visual mathematics

COURSE II STUDENT ACTIVITIES

This packet contains one copy of each Follow-up and of other activities used by individuals or pairs of students. Group activities and sheets are not included.

Visual Mathematics, Course II
by Linda Cooper Foreman and Albert B. Bennett Jr.
Student Activities

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Student Activities

Student Activities (continued)

LESSON 16 Connector Master A
Focus Master B
Focus Master C
Follow-up Student Activity 16.1

LESSON 17 Connector Master A (2 pages)
Focus Student Activity 17.1
Follow-up Student Activity 17.2

LESSON 18 Follow-up Student Activity 18.1

LESSON 19 Focus Master A
Focus Master B (optional)
Extended Project E (optional)
Focus Student Activity 19.1
Follow-up Student Activity 19.2

LESSON 20 Connector Master A (4 pages)
Connector Master B
Focus Master B
Follow-up Student Activity 20.1

LESSON 21 Connector Student Activity 21.1
Focus Master A
Focus Master B
Focus Student Activity 21.2
Focus Student Activity 21.3
Focus Student Activity 21.4
Focus Student Activity 21.5
Follow-up Student Activity 21.6

LESSON 22 Connector Student Activity 22.1
Connector Student Activity 22.2
Connector Student Activity 22.3
Connector Student Activity 22.4
Follow-up Student Activity 22.5

LESSON 23 Connector Student Activity 23.1
Focus Student Activity 23.3
Follow-up Student Activity 23.4

LESSON 24 Follow-up Student Activity 24.1

Student Activities (continued)

LESSON 25	Focus Master B
	Focus Master C
	Focus Master D
	Focus Master E
	Focus Student Activities 25.1
	Focus Student Activities 25.2
	Follow-up Student Activity 25.3
LESSON 26	Focus Student Activities 26.1
	Focus Student Activities 26.2
	Focus Student Activities 26.3 (2 pgs)
	Follow-up Student Activity 26.4
LESSON 27	Follow-up Student Activity 27.1
LESSON 28	Focus Master B
	Focus Master C
	Focus Master D
	Focus Master E
	Focus Master F
	Focus Master G
	Follow-up Student Activity 28.1
LESSON 29	Focus Master B
	Focus Master C
	Follow-up Student Activity 30.1
LESSON 30	Connector Master B
	Connector Student Activity 30.1
	Focus Master A
	Focus Student Activity 30.2
	Focus Student Activity 30.3
	Follow-up Student Activity 30.4
Grids:	2-cm Grid Paper
	1-inch Grid Paper
Tools:	Algebra Pieces
	Base Ten Area Pieces
	Blank Counting Pieces and N-Strips
	Cubical Dice Patterns
	Red and Black Counting Piece Masters
	Regular Polyhedra Patterns

Follow-up Student Activity 1.1

NAME _____ DATE _____

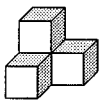
1 Write an explanation in your own words of the meaning of:

a) volume b) surface area c) dimensions

2 Explain how you think a unit of volume, a unit of surface area, and a unit of length are related and how they are different.

3 Build each of the following structures, and then determine the volume (V) and surface area (S) of each structure, assuming that 1 cube is a unit of volume.

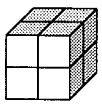
a)



V = _____

S = _____

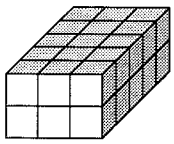
b)



V = _____

S = _____

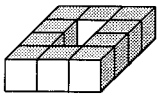
c)



V = _____

S = _____

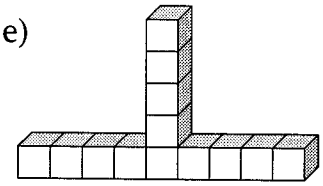
d)



V = _____

S = _____

e)



V = _____

S = _____

(Continued on back.)

Follow-up Student Activity (cont.)

4 Write a step-by-step explanation of the methods you used to count the volume of structure 3c) on the previous page.

5 Suppose that each of the 3 "arms" of structure 3e) grew to have 100 cubes. What would be the surface area of that structure? Write a step-by-step explanation of the methods you used to determine this.

6 Use cubes to build Structures a)-f) which are described below. Then, on another sheet, do the following for each structure:

- Sketch the structure (note: you may find more than 1 structure that satisfies each set of conditions). Next to each sketch record the surface area and volume of the structure.
- Cut out at least 1 jacket for the structure (remember, a jacket exactly covers the entire surface of a structure, with no gaps or overlaps). Label each jacket with the letter of the structure and staple or tape the jacket next to your sketch.

Structure a): This structure contains exactly 12 cubes and has a surface area less than 34.

Structure b): This structure has surface area 30. It is not a rectangular solid.

Structure c) and Structure d): These 2 rectangular solids have the same volume but different surface areas.

Structure e) and Structure f): These 2 rectangular solids have different volumes but the same surface area.

Follow-up Student Activity 2.1

NAME _____ DATE _____

1 In the class activity we discovered that solids with the same volume can have different surface areas, and that surface area can get infinitely large without changing the volume.

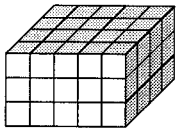
a) On another sheet, explain how the above can be. Use diagrams to illustrate your thinking.

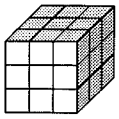
b) Is it possible for solids to have the same surface area but different volumes? If so, give examples. If not, tell why not.

2 Determine the dimensions, surface area, and volume of the rectangular solid shown at the right, based on each of the units given.

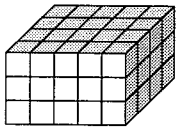
a) 
1 volume unit

Dimensions of solid _____
Surface area of solid _____
Volume of solid _____



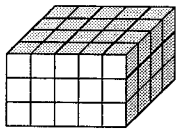
b) 
1 volume unit

Dimensions of solid _____
Surface area of solid _____
Volume of solid _____



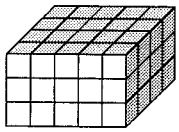
c)
1 linear unit

Dimensions of solid _____
Surface area of solid _____
Volume of solid _____



d) 
1 area unit

Dimensions of solid _____
Surface area of solid _____
Volume of solid _____



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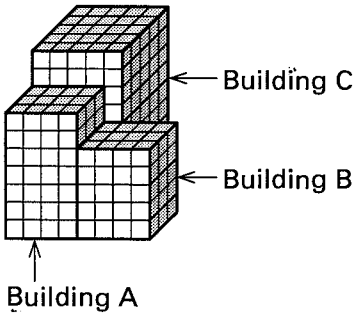
Follow-up Student Activity (cont.)

3 The diagram at the right shows a scale model of 3 buildings. There is no space between buildings and the edge of 1 small cube represents 5 meters. Each layer of cubes represents 1 floor of the buildings. Problems a) and b) that follow refer to these buildings.



a) Suppose a gallon of paint covers 45 square meters. How much paint is needed to paint the tops and sides of the 3 buildings (assuming there are no windows)? On another sheet, give a step-by-step explanation of the methods you use to determine this.

b) Lobbies, elevators, hallways, rest rooms, and storage rooms take up ⅓ of the floor space of each building. The remaining space is rented out as offices. For every 25 square meters of floor space rented, 1 parking space is provided in a nearby parking lot. How many parking spaces are needed for each building? On another sheet, explain the reasoning and methods you use to determine this.

4 Locate a small cereal box (or other similarly sized box) in your home. Then do the following:

a) On another sheet, make a sketch of your box, labeling all dimensions.

b) Using tape and 1-inch grid paper, create a jacket that will exactly cover the box, with no gaps or overlaps. Attach your jacket to this assignment.

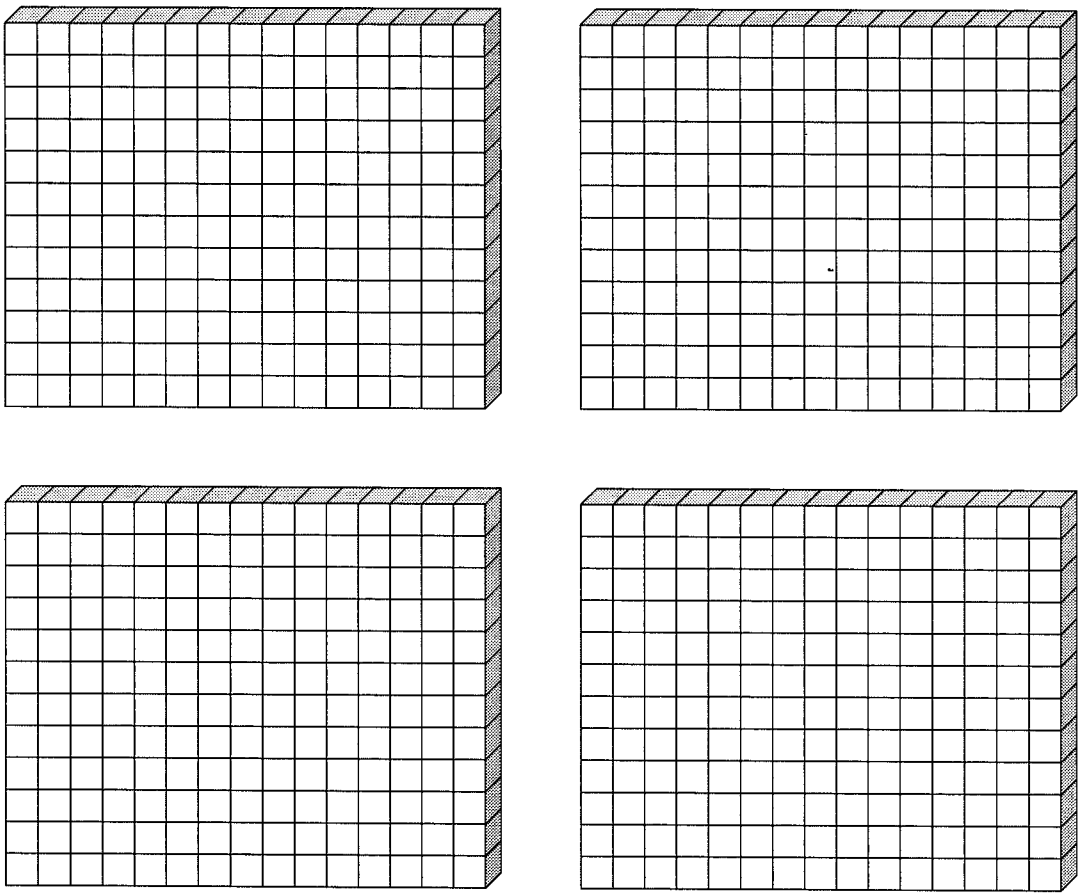
c) Use your jacket to determine the surface area, to the nearest square inch, of the box. Record that measurement on your jacket and write an explanation of your methods.

d) Suppose you need to design a carton (with a top) that will hold 12 of your boxes, with little or no wasted space. On another sheet of paper, draw a diagram to show what the carton will look like when it is flattened (i.e., before it is assembled).

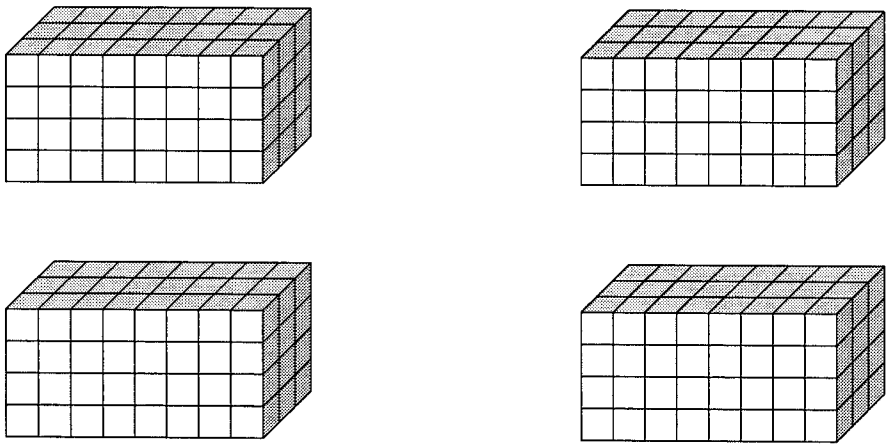
e) Now draw a diagram to show what the carton will look like after it is assembled, again labeling dimensions. Next to your diagram, explain how your boxes will be packed inside the carton, so it is clear they will all fit with little or no wasted space.

Connector Master A

a)



b)

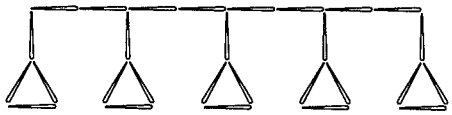


Follow-up Student Activity 3.1

NAME _____ DATE _____

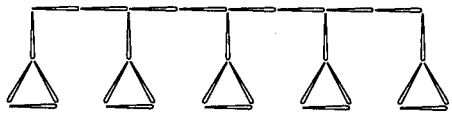
1 a) Describe in words two different methods, other than one-by-one counting, of "seeing" that the total number of toothpicks in the following figure is 28. "Loop" the toothpicks to show your thinking.

Method I



Method II

b) Suppose the row of toothpick arrangements above could be extended, and the row always contains a "hanging triangle" on each end. The row shown contains 5 triangles. Determine the number of toothpicks there will be if the figure is extended to a figure with 150 triangles. Explain your methods and loop the diagram to illustrate your thinking.



c) Write at least one formula for N, the total number of toothpicks in a row that contains T triangles.

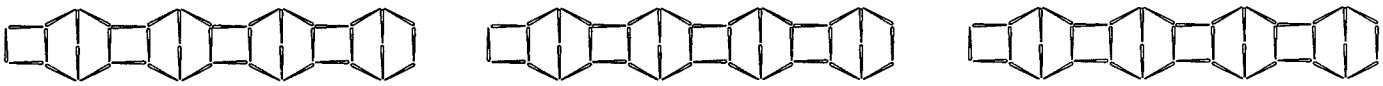
d) A certain row of the above toothpick arrangements contains 580 toothpicks. What is the number of triangles in the figure? Explain the methods you used to decide.

(Continued on back.)

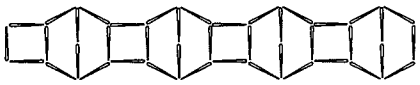
Follow-up Student Activity (cont.)

2 Suppose the row of toothpick arrangements shown below could be extended, and the row always begins with a square and ends with 2 trapezoids.

a) Write 3 different formulas for the total number of toothpicks in a row that contains 5 squares. Loop the diagrams below to show how your formulas work.



b) If a row is extended so that the total number of squares is 138, how many toothpicks will it have? Explain how you decided this. Mark the diagram to help show your methods.



c) If a row is extended so that it has a total of 731 toothpicks, how many trapezoids will it have? Explain your thought processes.

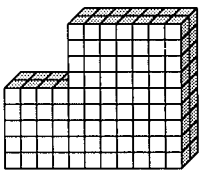
3 On another sheet, create a row of toothpick arrangements that can be extended. Write and answer several interesting questions about the row when it is extended.

Follow-up Student Activity 4.1

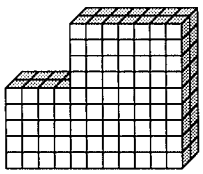
NAME _____ DATE _____

1 Mark each of the following arrangements of cubes to show a different method of "seeing" and counting (other than one-by-one) the volume of the arrangement. Then write an equation to represent each of your counting methods.

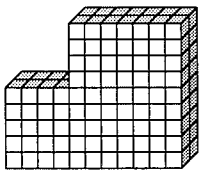
a)



V =

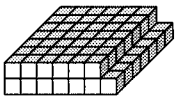


V=

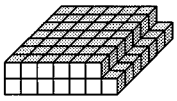


V=

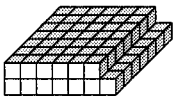
b)



V =

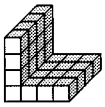


V=

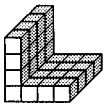


V=

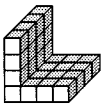
c)



V =



V=



V=

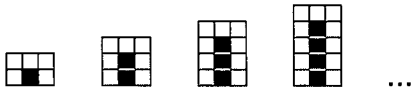
2 Find the surface area of the 3 different arrangements shown above and write equations to represent your methods of counting each surface area.

a) A = b) A = c) A =

(Continued on back.)

Follow-up Student Activity (cont.)

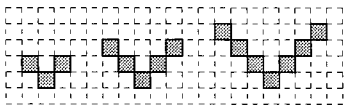
3 The first 4 arrangements in a sequence are shown below. Write your answers to a) and b) on another sheet of paper.

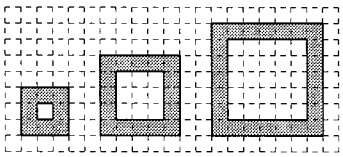


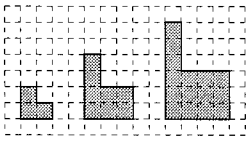
a) Assuming this pattern continues, how many white squares are there in the arrangement that has 100 black squares? Explain the reasoning you used to determine this.

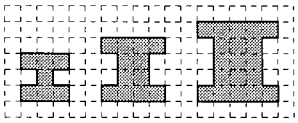
b) Is there an arrangement in the above pattern of black and white squares that has exactly 46 white squares? If so, which one and how can you be sure? If not, why not?

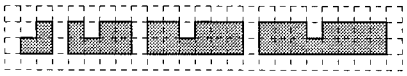
4 Each of a)-e) below shows the first 3 or 4 arrangements in a sequence.

a) 

b) 

c) 

d) 

e) 

On a sheet of ¼" grid paper, do the following for *at least 3* of the above sequences:

i) sketch the given arrangements and what you think is the most logical next arrangement in that sequence;

ii) write in words a description of what you imagine the 20th arrangement looks like, and tell how you used the first 3 arrangements to determine this;

iii) explain 2 different ways of "seeing" and counting (other than one-by-one) the squares in the 53rd arrangement;

iv) write at least one formula for S, the number of squares in the *n*th arrangement of the sequence.

Focus Student Activity 5.1

NAME _____ DATE _____

1 Fill in the missing numbers:

	Total No. of Pieces	No. of Red Pieces	No. of Black Pieces	Net Value
a)		6		+3
b)		2	7	
c)	12		5	
d)		4		0
e)	10			0
f)	7			+7
g)			8	+3
h)	13			−5

2 Suppose that:

Collection X contains 2 red and 7 black pieces;
Collection Y contains 8 red and 5 black pieces; and
Collection Z contains 7 red and 3 black pieces.

a) Record the net value of collection X:_____, Y:_____ , Z:_____.

b) Record the net value if collections X and Y are combined: _____.

c) Record the net value if collections Y and Z are combined: _____.

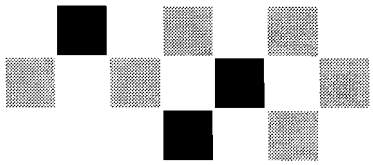
d) Record the net value if collection X and the opposite of collection Y are combined: _____.

Follow-up Student Activity 5.2

NAME _____ DATE _____

Counting pieces will be helpful for this activity. Remember that a minus sign indicates a red net value and a plus sign indicates a black net value.

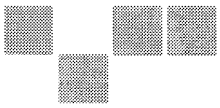
1 Sketch the collection that is the opposite of Collection A (shown below). Write the net value of both collections.



Collection A

Opposite of Collection A

2 Shown below are the red pieces from a collection whose net value is 5. Sketch the black pieces needed to complete the collection.



3 A certain counting piece collection has 12 pieces, 9 are red. Sketch the collection and write its net value.

4 A certain counting piece collection has 7 black pieces and a net value of 0. Sketch the total collection.

(Continued on back.)

Follow-up Student Activity (cont.)

5 A collection has 13 pieces and a net value of $^{-}3$. Draw a diagram of the collection and explain how you determined the number of red pieces in the collection.

6 Sketch 4 different collections with net value $^{-}4$.

7 Suppose collections A, B, and C contain the following counting pieces:

Collection A contains 3 red and 9 black pieces.
Collection B contains 11 red and 5 black pieces.
Collection C contains 8 red and 4 black pieces.

a) Record the net value of A_____, B_____, C_____.

b) Record the net value if collections A and B are combined_____.

c) Record the net value if all three collections are combined_____.

d) Record the net value if collection B and the opposite of Collection C are combined_____.

8 On another sheet of paper describe 4 interesting situations from life outside of school that can be modeled using the red and black counting pieces. Next to each situation sketch a counting piece model of the situation, and write two or more mathematical statements you can make by observing your model.

Follow-up Student Activity 6.1

NAME _____ DATE _____

1 Use your black and red pieces to find the following sums and differences. Then, on another sheet of paper, sketch diagrams to show your counting piece procedures and results.

a) $^{+}7 + {}^{-}4$

b) $^{-}3 + {}^{-}8$

c) $^{-}9 + {}^{+}5$

d) $^{-}6 + {}^{-}6$

e) $^{+}8 - {}^{+}5$

f) $^{-}5 - {}^{+}6$

g) $^{+}4 - {}^{-}7$

h) $^{-}3 - {}^{+}2$

2 For each of the following, imagine counting piece collections and mentally compute the answers. Then, on another sheet, write a sentence or two to explain your mental methods for each problem.

a) $^{+}45 + {}^{-}25 =$

b) $^{-}80 - {}^{+}55 =$

c) $^{-}35 - {}^{+}75 =$

d) $^{+}100 + {}^{-}42 =$

3 On another sheet, sketch a model of each of these situations and use your model to determine the requested information. Next to each sketch, write an equation (or equations) that represents the thought processes you used to find the answer.

a) The student store's profit/loss statement for last week showed the following: Monday, profit, \$3; Tuesday, loss, \$5; Wednesday, loss, \$7; Thursday, broke even; Friday, profit, \$6. The students had set a goal of earning \$45 profit for the week. Determine how close they are to their goal.

b) Between midnight and noon a submarine cruises at $^{-}200$ feet, then dove down 150 feet further, climbed up 115 feet, dove 180 feet, and finally climbed up 100 feet. Determine the difference between the highest and lowest positions of the submarine during this 12 hour period.

(Continued on back.)

Follow-up Student Activity (cont.)

4 Teach an adult how to add and subtract integers by completing the following steps. Ask the adult to read Problems 4-6 on this sheet, so they will know what you have planned and their role.

- Show how to use the red and black counting pieces and the meanings of addition and subtraction to answer these problems:

a) $^{-}3 + {}^{+}7 =$ c) $^{-}2 - {}^{-}2 =$ e) $^{+}4 - {}^{-}3 =$

b) $^{-}5 + {}^{-}4 =$ d) $^{-}6 - {}^{-}5 =$ f) $^{-}8 - {}^{+}2 =$

- Be sure to carefully model and explain your methods so the adult can understand *why* you do what you do.
- Tell some generalizations that we discussed in class or conjectures you have made. Try to convince the adult why you think these work.
- If you get stuck on a problem or idea, ask the adult *not* to show you how to solve it. Instead, leave that problem and move on. Later, try the problem again. If you are still stuck, write down your questions and "stuck points" to bring back to class.

5 Explain to the same adult how you can *imagine* black and red counting pieces to help you mentally compute the following:

a) $^{+}27 + {}^{-}19$ b) $^{-}137 + {}^{-}63$ c) $^{-}25 - {}^{-}75$

6 On another sheet of paper, do the following:

a) Tell the name of the adult you taught and how much time you spent teaching.

b) Tell what generalizations or conjectures you shared with them.

c) Describe any problems you couldn't solve, or ideas that were difficult to explain.

d) Describe any AHA!s or insights that you had while you were teaching or preparing to teach.

e) Have the adult write at least two "I appreciate..." statements about your presentation. Attach those to your assignment.

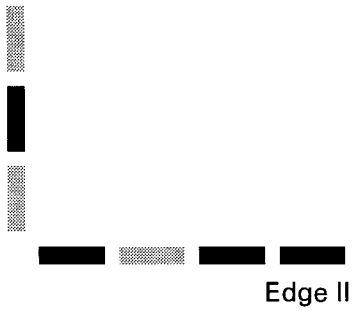
Follow-up Student Activity 7.1

NAME _____ **DATE** _____

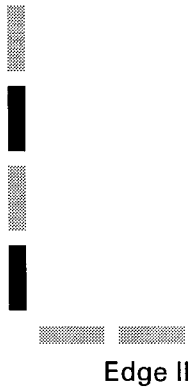
Write your responses to the following problems on separate paper. State each problem next to your responses. You may find it helpful to use your red and black counting pieces to form the arrays before making sketches.

1 Sketch the red and black counting piece arrays determined by the following collections of edge pieces.

a) Edge I



b) Edge I

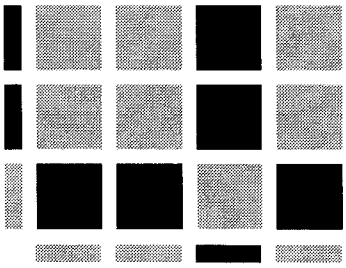


2 Label the net value of each edge and each array in Problem 1.

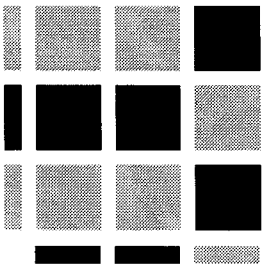
3 Sketch 2 different arrays, with edges, so that Edge I of each array has net value $^{-}4$, Edge II of each array has net value $^{-}3$, and the 2 arrays contain different numbers of counting pieces. On your sketches record the net value of each array.

4 For each of the following arrays, sketch the minimal array whose edges have the same net values as the edges of the array shown.

a)



b)



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Follow-up Student Activity (cont.)

5 For each of the following integers, sketch all the different possible minimal arrays of counting pieces, with edges, so the net value of the array is the given integer. Label the net value of each array and the net value of each edge. (Assume arrays are different if their edges have different net values.)

a) $^{+}18$ b) $^{-}9$ c) $^{-}13$ d) $^{+}16$ e) $^{-}24$

6 Sketch 3 different arrays whose net values are zero. Include the edges of each array in your sketches.

7 Sketch diagrams of the following Mystery Minimal Arrays, including their edges. Label the net value of each array and the net value of its edges. In some cases there is more than 1 correct array; try to find all possibilities. In other cases, it is not possible to form the array; if so, explain why.

a) Mystery Minimal Array A has net value $^{-}15$. One edge of this array has net value $^{+}5$.

b) The edges of Mystery Minimal Array B have values $^{-}7$ and $^{-}2$.

c) Mystery Minimal Array C contains a prime number of black counting pieces. It contains more than 17 pieces and less than 36 pieces.

d) The number of red counting pieces in Mystery Minimal Array D is both an even number and a prime number.

e) There are an odd number of black squares in Mystery Minimal Array E. One edge of the array has net value $^{+}2$.

f) Mystery Minimal Array F is square and one edge has value $^{+}7$.

g) The net value of Mystery Minimal Array G is $^{-}42$ and one edge has value $^{-}14$.

h) One edge of Mystery Minimal Array H is red and the other edge is black. The array has net value $^{+}12$.

8 Explain in your own words what you think an integer is. Give several examples of numbers that are integers and several that are not integers.

Follow-up Student Activity 8.1

NAME _____ **DATE** _____

Write your responses to the following problems on separate paper. State each problem with your responses.

1 Sketch a minimal array with edge pieces to illustrate each product and quotient below. Label each array to show the net values of the array and its edges.

a) $^{-}5 \times {}^{-}4$ c) $2 \times {}^{-}3$ e) ${}^{-}20 \div 4$ g) $18 \div {}^{-}9$

b) 7×4 d) ${}^{-}6 \times 2$ f) ${}^{-}20 \div {}^{-}2$ h) $20 \div 4$

2 Write 1 or 2 paragraphs that explain how the signs of two signed numbers are related to the sign of their product and the sign of their quotient. Make diagrams of counting pieces to support your explanation.

3 Use diagrams of red and black counting pieces and brief explanations to show why you think each of the following must be true:

a) multiplication of any number by 0 gives a product of 0,

b) $7 \div 0$ is not possible,

c) $0 \div 0$ has an infinite number of solutions (and therefore is not allowed).

4 Use your red and black counting pieces to show an adult how to multiply and divide signed numbers. Before you start, ask the adult to read Problems 4-6 on this assignment, so they will know what you have planned, and so they will know their role.

Remember, if you get stuck on a problem or idea, ask the adult *not* to show you how to solve it. Instead, leave that problem and move on. Later, try the problem again. If you are still stuck, write down your questions and "stuck points" to bring back to class.

(Continued on back.)

Follow-up Student Activity (cont.)

a) Make up 2 or 3 example arrays, with edges, and show how edge pieces are used to indicate whether to "flip" over a row or column.

b) Show how the area method of multiplication works to find the product $^{+}3 \times {}^{+}5$.

c) Show how to use counting pieces to determine these products:

i) $^{-}2 \times {}^{+}4$ ii) $^{-}4 \times {}^{-}3$ iii) $^{+}5 \times {}^{-}2$

d) Show how the area method of division works to find the quotient $^{+}18 \div {}^{+}3$.

e) Show how to use counting pieces to find these quotients:

i) $^{-}24 \div {}^{+}6$ ii) $^{-}15 \div {}^{-}5$ iii) $^{+}14 \div {}^{-}7$

f) Tell some generalizations about products and quotients of signed numbers that we discussed in class, or conjectures you have made. Try to convince the adult why you think these work.

5 Explain to the same adult how you mentally compute the following:

a) $^{-}17 \times {}^{-}9$ b) $12 \times {}^{-}15$

6 Write a summary of your "teaching experience" as follows:

a) Tell the name of the adult you taught and how much time you spent teaching.

b) Tell what generalizations or conjectures you shared with them.

c) Describe any problems you couldn't solve or ideas that were difficult to explain.

