 (pages 25–26, optional)

Skills

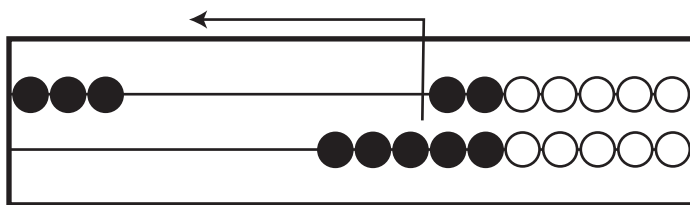
- ★ Discover strategies to solve unknown or missing addend problems.
- ★ Develop problem-solving strategies.
- ★ Build fluency with facts to ten.

Note Rekenrek activities like this can be extended on the ten row rekenrek by using the 1–100 number cards, pages 25–31. Have students represent the numbers using the fewest possible bead moves.

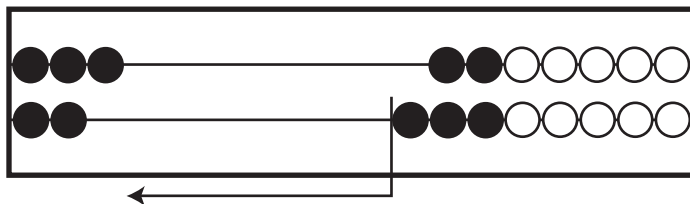
Instructions for It Takes Two to Build a Number

Ask a student to be your partner. Tell the class that you and your partner are going to build the number 5 as a team. You will be in charge of moving beads on the top row of the rekenrek and your student partner will move beads on the bottom row.

Teacher to Partner *I am going to slide 3 beads to the left on the top row. Now in one move, you slide beads on the bottom row to build the number 5.*



Shelby *I slid 2 beads on the bottom row with one move!*



Teacher *Now I would like all of you to pair up with a partner. I'll choose a number from the number cards. You will then build that number with your partner. (Begin with cards 1–10, or 1–20 if you are practicing facts to 20).*

Activity 4 It Takes Two to Build a Number (cont.)

Choose a couple cards to further model this activity before inviting the class to build the numbers on their rekenreks. Tell students the number of beads on the top row should be less than the number chosen. For instance, if the number card chosen is 8, the first student might move 5 beads on the top. Her partner would then move 3 beads on the bottom.

Optional: Building numbers 11–20

Use the 11–20 number cards to practice facts to 20. Help students see that certain row combinations require a minimal number of beads on one row to build numbers larger than 10.

***Teacher** I'm going to move 4 beads on the top row. You slide beads on the second row to make the number 16.*

***Michael** That won't work. I don't have enough beads.*

***Sharon** You have to move more beads on the top.*

***Kelly** Move 6 beads on the top. Then the 10 on the bottom will make 16.*

.....
Extension If you have a ten row rekenrek, challenge those students who are ready to represent numbers larger than 20. Have students work in pairs or small groups, taking turns drawing a number from the 1–100 number cards and building the number together or individually.
.....

Activity 5

Build a “Doubles” Number

Grade Levels

K–2

Common Core Standards

K.OA.1, K.OA.2, K.OA.3, K.OA.4,
K.OA.5, 1.OA.3, 1.OA.4, 1.OA.5,
1.OA.6, 1.OA.8, 2.OA.2, 2.OA.3

You Will Need

- ★ One rekenrek per student
- ★ One rekenrek for teacher demonstration
- ★ Even Ten Frame Cards 2, 4, 6, 8, 10 shown in different combinations (page 32; Run on cardstock or a transparency and cut cards apart.)
- ★ Optional Odd Ten Frame Cards 1, 3, 5, 7, 9 shown in different combinations (page 33; Run on cardstock or a transparency and cut cards apart.)

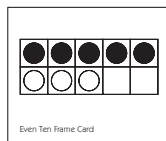
Skills

- ★ Add one-digit numbers to find sums to 20.
- ★ Use 5’s and 10’s to anchor addition and subtraction facts.
- ★ See the usefulness of doubles.

Instructions for Build a “Doubles” Number

Begin with all the beads to the right on the rekenrek. Display an even ten frame card and ask students to build the number by moving beads to the left. Have students check and compare their results with their neighbors. Then ask for volunteers to share different ways to build the number.

Teacher Here is a Ten Frame Card. Think of different ways to show this number on your rekenrek.



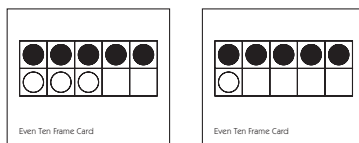
Ben I moved 5 red beads and 3 white beads.

Kathie I saw eight on the card so I made 4 red on top and 4 red on the bottom.

Florence That’s a doubles problem. $4 + 4 = 8$.

Repeat with several cards. Challenge students to build the numbers as doubles with equal numbers of beads on both rows.

Teacher Now I am going to display two Ten Frame cards. Can you make this a double using your rekenrek?



Sharon I moved over eight beads on the top and six on the bottom. To make a double I slid one bead on top back to the right and slid one bead on the bottom over to the left. Now I have 7 plus 7 which is the same as 8 plus 6!

Activity 5 Build a “Doubles” Number (cont.)