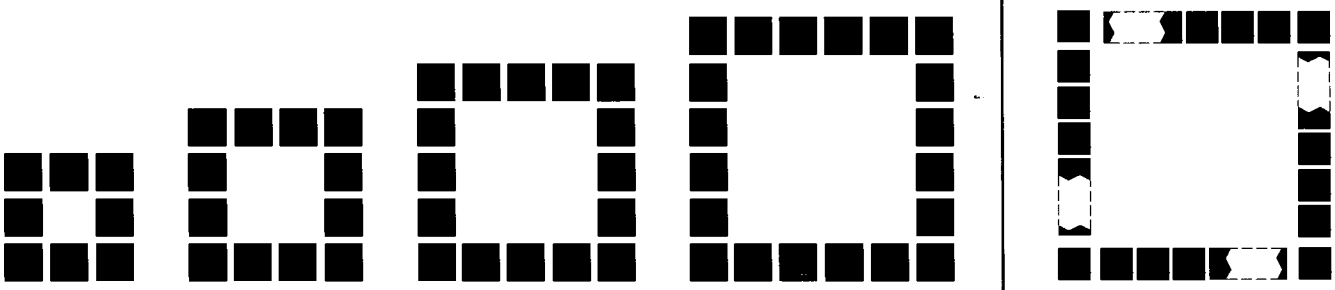
d) Describe any AHA!s or insights that you had while you were teaching or preparing to teach.

e) Have the adult write at least 2 "I appreciate..." statements about your presentation. Attach these to your assignment.

Connector Master A

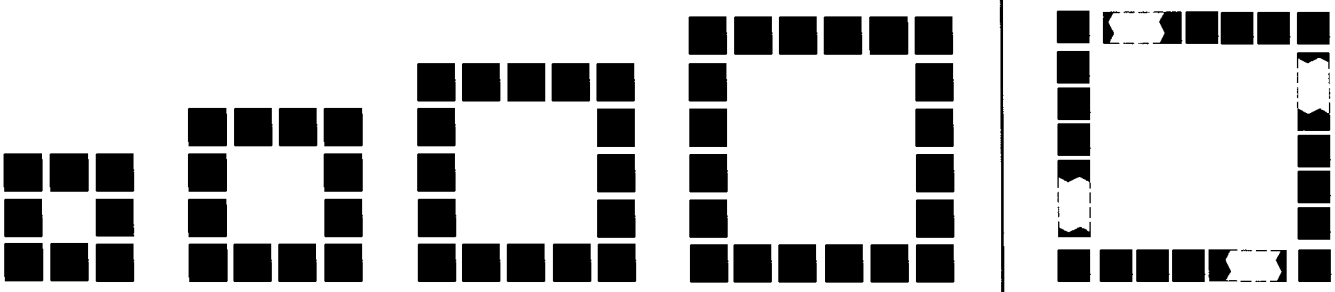
Arrangement

1st 2nd 3rd 4th 20th



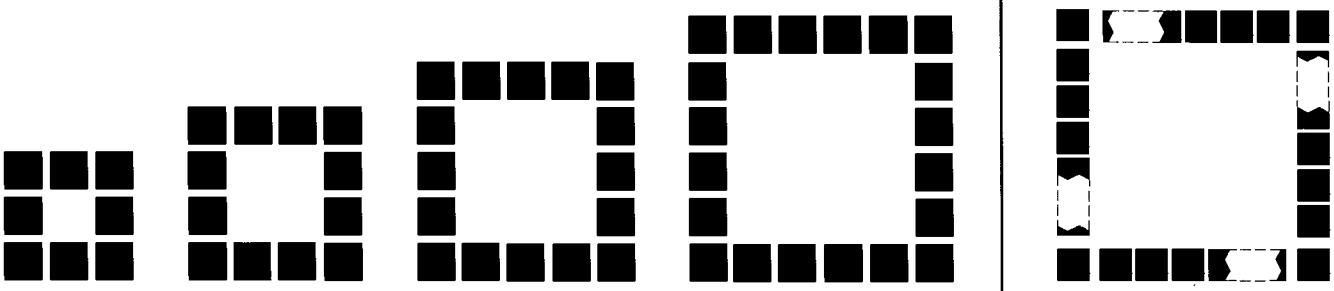
Arrangement

1st 2nd 3rd 4th 20th

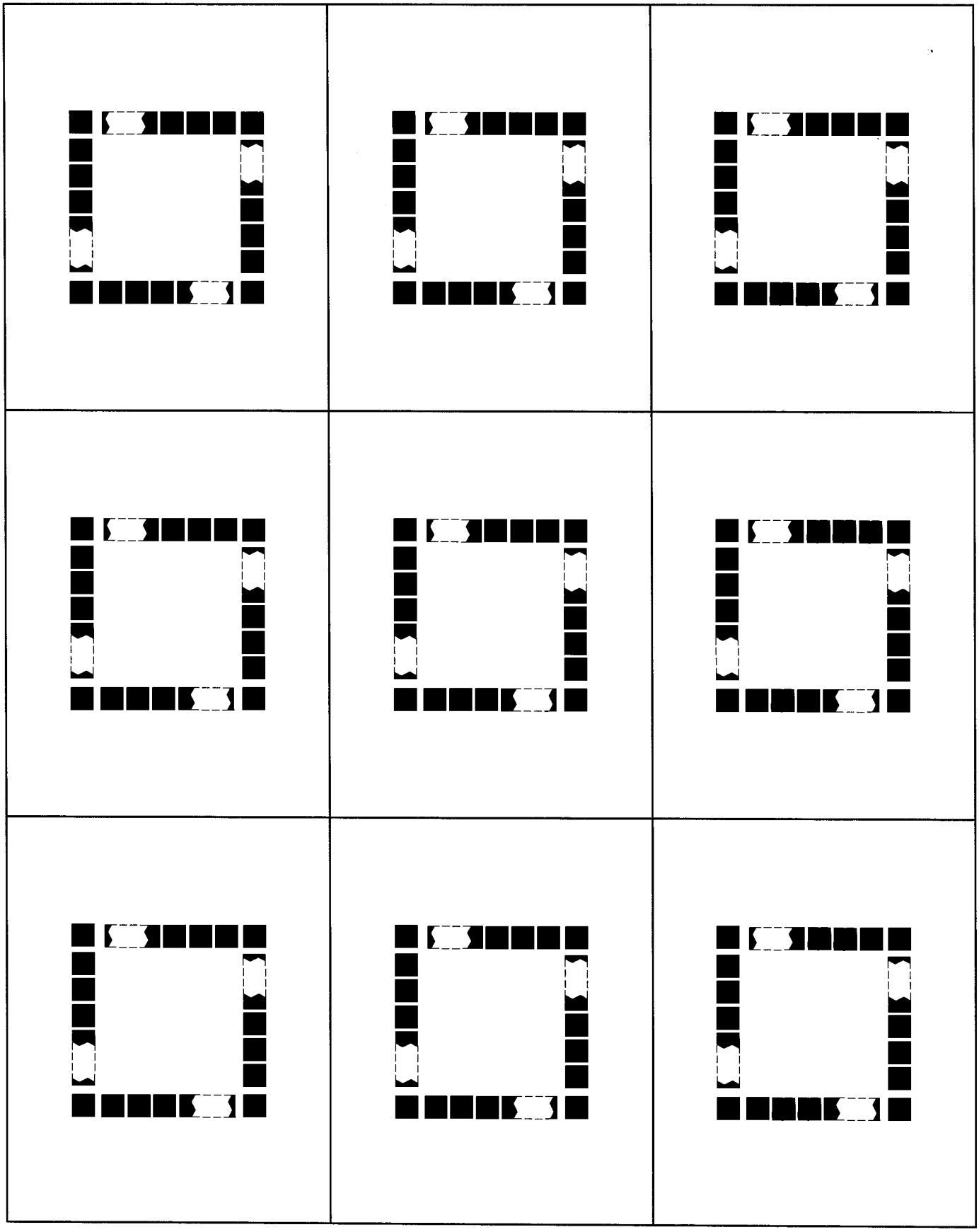


Arrangement

1st 2nd 3rd 4th 20th



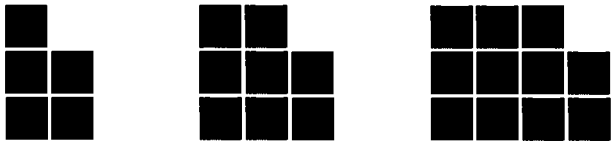
Connector Master B



Focus Student Activity 9.2

NAME _____ **DATE** _____

a) Draw the next arrangement in the following sequence:



1st 2nd 3rd 4th

Answer the following on separate paper. Be sure each problem is shown with your answer.

b) Describe, using words only, the 25th arrangement of the above sequence, so that someone reading your description could accurately build it. How many tile does it contain?

c) Make 3 copies of a rough sketch of the *n*th arrangement. Loop your sketches to show 3 different methods of "seeing" and counting (other than one by one) the tile in the *n*th arrangement. Then, based on your counting methods, next to each sketch write a formula for *T*, the number of tile in the *n*th arrangement.

d) A certain arrangement contains 500 tile. Michael used a sketch of the *n*th arrangement to help him determine which arrangement this is, and he didn't "guess and check" numbers. Show and/or explain how you think Michael could have reasoned from a sketch.

e) Show how to use the visual method you described for d), or a different visual method, to find which arrangement contains 212 tile.

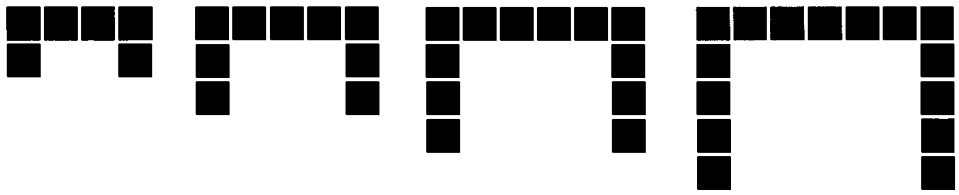
f) On a sheet of ¼" grid paper, sketch and label a coordinate graph to show the number of tile in the first 8 arrangements. Next to your graph, write several observations about relationships you notice in the graph, and predict the ordered pair that is associated with the 100th arrangement.

g) (Challenge) Two arrangements together contain 160 tile. One of the arrangements contains 30 more tile than the other. Show and explain how to determine which arrangements these are, by reasoning from sketches of the arrangements and without using guess and check methods.

Follow-up Student Activity 9.3

NAME _____ DATE _____

1 Examine the following sequence of tile arrangements.



Answer the following on separate paper:

a) Sketch 3 copies of the 5th arrangement in this sequence. Loop each sketch to show a different way of seeing and counting (other than one by one) the number of tile in the arrangement. Under each sketch, write a number statement to represent your counting methods.

b) Describe, in words only, the 50th arrangement, so the reader could accurately build it. How many tile does it contain?

c) On another sheet make 3 or more copies of a rough sketch of the *n*th arrangement. Loop each diagram to show different methods of seeing and counting the number of tile in the *n*th arrangement. Based on your counting methods, next to each sketch write a formula for *T*, the number of tile in the *n*th arrangement.

d) On a sheet of ¼" grid paper, draw a coordinate graph showing the number of tile in the first 6 arrangements of the above sequence. Then write at least 3 observations about mathematical relationships you notice in your graph and tell the ordered pair associated with the 75th arrangement.

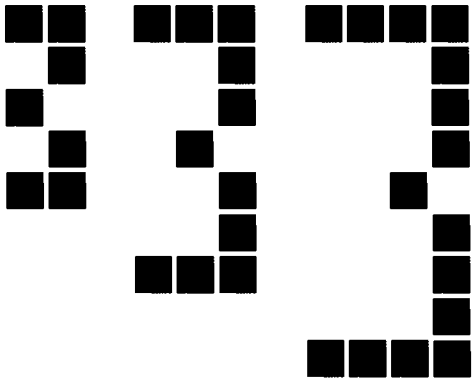
(Continued on back.)

Follow-up Student Activity (cont.)

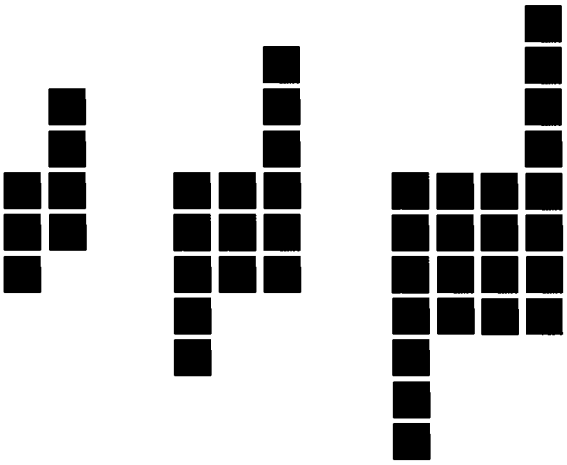
2 Copy each of these sequences of tile arrangements on a separate sheet of paper. Next to each sequence, write as many mathematical observations as you can about the sequence. Include formulas and graphs. Loop diagrams of the *n*th arrangement to show how your formulas work.

a)

b)



c)

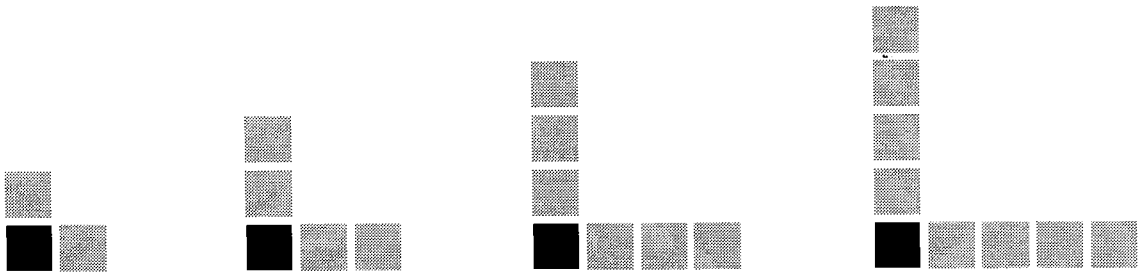


d)

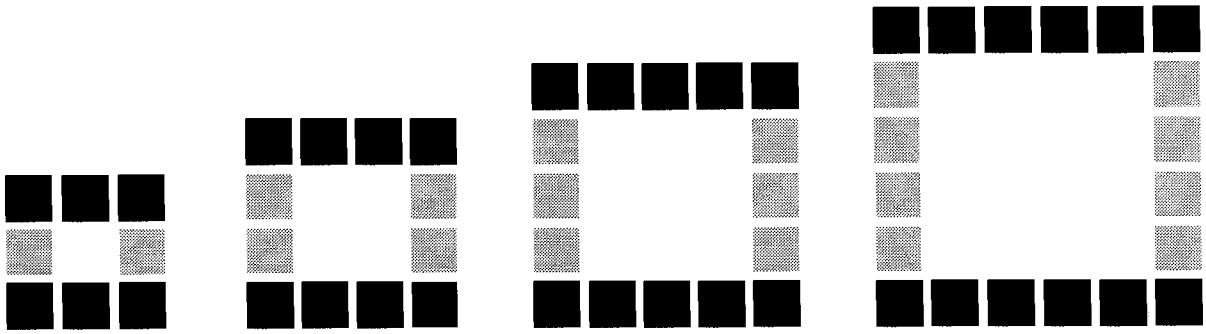
Focus Master A

1)

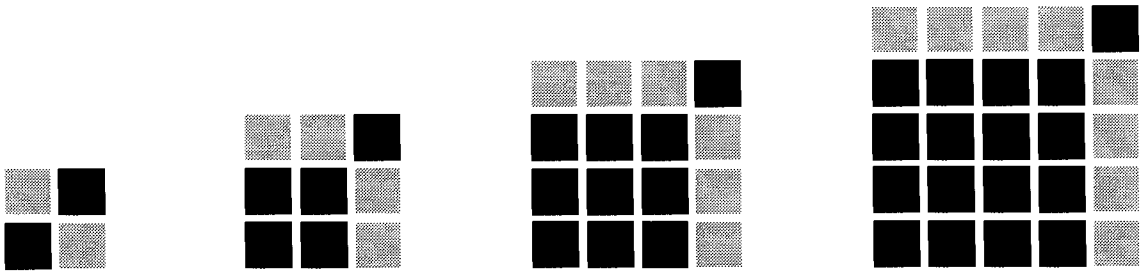
2)



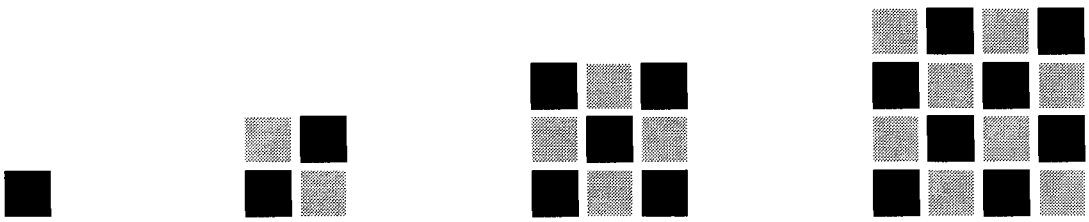
3)



4)



5)



Focus Master C

Situations Involving Patterns

Each of Situations 1-7 on the next page can be represented by a sequence of arrangements of red and/or black tile. Do the following for each situation assigned by your teacher.

a) Form the first 4 or 5 arrangements in a sequence representing that situation. Then sketch these arrangements on a sheet of paper and label the value of each arrangement. Explain how your diagram is related to the situation.

b) Describe what the 25th arrangement would look like, and tell what its value represents in terms of the situation.

c) Write a formula to show the value of the nth arrangement.

d) Draw a coordinate graph showing $v(n)$ for n equal to 1, 2, 3, 4, and 5.

e) Predict what the shape of the graph would be if you graphed $v(n)$ for n equal to 1 through 100. Explain your reasoning.

Focus Master C (page 2)

1) Tyrone earns $2 for every seedling tree that he digs and wraps at a nursery.

2) Gabriela's teacher gives a prize to the first student who predicts the daily secret number. Gabriela noticed a pattern in the secret numbers: triple the day of the month and subtract 4.

3) Marissa earns $4 per hour for babysitting. For every $4 she earns, she gives her mom $2 to repay money she borrowed to buy a new bike.

4) Each year on her birthday Meghan's grandpa gives her 4 times as many dollars as her age. She always puts ¾ of this in the bank.

5) Sasha has a computer game that requires him to start by picking a number. That number is used to determine the number of points he is assigned at the start of the game. The first coordinate of each of the following ordered pairs is a number he picked, and the second coordinate tells the points he was assigned for that number: (1, 5), (2, $^{-}$8), (3, 11), (10, $^{-}$32), (25, 77), (100, $^{-}$302).

6) To baby-sit Jason charges a $5 fee plus $1 per hour. Winston charges a $1 fee plus $5 per hour. Each of them rounds up fractional parts of hours (e.g., 2 hours and 10 minutes is charged as 3 hours). (Hint: use 2 sequences and 2 graphs to represent this situation.)

7) Sean's aunt said he could choose between two formulas for deciding how she would pay him for picking apples: $P = 5n - 1$ or $P = 3n + 5$, where P is the amount of pay he will receive and n is the number of boxes of apples he picks.

8) You make up a situation that could be represented by a sequence of arrangements. Complete parts a)-e) from the previous page regarding your situation.

Follow-up Student Activity 10.1

NAME _____ **DATE** _____

1 For each of sequences i)-v) below, do the following on a sheet of ¼" grid paper:

a) Sketch the first 5 arrangements in the sequence.

b) Write a description, using words only, of the 50th arrangement in the sequence so someone reading your description could form the arrangement with no other information.

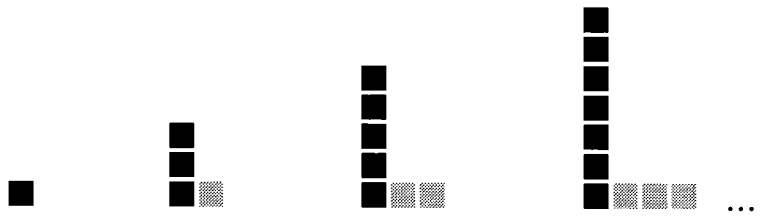
c) Write at least 2 different algebraic formulas for $v(n)$, the net value of the nth arrangement. Make sketches of the nth arrangement and "loop" your sketches to show how you "see" each formula.

d) Make a coordinate graph showing $v(n)$ for the first several arrangements in the sequence. Under your graph, describe what you think would be the shape of the graph if you plotted $v(n)$ for n equal to 1 through 100. Explain your reasoning.

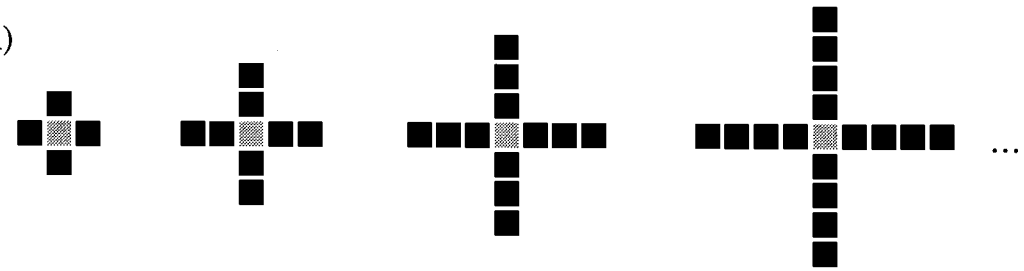
e) Next to your graph, tell what would be the ordered pair of the 37th and 200th arrangements in the sequence, and show how you determined these.

f) Near your graph describe relationships you see between the sequence and its graph.

i)



ii)



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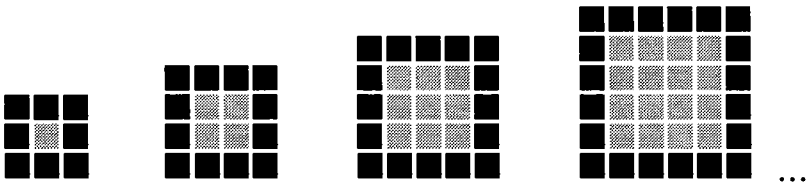
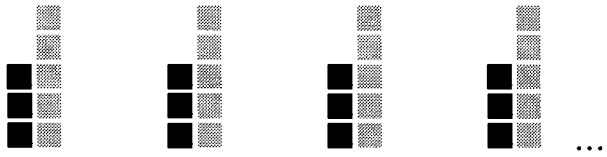
Follow-up Student Activity (cont.)

iii)



iv)

v)



2 Suppose the value $v(n)$ of the nth arrangement in a sequence is $5n + {}^{-}3$. Do the following on a sheet of grid paper:

a) Sketch what you think could be the first 4 arrangements in the sequence.

b) Under the sequence, make a coordinate graph of $v(n) = 5n + {}^{-}3$.

c) Next to your graph, write several observations about the sequence and its graph.

3 Using your red and black counting pieces, form the first 4 arrangements in one of Sequences i)-v) from Problem 1. Get an adult to share with you how they "see" the 20th arrangement in the sequence. Then share with them ways you see the 20th arrangement and how you see formulas for the nth arrangement. If they are interested, repeat for other sequences from Problem 1.

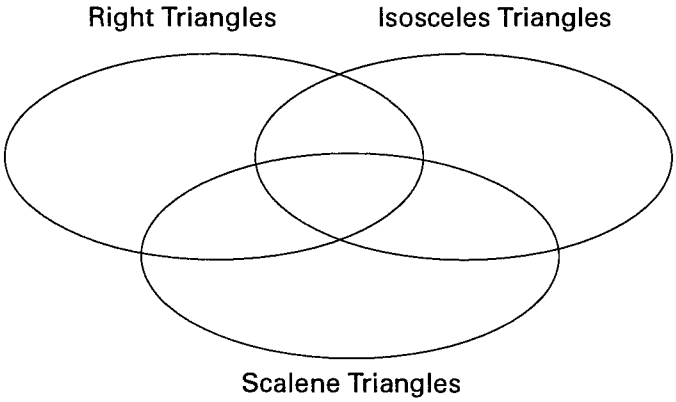
On another sheet, write a summary of what happened. (Hint: it will help to pick an adult that you have taught about operations with the red and black pieces; otherwise, you will need to teach them about the pieces first.)

Follow-up Student Activity 11.1

NAME _____ **DATE** _____

1 On a sheet of 16-pin Geoboard Recording Paper, record 20 non-congruent triangles, 1 triangle on each geoboard. Write the area of each triangle on the geoboard. Be sure to identify your area unit.

2 Classify each of the 20 triangles in Problem 1 by writing the letter of the triangle in the appropriate subregion of the following Venn diagram.



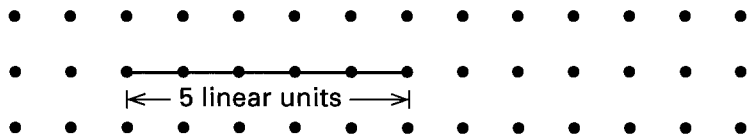
Write several observations about relationships illustrated by your diagram.

3 On another sheet of paper, sketch at least 2 different Venn Diagrams containing 2 or more loops to illustrate other ways of sorting and classifying the triangles you formed in Problem 1. Label the classification of each loop and write the letters of the triangles in the appropriate subregions of your diagram.

(Continued on back.)

Follow-up Student Activity (cont.)

4 On a sheet of Geoboard Dot Paper, sketch at least 6 different triangles that have the base shown below and have area 7½ square units. How many of these triangles do you think there are? Explain.

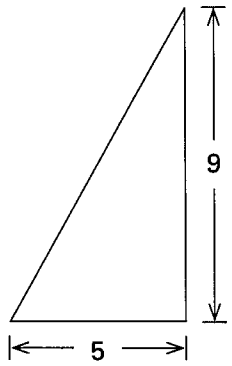


5 On Geoboard Dot Paper, sketch 10 noncongruent *shapes* that have area 3 square units, using one small square as the area unit. Tell how many other shapes with area 3 you think there are and why you feel this way.

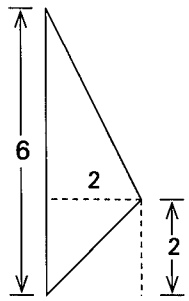
6 On a sheet of 25-pin Geoboard Recording Paper, sketch all the possible noncongruent isosceles geoboard triangles. On each triangle, write its area.

7 The following triangles were "lifted" from geoboard dot paper and then reduced. Record the area of each triangle. Draw lines on each diagram so it is possible to "see" how you determined the area and why that area is correct. Show any computations that you made. If needed, add a few words to clarify your methods. (Note: drawings aren't necessarily to scale, but a few measurements are given as hints.)

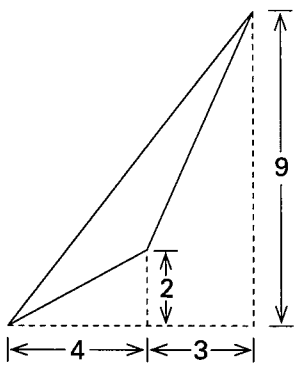
a)



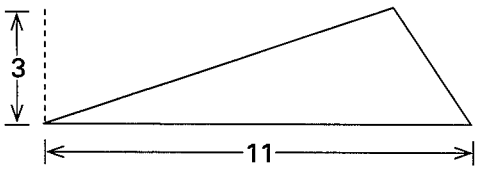
b)



c)

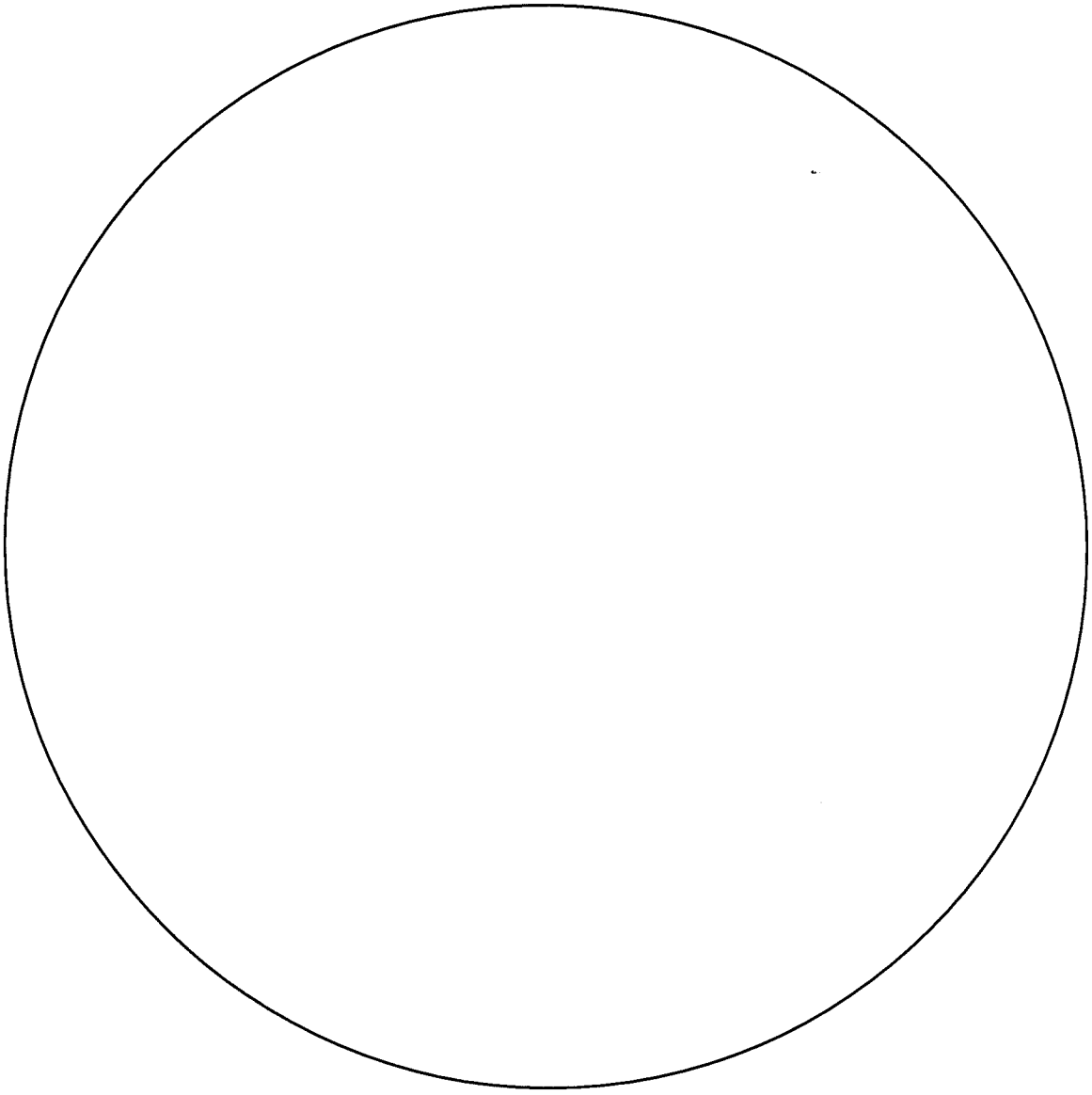


d)



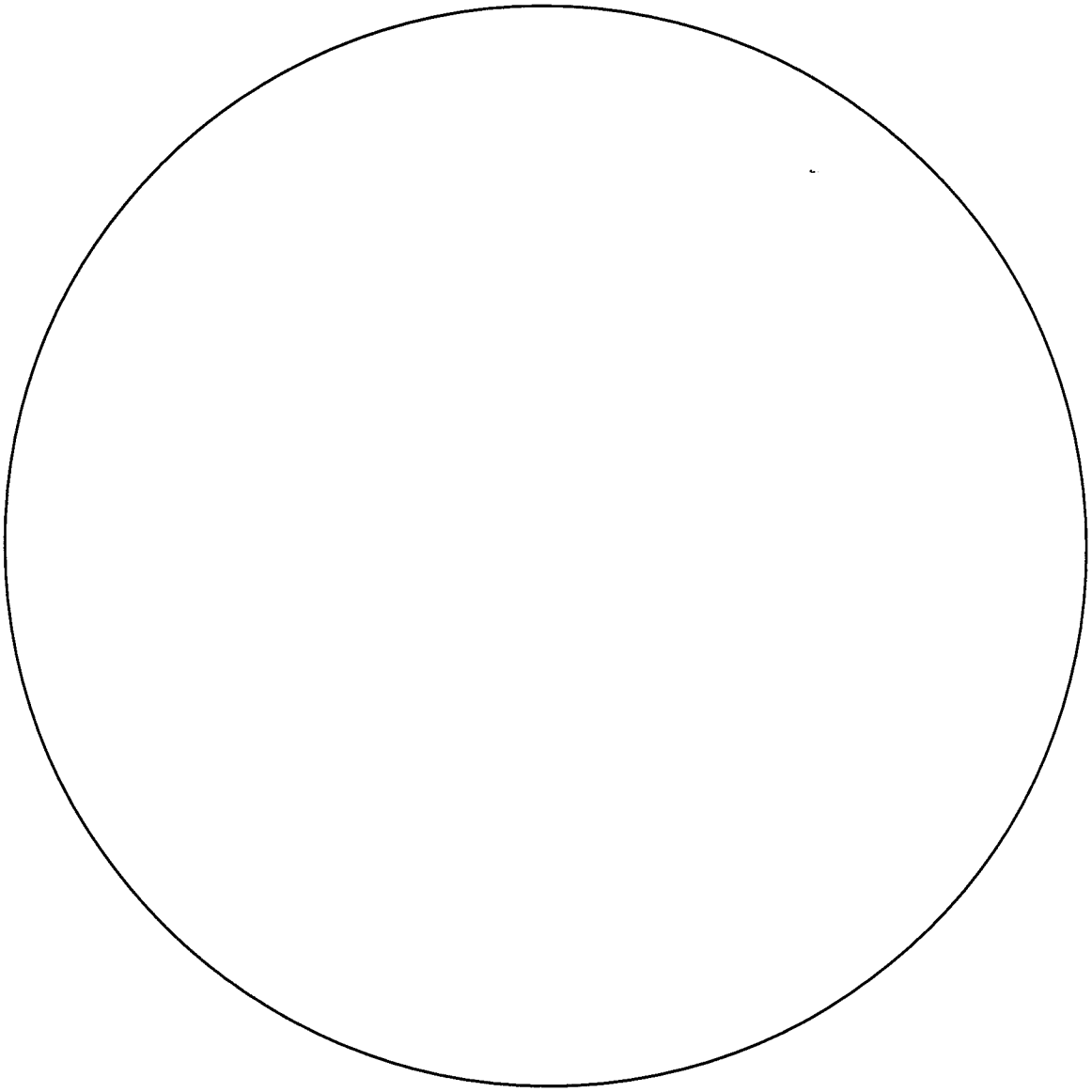
Focus Master A

Circle Pattern

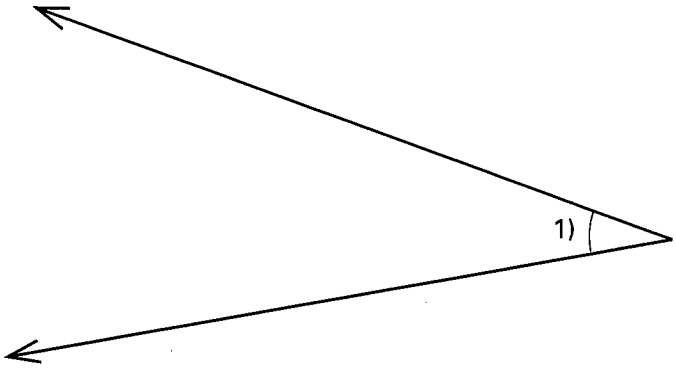


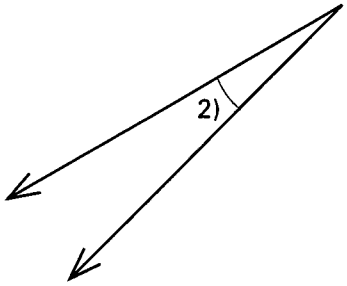
Focus Master A

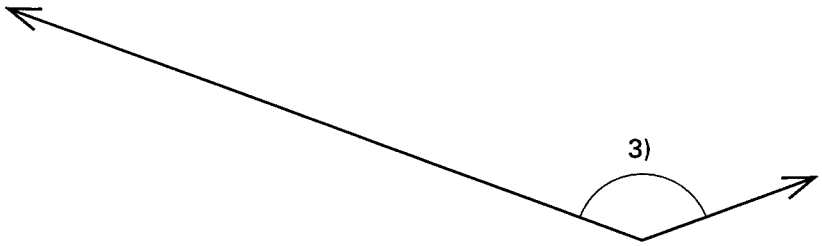
Circle Pattern

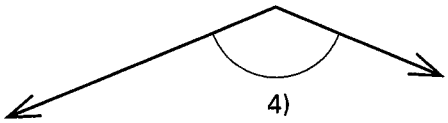


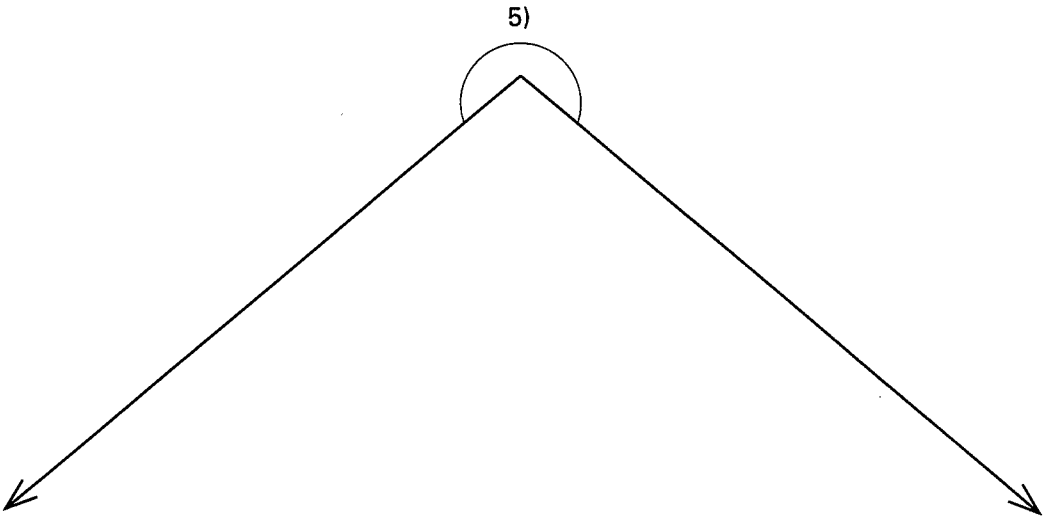
Focus Master B











Focus Student Activity 12.1

NAME _____ **DATE** _____

Complete Columns A, B, and C for Angles 1-5 on Focus Master B. Do not complete Column D until you get a Standard Protractor from your teacher.

Angle	**Column A** Type of Angle	**Column B** "Eyeball" Prediction of Angle Measure	**Column C** Folded Circle Protractor Measurement	**Column D** Standard Protractor Measurement
1				
2				
3				
4				
5				

Follow-up Student Activity 12.2

NAME _____ DATE _____

Complete this assignment on separate paper.

1 For each of the conditions listed below do the following:

- use a standard protractor and straightedge to make drawings which satisfy the conditions, if possible; if it is not possible to satisfy the conditions, explain why;
- label the measure of each angle you draw and show or explain how your drawing satisfies the conditions.

a) Two acute angles such that the sum of their measures equals 90°. (Angles whose sum is 90° are called *complementary angles*.)

b) An acute angle and an obtuse angle such that the sum of their measures equals 180°. (Angles whose sum is 180° are called *supplementary angles*.)

c) Two acute angles which are supplementary angles.

d) An acute angle and an obtuse angle such that the measure of the obtuse angle is 3 times the measure of the acute angle.

e) A quadrilateral with 2 obtuse and 2 acute angles.

f) Two adjacent complementary angles (*adjacent* angles share a side and have the same vertex).

g) A triangle with 2 obtuse angles and 1 acute angle.

h) Two angles such that the measure of one is twice the measure of the other and the sum of their measures is 90°.

i) Two adjacent supplementary angles such that one of them is a right angle.

j) Two acute angles such that the measure of one angle is 82° greater than the measure of the other angle.

(Continued on back.)

Follow-up Student Activity (cont.)

2 For each of a)-f), use your ruler and protractor to construct a polygon, if possible, that satisfies the conditions. Address the 3 points below in your answers.

- if more than one polygon can be formed, tell how many you think are possible and why;
- if impossible, explain why you think this is so;
- record the length of each segment and the measure of each angle in your drawings.

a) a triangle with a 3-cm side between angles of 43° and 64°

b) an acute triangle with angles of 47° and 32°

c) an isosceles trapezoid with exactly 1 right angle

d) a triangle with sides of length 6 cm, 8 cm, and 10 cm

e) a quadrilateral whose angles are all different measures

f) a hexagon whose angles are all equal

3 For each of the following, complete the statement so that it is true. Be sure to label your diagrams.

a) An infinite number of triangles satisfy these conditions... and here are 2 such triangles...

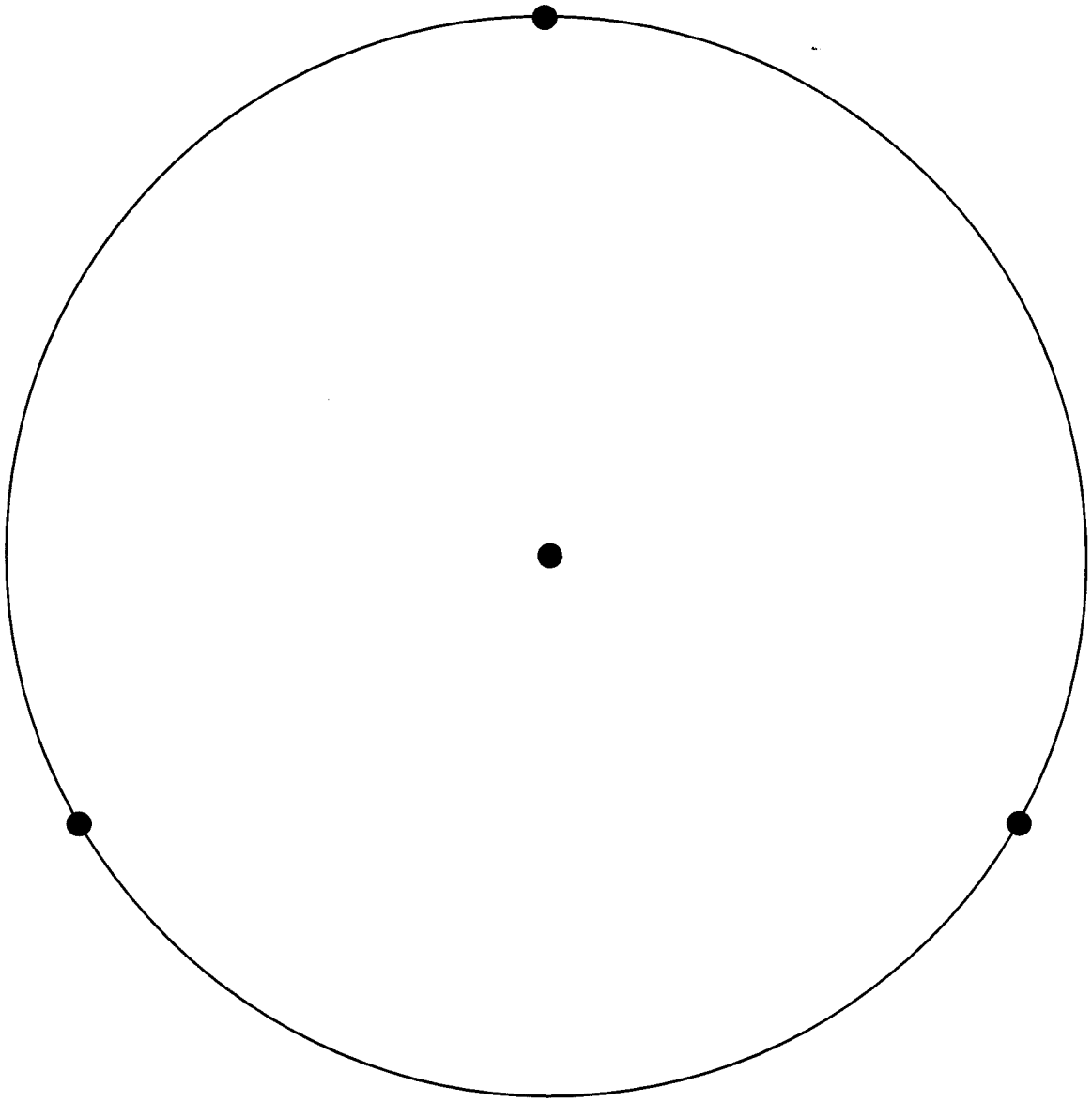
b) These conditions describe a quadrilateral that is impossible to form... and here is why...

c) Exactly 1 pentagon satisfies these conditions... and here is a diagram of the pentagon...

4 Yuji is about to move to this country, and does not know the meaning of the word "angle" or how to measure and construct angles and polygons. Write a letter that will help Yuji understand what an angle is, what the different types of angles are, and how to measure and construct angles and polygons. Be sure to include diagrams in your letter to Yuji.

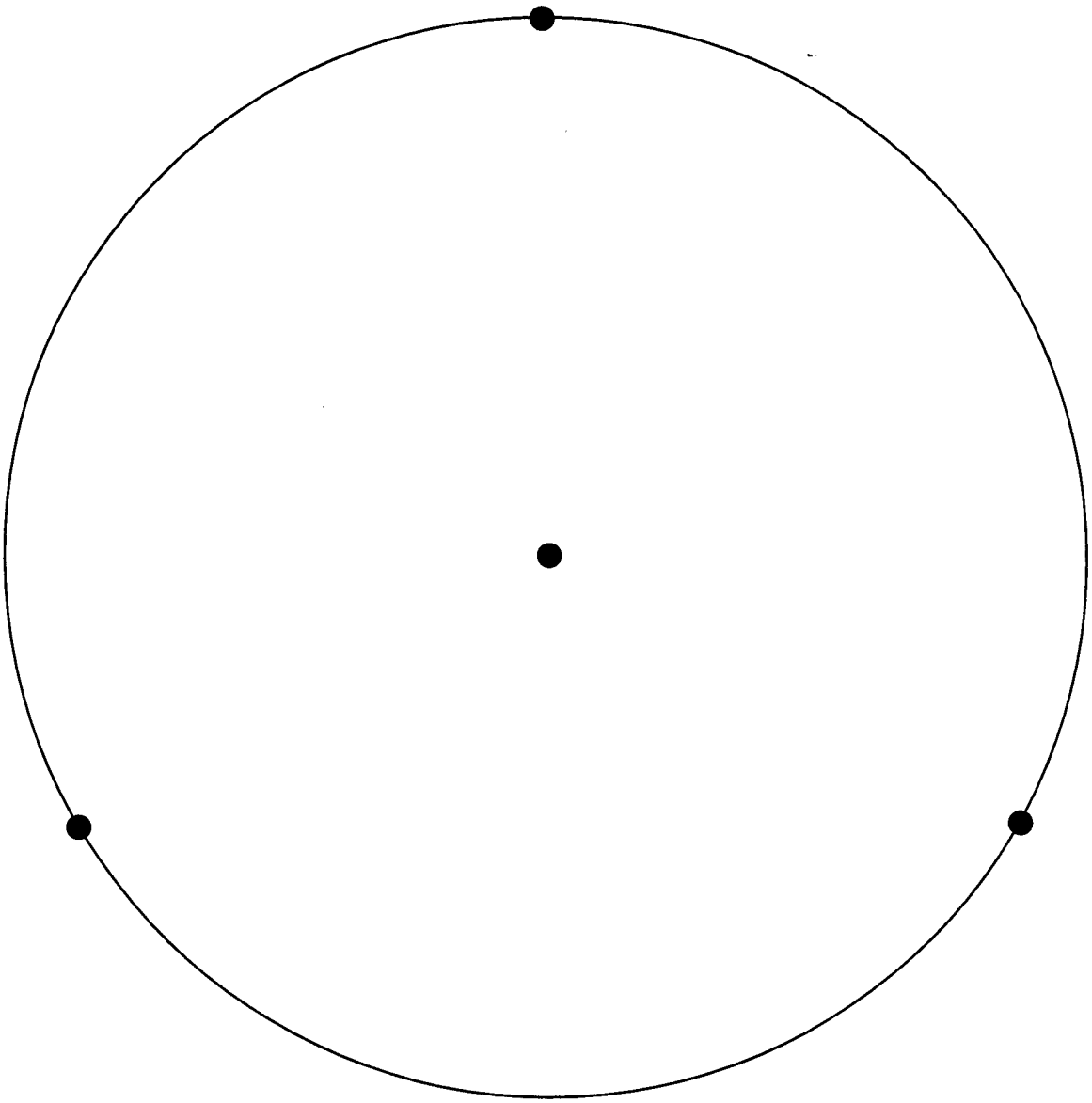
Connector Master A

Circle Pattern



Connector Master A

Circle Pattern



Follow-up Student Activity 13.1

NAME _____ **DATE** _____

1 Cut 3 sheets of 8½" × 11" paper in half. For each of the following sets of polygons, fold a half-sheet one or more times so that when the sheet is unfolded the creases form the given polygons and no others. Attach your folded sheets to this assignment and label each sheet to indicate which polygons you have formed.

a) 4 congruent triangles

b) 32 congruent rectangles

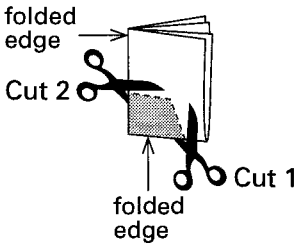
c) 2 congruent trapezoids and 2 congruent triangles

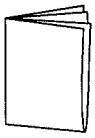
d) 2 isosceles right triangles and 1 rectangle

e) 6 congruent triangles

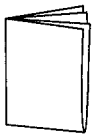
f) 2 isosceles triangles and a pentagon

2 For each of a)-f) below, fold a ¼-sheet of paper in half twice so the folds are perpendicular to each other. Make 2 straight cuts from the folded edges to an inside point (an example is shown at the right) so that the cutout (shaded) section unfolds to form the given polygon. Carefully mark each diagram below to show the location of your Cut 1 and Cut 2 for that polygon. Label your cutout polygons and attach them to this assignment.

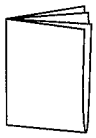


a) 

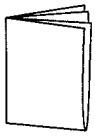
regular octagon

c) 

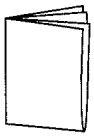
regular hexagon

e) 

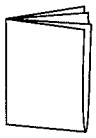
square

b) 

concave octagon

d) 

concave hexagon

f) 

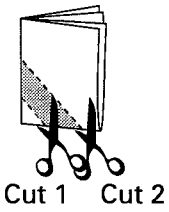
nonsquare rectangle

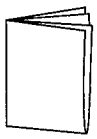
(Continued on back.)

Follow-up Student Activity (cont.)

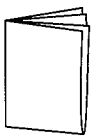
3 Challenge (complete on another sheet): for each of polygons a)-f) from Problem 2, tell the *range* of angle measures for Cut 1 and Cut 2 so as to form *all* the possible polygons of that type. If any lengths also matter, explain.

4 Several different types of "frames" can be formed by double folding a sheet and making 2 straight cuts across the folded edges, as shown in the diagram at the right. For each of the following, carefully sketch the lines of Cut 1 and Cut 2 that will produce the type of frame listed, and then sketch the unfolded frame. If any specific angles or lengths are important parts of your frame constructions, explain.

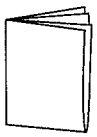


a) 

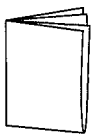
a square inside a square

b) 

a nonsquare rhombus inside a nonsquare rhombus

c) 

a nonsquare rhombus inside a square

d) 

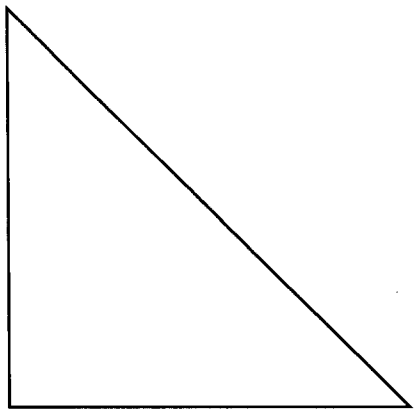
a square inside a nonsquare rhombus

e) Challenge: On another sheet, write instructions using words only to describe the locations of Cut 1 and Cut 2 that will produce *all* of the possible frames that are a square inside a nonsquare rhombus. The reader should be able to produce the frames by following your written instructions and without any additional clues.

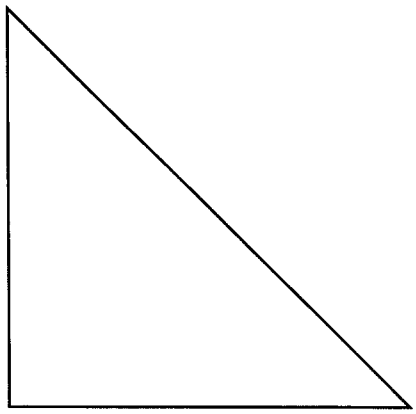
5 Experiment with methods of creating frames from a double-folded sheet using *more than* 2 cuts. For each new frame that you form, sketch the folded sheet with the location of your cuts carefully marked, name the frame (e.g., square inside a square), and sketch the unfolded frame. Be sure to indicate requirements for angles and lengths, if any.

Focus Master A

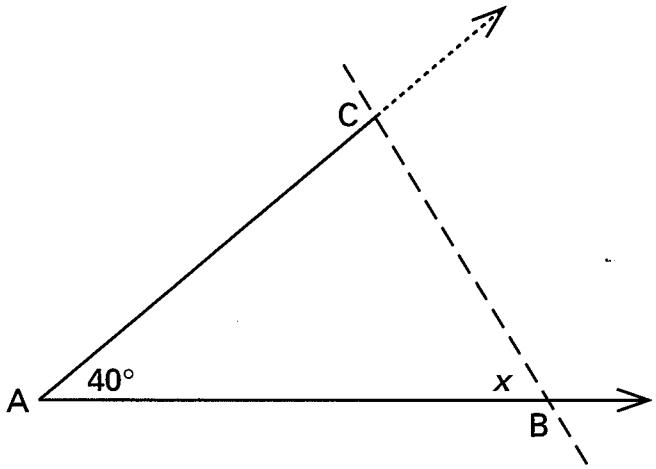
a) Place the mirror on the isosceles right triangle so you can "see" a square. Write instructions that someone else could follow to accomplish this.

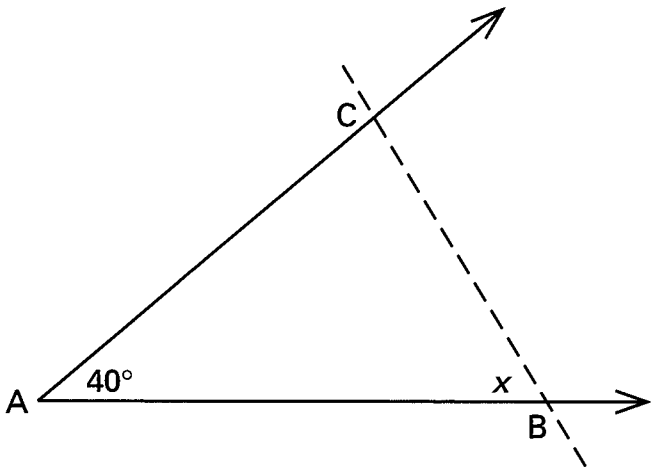


b) Place the mirror on the isosceles right triangle so you can "see" a rhombus that is not a square. Write instructions that someone else could follow to accomplish this.



Focus Master D





Focus Student Activity 14.1

NAME _____ **DATE** _____

	Column 1 **Right Triangle**	Column 2 **Parallelogram**	Column 3 **Hexagon**
Square			
Rhombus (not a square)			
Concave Kite			
Convex Kite (not a rhombus)			
Parallelogram (not a rhombus)			
Isosceles Triangle			
Equilateral Triangle			
Pentagon			
Hexagon			
Decagon			

Focus Student Activity 14.1 (page 2)

Mirrors on Triangles

1 If, for a shape listed on page 1, a mirror can be placed upright on an isosceles right triangle so that shape can be seen, write "Yes" in the appropriate space in Column 1, page 1. Indicate on one of the triangles on page 3 where the mirror should be placed to see the shape and write the name of the shape beneath the triangle.

2 In the remaining spaces in Column 1, page 1, explain briefly why you believe the mirror cannot be placed to see the shape.