Tom I slid six over on the top and six over on the bottom. I needed to slide two more over so I slid one on top and one on the bottom and saw the double 7 plus 7 equals 14.

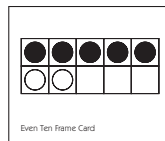
Glenda I saw that the cards had 8 and 6. To make them the same I took one away from the 8 and added it to the 6. That gave me 7 and 7. Then I moved 7 beads over on the top and 7 on the bottom.

Gill I saw two 5's, one 3, and one 1. That's 14 altogether. I know that two 7's make 14 so that's how I made mine, 7 on top and 7 on the bottom. It's symmetrical.

Optional Extension: Odd Ten Frame Cards

Use the Odd Ten Frame Cards to explore the concept of doubles plus one.

Teacher Here's a ten frame card that presents a new challenge. Can you build it as a double?



Michael No. It has to be 6 or 8.

Sharon The closest I can come is to put 3 on the top and 4 on the bottom.

Kelly 7 is an odd number. Odd numbers can't have doubles.

Kip I think that odd numbers will always have doubles plus 1, like 3 plus 3 plus 1.

Jessica Yes, one row will always be 1 more than the other.

Teacher Let's try another one....

When students seem comfortable with the Odd Ten Frames, mix them with the Even Ten Frame cards and ask them to identify if a card is even or odd.

Activity 6

Show the Sum

Grade Levels

K–2

Common Core Standards

K.OA.1, K.OA.2, K.OA.3, K.OA.4,
K.OA.5, 1.OA.3, 1.OA.4, 1.OA.5,
1.OA.6, 1.OA.8, 2.OA.2, 2.OA.3

You Will Need

- ★ One rekenrek for each student
- ★ Optional rekenrek for teacher demonstration
- ★ Classroom whiteboard or chart to record various computational strategies
- ★ 2 dice or wooden cubes labeled with the following numbers—Die One: 5, 6, 7, 8, 9, 10; Die Two: 4, 5, 6, 7, 8, 9

Skills

- ★ Develop and demonstrate computational fluency strategies for addition facts to 20.
- ★ Add two 1-digit numbers.
- ★ Use 5's and 10's as anchors to solve addition facts to 20.
- ★ Explore multiple strategies for adding two numbers.

Instructions for Show the Sum

Invite a student volunteer to roll the two dice to determine two numbers to add. Students use their rekenrek to share their strategies for calculating the sum.

Teacher *Karen's roll turned up 9 and 6. Show me 9 plus 6 on your rekenrek. What is the sum?*

Shelby *I moved 5 red and 4 white on the top to show 9. Since I know 5 plus 1 is 6, on the bottom I moved all 5 red beads and one more white one. They all add up to 15.*

Record the number sentences on a chart or whiteboard to represent each student's strategy:

$$5 + 4 = 9$$

$$5 + 1 = 6$$

$$9 + 6 = 15$$

Hannah *I know 1 less than 10 is 9 so I moved all the beads except for 1 over. Then I moved all 10 on the bottom row and slid four back to show 6. I think I made more slides than Shelby, but we got the same answer.*

$$10 - 1 = 9$$

$$10 - 4 = 6$$

$$9 + 6 = 15$$

Emmy *I started with 10 on the top and 5 on the bottom. Because 9 is one less than 10 and 6 is one more than 5, I just moved one from the top to the bottom. 15 is the answer and what I started with!*

$$10 - 1 = 9$$

$$5 + 1 = 6$$

$$9 + 6 = 15$$

Repeat the roll of the dice several times, making sure to have several students share various ways to calculate a sum.

Activity 7

Roll Three for 20 or Bust!

Grade Levels

1–2

Common Core Standards

1.OA.2, 1.OA.3, 1.OA.4, 1.OA.5,
1.OA.6, 1.OA.8, 2.OA.2

You Will Need

- ★ One rekenrek for each student
- ★ Optional rekenrek for teacher demonstration
- ★ Classroom whiteboard or chart to record various computational strategies and number sentences
- ★ Several sets of 3 dice or wooden cubes labeled with the following numbers—Red Die: 0, 1, 2, 3, 4, 5; Blue Die: 2, 3, 4, 5, 6, 7; Green Die: 4, 5, 6, 7, 8, 9

Skills

- ★ Develop and demonstrate computational fluency strategies for addition facts to 20.
- ★ Add three 1-digit numbers.
- ★ Use 5's and 10's as anchors to solve addition facts to 20.

Instructions for Roll Three for 20 or Bust!

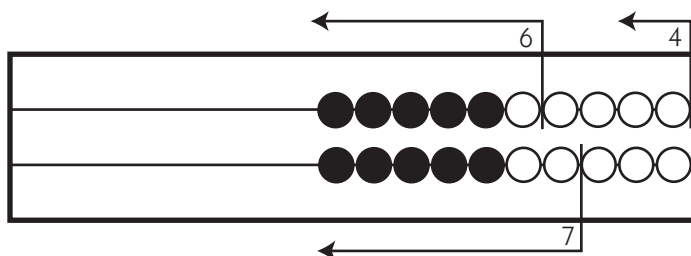
The object of this game is to obtain a sum close to or equal to 20 without going over.