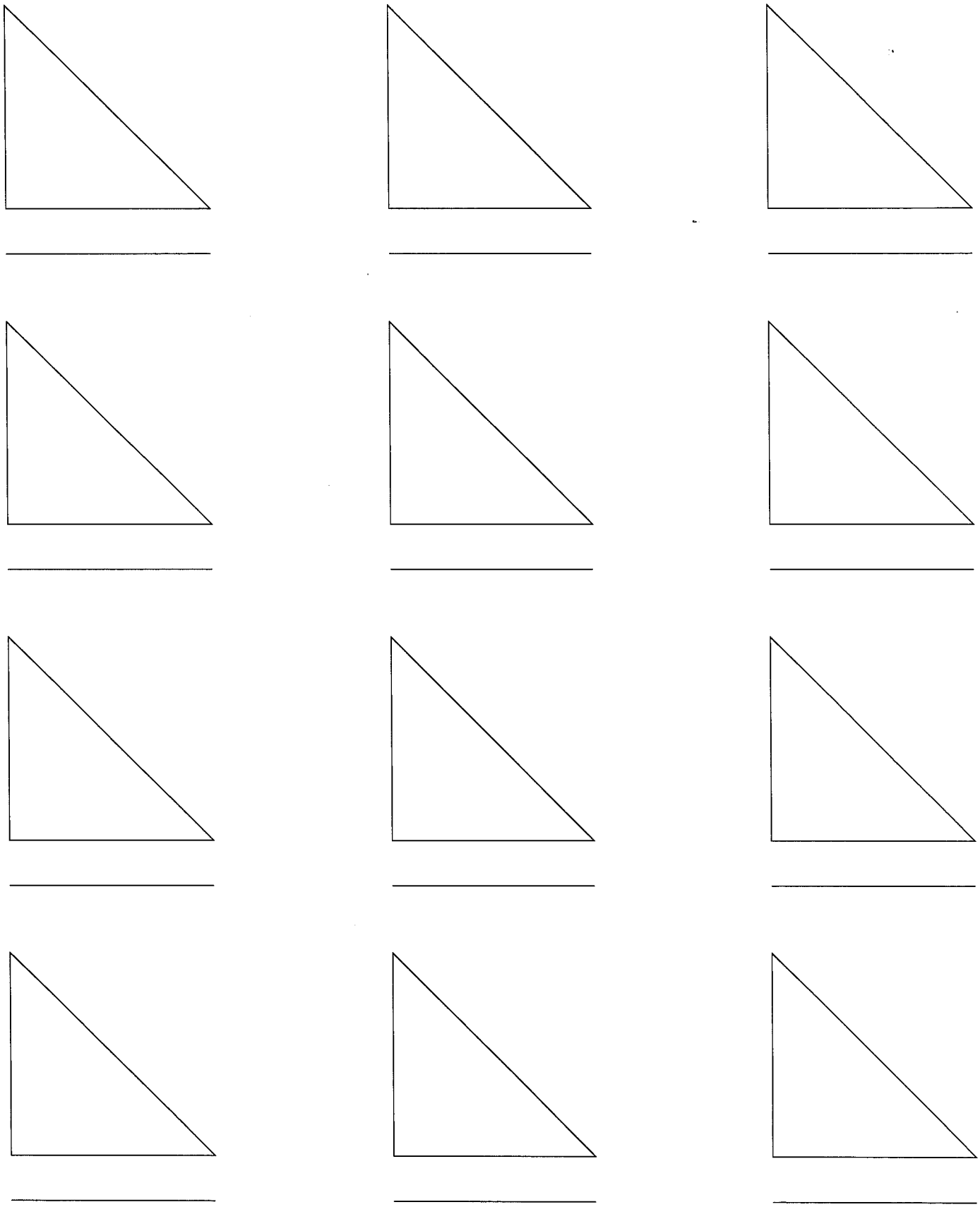
3 Pick 3 shapes, other than a rhombus or square, and in the spaces below write (using words only) directions for placing a mirror on an isosceles right triangle so those shapes can be seen.

a) To create a _____, place the mirror on an isosceles right triangle as follows:

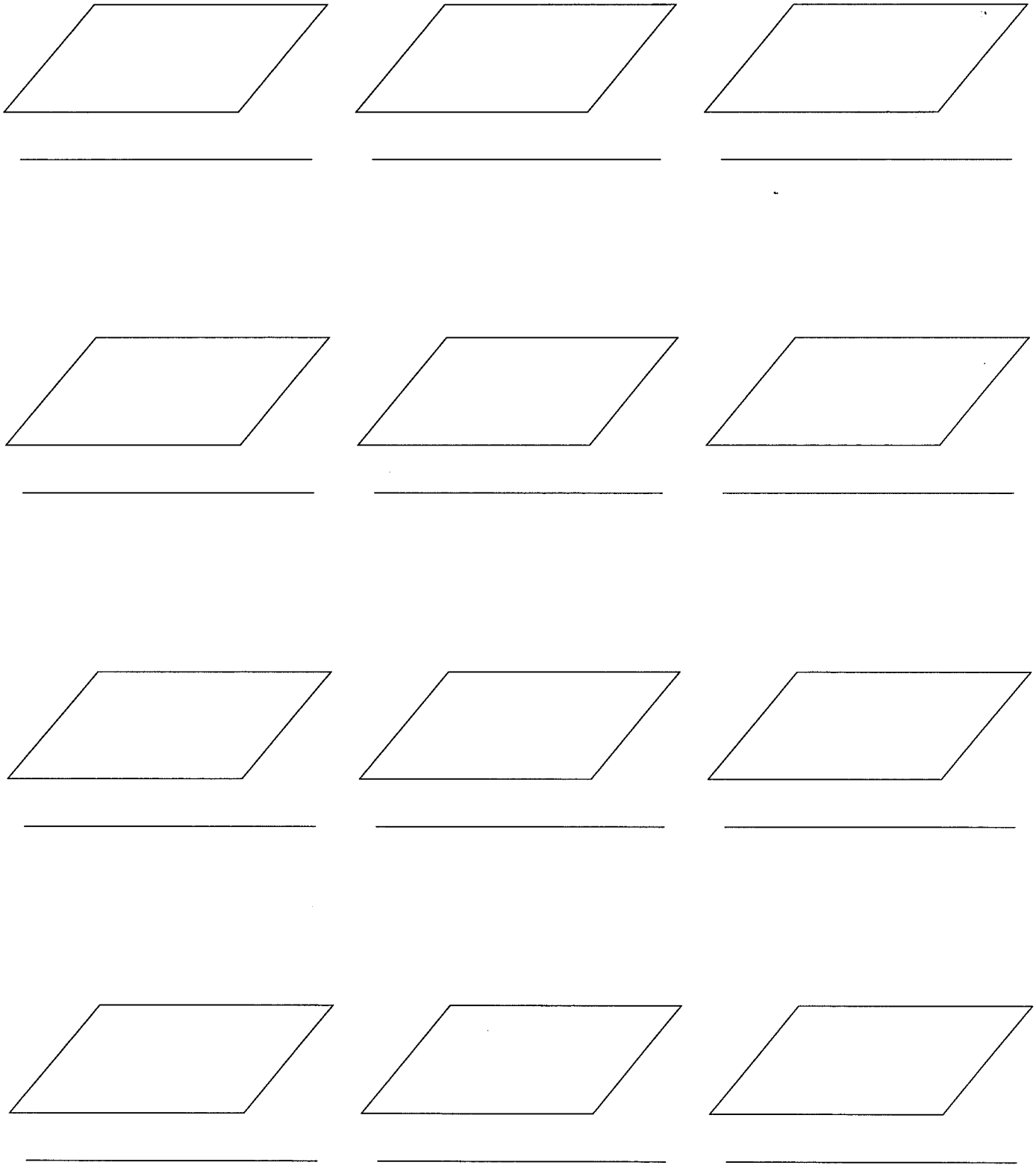
b) To create a _____, place the mirror on an isosceles right triangle as follows:

c) To create a _____, place the mirror on an isosceles right triangle as follows:

Focus Student Activity 14.1 (page 3)



Focus Student Activity 14.1 (page 4)



Focus Student Activity 14.1 (page 5)

Mirrors on Parallelograms

1 If, for a shape listed on page 1, a mirror can be placed upright on a parallelogram so that shape can be seen, write "Yes" in the appropriate space in Column 2, page 1. Indicate on one of the parallelograms on page 5 where the mirror should be placed to see the shape and write the name of the shape beneath the parallelogram.

2 In each remaining space in Column 2, page 1, explain briefly why you believe the mirror cannot be placed to see the shape.

3 Pick 3 shapes and in the space below write (using words only) directions for placing a mirror on a parallelogram so those shapes can be seen.

a) To create a _____, place the mirror on an parallelogram as follows:

b) To create a _____, place the mirror on an parallelogram as follows:

c) To create a _____, place the mirror on an parallelogram as follows:

Focus Student Activity 14.1 (page 6)

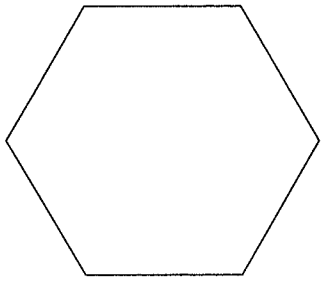
Imagining Mirrors on Hexagons

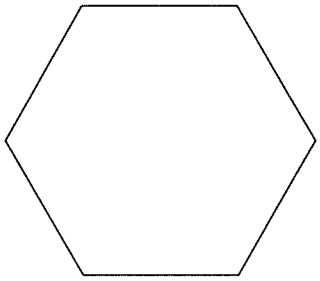
Do the following without using a mirror. Rather, *imagine* placing a mirror upright on a regular hexagon, such as those on page 7 of this activity, and moving it about.

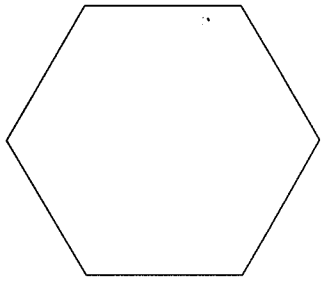
1 If, for a shape listed on page 1, you think a mirror can be placed upright on a hexagon so that shape can be seen, write "Yes" in the appropriate space in Column 3, page 1. Indicate on one of the hexagons on page 7 where you think the mirror should be placed to see the shape and write the name of the shape beneath the hexagon.

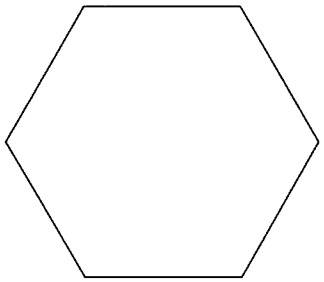
2 In the remaining spaces in Column 3, page 1, indicate briefly why you believe the mirror cannot be placed to see the shape.

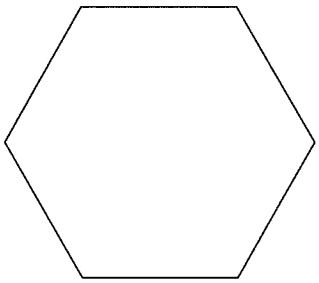
Focus Student Activity 14.1 (page 7)

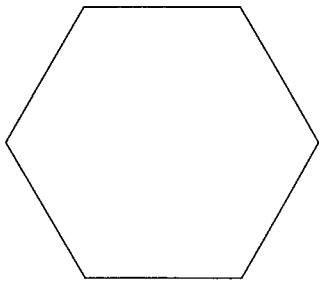


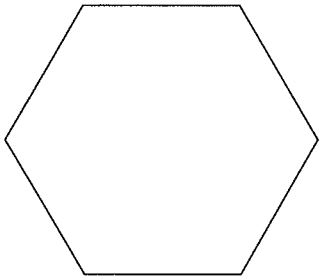


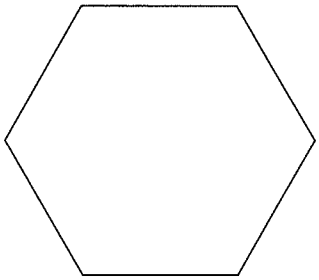


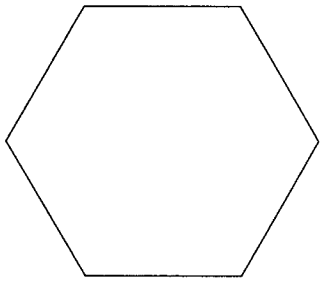


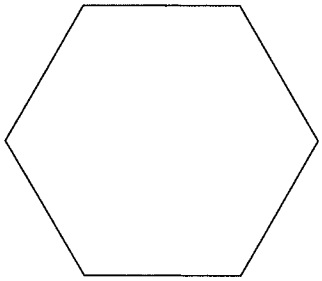


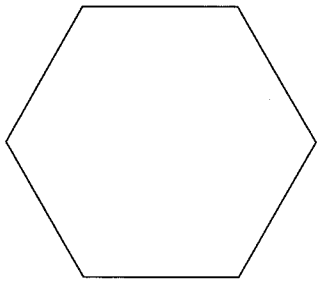


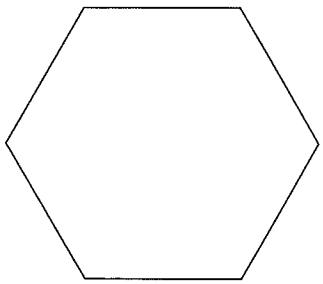










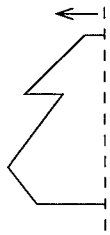


Follow-up Student Activity 14.2

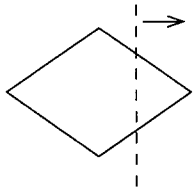
NAME _____ **DATE** _____

1 The dotted line in each of the following diagrams represents the position of a mirror (assume the reflective side faces the direction of the arrow). Complete each diagram, as accurately as possible, to show the reflection you would see in the mirror.

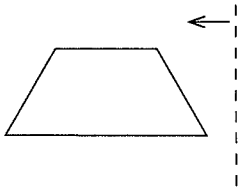
a)



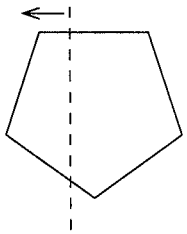
b)



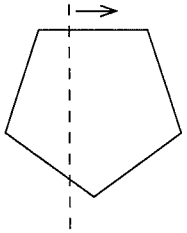
c)



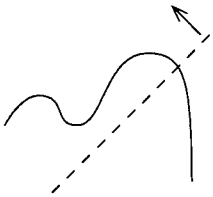
d)



e)



f)



2 Add the missing steps needed to create "adequate" instructions for using the mirror to form an isosceles triangle.

Step 1. Draw an acute angle.

You will then see an isosceles triangle.

Explain whether your instructions above are "redundant" and why.

(Continued on back.)

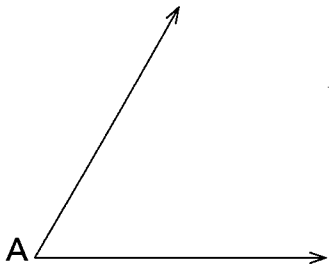
Follow-up Student Activity (cont.)

3 For a)-f) below, do the following without using a mirror:

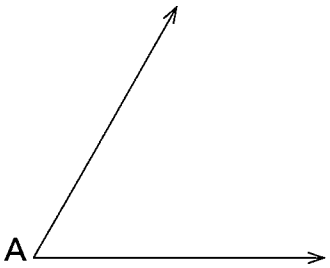
- Draw a dotted line to show where (if possible) you would position a mirror across the sides of Angle A so as to see the given shape.
- If you think it is impossible, explain why.
- If you think the shape can be seen by positioning the mirror in more than one way, describe all other positions.

After completing the above steps, use a mirror to check your predictions. If you no longer agree with your predictions, explain your new ideas, but don't erase your original predictions. (Draw any adjusted mirror lines with another color and in that same color make additions or changes, if needed, to your explanations.)

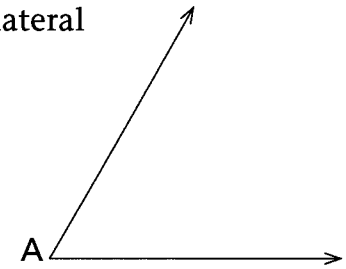
a) a triangle



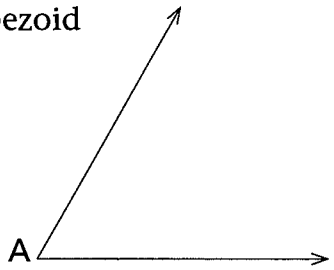
d) a rectangle



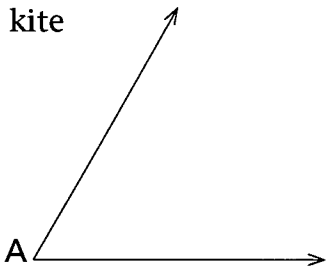
b) a quadrilateral



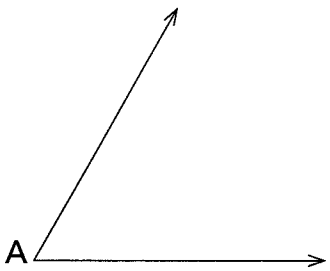
e) an isosceles trapezoid



c) a concave kite

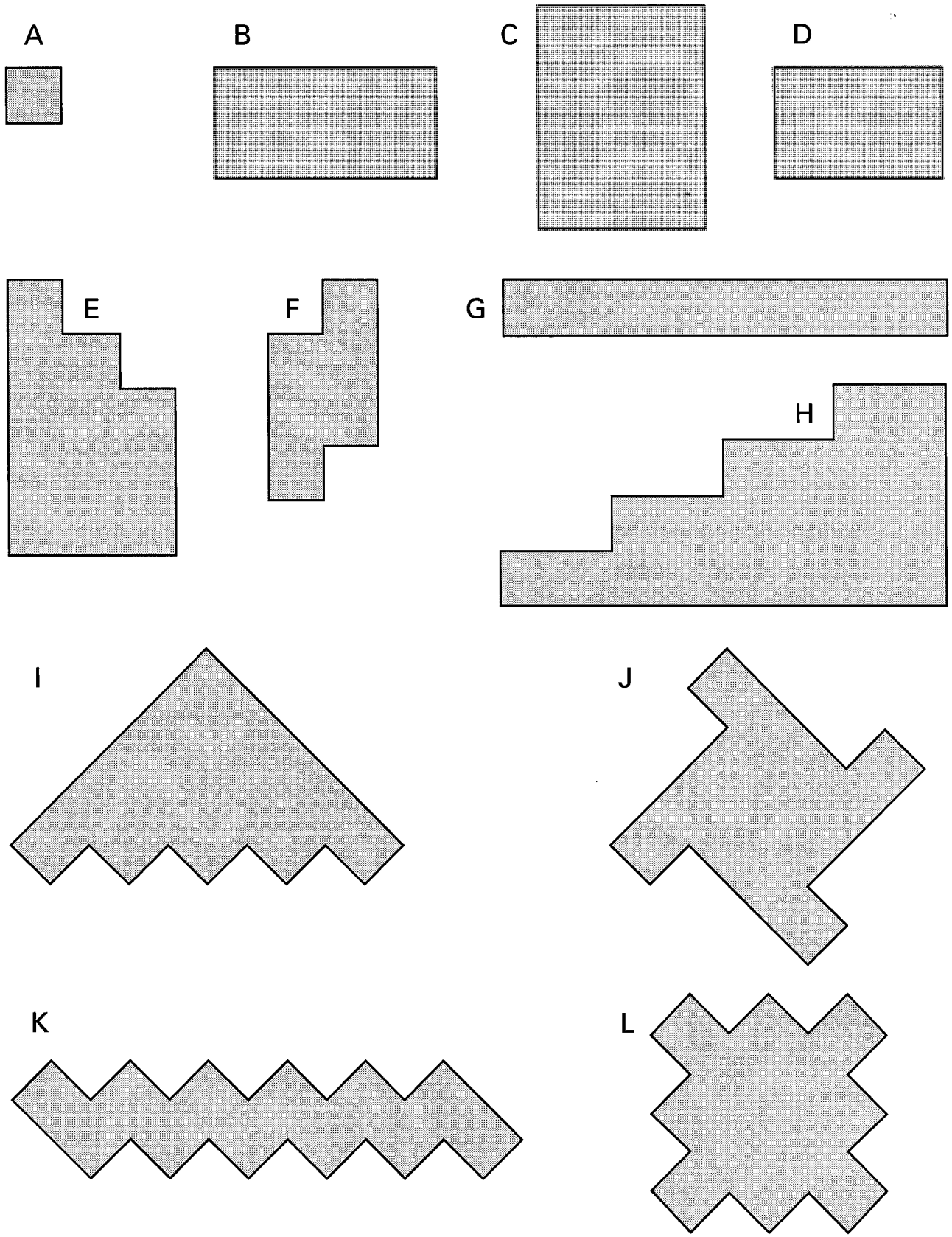


f) a convex kite

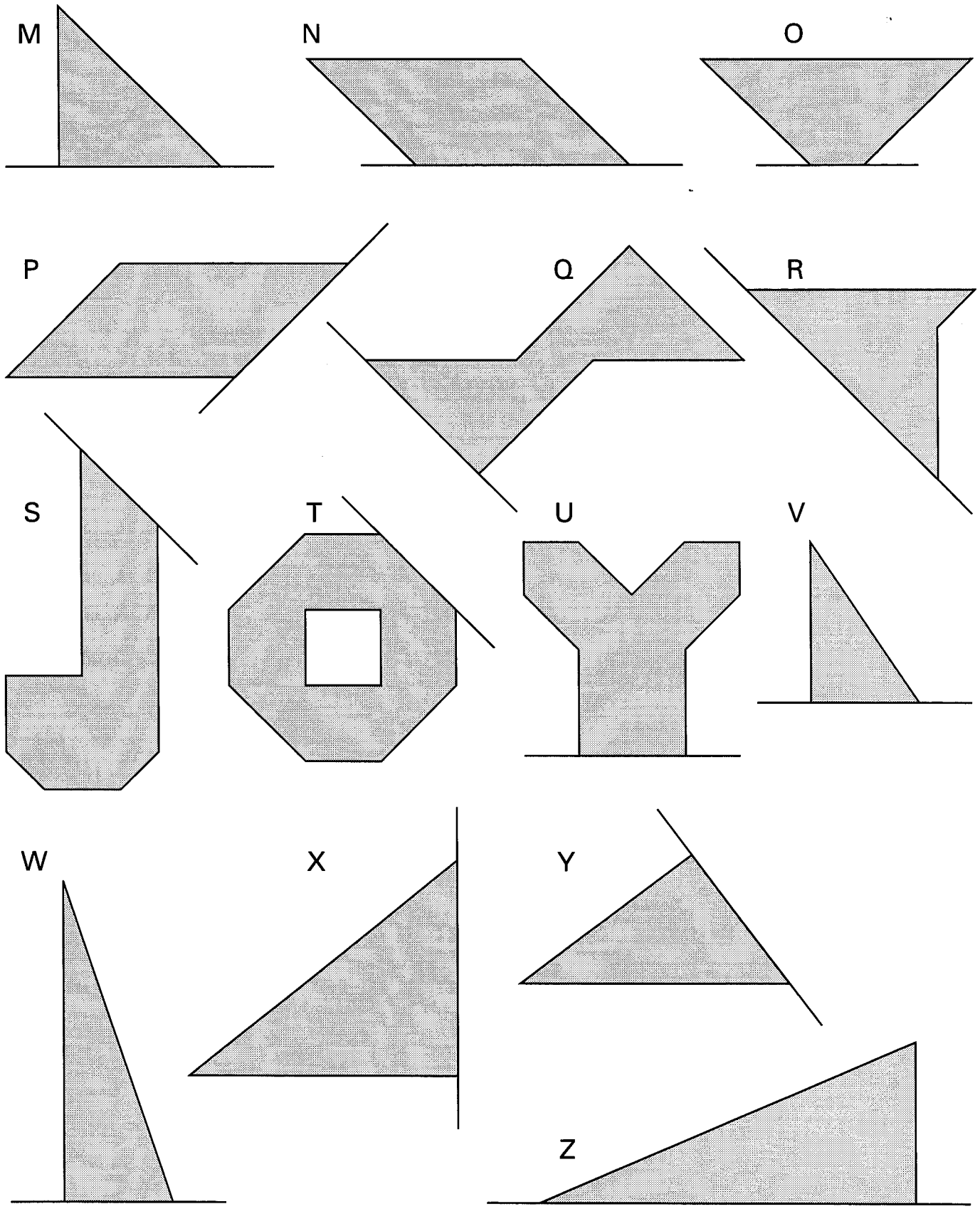


4 On separate paper, show (and name) all the different polygons that can be formed using a mirror on an *obtuse angle*. Include diagrams that show the mirror position on the angle and the reflection in the mirror. Then repeat for a *right angle*.

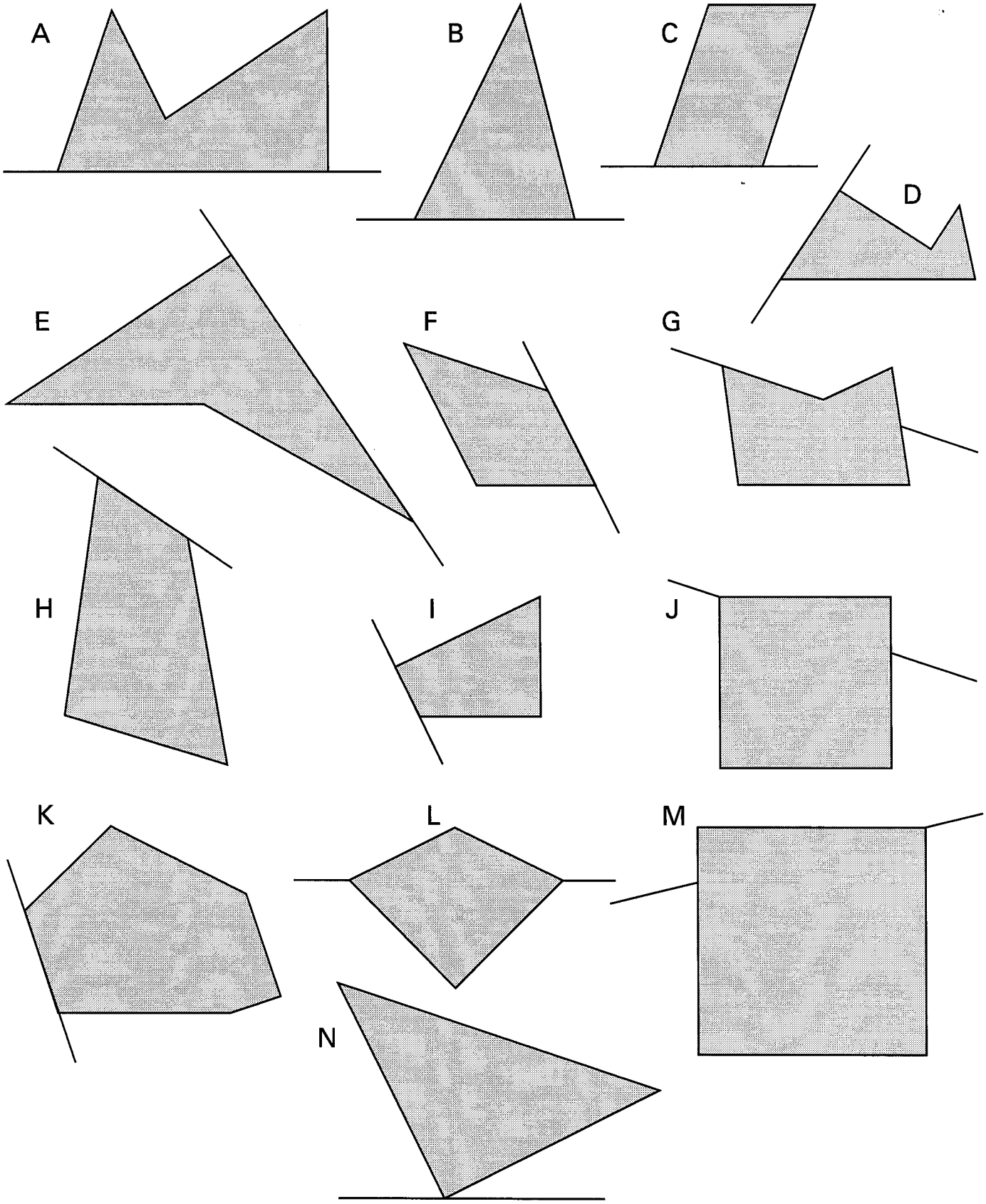
Connector Master B



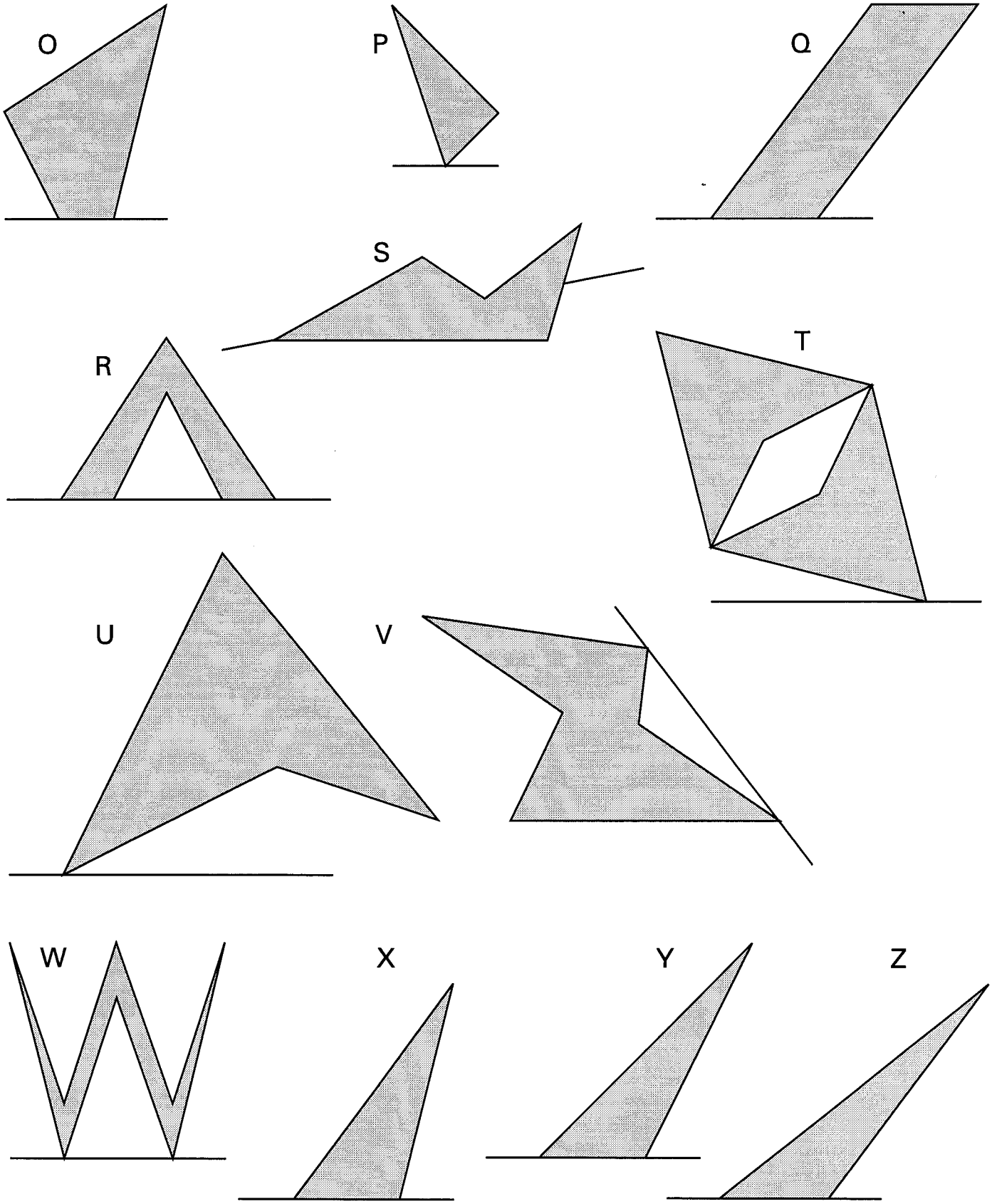
Focus Master A



Focus Master B



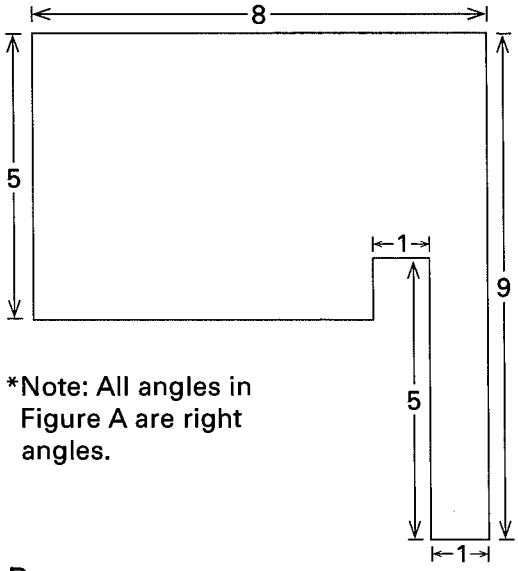
Focus Master C



Focus Student Activity 15.1

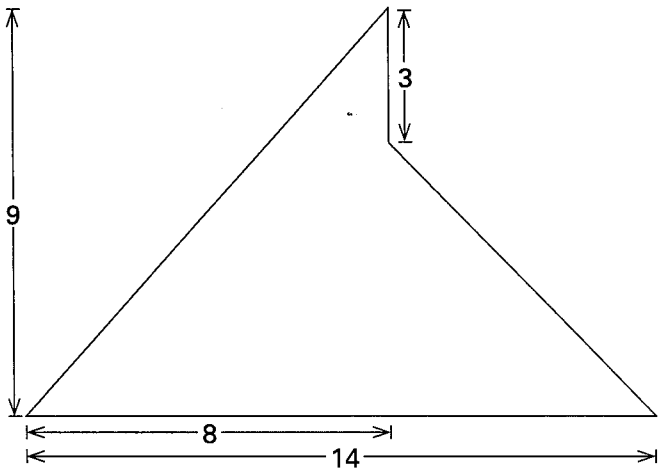
NAME _____ **DATE** _____

A*

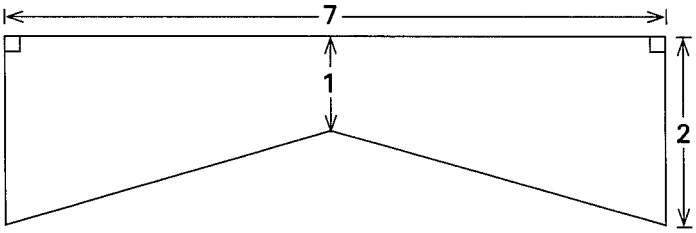


*Note: All angles in Figure A are right angles.

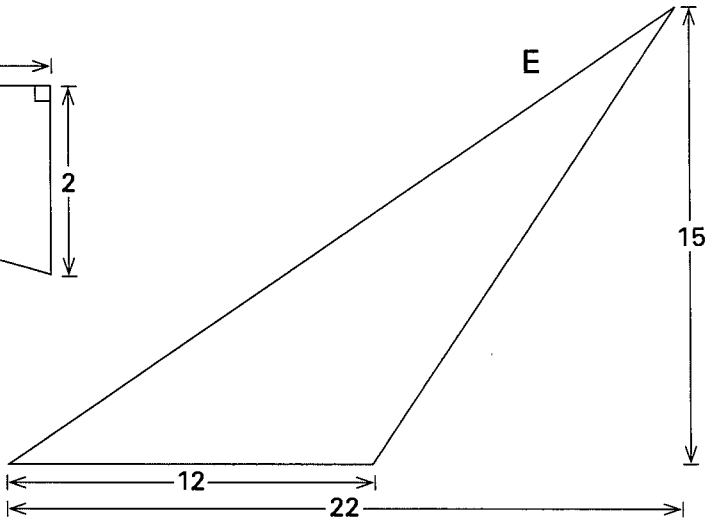
D



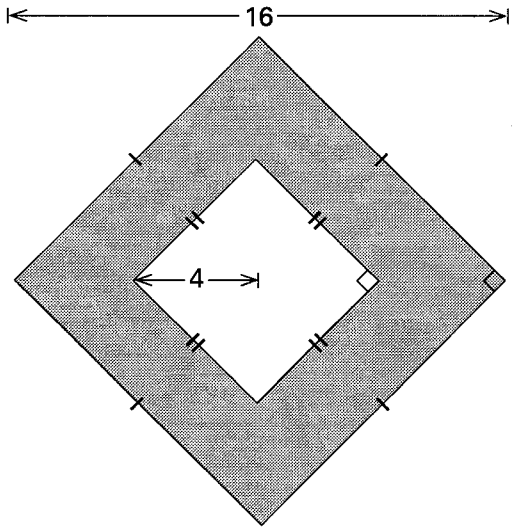
B



E



C*

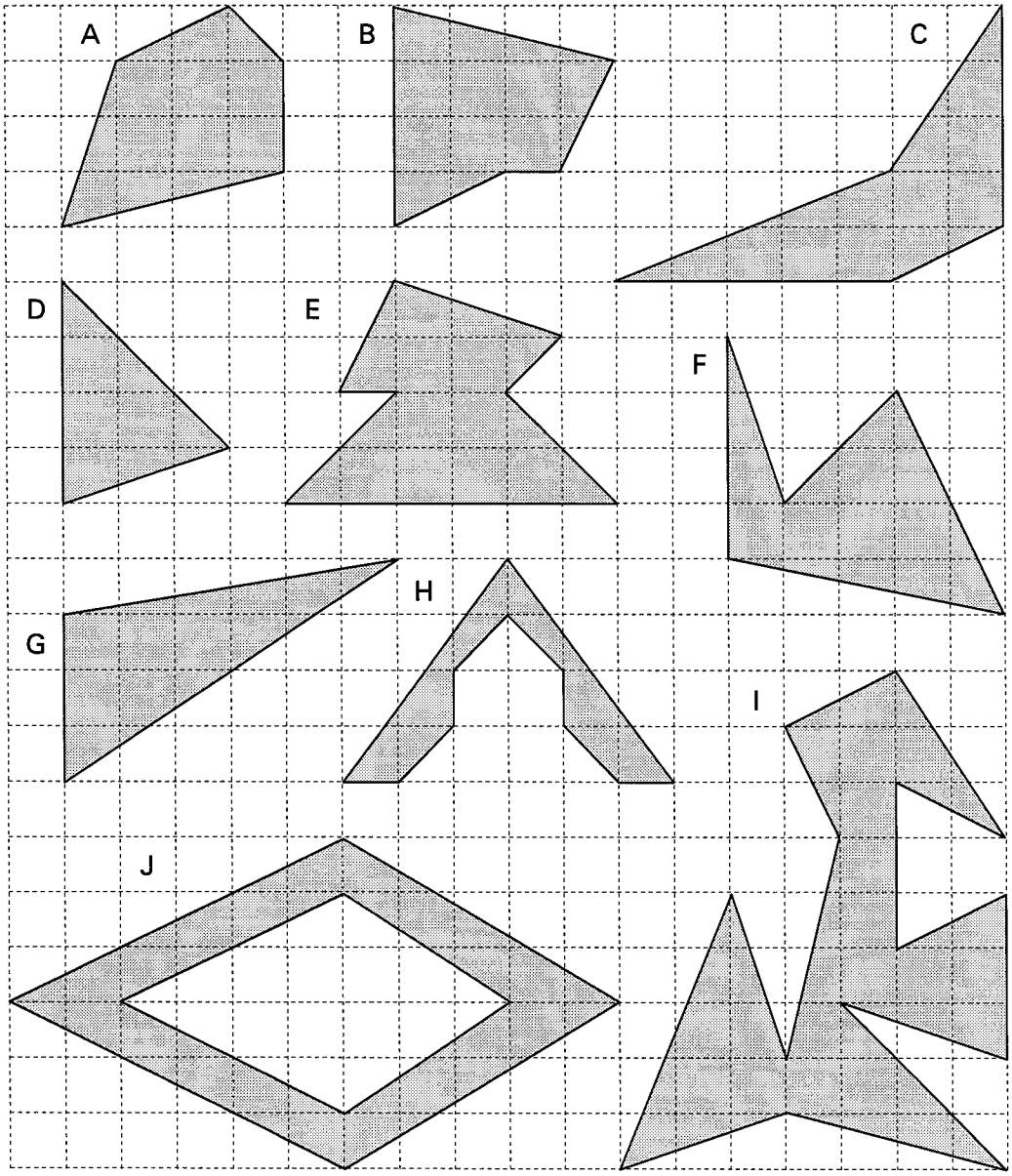


Find area of shaded part.
*Note: ' and " marks indicate which sides are congruent.

Follow-up Student Activity 15.2

NAME _____ **DATE** _____

1 Determine the area of the shaded part of each figure below, assuming 1 small square has area .01 square unit. Mark the diagrams to show your thought processes. In the column at the right, express each area as a fraction, decimal, and percent.



A _____

B _____

C _____

D _____

E _____

F _____

G _____

H _____

I _____

J _____

(Continued on back.)

Follow-up Student Activity (cont.)

2 On ¼" grid paper, do the following for each of a)-k) below:

- sketch a polygon that satisfies the given conditions (choose an area unit so that someone reading your paper can accurately count the area of each shape)
- label the polygon with its letter a)-k) and label its area as a fraction, decimal, and percent
- identify the area unit you used for that polygon

a) A rectangle of area .33 square units.

b) A nonrectangular parallelogram of area 1.5 square units.

c) 2 different right triangles of area 10 square units.

d) 3 different nonright triangles, each having area 10 square units.

e) A trapezoid that covers 7% of 1 area unit.

f) An isosceles triangle of area .06 square units.

g) 4 different triangles, each having area 3½ square units.

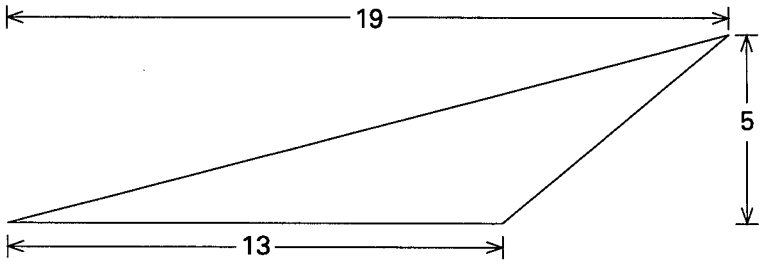
h) A square with area 49% of 1 area unit.

i) A rectangle with area equal to ¾ of 2 area units.

j) A convex kite with area greater than 30 square units.

k) A concave kite with area greater than 50% of 15 square units.

3 The triangle shown below was drawn on a grid, and then the grid lines were removed and the diagram was reduced. Using the given lengths, determine the area of the triangle. Explain each step of the methods that you used, and add lines to the drawing to provide a visual "proof" that your answer must be correct.



Connector Master A

Sums Game

Player 1 randomly draws 2 markers from Sack A, computes the sum of the marker numbers, tallies the sum on a score card, replaces the markers in the sack, and shakes the sack. Then Player 2 repeats this procedure, recording on a separate score card. Players continue alternating turns.

The winner of this game is the first person to obtain each different sum at least once *or* to obtain any single sum 6 times.

Score Card – Player 1 Sums

2	3	4	5	6	7	8

Score Card – Player 2 Sums

2	3	4	5	6	7	8

Focus Master B

Game 2 – Products Game

a) Player 1 randomly selects 2 markers from Sack A, computes and records the *product* of the numbers on the 2 markers, and returns the markers to the sack.

Player 1 earns a point if the product is less than or equal to 4; Player 2 earns a point if the product is greater than 4.

b) Player 2 selects 2 markers from the sack and points are assigned as described above.

c) Players continue alternating turns until 1 player earns 5 points.

Game 3 – 20 Points Game

Player 1 and Player 2 take turns shaking and randomly flipping the 3 markers together and recording the color and letter showing on each marker. On each flip, 1 point is assigned to either Player 1 *or* Player 2 according to the following criteria:

Player 1 earns 1 point if:

i) the green marker shows an A, or
ii) the red and blue both show an A, or
iii) all 3 markers show an A.

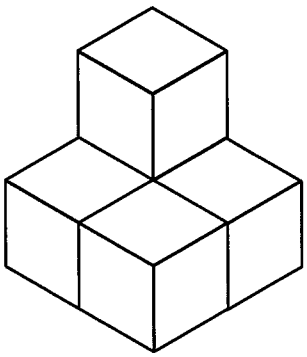
Player 2 earns 1 point in all other cases.

Players alternate turns until one player earns 20 points.

Focus Master C

Game 4 – Weird Die Game

Player 1 and Player 2 take turns randomly tossing the Weird Die.



On any toss if the die rests on a base that has either 3 squares or 4 squares, then Player 2 earns 2 points. If the die rests in any other way, then Player 1 earns 3 points.

The winner is the person with the most points after 1000 tosses.

Follow-up Student Activity 16.1

NAME _____ **DATE** _____

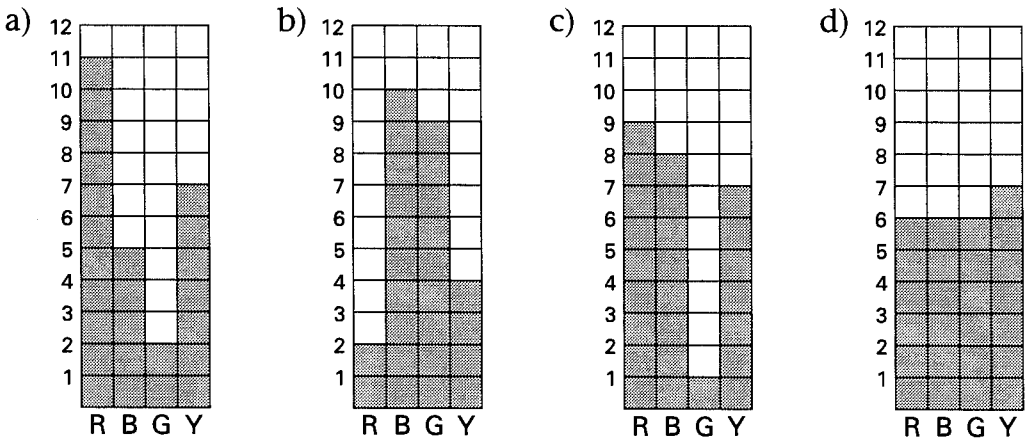
Write your responses to each of these problems on a separate sheet of paper. Be sure the reader of your paper knows what the problem is asking from you.

1 Imagine that the following 12 squares are placed in a container and Player 1 and Player 2 play a game by selecting markers. Each player in turn takes a marker, records its color, and returns it to the container.

blue	red	blue	red
red	yellow	red	yellow
yellow	blue	green	red

Player 1 wins by selecting a red marker and Player 2 wins by selecting a marker that is not red. Do you think this is a fair game? Explain your reasoning and use probability and graphs to support your position. If you don't think it's fair, tell how it could be revised to be fair.

2 Classify each of the following bar graphs as LIKELY, SOMEWHAT LIKELY, or UNLIKELY representations of 25 selections from the markers in Problem 1. Explain how you decided each classification.



(Continued on back.)

Follow-up Student Activity (cont.)

3 Cut out 8 squares (use cardstock or cardboard). Color 4 squares blue and number them 1, 2, 3, and 4. Color the remaining 4 squares red and number them 1, 2, 3, and 4. Place the squares in a container.

1	1	2	2	3	3	4	4

Play the following game with another student or someone in your family:

• Players 1 and 2 take turns randomly selecting 2 markers from the container, recording the color and number of each marker, and then replacing the markers in the container.

• Regardless who makes the draw, Player 1 wins if the 2 markers are of different colors and Player 2 wins if they are the same color.

Do you think this game is fair? On another sheet of paper, write your conclusions and write a convincing argument (including graphs, diagrams, and experimental and theoretical probability) to support your viewpoint. If you think it is unfair, tell how you would change it to be fair.

4 A game of Bingo is going to be played where a Bingo caller randomly selects pairs of the following numbers from a sack, announces their product, and returns the numbers to the sack.

1	1	2	2	3	3	4	4	5	5	6	6

Each player places a marker over a number on their Bingo card if the number equals the product announced by the caller. Play continues until a player has covered all 9 numbers on their card.

Fill in the card at the right with 9 different numbers that you feel will make a winning Bingo card for the game described above. Then, on another sheet, write a detailed and convincing argument to show why your card is likely to be a winning card.

B	I	N	G	O

Connector Master A

Student _____

Class Survey

Complete the following survey about yourself.

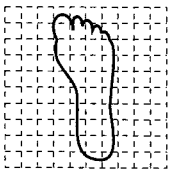
1. What is your gender, male or female?_____

2. How many pets do you have?_____

3. What is your birthday (month, day, year)?_____

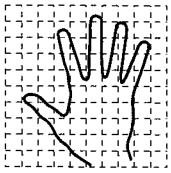
4. What is the number of people living in your household?_____

5. On 1-cm grid paper, trace your right footprint, without a shoe. Determine a close approximation (in square centimeters) of the area of your footprint. _____ sq. cm

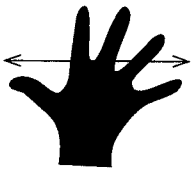


6. To the nearest centimeter, how long is your right foot without a shoe?_____cm

7. On 1-cm grid paper, trace your right handprint (fingers spread). Determine a close approximation (in square centimeters) of the area of your handprint. _____ sq. cm



8. To the nearest centimeter, what is the length of your hand span (tip of thumb to tip of little finger when hand is spread its widest)?_____cm



9. To the nearest centimeter, what is your arm span (finger tip to finger tip when your arms are spread their widest)?_____cm

Connector Master A (cont.)

10. To the nearest centimeter, how far can you blow a peanut (in its shell)? Record your distances for 3 blows.

    _____cm _____cm _____cm

11. What is your shoe size?_____

12. What color are your eyes?_____

13. What is the color of your hair?_____

14. How many peanuts (in their shells) do you *predict* you can you hold in your right hand?_____

15. How many peanuts (in their shells) did you *actually* hold in your right hand?_____

16. What is your favorite color?_____

17. On the average and to the nearest hour, how many hours do you think you spend watching television during each 7-day week of the school year?_____hrs.

18. On the average and to the nearest hour, how many hours do you think you spend doing homework during each 7-day week of the school year?_____hrs.

19. What, if any, is your favorite sport to play?_____

20. What, if any, is your favorite sport to watch?_____

Focus Student Activity 17.1

NAME _____ DATE _____

Form columns of cubes that model the situations described below. Then sketch and label the columns you form. Next to your diagram, write mathematical observations about the situation you have modeled.

a) There are 5 columns of cubes. The heights of the columns, which represent the ages of 5 children in a family, vary from 3 to 11 and their mean height is 7.

b) These 4 columns have heights which represent the number of hours 4 students spent on homework last week. The range of heights of the columns is 7. No column has a height that is equal to the mean height.

c) The mean, mode, and median heights of these 7 columns are all the same and the range of their heights is 6. The heights of the columns represent the numbers of hours 7 students spent on homework one week.

(Continued on back.)

Focus Student Activity 17.1 (cont.)

d) The mean height of these 8 columns is 8.25, and no column has a height equal to the median height. The heights of these columns represent points scored by soccer players last season.

e) There are 3 different modes of the heights of these 8 columns, and 2 of the columns have the same heights as the median height. The heights of these columns represent the number of movies 8 students watched during the summer.

f) There are 10 columns of cubes. Four of the columns have a mean height of 5. The mean height of the remaining 6 columns is 8. The heights of these columns represent the number of fish Kyle caught on 10 fishing trips.

g) The heights of these 7 different columns represent the numbers of points Jennifer scored in 7 basketball games last month. These columns have a mean height of 3. During 7 games this month she doubled each of her last month's scores.

Follow-up Student Activity 17.2

NAME _____ DATE _____

Write your responses to all these problems on other paper.

1 Draw diagrams of columns of cubes that satisfy each of the following conditions. Label the height of each column and show or explain how your diagram satisfies the conditions given.

a) If one of these 6 columns is removed, the mean height of the columns changes from 12 to 4.

b) These 6 columns are all different heights; none of their heights is equal to the mean or median height. The range of their heights is 9.

c) These 7 columns have a mean height of 6, and 20 is an outlier.

d) The heights of these 8 columns have 2 modes, the median is greater than the mean, and their range is 11.

e) The total height of these 10 columns is 65. The mean, median, and mode heights of these columns are all the same. The range of their heights is 8.

f) Columns of cubes were used to determine the mean of these numbers 5, 6, 6, 0, 3, 2, 9, 0, 5, 1, 7.

g) The mean height of these columns is greater than the mean height of the tallest and shortest columns.

h) The mean height of these columns is less than the mean height of the tallest and shortest columns.

2 Tell what else (as much as you can) you think must be true about each of the sets of numbers described below.

a) The mean of the 5 numbers in Set A is 7.2.

b) The median of the numbers in Set B is 6.5.

c) The mode of the numbers in Set C is 12.

(Continued on back.)

Follow-up Student Activity (cont.)

3 Show how it is possible to solve the following problems by using sketches of black and/or red counting pieces. (This means to sketch the relationships and use your sketch to tell you what to compute, not to solve the problem and then sketch.) Explain how you interpret the meaning of average in each problem.

a) The low temperature for each of 5 consecutive days in Durham was $^{-}7$, $^{-}4$, $^{-}8$, $^{-}11$, and $^{-}5$ degrees. What was the average low temperature for those 5 days?

b) In another town, the record average low temperature for any 5-day period was $^{-}2$. During the first 4 days of the record 5-day period the daily temperatures were $^{-}3$, $^{+}4$, $^{-}7$, and 0. What was the temperature on the 5th day?

c) The average low temperature during 4 days last winter was $^{-}3$. What was the low temperature on the 5th day if the 5-day average low temperature was $^{+}2$?

d) Kerry earned the following test scores: 86, 92, 88, and 84. What must she earn on the next test in order to raise her average score to 90? (Hint: make a rough sketch to represent large numbers.)

4 One important use of averages is to compare different sets of data. Using the data from the class spreadsheet, compare the "average boy" in the class to the "average girl" in the class. Prepare a poster or other display that illustrates and explains your conclusions and your methods of reaching each conclusion. Your explanations and diagrams should reveal:

- your understanding of mean, median, mode, range, and outliers (how to determine these, what they are, when they are and aren't appropriate to use, etc.);
- your ability to appropriately organize and represent data using the methods explored in class;
- your understanding of the information revealed by the shape and spread of data.

Follow-up Student Activity 18.1

NAME _____ DATE _____

For this assignment, you need a copy of the Class Spreadsheet showing the data collected in the Lesson 17 class survey.

1 Discuss your ideas about the similarities and differences, and advantages and disadvantages of the following types of graphs:

line plot

bar graph

scatter plot

circle graph

stem and leaf plot

2 Using the data on the Class Spreadsheet, create a scatter plot of the predicted vs actual number of peanuts students could hold in the hands. Describe how well you think the class as a whole predicted the numbers of peanuts they could hold? Is there a relationship between the predicted and actual numbers? Use evidence about the shape and spread of the data points on your scatter plot to support your conclusions.

(Continued on back.)

Follow-up Student Activity (cont.)

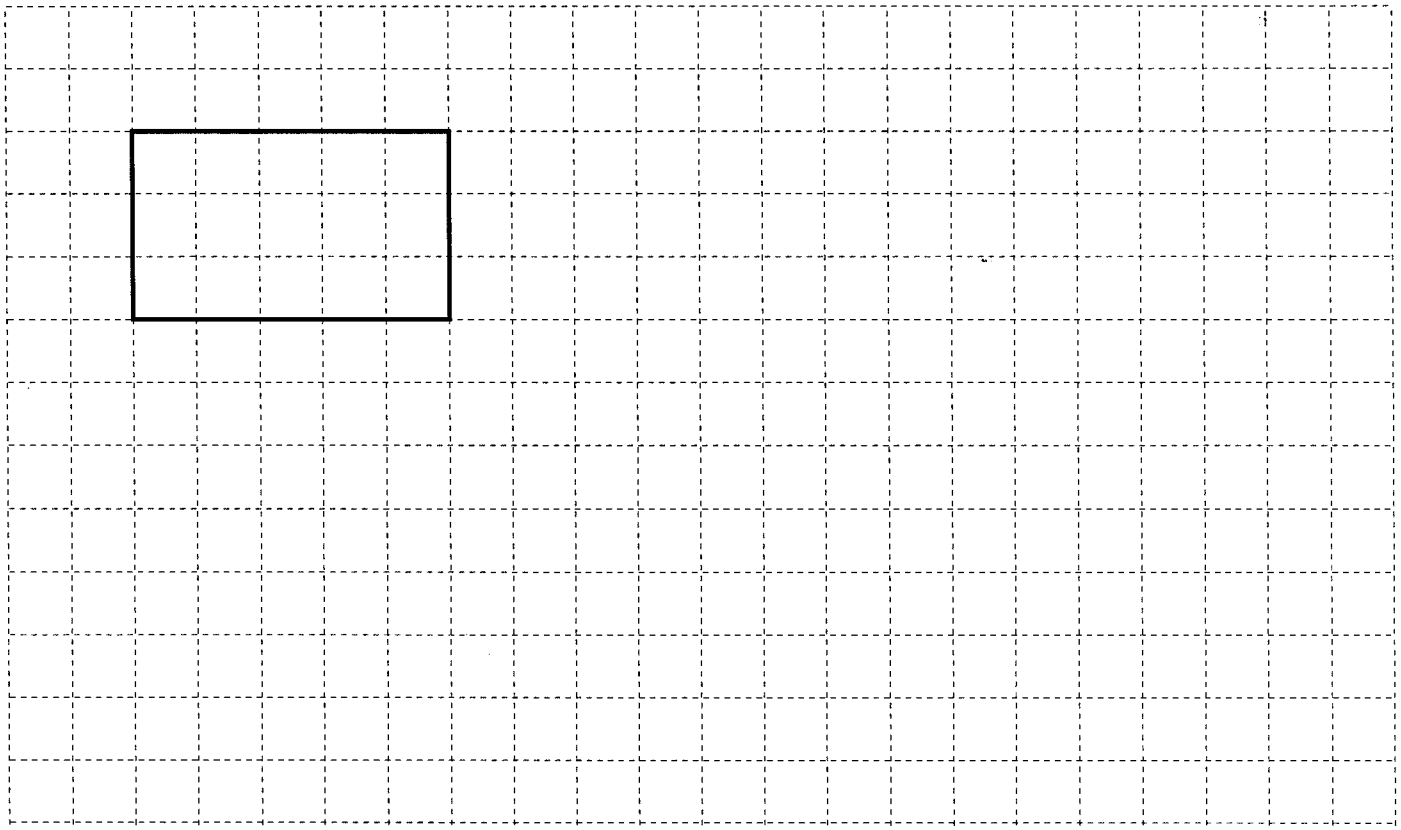
3 Pick another 2 columns of data from the Class Spreadsheet that you feel may be related. Make a scatter plot of the data in those 2 columns. Next to your graph, explain what you expected the relationship to be, and then tell whether you think your graph supports or contradicts your prediction. Explain what you "see" in the shape and spread of the graph that leads you to feel this way.

4 Now pick 2 columns of data that you predict should not be related, and make a scatter plot of that data. Next to your graph explain whether and why your scatter plot supports or contradicts your prediction.

5 Using a back-to-back stem and leaf plot, graph the data from the columns "area of hand" and "area of foot." Next to your graph, explain what information is revealed by the shape, center(s), and spread of the data.

6 Examine the Class Spreadsheet more closely. List here several "I wonder..." statements regarding relationships or conclusions that might be revealed by graphing and analyzing the data from a variety of points of view.

Focus Master A



Focus Master B

Lindsay's Conjectures

In class one day, Lindsay made the following conjectures:

"I think that any time you multiply a number by 10, the answer is the digits of the number with 1 zero placed to the right of the digits. When you multiply the number by 100, the answer is the digits of the number with 2 zeroes placed after the digits. When you multiply the number by 1000, the answer is the digits of the number with 3 zeroes placed after the digits. And so on."

"I also think that any time you multiply a number by .1 the answer is the digits of the number with the decimal moved one place to the left of its position in the original number. If you multiply the number by .01, the answer is the digits of the number with the decimal moved 2 places to the left. If you multiply the number by .001, the answer is the digits of the number with the decimal moved 3 places to the left. And so on."

a) Do you agree or disagree with Lindsay's conjectures?

b) What does Lindsay mean by "and so on" in her conjectures? Be specific.

c) Explore some ways to generalize about multiplication
by 20, 200, 2000...;
by 30, 300, 3000...;
by 70, 700, 7000...;
and so on.