- Roll the three dice to determine the first three numbers to add.
- Tell students to use their rekenreks to share their strategy for calculating the sum of the three numbers rolled.
- Students must then decide to stay put or roll 1, 2, or 3 dice to add more to the sum they have so far to come close or equal to 20 without going over.

Model the activity before inviting students to play this game in small groups. Roll the dice and ask the class to help you determine the sum of the three numbers using their rekenrek. Ask student to share their strategies. Use the whiteboard or a chart to record their thinking.

Teacher I rolled a 6 and a 7 and a 4. How should we add these three numbers on the rekenrek?

Ellie Move 6 and then 4 over. That makes 10. Then on the bottom move 5 red and 2 white which is 7. 10 plus 7 equals 17. That is a fast ten fact!



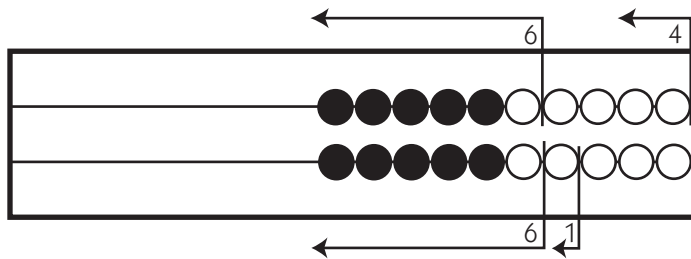
$$6 + 4 = 10$$

$$5 + 2 = 7$$

$$10 + 7 = 17$$

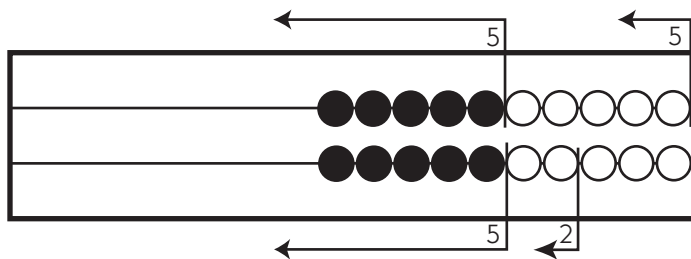
Activity 7 Roll Three! For 20 or Bust! (cont.)

Reba Slide 6 on top and double it by sliding 6 on the bottom. Then slide 4 over on top and 1 more on the bottom. That's also 17.



$$6 + 6 + 4 + 1 = 17$$

Ernesto I think it would be easy to move the beads over in fives. I know in my head that the answer is 17, so I would slide 5, and then 5 more to make 10, and then 5 more to make 15. You still need 2 more white beads to make 17.



$$5 + 5 + 5 = 15$$

$$15 + 2 = 17$$

Teacher So we've determined that $6 + 7 + 4$ equals 17. Do we want to roll more dice to get closer to 20?

Carlin Yes, but only roll one and use the red one. It has the smallest numbers.

Teacher Ok. We rolled a 4.

Marci That's too much! $17 + 4$ is more than 20. We went bust!

Have students break into pairs or small groups and play the game. Be sure there is a set of dice for each group. Watch for groups that get close to 20 and ask them to show you their sum on the rekenrek.

Activity 8

Target Think Addition

Grade Levels

1–2

Common Core Standards

1.OA.3, 1.OA.4, 1.OA.5, 1.OA.6,
1.OA.8, 2.OA.2

Skills

- ★ Use the “Think Addition” strategy to solve subtraction facts to 20.
- ★ Combine numbers to find a target number.
- ★ Decompose and compose numbers.

You Will Need

- ★ Set of Number Cards 11–20 to use as Target Numbers (pages 25 and 26; Run on cardstock and cut apart.)
- ★ Set of Number Cards 1–10 to use as Starting Numbers (pages 25 and 26; Run on cardstock and cut apart.)
- ★ Two containers—one for holding “Starting” number cards 1–10, one for holding “target” number cards 11–20
- ★ One rekenrek per student
- ★ Optional rekenrek for teacher demonstration

.....
Note Subtraction as “Think Addition” is a significant strategy for recalling subtraction facts.

Instructions for Target Think Addition

- Draw a Starting Number Card from a container of number cards 1–10. Ask students to build the number on the rekenrek.
- Next draw a Target Number Card from a container of number cards 11–20. Tell students that number is the target they want to get to.
- Ask students to think of how many beads they must add to the Starting Number to equal the Target Number. Before they move any beads, ask students to share how many they will move. Then have them make the move to create the Target Number. Invite students to share their methods.

This mental activity leads to the use of a “think addition” strategy rather than a “count what is left” strategy when solving subtraction problems. It reinforces the concept of anchor numbers and seeing combinations rather than counting by ones.