And explore some ways to generalize about multiplication
by .2, .02, .002, .0002...;
by .3, .03, .003...;
by .7, .07, .007...;
and so on.

d) Investigate division by powers and multiples of 10. What conjectures or generalizations can you make?

Remember to support each of your conclusions with valid mathematical arguments and visual "proofs" using base ten pieces that will convince your audience your conclusions are correct. Use Extended Project E as a guide for deciding what to put in your report.

Extended Project E – Problem Summary

The purposes of this project are:

- to communicate your mathematical problem solving methods and reasoning about the attached problem;
- to reveal as much as possible about your understanding of the mathematics involved in the problem.

Please include the following in your Problem Summary:

1. **Your notes.** As you work on the attached problem, please save *everything* you do. Monitor your thoughts and important moments in your problem-solving process by using words, pictures, or symbols to keep track of the following:
   - how you get started
   - diagrams and models that you use
   - your hunches and conjectures
   - what helps your understanding
   - ideas you try that work and that don't work
   - your frustrations
   - places you get stuck
   - what excites you or makes you curious
   - your questions
   - ways you get unstuck
   - assumptions you make about the problem
   - help or ideas you get from discussing the problem with other people
   - feelings that surface while you work
   - your AHA!s
2. **One of the three options listed on the back of this sheet.**
3. **Your reflections** (written after completing 1 and 2 above):
   - the mathematics you learned or understand more clearly as a result of exploring this problem.
   - any new feelings or observations you have about yourself as a problem-solver.
   - at least one mathematical idea or question which was prompted by this investigation and which you feel would be interesting to think about some more.
   - the quality of your work, based on the criteria in the Follow-up Assessment Form.

This project is due on _____ and is to be completed individually, although it is appropriate to discuss ideas with other students working on the same problem. Prior to beginning your work on this project, please schedule three progress check meetings with your teacher.

Extended Project E – Problem Summary (cont.)

Option A Write a sequential summary of your thoughts and methods during the problem solving process, based on your notes, and give evidence that:

a) your conclusions are correct.
b) you have considered the important mathematics involved in the problem.
c) you are able to extend your methods and conclusions to related problems or ideas.
d) you recognize issues that might cause another person to have difficulty with this problem.

Option B You choose to explore a different but related problem. Write a sequential summary of your thoughts, methods, and conclusions during the problem solving process, based on your notes, and give evidence that:

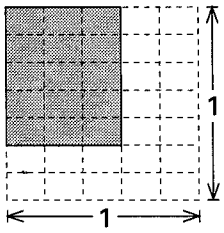
a) your new problem is worthwhile and your reasons for investigating the problem or idea are important.
b) your conclusions are correct.
c) you have considered the important mathematics involved in the problem.
d) you are able to extend and/or adapt your methods and conclusions.
e) you recognize issues that could cause another person to have difficulty with the problem.

Option C You worked intently on the problem, but you are stuck and not interested in pursuing the problem any further at this time. Provide a detailed sequential summary of your thoughts, methods, and conclusions during the problem solving process, based on your notes, and explain your reasons for abandoning the problem for now.

Focus Student Activity 19.1

NAME _____ **DATE** _____

1 What product is illustrated by the shaded rectangle?



2 Suppose that each of the squares shown below is 1 area unit. Sketch each of the given products on the squares so that it is possible to "see" the answer in your diagram.

a) $\frac{1}{2} \times \frac{6}{8}$

b) $\frac{1}{7} \times \frac{7}{9}$

c) $\frac{1}{6} \times \frac{10}{12}$

d) $\frac{5}{6} \times \frac{6}{7}$

e) $\frac{7}{10} \times \frac{9}{10}$

f) $\frac{4}{5} \times \frac{2}{3}$

g) $\frac{7}{10} \times \frac{3}{5}$

h) $\frac{3}{6} \times \frac{8}{10}$

i) $.6 \times .4$

(Continued on back.)

Focus Student Activity (cont.)

3 Study each diagram you made on the previous page and verify that you can "see" the repeated addition representation of each product in your area representation. Write an explanation of how you see this in Problem 2d).

4 Record 2 different division problems represented by each diagram in Problem 2.

a)

b)

c)

d)

e)

f)

g)

h)

i)

5 On another sheet of paper, show how it is possible to adapt the methods used in Problems 1 and 2 to find $2\frac{3}{5} \times 1\frac{5}{7}$ and 1.7×3.9.

6 Complete the following thought starter:

Here are my generalizations, conjectures, observations, questions, and/or AHA!s related to this activity...

Follow-up Student Activity 19.2

NAME _____ DATE _____

Answer these problems on separate paper.

1 Use diagrams and brief notes to explain the *meanings* of the operations addition, subtraction, multiplication, and division. Be sure to show all the different meanings of any operations that can be interpreted in more than one way.

2 In class you have explored a variety of methods of solving fraction, decimal, and whole number computations. For each of problems a)-l) below do the following:

i) Write a step-by-step explanation of the method (or methods) that you are most comfortable using to solve the problem.

ii) Include diagrams and explanations so it is clear *why* your method works.

a) $\frac{5}{6} + \frac{3}{8}$	e) $3.7 + 1.8$	i) $46 + 29$
b) $\frac{5}{6} - \frac{3}{8}$	f) $3.7 - 1.8$	j) $46 - 29$
c) $\frac{5}{6} \times \frac{3}{8}$	g) 3.7×1.8	k) 46×29
d) $\frac{5}{6} \div \frac{3}{8}$	h) $3.7 \div 1.8$	l) $46 \div 29$

3 For each of computations a)-h) in Problem 2, write a word problem whose solution requires the computation.

(Continued on back.)

Follow-up Student Activity (cont.)

4 The method of forming *equal sums* can be used to change numbers so their sum is the same but the numbers are easier to add.

a) Explain how and why you think this method works (give several examples using fractions, decimals, and whole numbers).

b) Write some special "tips" about when to use this method, and/or describe important points to keep in mind about the method.

5 The method of forming *equal differences* can be used to change numbers so their difference is the same but the numbers are easier to subtract.

a) Explain how and why you think this method works (give several examples using fractions, decimals, and whole numbers).

b) Write some special "tips" about when to use this method, and/or describe important points to keep in mind about the method.

6 Regarding fraction, decimal, and whole number operations, complete the following thought starters (give detailed responses with examples):

a) I know...

b) I understand...

c) What puzzles me...

d) I need/want to know more about...

e) The really BIG IDEAS to keep in mind are...

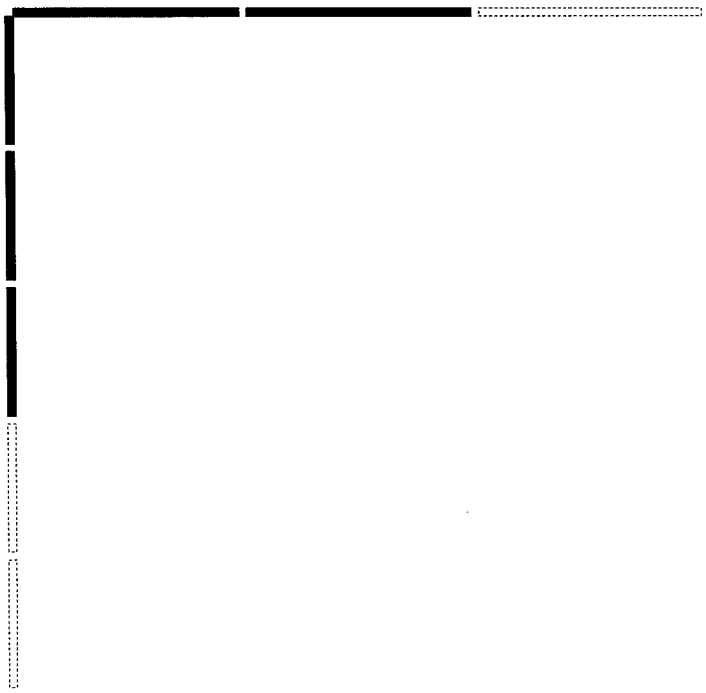
f) For each of the 4 basic operations with fractions, decimals, and whole numbers, here are examples of *hard* problems I can solve:

g) Here are examples of problems I wish I could solve:

h) The generalizations I have made and feel proud about are:

Connector Master A

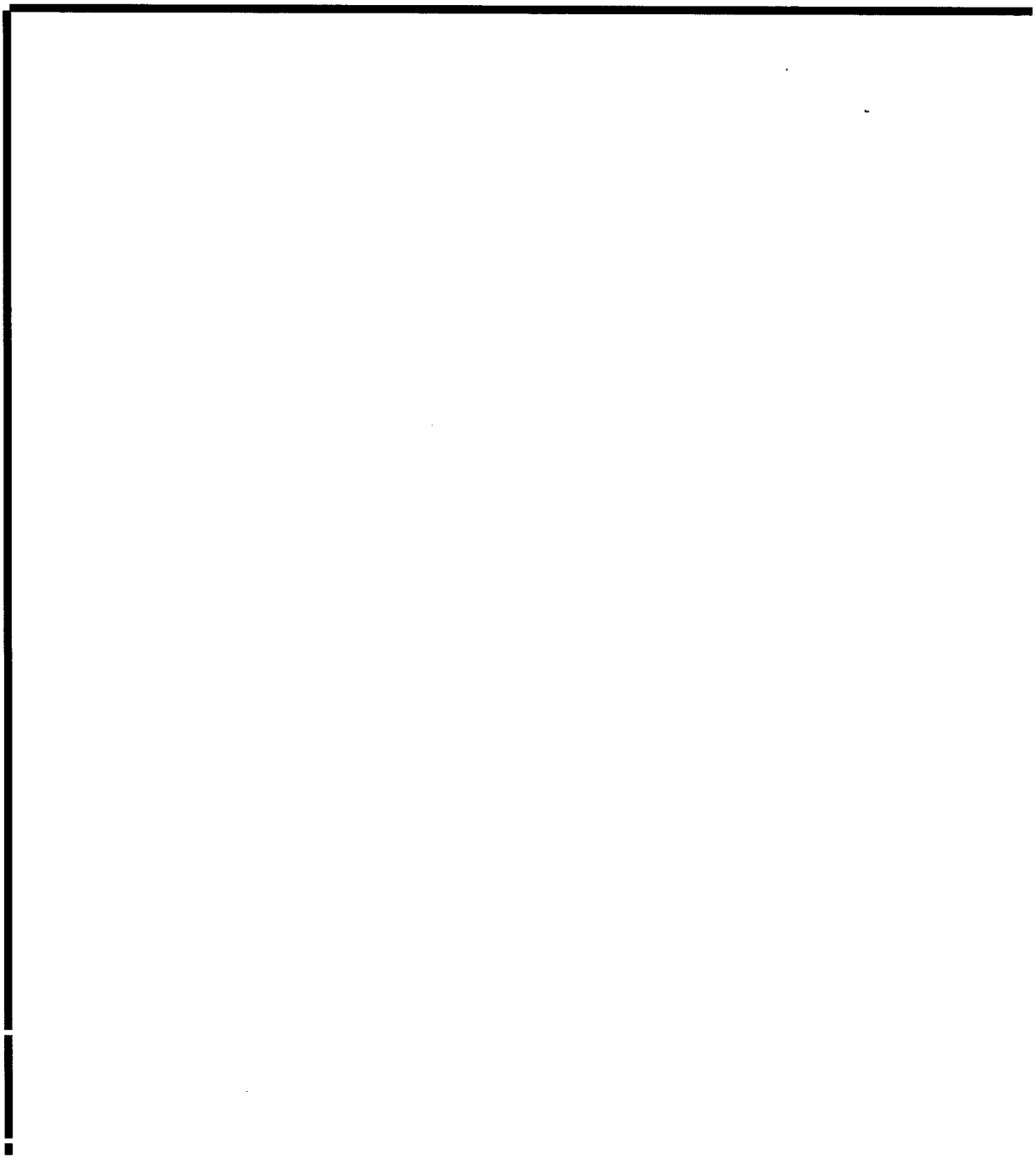
a) I am thinking of a fraction product and here are the edges I see in my mind...



b) I am thinking of a decimal product and here are the edges I see in my mind...

Connector Master A (page 2)

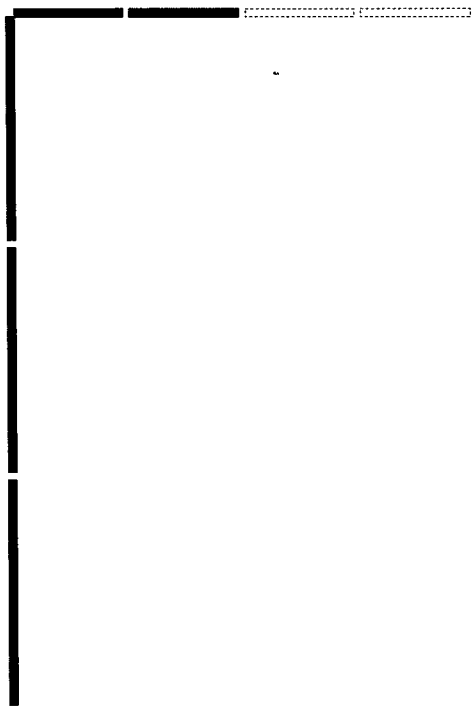
c) I am thinking of a whole number product and here are the edges I see in my mind...



Connector Master A (page 3)

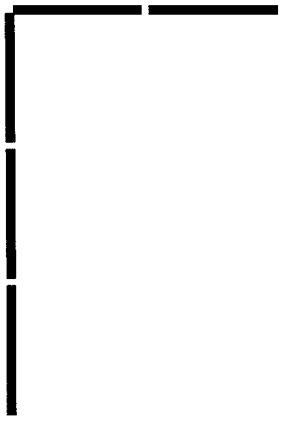
d) I am thinking of a decimal product and here are the edges I see in my mind...

e) I am thinking of a fraction product and here are the edges I see in my mind...



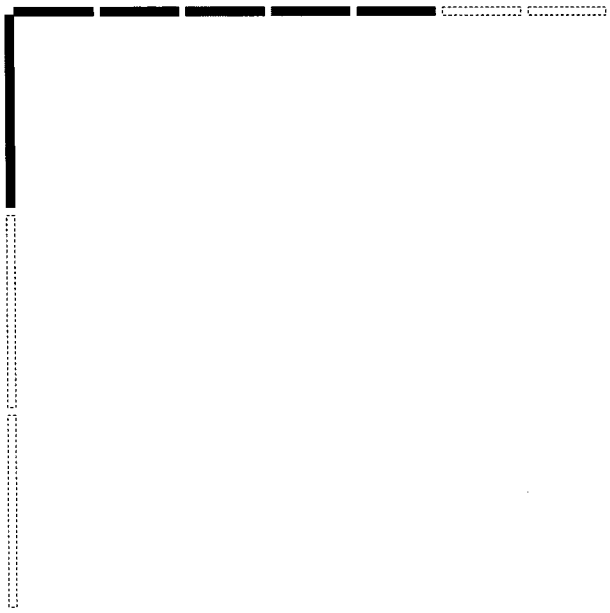
f) I am thinking of a product and here are the edges I see in my mind...

g) I am thinking of a product and here are the edges I see in my mind...



Connector Master A (page 4)

h) I am thinking of a fraction product and here are the edges I see in my mind...



i) I am thinking of a product and here are the edges I see in my mind...

Connector Master B

For each of the following products:

i) make a sketch that shows how to use the repeated addition method of multiplication to determine the answer;

ii) record any conjectures or generalizations you make while you work.

a) $\frac{1}{8} \times 8$

b) $\frac{5}{6} \times 6$

c) $\frac{3}{5} \times 5$

d) $\frac{2}{3} \times 3$

e) $4 \times \frac{3}{4}$

f) $2\frac{1}{4} \times \frac{4}{9}$

g) $\frac{3}{7} \times 7$

h) $9 \times \frac{3}{9}$

i) $5 \times \frac{4}{5}$

j) $\frac{9}{10} \times 10$

k) $\frac{1}{9} \times \frac{9}{10}$

l) $\frac{3}{5} \times \frac{5}{8}$

m) $\frac{5}{6} \times \frac{6}{7}$

n) $\frac{7}{8} \times \frac{8}{10}$

o) $\frac{3}{4} \times \frac{4}{3}$

p) $\frac{3}{8} \times \frac{8}{3}$

q) $\frac{12}{20} \times \frac{20}{12}$

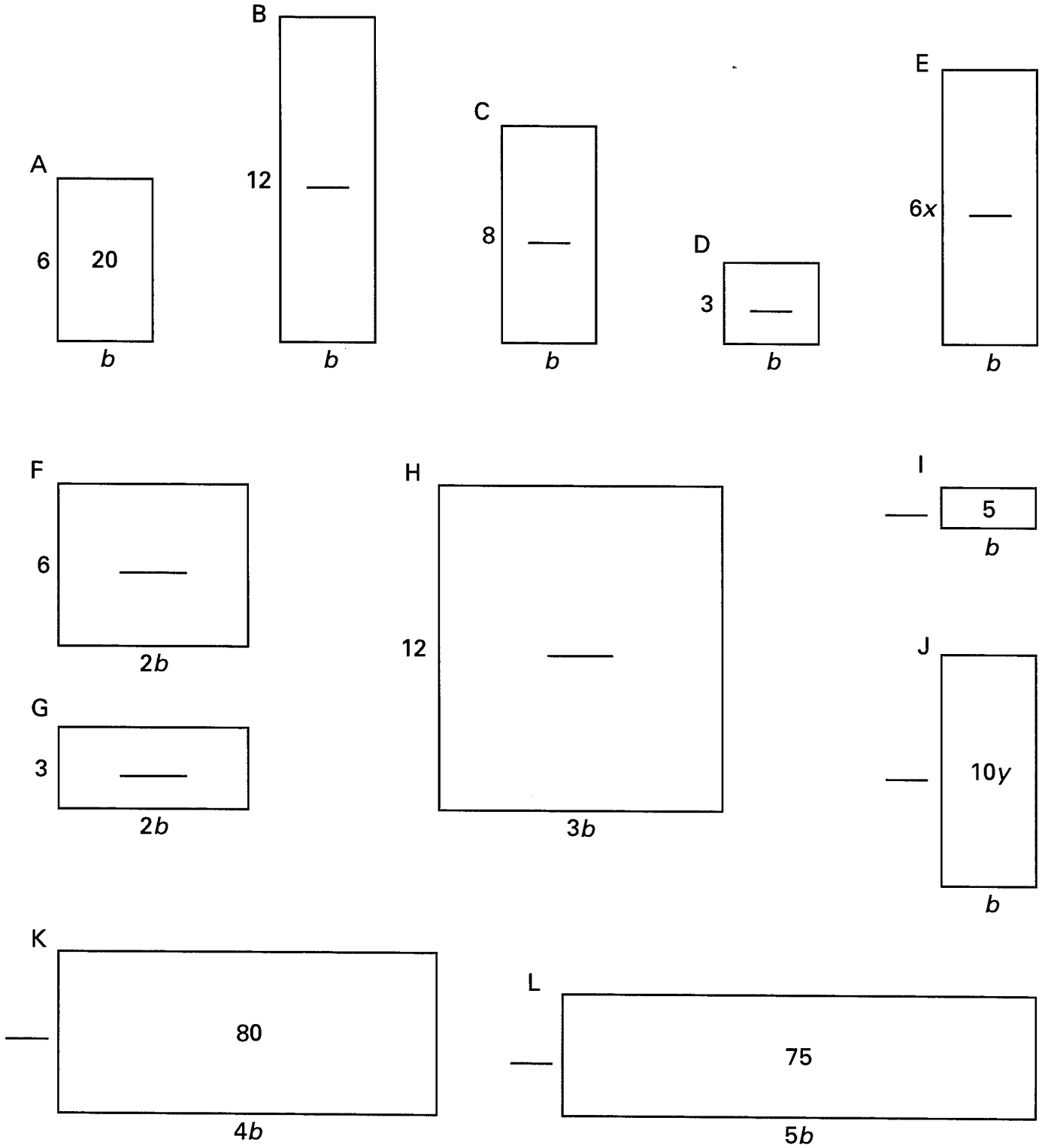
r) $1\frac{9}{8} \times 8$

s) $.7 \times 10$

t) $.08 \times 100$

Focus Master B

Rectangle A has area 20, height 6 and base *b*. For the other rectangles, either the area or a dimension is missing. Fill in the missing area or the missing dimension with either a numerical value or an algebraic expression that does not involve *b*.



Follow-up Student Activity 20.1

NAME _____ **DATE** _____

Answer the following on separate paper.

1 Forming equal quotients and equal products are useful methods of solving multiplication and division problems. Explain each method and give several examples to illustrate how each one works.

2 Show how to use *rectangle maneuvers* to solve each of the following computations. Add a few words to your diagrams to help the reader understand the steps you used.

a) 24×120
b) 18×35
c) 56×99
d) 1.5×66
e) $\frac{1}{5} \times \frac{2}{3}$
f) 3.1×4.2
g) $160 \div 44$
h) $36 \div 1.8$
i) $2.1 \div .7$
j) $\frac{3}{5} \div \frac{2}{3}$

3 Sketch diagrams that show your step-by-step procedures for using rectangle maneuvers to compute the following:

a) $\frac{3}{4} + \frac{5}{8}$
b) $\frac{3}{8} + \frac{1}{6}$
c) $\frac{7}{9} - \frac{1}{3}$
d) $\frac{4}{3} - \frac{5}{4}$
e) $\frac{7}{5} - \frac{3}{4}$

4 Silas claims he can solve all of the following problems *mentally* by picturing rectangle maneuvers in his mind. For each of the following, explain and sketch the rectangle maneuvers you think Silas would imagine.

a) 17×19
b) $840 \div 14$
c) $640 \div 8$
d) 4.2×2.1
e) $3600 \div 900$
f) 2.5×1.4
g) $1260 \div 18$

(Continued on back.)

Follow-up Student Activity (cont.)

5 Record several problems not already on this assignment that you can solve mentally by picturing rectangle maneuvers in your mind's eye. Then draw diagrams to show your methods.

6 Challenge. Write a set of procedures for using rectangle maneuvers to:

a) compute $a/b + c/d$ where a, b, c, and d are whole numbers greater than zero.

b) compute $a/b - c/d$ where a, b, c, and d are whole numbers greater than zero.

7 In class you have explored many ways to operate with fractions, decimals, and whole numbers. Complete the following thought starters, describing methods other than using a calculator. For each statement that you write, make up problems different from any on this assignment and show how your methods work.

a) My favorite method(s) of adding fractions...

b) To me the easiest method(s) of subtracting fractions...

c) The method(s) I use most often to multiply fractions...

d) I am most confident using the following method(s) of dividing fractions...

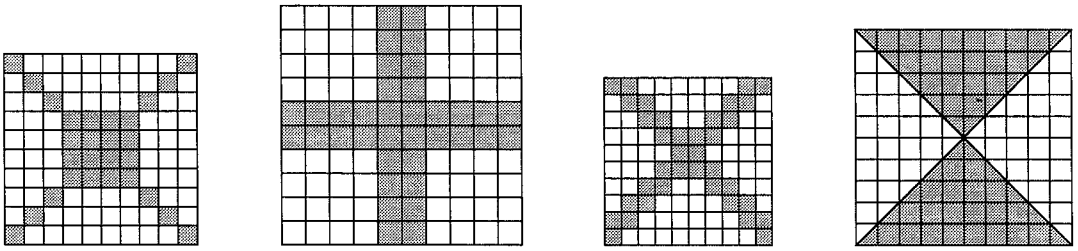
e) When multiplying whole numbers and/or decimals I have the most difficulty when...

f) Here is how I decide whether my answer to a division problem is reasonable...

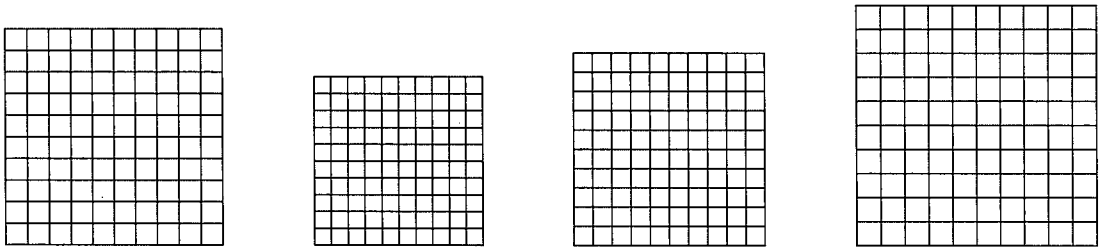
Connector Student Activity 21.1

NAME _____ DATE _____

1 Determine what percentage of each grid is shaded.



2 Shade the indicated percentage of each grid.



25% 9% .5% 60½%

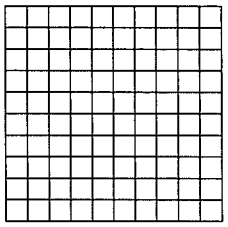
3 Suppose the grid shown here has a value of 300 and this value is distributed evenly among the small squares? Record your methods of determining the following:

a) What is the value of 10 small squares?

b) Shade and determine the value of 15% of the grid.

c) What is the value of 87.5 small squares.

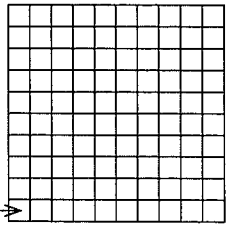
300



4 Suppose each small square on this grid has a value of 1.6. Record your methods of determining the following:

a) What is the value of the whole grid?

b) How many small squares would have a value of 56?



1.6 →

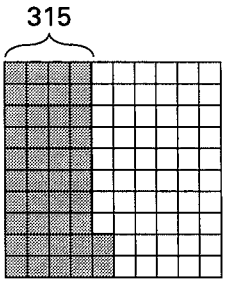
(Continued on back.)

Connector Student Activity (cont.)

c) 56 is what percent of the whole grid?

d) What is the value of 90% of the whole grid?

5 Suppose the shaded part of this grid has a value of 315 and this value is spread evenly among the shaded squares. Show your methods and reasoning for each of the following:



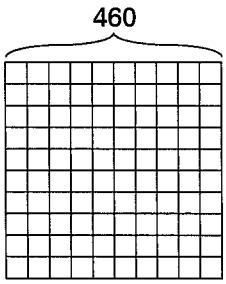
a) What is the value of 1 small square?

b) What is the value of the whole grid?

c) 315 is what percent of the whole grid?

d) What is the value of the unshaded part of this grid?

6 Suppose the following grid has a value of 460 and this value is spread evenly among the small squares. Show your methods and reasoning for each of the following:



a) What percent of the grid would have a value of 345?

b) What is the value of 27% of the grid?

7 Problems a)-c) below refer to the following line segment. Record your methods of determining each answer.

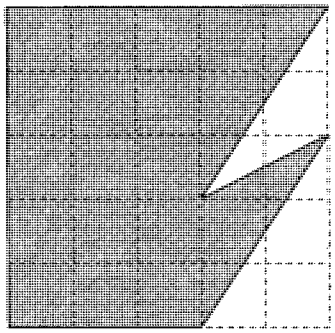
a) What is the total length of segment AB?

b) How long is 17% of segment AB?

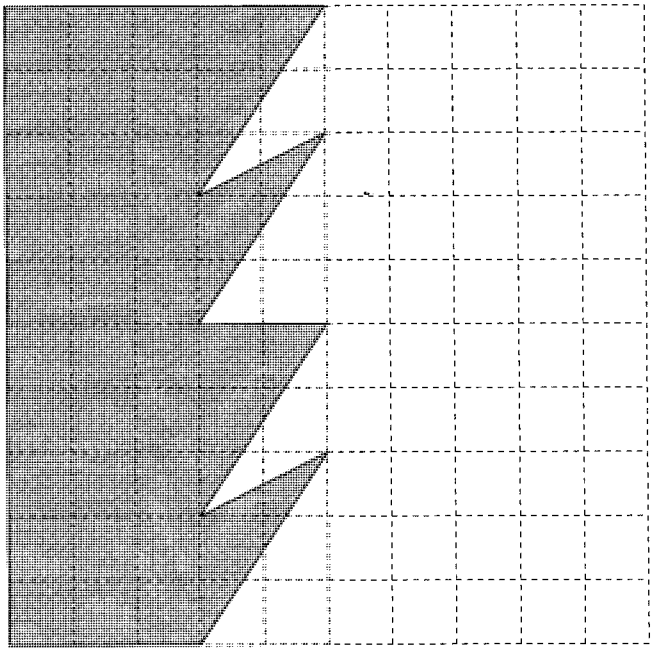
c) What percent of AB is 35 linear units?