Teacher *I'm going to draw our Starting Number—it's 4. Build that number on your rekenrek.*

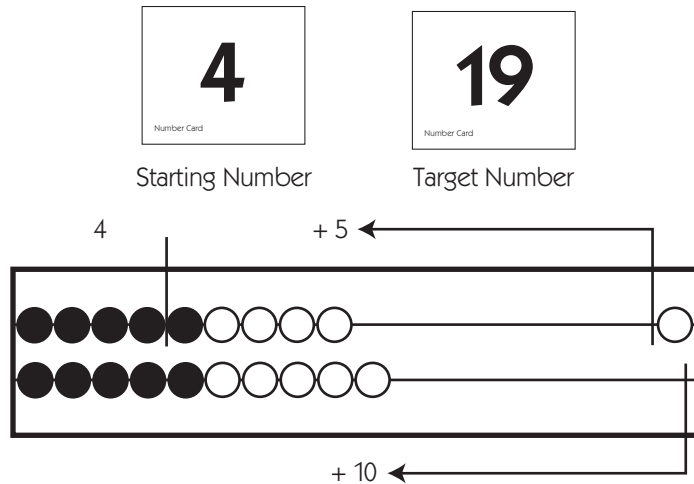
Teacher *Now let's choose our Target Number. We drew 19! Think for a moment before you move any beads. How many will you need to move over so that you have the target number 19? Remember, we want to use as few moves as we can.*

Charles *I would move the whole bottom 10 over first and that would make 14. Then I would move 5 more over on the top and that would make the target number 19.*

Teacher *That's pretty good, only two moves. So how many beads did Charles move all together?*

Terri *He moved 15. So we had to add 15 to 4 to hit the target 19. $4 + 15$ is 19.*

Activity 8 Target Think Addition! (cont.)



Repeat this activity several times with new Starting and Target numbers.

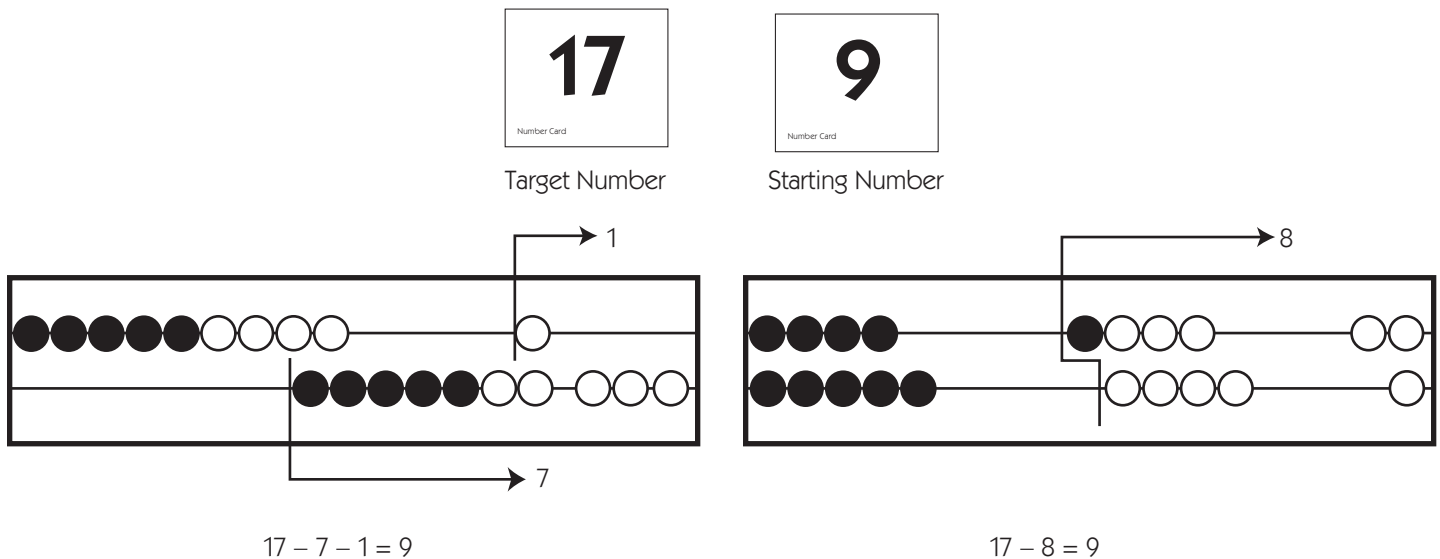
Extend to Subtraction

To move students into subtraction, draw the cards in a different order. Start by drawing a Target Number (11–20). Ask the class to build it on their rekenrek. Then draw a Starting Number Card (0–10). Ask students to think of how many beads they will need to remove from the Target Number (move back to the right) to get back to the Starting Number.

Teacher This time let's start with a Target Number. I've drawn a target of 17. Build that on the rekenrek. (Pause) Now let's draw a Starting Number—9. Think for a moment. How many beads will you need to remove from the Target Number to get back to the Starting Number 9?

Samantha To get to 9 I'm going to have to move 7 back on the bottom and 1 more on top, that's 8 I have to move to get to 9. $17 - 8$ is 9.

Kelly I started with 17 as doubles plus 1. To get to 9, I took double 4's away. $17 - 8$ is 9.



Activity 9

Tens or Ones

Grade Levels

1–2

Common Core Standards

1.OA.2, 1.OA.3, 1.OA.4, 1.OA.5,
1.OA.6, 1.OA.8, 1.NBT.2, 2.OA.2

Skills

- ★ Compose and decompose numbers by tens or ones.
- ★ Demonstrate fluency with addition and subtraction facts.
- ★ Use strategies to demonstrate fluency.

You Will Need

- ★ Double set of number cards 11–19 (page 25 and 26; Run on cardstock and cut apart.)
- ★ Run Away Ones Cards (page 35; Run on cardstock and cut apart.)
- ★ Take Away Tens Cards (page 35; Run on cardstock and cut apart.)
- ★ Two containers from which to draw cards—one for holding number cards 11–19, one for holding Take Away Tens and Run Away Ones cards
- ★ One rekenrek for every student

Instructions for Tens or Ones

Begin by drawing one of the Number Cards 11–19 from a container. Display the chosen card to the class and ask them to build the number on their rekenreks.

Next draw a card from the second container holding the Take Away Tens and Run Away Ones Cards. Have the class perform the action listed on the card by taking away either ten or the ones from the number on their rekenreks. Be sure to look for different strategies and invite student sharing.

Teacher *Our first number card is 19, and it looks like each of you has it built on your rekenrek. Now let's draw from the second container—a Take Away Tens card. What should we do?*

Glenda *Move ten beads back to the right! That leaves 9! I just moved the top row of ten and that left nine on the bottom! $19 - 10$ is 9.*

Repeat the process several times by drawing additional cards, asking students to build and subtract either tens or ones from the number on their rekenrek. Play until all Number Cards are drawn.

Teacher *Carol has drawn our next number card—16. Everyone build 16.*

Charlotte *I put 8 on top and 8 on the bottom. Doubles!*

Carl *I slid 10 over on top and 6 on the bottom. I think that's the quickest way.*

Teacher *Carol, pull a card out of our second container—it's a Take Away Ones Card.*

Carol *We have to take away all the ones from 16. That's 6. So take 6 away.*

Charlotte *I took 6 away on the bottom and that left 8 and 2. I know 8 and 2 is 10.*

Carl *I just moved the 6 on the bottom and ten are left on top! We both got 10!*

Carol *So our number sentence is $16 - 6$ is 10.*

Activity 10

Count and Compare

Grade Levels

1–2

Common Core Standards

1.OA.3, 1.OA.4, 1.OA.5, 1.OA.6,
1.OA.8, 2.OA.2

Skills

- ★ Demonstrate fluency with addition and subtraction facts.
- ★ Use the rekenrek model and mental strategies to solve addition and subtraction facts.

You Will Need

- ★ One rekenrek per student
- ★ Optional rekenrek for teacher demonstration
- ★ Count and Compare Gameboard (page 36; Run a half class set on cardstock and on optional transparency.)
- ★ Set of Count and Compare Cards (pages 37– 43; Run a half class set on cardstock and on an optional transparency. Cut apart.)
- ★ Half class set of clear overlay spinners or pencils and paperclips to use as spinners

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Note You may wish to model this game at the overhead or play teacher against class. If so, run transparencies of the items listed above. If you would like the students to play in pairs, make copies of the gameboard and cards for every two students. Below are the directions for the student partner game.

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Instructions for Count and Compare

Distribute the Count and Compare Gameboard and a set of Count and Compare Cards to each pair of students. Ask them to shuffle their decks of cards and place the cards on top of the gameboard. Make sure that each student has a rekenrek.

To begin the game, tell each student to draw one fact card from the top of the pile and use their rekenrek to find the difference or sum for the card drawn.

Ask students to share their strategies for determining solutions for their cards. Once both partners have agreed on their solutions, tell them to decide which card is more and which card is less. Have them place each card in the appropriate More or Less box on the game board. If the solutions on both cards are equal, tell them to place both cards in the equal box and draw two more cards until they have one solution that is more and one that is less.

Using a clear spinner or pencil and paper clip, tell them to spin the More Or Less Spinner to decide who gets to take both or all the cards. (If cards are in the equal box, the winner gets those cards as well.) If the spinner lands on More, the person who drew the card with the greater sum or difference gets to take the cards. If the spinner lands on Less, the person who drew the card with the smaller solution gets the cards.

Activity 10 Count and Compare (cont.)

Have students repeat the game until they are out of cards or until time runs out. Tell each student to count the number of cards they each collected. Then tell the players with the most cards to place their stacks of cards in the More box and those with the fewest cards to put their cards in the Less box. Have them spin the spinner one more time to decide who wins the game!

Number Cards page 1 of 7

1 Number Card	2 Number Card	3 Number Card	4 Number Card
5 Number Card	6 Number Card	7 Number Card	8 Number Card
9 Number Card	10 Number Card	11 Number Card	12 Number Card
13 Number Card	14 Number Card	15 Number Card	16 Number Card

Number Cards page 2 of 7

17 Number Card	18 Number Card	19 Number Card	20 Number Card
21 Number Card	22 Number Card	23 Number Card	24 Number Card
25 Number Card	26 Number Card	27 Number Card	28 Number Card
29 Number Card	30 Number Card	31 Number Card	32 Number Card

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33 Number Card	34 Number Card	35 Number Card	36 Number Card
37 Number Card	38 Number Card	39 Number Card	40 Number Card
41 Number Card	42 Number Card	43 Number Card	44 Number Card
45 Number Card	46 Number Card	47 Number Card	48 Number Card

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49 Number Card	50 Number Card	51 Number Card	52 Number Card
53 Number Card	54 Number Card	55 Number Card	56 Number Card
57 Number Card	58 Number Card	59 Number Card	60 Number Card
61 Number Card	62 Number Card	63 Number Card	64 Number Card