Focus Master A

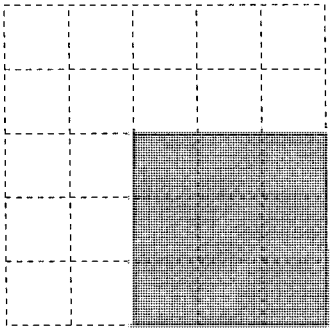
A



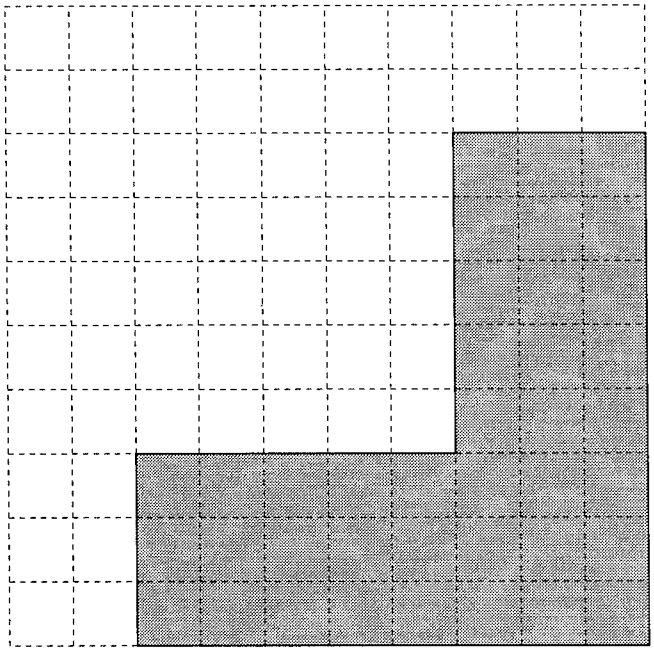
B



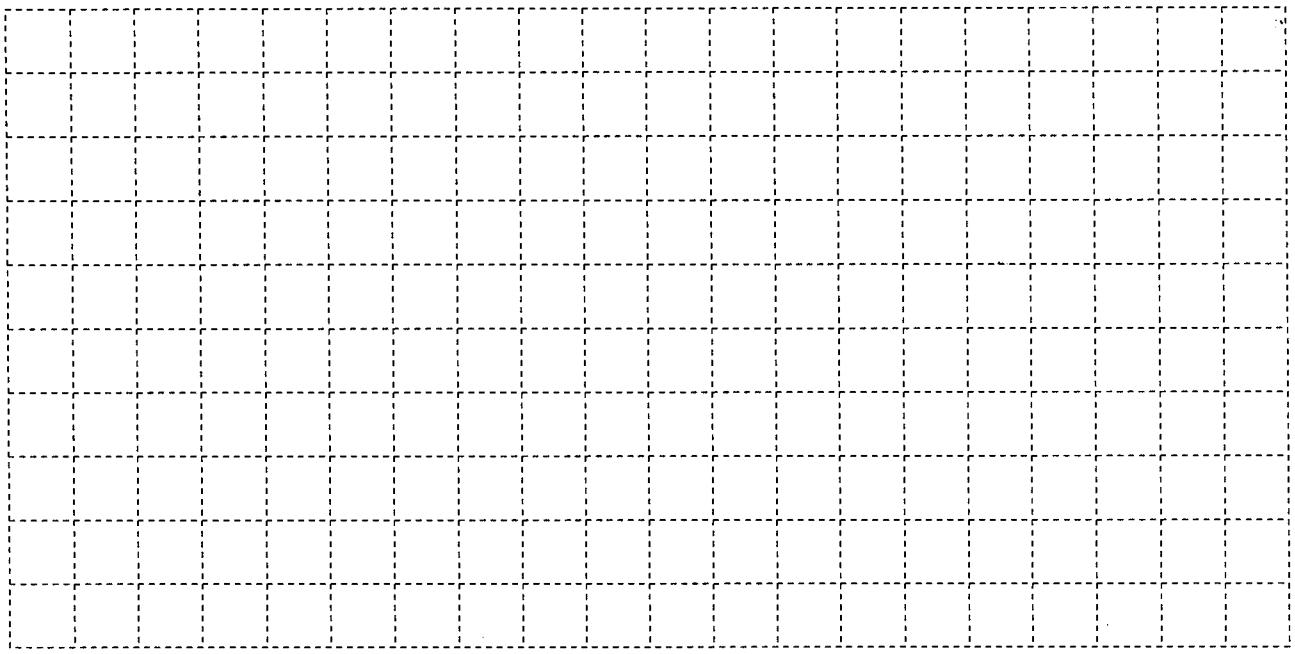
C

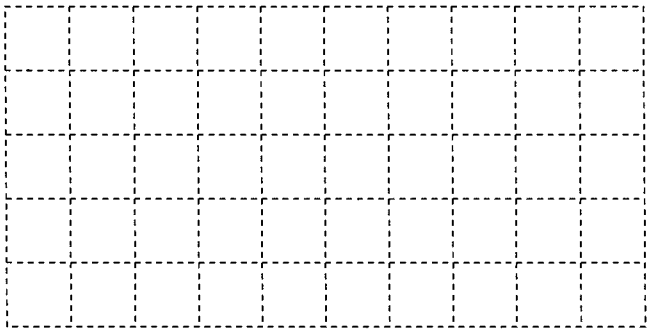


D



Focus Master B



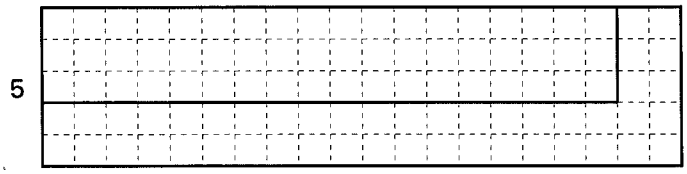


Focus Student Activity 21.2

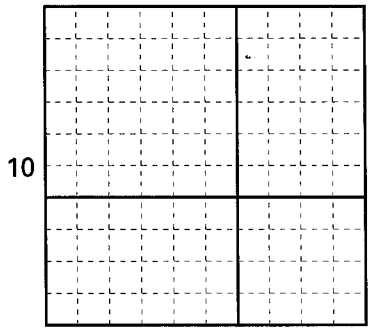
NAME _____ **DATE** _____

Find what percent the area of each subregion is of the entire region.

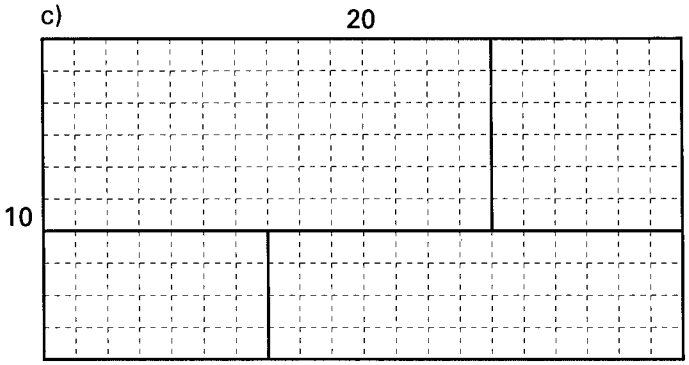
a)



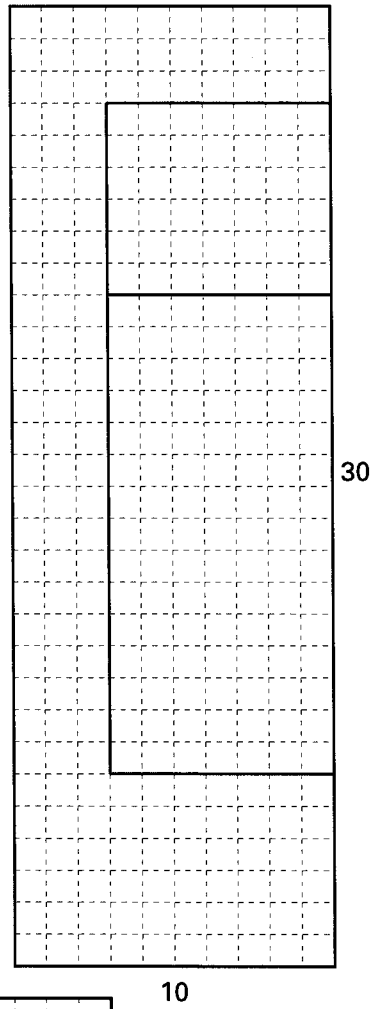
b)



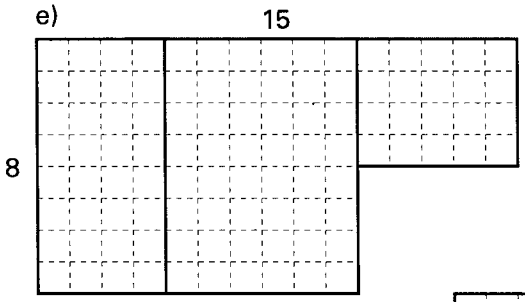
c)



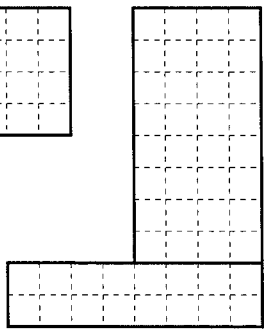
d)



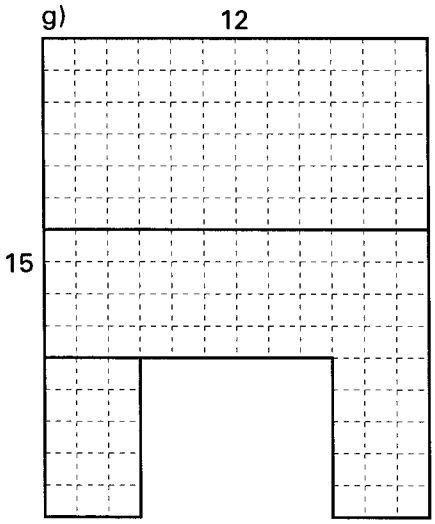
e)



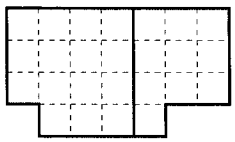
f)



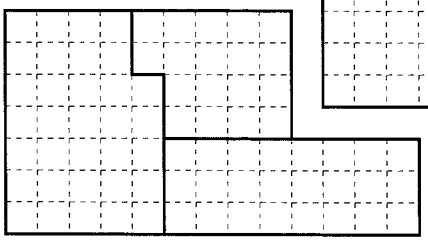
g)



h)



i)

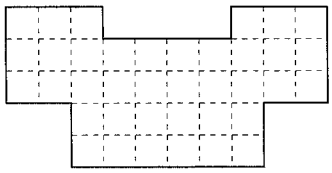


Focus Student Activity 21.3

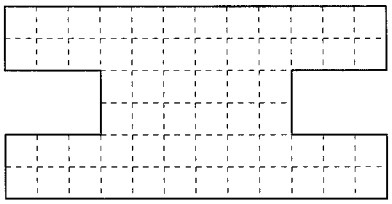
NAME _____ **DATE** _____

1 For each of the following regions, shade in the given percentage of its area.

a) 50%

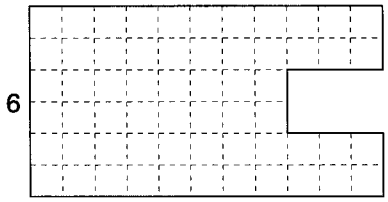


b) 75%



12

c) 20%



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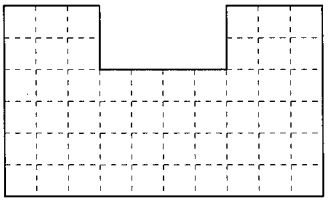
11

d) 33⅓%



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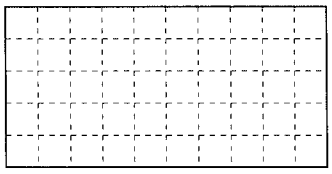
2 Shade in 30% of the area of each of the following regions.

a)

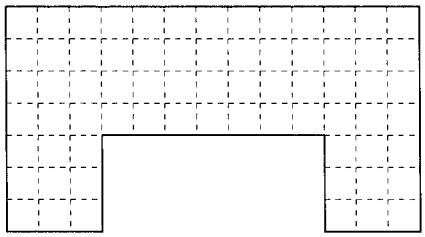


5

10

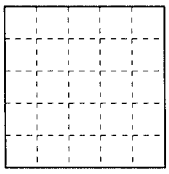
b)

13



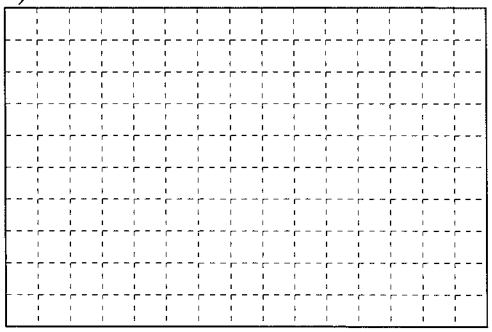
7

c)



3 Shade in 72% of the area of the following regions.

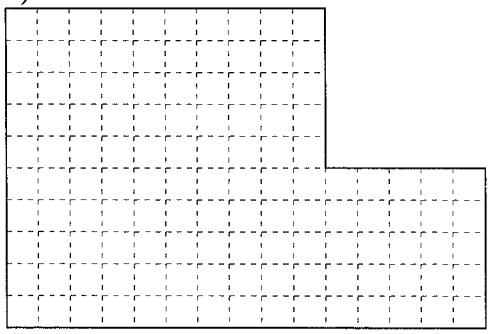
a)



10

15

b)



10

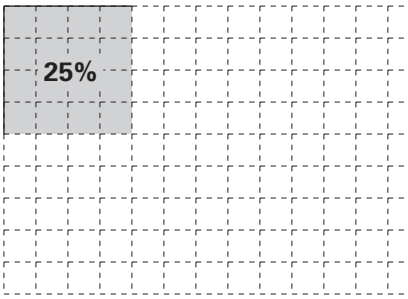
15

Focus Student Activity 21.4

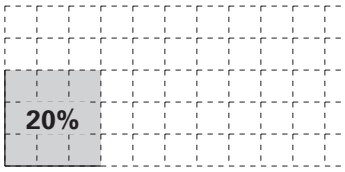
NAME _____ **DATE** _____

1 Complete the rectangle so the shaded region is the given percentage of its area.

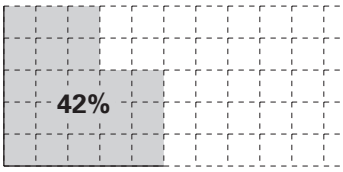
a)



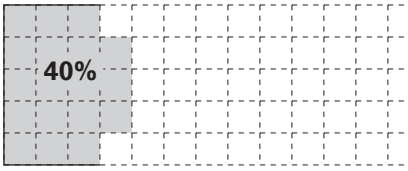
b)



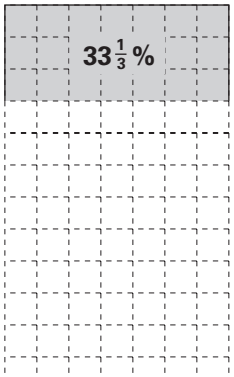
c)



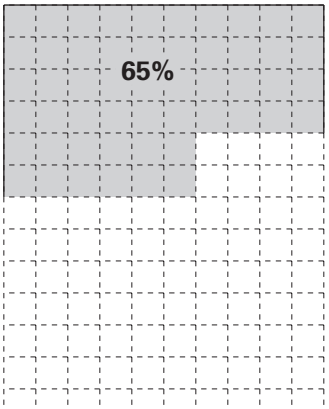
d)



e)

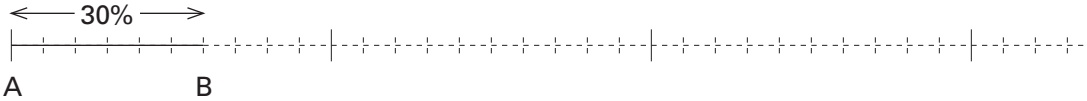


f)

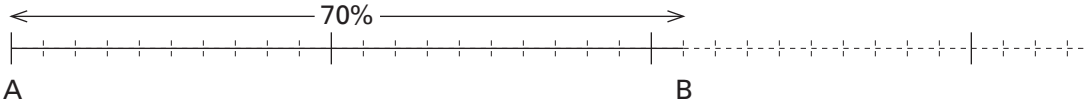


2 In each of the following, AB is part of a line segment AC. Complete the line segment AC, assuming AB is the given percentage of its length.

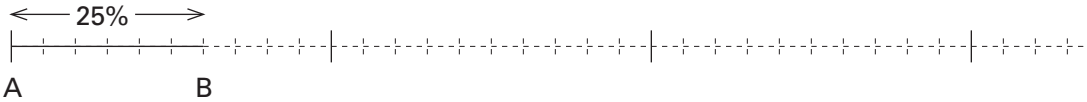
A



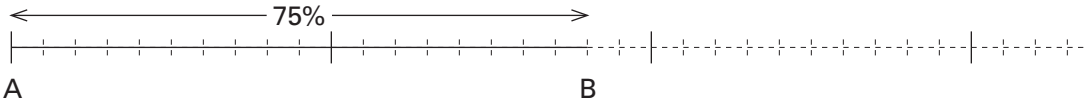
B



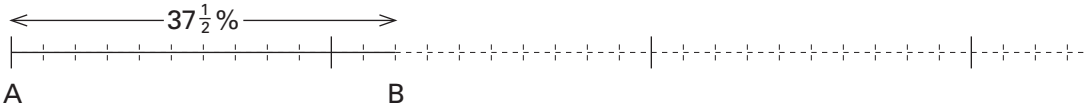
C



D



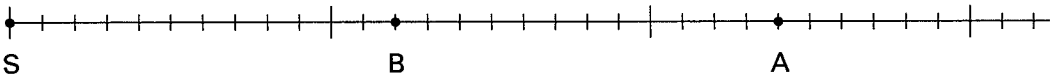
E

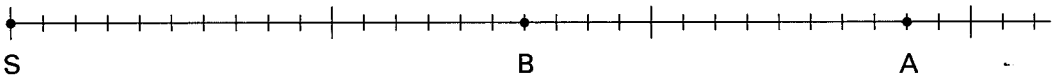


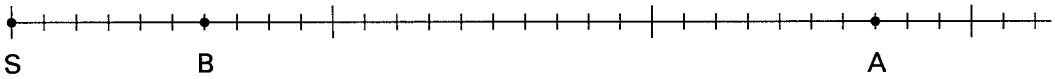
Focus Student Activity 21.5

NAME _____ **DATE** _____

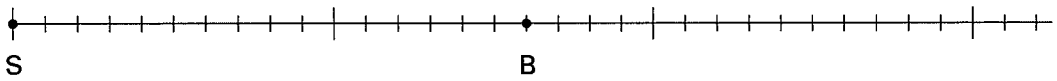
1 For each of the following find what percent length SA is of length SB.

a) 

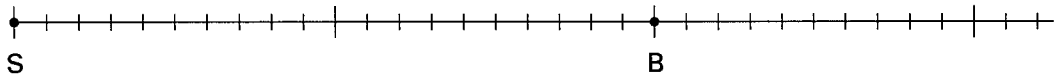
b) 

c) 

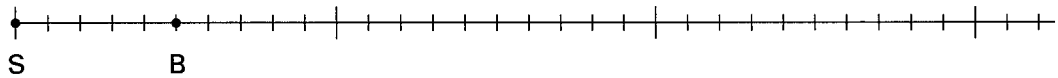
2 Find point A if the length of SA is 150% of the length of SB.



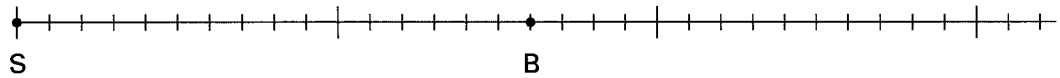
3 Find point A if the length of SA is 135% of the length of SB.



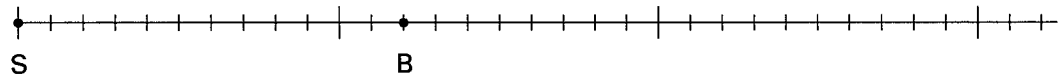
4 Find point A if the length of SA is 460% of the length of SB.



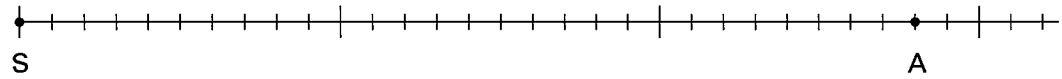
5 Find point A if SA is 25% longer than SB.



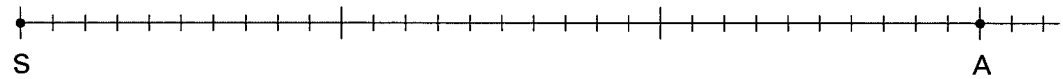
6 Find point A if the length of SA is the length of SB increased by 125%.



7 Find point B if the length of SA is 200% of the length of SB.



8 Find point B if the length of SA is 120% of the length SB.

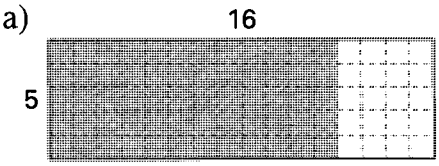


Follow-up Student Activity 21.6

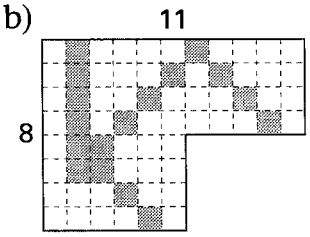
NAME _____ DATE _____

1 Determine what percent of the area of each region is shaded.

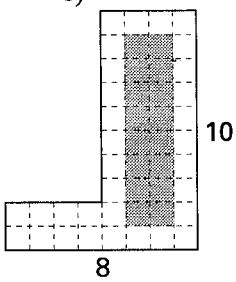
a)



b)

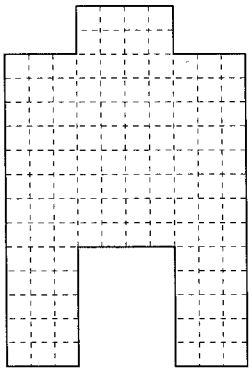


c)

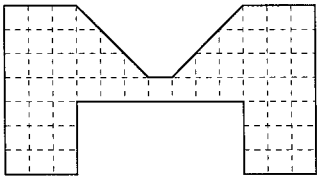


2 Shade the given percentage of the area of each of the following regions. On another sheet, explain how you decided the amount of each to shade.

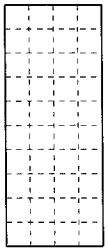
a) 75%



b) 75%

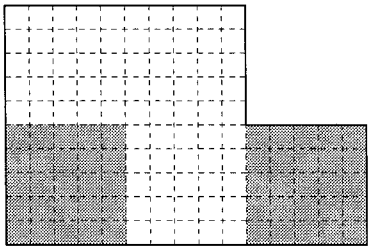


c) 40%

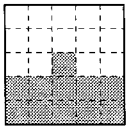


3 Which of the following regions has the greatest portion of its area shaded? Explain your reasoning.

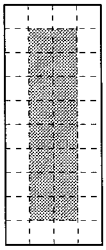
Region I



Region II



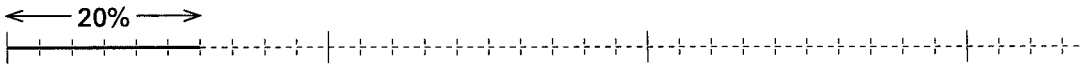
Region III

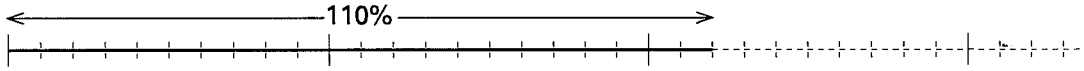


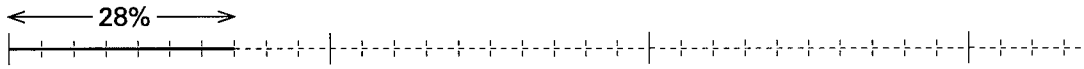
(Continued on back.)

Follow-up Student Activity (cont.)

4 In each of the following, part of a line segment is a given percentage of the whole line segment. Find the whole line segment and mark its end point. Mark the diagram to show your methods.

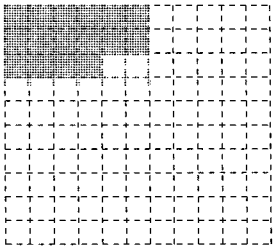
a) 

b) 

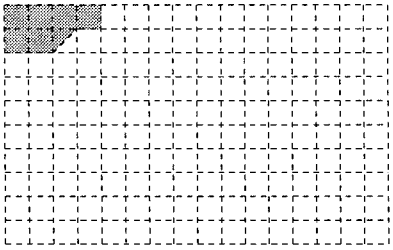
c) 

5 Complete the figure so the shaded region is the given percentage of its area.

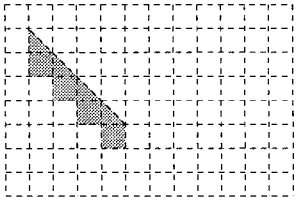
a) 25% of a square



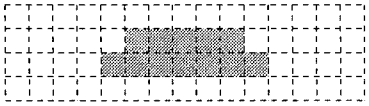
b) 12½% of a rectangle



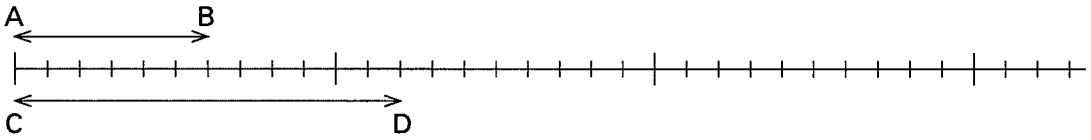
c) 33⅓% of a right triangle



d) 75% of a nonrectangular parallelogram



6 In each of the following, what percentage of the length of AB is the length of CD?

a) 

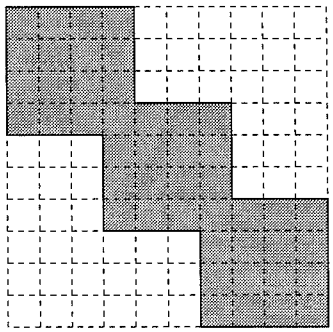
b) 

7 On another sheet, explain in your own words the meaning of percent. Use diagrams and examples to support your explanations.

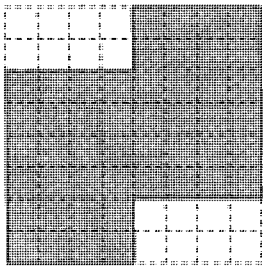
Connector Student Activity 22.1

NAME _____ **DATE** _____

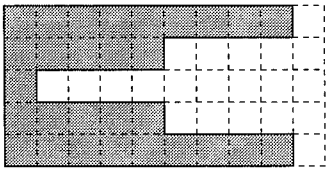
a)



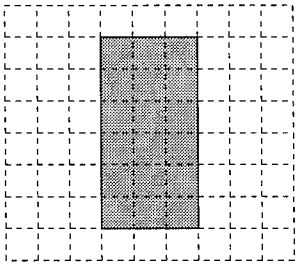
b)



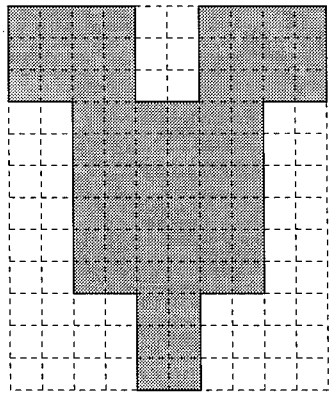
c)



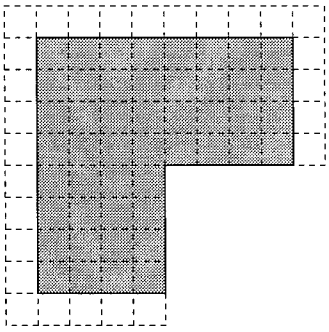
d)



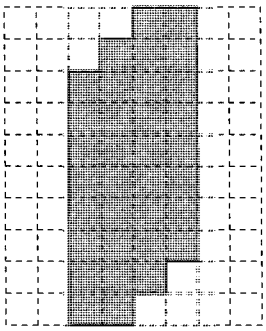
e)



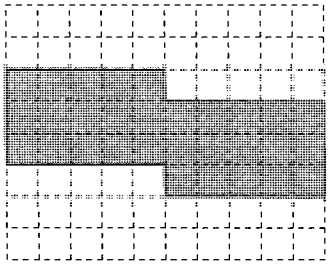
f)



g)



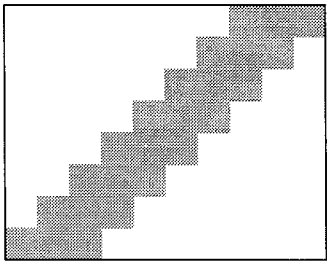
h)



Connector Student Activity 22.2

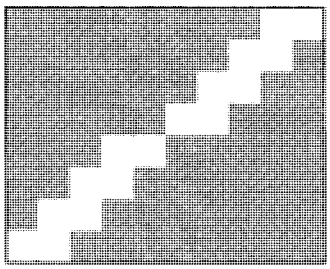
NAME _____ **DATE** _____

a)



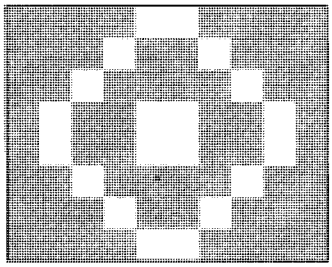
.3 of the area is shaded.

b)



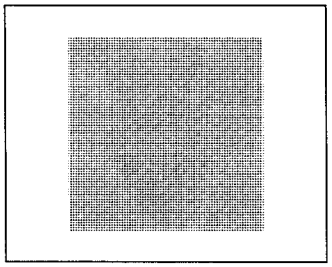
80% of the area is shaded.

c)



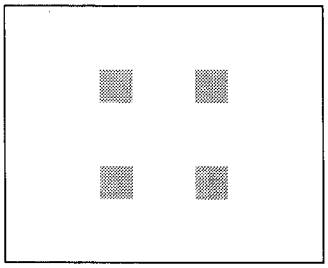
75% of the area is shaded.

d)



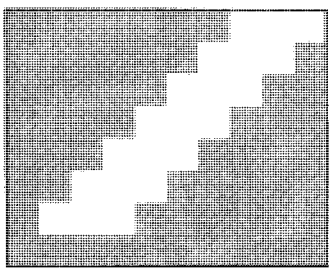
.45 of the area is shaded.

e)



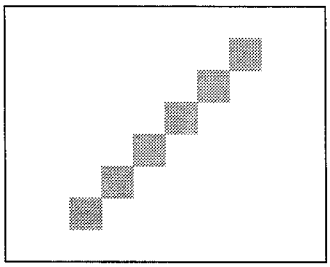
5% of the area is shaded.

f)



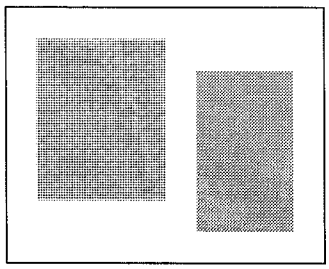
.725 of the area is shaded.

g)



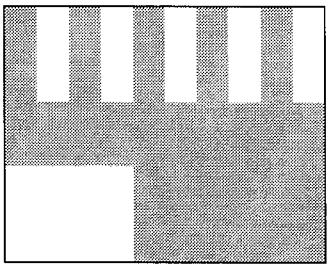
.075 of the area is shaded.

h)



.4375 of the area is shaded.

i)

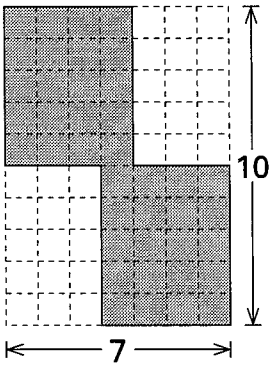


66¼% of the area is shaded.