Number Cards page 5 of 7

65 Number Card	66 Number Card	67 Number Card	68 Number Card
69 Number Card	70 Number Card	71 Number Card	72 Number Card
73 Number Card	74 Number Card	75 Number Card	76 Number Card
77 Number Card	78 Number Card	79 Number Card	80 Number Card

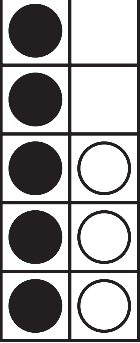
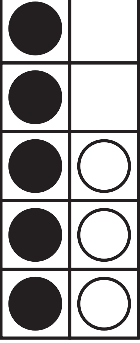
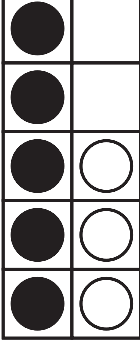
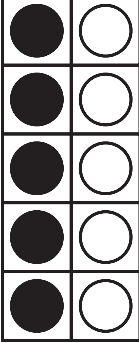
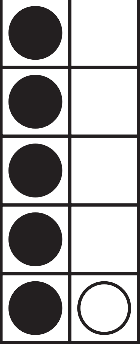
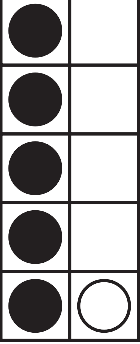
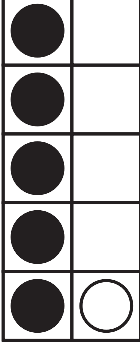
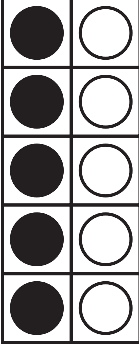
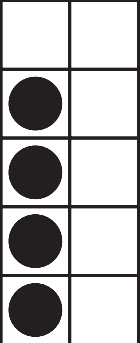
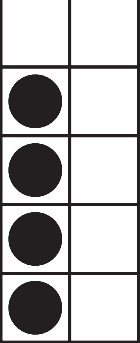
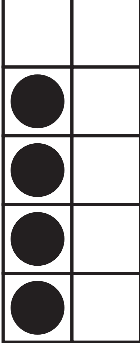
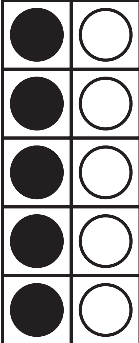
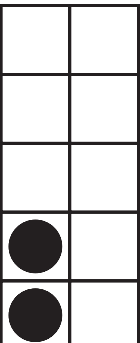
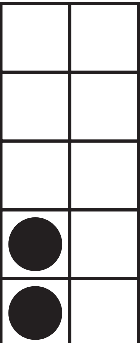
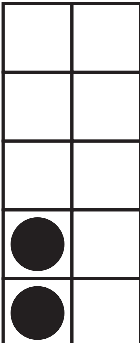
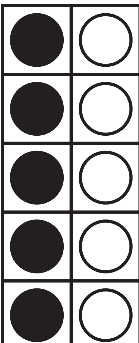
Number Cards page 6 of 7

81 Number Card	82 Number Card	83 Number Card	84 Number Card
85 Number Card	86 Number Card	87 Number Card	88 Number Card
89 Number Card	90 Number Card	91 Number Card	92 Number Card
93 Number Card	94 Number Card	95 Number Card	96 Number Card

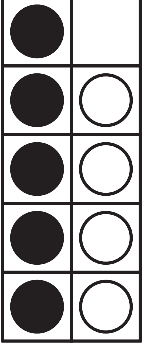
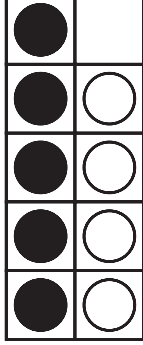
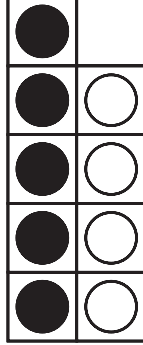
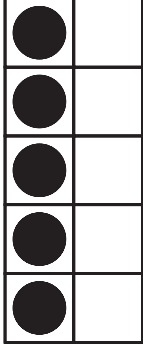
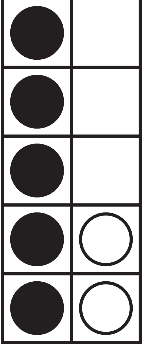
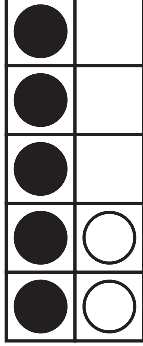
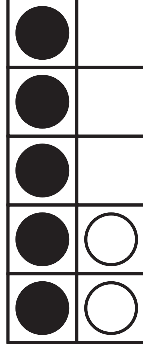
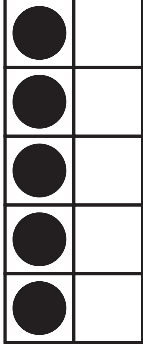
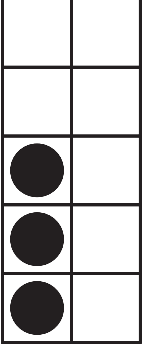
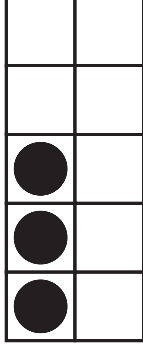
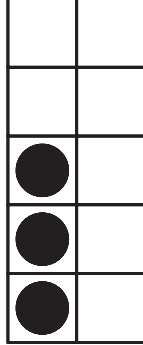
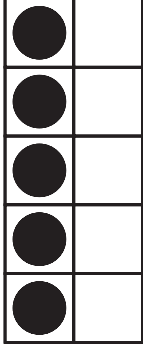
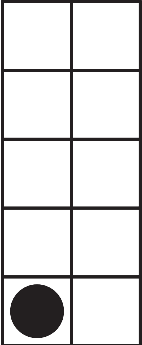
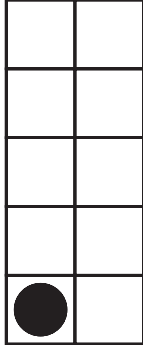
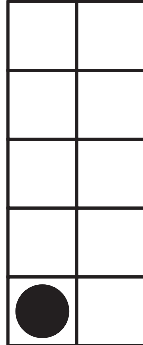
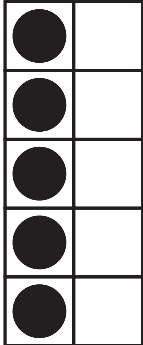
Number Cards page 7 of 7

97 Number Card	98 Number Card	99 Number Card	100 Number Card
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Even Ten Frame Cards

	Even Ten Frame Card		Even Ten Frame Card		Even Ten Frame Card		Even Ten Frame Card
	Even Ten Frame Card		Even Ten Frame Card		Even Ten Frame Card		Even Ten Frame Card
	Even Ten Frame Card		Even Ten Frame Card		Even Ten Frame Card		Even Ten Frame Card
	Even Ten Frame Card		Even Ten Frame Card		Even Ten Frame Card		Even Ten Frame Card

Odd Ten Frame Cards

	Odd Ten Frame Card		Odd Ten Frame Card		Odd Ten Frame Card		Odd Ten Frame Card
	Odd Ten Frame Card		Odd Ten Frame Card		Odd Ten Frame Card		Odd Ten Frame Card
	Odd Ten Frame Card		Odd Ten Frame Card		Odd Ten Frame Card		Odd Ten Frame Card
	Odd Ten Frame Card		Odd Ten Frame Card		Odd Ten Frame Card		Odd Ten Frame Card

Run Away Ones Cards

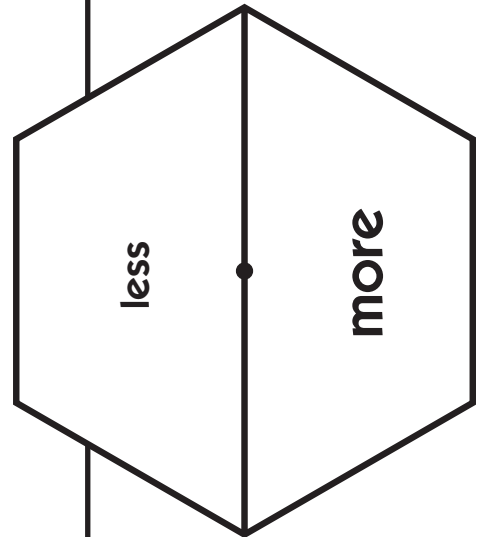
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<p>Run Away Ones</p> <p>Run Away Ones Card</p>	<p>Run Away Ones</p> <p>Run Away Ones Card</p>	<p>Run Away Ones</p> <p>Run Away Ones Card</p>	<p>Run Away Ones</p> <p>Run Away Ones Card</p>

Take Away Tens Cards

<p>Take Away Tens</p> <p>Take Away Tens Card</p>	<p>Take Away Tens</p> <p>Take Away Tens Card</p>	<p>Take Away Tens</p> <p>Take Away Tens Card</p>	<p>Take Away Tens</p> <p>Take Away Tens Card</p>
<p>Take Away Tens</p> <p>Take Away Tens Card</p>	<p>Take Away Tens</p> <p>Take Away Tens Card</p>	<p>Take Away Tens</p> <p>Take Away Tens Card</p>	<p>Take Away Tens</p> <p>Take Away Tens Card</p>

Count & Compare Gameboard

Less	Equal	More
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Count & Compare Cards, page 1 of 7

$\begin{array}{r} 11 \\ - 5 \\ \hline \end{array}$	Count & Compare Card	$\begin{array}{r} 12 \\ - 4 \\ \hline \end{array}$	Count & Compare Card
$\begin{array}{r} 11 \\ - 6 \\ \hline \end{array}$	Count & Compare Card	$\begin{array}{r} 12 \\ - 5 \\ \hline \end{array}$	Count & Compare Card
$\begin{array}{r} 11 \\ - 7 \\ \hline \end{array}$	Count & Compare Card	$\begin{array}{r} 14 \\ - 5 \\ \hline \end{array}$	Count & Compare Card
$\begin{array}{r} 11 \\ - 8 \\ \hline \end{array}$	Count & Compare Card	$\begin{array}{r} 12 \\ - 7 \\ \hline \end{array}$	Count & Compare Card

Count & Compare Cards, page 2 of 7

$\begin{array}{r} 12 \\ - 8 \\ \hline \end{array}$	Count & Compare Card	$\begin{array}{r} 13 \\ - 7 \\ \hline \end{array}$	Count & Compare Card
$\begin{array}{r} 13 \\ - 4 \\ \hline \end{array}$	Count & Compare Card	$\begin{array}{r} 13 \\ - 8 \\ \hline \end{array}$	Count & Compare Card
$\begin{array}{r} 13 \\ - 5 \\ \hline \end{array}$	Count & Compare Card	$\begin{array}{r} 14 \\ - 5 \\ \hline \end{array}$	Count & Compare Card
$\begin{array}{r} 13 \\ - 6 \\ \hline \end{array}$	Count & Compare Card	$\begin{array}{r} 12 \\ - 7 \\ \hline \end{array}$	Count & Compare Card

Count & Compare Cards, page 3 of 7

$\begin{array}{r} 14 \\ - 8 \\ \hline \end{array}$	Count & Compare Card	$\begin{array}{r} 15 \\ - 8 \\ \hline \end{array}$	Count & Compare Card
$\begin{array}{r} 14 \\ - 9 \\ \hline \end{array}$	Count & Compare Card	$\begin{array}{r} 15 \\ - 9 \\ \hline \end{array}$	Count & Compare Card
$\begin{array}{r} 15 \\ - 6 \\ \hline \end{array}$	Count & Compare Card	$\begin{array}{r} 16 \\ - 7 \\ \hline \end{array}$	Count & Compare Card
$\begin{array}{r} 15 \\ - 7 \\ \hline \end{array}$	Count & Compare Card	$\begin{array}{r} 16 \\ - 9 \\ \hline \end{array}$	Count & Compare Card

Count & Compare Cards, page 4 of 7

$\begin{array}{r} 8 \\ + 6 \\ \hline \end{array}$	Count & Compare Card	$\begin{array}{r} 10 \\ + 5 \\ \hline \end{array}$	Count & Compare Card
$\begin{array}{r} 8 \\ + 7 \\ \hline \end{array}$	Count & Compare Card	$\begin{array}{r} 6 \\ + 5 \\ \hline \end{array}$	Count & Compare Card
$\begin{array}{r} 8 \\ + 8 \\ \hline \end{array}$	Count & Compare Card	$\begin{array}{r} 6 \\ + 7 \\ \hline \end{array}$	Count & Compare Card
$\begin{array}{r} 17 \\ - 8 \\ \hline \end{array}$	Count & Compare Card	$\begin{array}{r} 17 \\ - 9 \\ \hline \end{array}$	Count & Compare Card

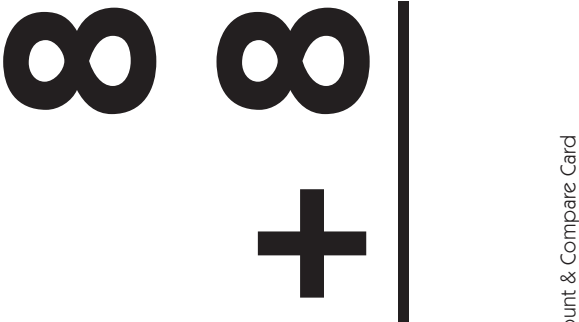
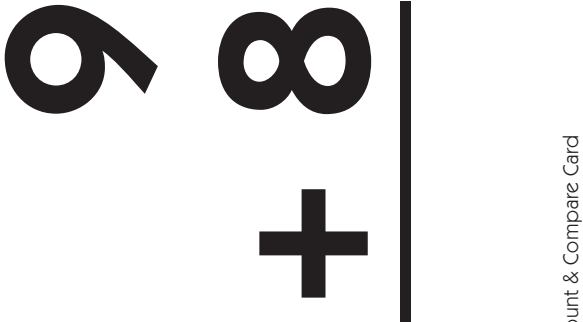
Count & Compare Cards, page 5 of 7

$\begin{array}{r} 11 \\ + 4 \\ \hline \end{array}$	Count & Compare Card	$\begin{array}{r} 11 \\ + 8 \\ \hline \end{array}$	Count & Compare Card
$\begin{array}{r} 11 \\ + 5 \\ \hline \end{array}$	Count & Compare Card	$\begin{array}{r} 10 \\ + 5 \\ \hline \end{array}$	Count & Compare Card
$\begin{array}{r} 11 \\ + 6 \\ \hline \end{array}$	Count & Compare Card	$\begin{array}{r} 10 \\ + 7 \\ \hline \end{array}$	Count & Compare Card
$\begin{array}{r} 8 \\ + 5 \\ \hline \end{array}$	Count & Compare Card	$\begin{array}{r} 6 \\ + 4 \\ \hline \end{array}$	Count & Compare Card

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$8 \quad 9$ $+ \quad $	Count & Compare Card	$9 \quad 9$ $+ \quad $	Count & Compare Card
$9 \quad 6$ $+ \quad $	Count & Compare Card	$9 \quad 7$ $+ \quad $	Count & Compare Card
$9 \quad 7$ $+ \quad $	Count & Compare Card	$9 \quad 9$ $+ \quad $	Count & Compare Card
$6 \quad 8$ $+ \quad $	Count & Compare Card	$11 \quad 7$ $+ \quad $	Count & Compare Card

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 <p>Count & Compare Card</p>	 <p>Count & Compare Card</p>

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