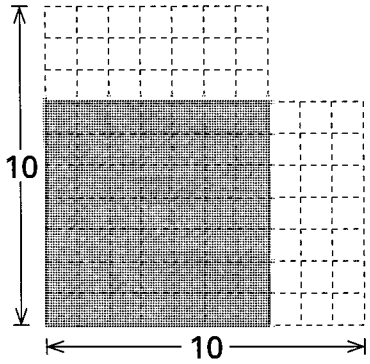
Connector Student Activity 22.3

NAME _____ **DATE** _____

a)



b)



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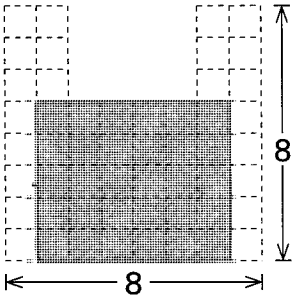

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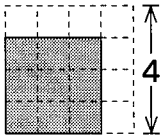
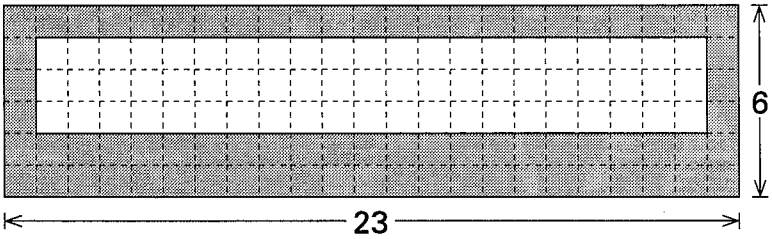
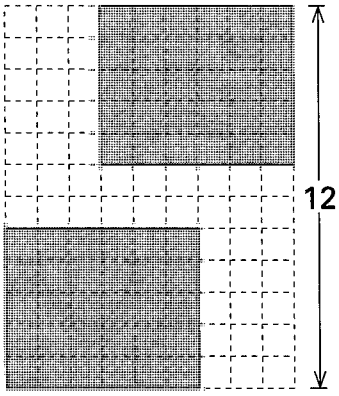


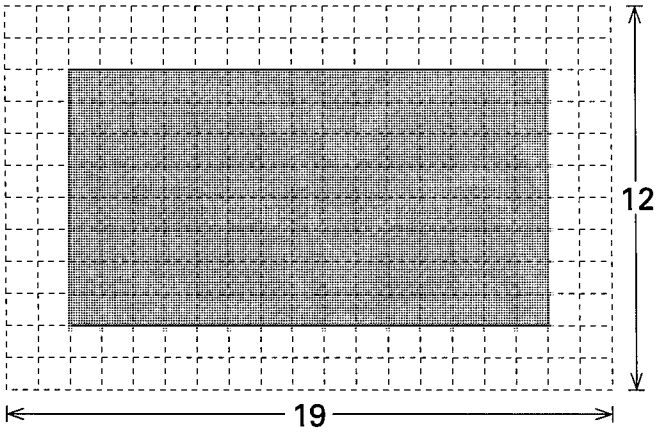
e)



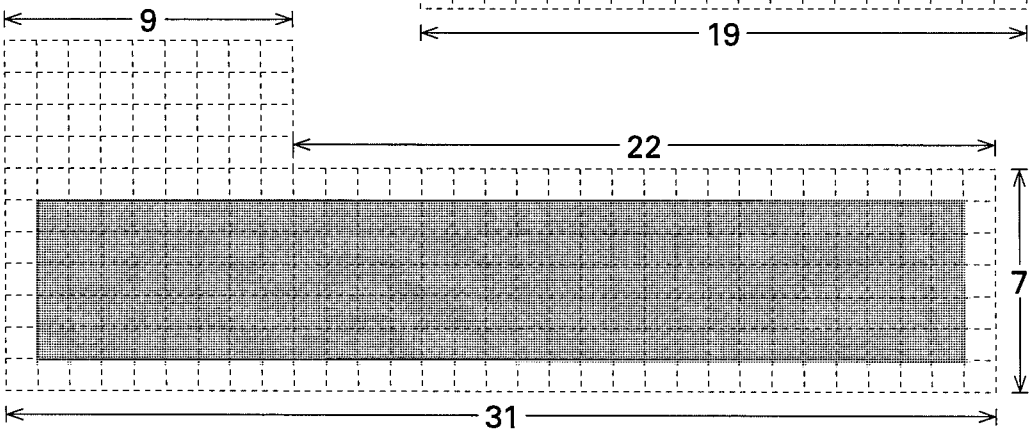
f)



g)



h)

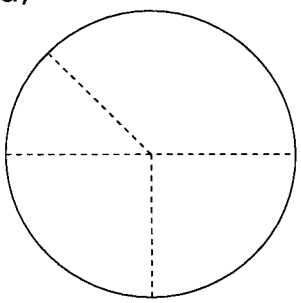


Connector Student Activity 22.4

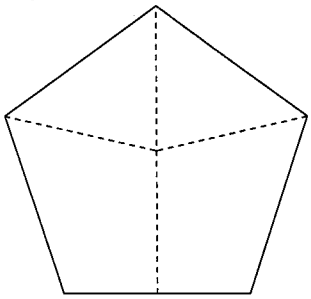
NAME _____ **DATE** _____

What percent is the area of each subregion of the area of the entire region?

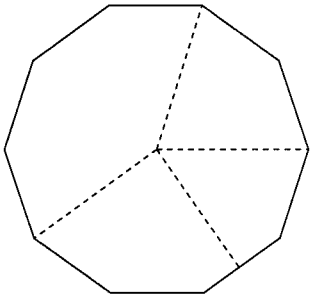
a)



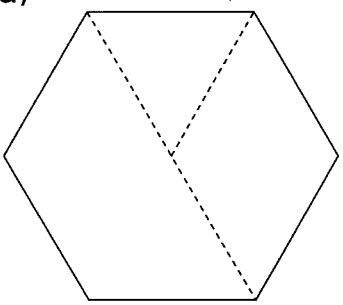
b)



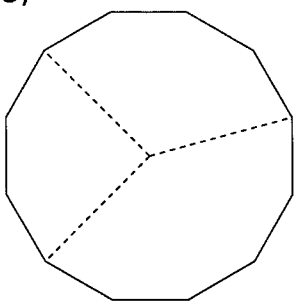
c)



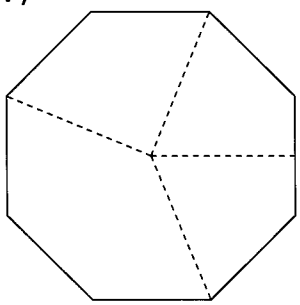
d)



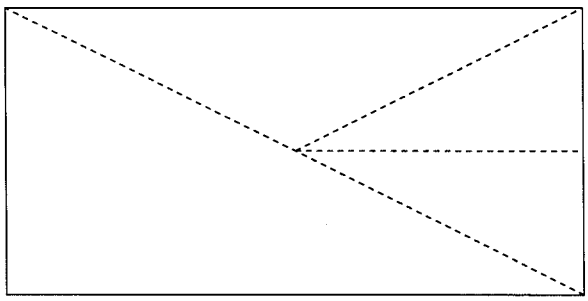
e)



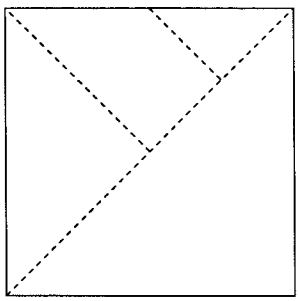
f)



g)



h)

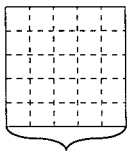


Follow-up Student Activity 22.5

NAME _____ DATE _____

Show all of your explanations and work on separate paper.

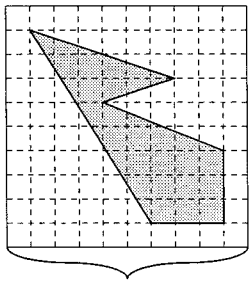
1 Suppose the square at the right has value 300 and this value is spread evenly among all the small squares.



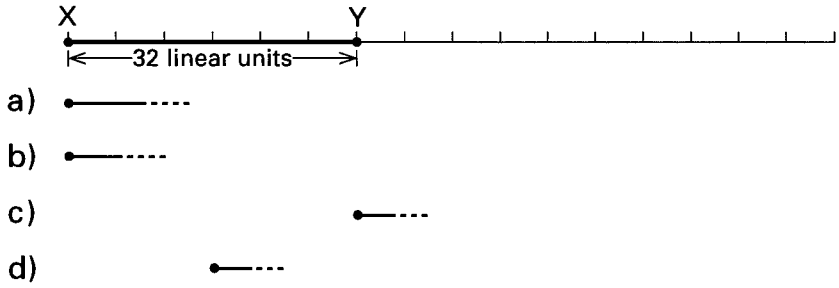
a) Shade 39% of the large square and explain how you determined how much to shade.

b) Determine the value of the part you shaded. Explain how you determined this value.

2 The square at the right has value 52 and this value is spread evenly among all the small squares. Determine the value of the shaded part and explain your methods.



3 In the diagram below, one endpoint of each of Segments a)-d) is sketched. Finish each segment so that statements a)-d) are true. Record the length of each completed segment.



a) The length of Segment a) is 150% of the length XY.

b) The length of Segment b) is $66\frac{2}{3}$% of the length of Segment XY.

c) Segment c) is 225% as long as Segment b).

d) Segment d) is $\frac{7}{9}$ as long as Segment c).

e) What percent of the length of Segment c) is the length of Segment a)? Explain how you decided this.

f) What percent of the length of Segment d) is the length of Segment c)? Explain how you decided this.

(Continued on back.)

Follow-up Student Activity (cont.)

4 Sketch diagrams that show the mathematical relationships in each of these problems. Use your diagrams (and whatever calculating option you choose) to determine the answers to the questions. Mark your diagrams to illustrate your thought processes and record all computations you do. (Remember the point here is *not* to find an answer and draw a picture of it, but rather to use the picture to solve the problem!)

a) Corrina has paid $880 towards a $2000 loan. What percent of the loan has she paid?

b) After a discount of 70% the price of a bracelet is $18. What was the original price of the bracelet?

c) The price of a sleeping bag increased by 28%. Its original price was $154. What is the increased price of the bag?

5 A department store had a "⅓ off everything in the store" sale. Collin, Jillian, Martina, and Rocco each made purchases at the sale. Which, if any, of the following statements, are correct and which, if any, are incorrect? Write brief explanations and diagrams to illustrate your reasoning about each statement.

a) Collin said he paid 66⅔% of the original price for his purchase.

b) Martina said that the original price was 150% of what she paid.

c) Jillian said the original price was 33⅓% more than what she paid.

d) Rocco said that he could buy 6 shirts on sale for the price of 4 shirts when they were not on sale.

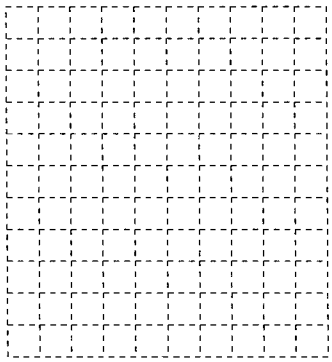
6 The average gratuity (tip) given for service in a restaurant is 15% of the bill. Interview 3 different people regarding the methods they use to calculate a 15% gratuity on $40; on $78; and on $101.67. Describe their methods.

7 Explain how you would *mentally calculate* a 15% gratuity for each of the following bills: $24.00; $19.97; $62.25.

Connector Student Activity 23.1

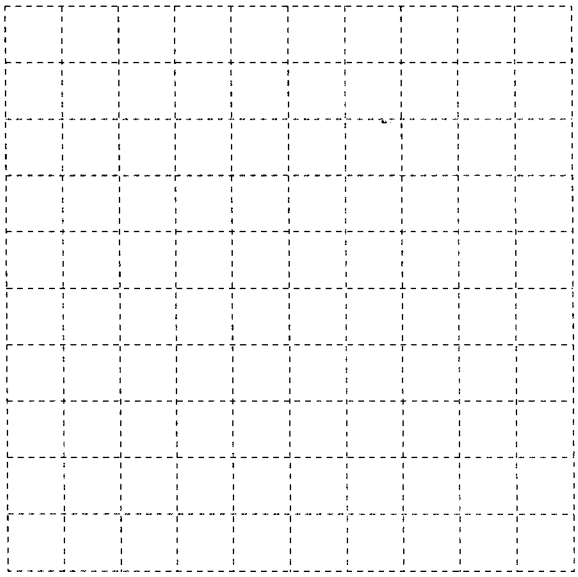
NAME _____ **DATE** _____

Grid A



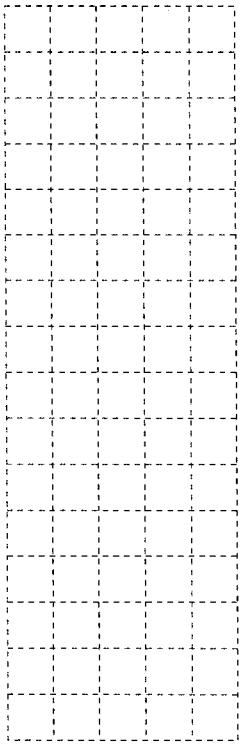
4 shaded squares for every 7 unshaded squares

Grid B



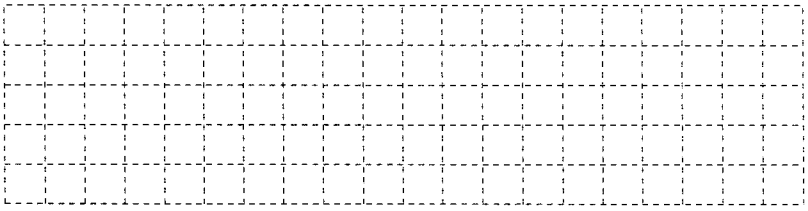
10 shaded squares for every 16 squares

Grid C



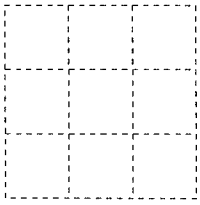
5 unshaded for every 3 shaded

Grid D



27 unshaded for every 40 squares

Grid E

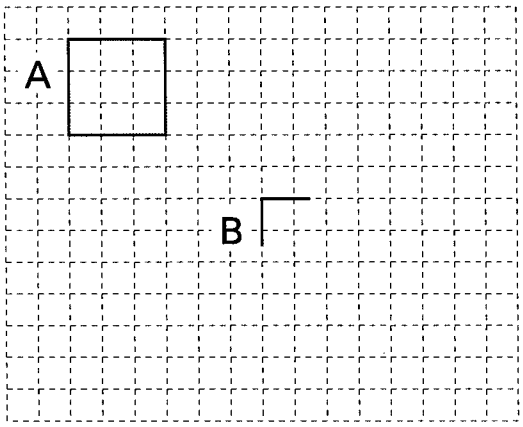


1 shaded for every 3 unshaded

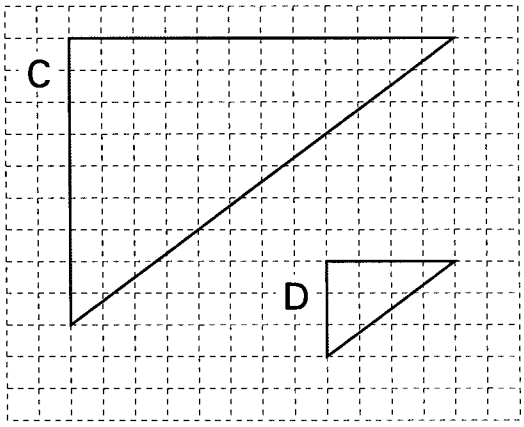
Focus Student Activity 23.3

NAME _____ **DATE** _____

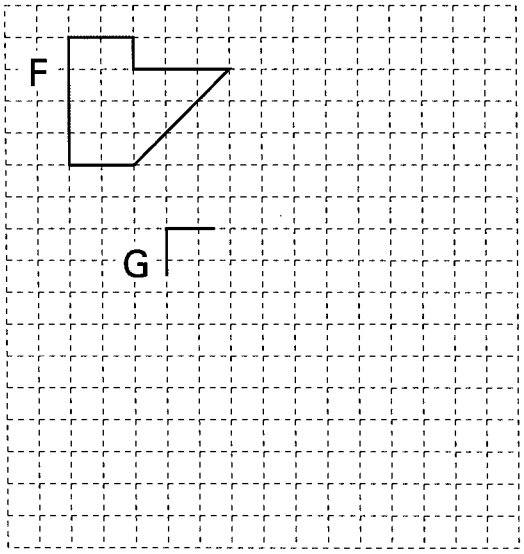
a) Complete Square B so the ratio of the length of its side to the side of Square A is 2 to 1. What is the ratio of the area of Square B to the area of Square A?



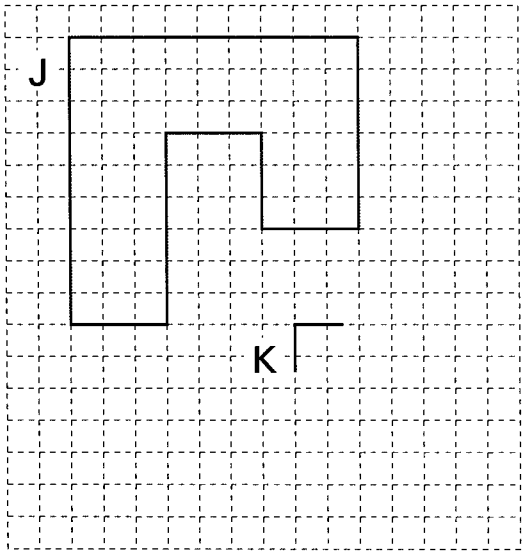
b) What is the ratio of the perimeter of Triangle C to the perimeter of Triangle D? What is the ratio of the area of Triangle C to the area of Triangle D?



c) Complete Figure G so it has the same shape as Figure F and the ratio of the perimeter of F to the perimeter of G is 1 to 2. What is the ratio of the area of F to the area of G?



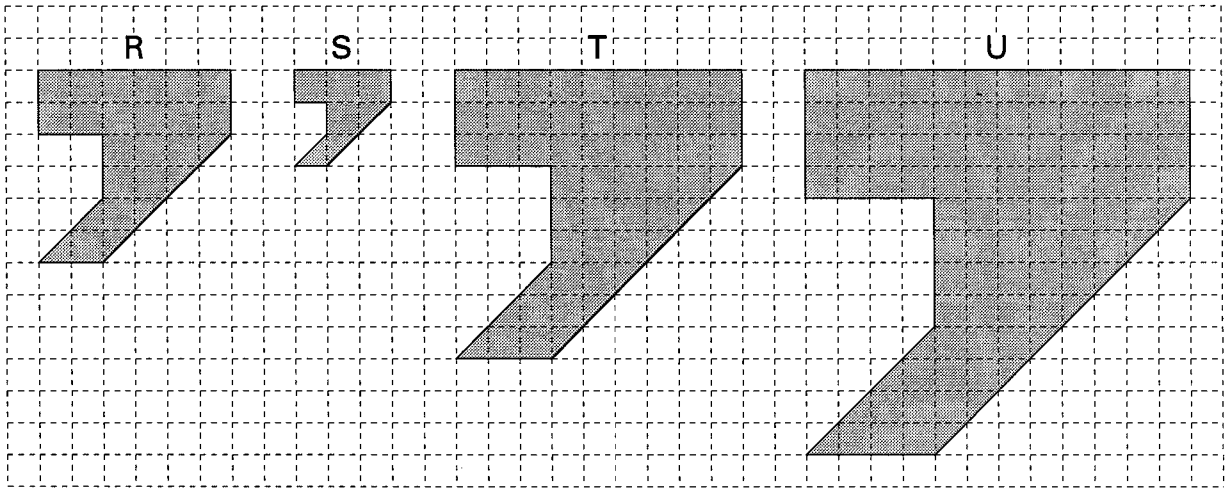
d) Complete Figure K so it has the same shape as Figure J and the ratio of the perimeter of J to the perimeter of K is 3 to 2. What is the ratio of the area of J to the area of K?



Follow-up Student Activity 23.4

NAME _____ **DATE** _____

1 Use these figures to answer the questions below.



a) What is the simplest ratio of the perimeters for each pair of the following figures (note "simplest ratio" means there is no equivalent ratio using smaller whole numbers).

R to S _____ S to R _____ S to T _____
R to T _____ T to U _____ R to U _____

b) What is the simplest ratio of the areas for each pair of the following figures?

R to S _____ S to R _____ S to T _____
R to T _____ T to U _____ R to U _____

c) Make some mathematical observations about relationships you notice in a) and b) above.

(Continued on back.)

Follow-up Student Activity (cont.)

2 In your own words, explain the meaning of the term *ratio* and the term *proportion*. Give some examples to illustrate your explanations.

3 For each of the following, on another sheet of paper, show how to use a diagram or sketch to determine the missing information. It is okay to use a calculator, but the point here is to use a diagram of the relationships to determine what to compute. Next to your diagram show any computations that you do and circle ones you do with the calculator. If needed to help the reader "see" your thinking, add a few words or phrases to help explain how you used your diagram to know what to compute.

a) The ratio of girls to boys in the 7th grade at Portsmouth Middle School last year was 5 to 6. There were 72 boys and _____ girls.

b) Last year's sales receipts in a local music store showed that for every 7 country western tapes that were sold, 8 rock and roll tapes were also sold. A total of 1799 country western tapes were sold and _____ rock and roll tapes were sold.

c) A bag is filled with green and blue marbles so that for every 8 green marbles there are 12 blue ones. There are 108 blue marbles and _____ total marbles in the bag.

d) The ratio of private school students to public school students in a city is 2 to 17 and there is a total of 15,922 students from both types of schools. There are _____ more public school students than private school students.

Follow-up Student Activity 24.1

NAME _____ DATE _____

Write your answers to all of these problems on separate paper.

1 Ten different students each completed 20 random draws from a bag of 60 black and white balls (a ball was replaced in the bag after it was drawn). Here is the data they collected:

1. 13 black, 7 white
2. 17 black, 3 white
3. 13 black 7 white
4. 12 black, 8 white
5. 7 black, 13 white
6. 13 black, 7 white
7. 16 black, 4 white
8. 10 black, 10 white
9. 12 black, 8 white
10. 15 black, 5 white

a) Write several "I notice..." statements about the above data.

b) Describe what you think is a likely distribution of black and white balls in the bag. Use mathematical evidence to support your opinion.

c) Assuming you cannot look in the bag and balls must be replaced after each draw, describe what you would do to determine its contents with more confidence than you feel about your prediction from b). Explain why this procedure would give you more confidence in your conclusion.

2 The following chart shows the frequency of each color (blue, green, red, yellow, orange, and brown) in six 8-ounce bags of M&M's® Plain Chocolate Candies.

Sample #	BL	G	R	Y	O	BR	Total
1	38	39	56	41	11	75	260
2	47	35	50	33	17	79	261
3	28	29	50	52	21	77	257
4	19	27	58	59	30	65	258
5	35	38	47	47	13	80	260
6	38	31	48	46	10	85	258

(Continued on back.)

Follow-up Student Activity (cont.)

a) Write several "I notice..." statements about the M&M's® data.

b) What changes a lot and what changes very little across the M&M's® samples? Explain what you think could contribute to these changes and regularities.

c) Use the M&M's® data to make a prediction regarding what you think is the probable *range* of percentages of each color that the candy company puts in a bag of M&M's®. Create a report that presents your conclusions, a convincing and valid mathematical argument that supports your conclusions, and an explanation of your level of confidence in your conclusions. Use this report to demonstrate:

- your understanding of methods of making meaningful and reliable predictions from samples;
- your understanding of graphing and statistical techniques and your ability to use them appropriately;
- the quality and effectiveness of your mathematical communication and reasoning.

3 Stefan has a 4-oz. bag of candy which contains pieces of the following colors: Blue, 12; Green, 18; Red, 24; Yellow, 27; Orange, 9; and Brown, 38.

a) Choose one of the following labels, **Likely, Unlikely, Not Sure,** or **Impossible**, to indicate your feelings about whether Stefan has a bag of M&M's®. Explain the reasoning behind your choice.

b) For each of the labels, **Likely, Unlikely, Not Sure,** or **Impossible**, that you did not choose in a), describe the contents of a bag of candy which you feel is appropriate for that label.

Focus Master B

Data Collection and Analysis Report

1. Design and carry out a survey to get information about your research question.

2. Submit a written report with the following 6 sections:

a) The question(s) you hoped to answer with your survey data.

b) How you collected data about your question(s), including:

- copies of all surveys and interview questions,
- a detailed description of the population you sampled,
- a detailed description of how, when, and where you sampled,
- how you tried to eliminate sources of bias from your sampling procedures.

c) An organized visual display of your data. This should show your knowledge of a variety of graphing techniques.

d) An in-depth, written statistical analysis of your data. This is the "heart" of your report. Diagrams, graphs, and explanations should reveal as much as possible about your knowledge.

e) Your conclusions—the answer(s) to your research question(s).

f) Your reflections about the above process, including:

- what you learned about yourself as a mathematician,
- what you learned about mathematics,
- new questions that were prompted by your research results,
- a thorough self-analysis of the quality of your work.

3. Make a 10-15 minute oral presentation of your report and display.

Focus Master C

Survey Project Time lines

	Date Due	Date Completed	Teacher Initials
a) Choose survey topic and questions to research.	_____	_____	_____
b) Write draft of survey and plans for conducting the survey; get peer feedback about survey and plans on Focus Master D.	_____	_____	_____
c) Finalize survey and data collection plans.	_____	_____	_____
d) Gather data; organize and prepare visual displays of data.	_____	_____	_____
e) Complete data analysis and first draft of report and reflections (see Focus Master B).	_____	_____	_____
f) Rehearse oral presentation and get peer feedback on Focus Master E. Edit report as needed.	_____	_____	_____
g) Give oral presentation and collect feedback on Focus Master E. Turn in final report.	_____	_____	_____

Focus Master D

Survey and Data Collection Plan—Feedback Form

Feedback for _____ by _____

Survey

a) Are the questions easy to understand? If not, what wording is confusing?

b) Is the survey likely to produce biased responses? If so, how?

c) Will the survey questions help the surveyor answer their research question? Explain.

d) Comments:

Data Collection Plan

a) Are methods of choosing the sample population appropriate?

b) Is the sample size reasonable?

c) Will the sample be representative of the population? If not, explain.

d) Comments:

Focus Master D

Survey and Data Collection Plan—Feedback Form

Feedback for _____ by _____

Survey

a) Are the questions easy to understand? If not, what wording is confusing?

b) Is the survey likely to produce biased responses? If so, how?

c) Will the survey questions help the surveyor answer their research question? Explain.

d) Comments:

Data Collection Plan

a) Are methods of choosing the sample population appropriate?

b) Is the sample size reasonable?

c) Will the sample be representative of the population? If not, explain.

d) Comments:

Focus Master D

Survey and Data Collection Plan—Feedback Form

Feedback for _____ by _____

Survey

a) Are the questions easy to understand? If not, what wording is confusing?

b) Is the survey likely to produce biased responses? If so, how?

c) Will the survey questions help the surveyor answer their research question? Explain.

d) Comments:

Data Collection Plan

a) Are methods of choosing the sample population appropriate?

b) Is the sample size reasonable?

c) Will the sample be representative of the population? If not, explain.

d) Comments:

Focus Master E

Oral and Written Report Feedback Form

Feedback for _____ by _____

Please record an **X** on each line to show your ratings.

1. Purpose for the survey (i.e., what is their research question?).

0	1	2	3	4
not given		clear		very clear

2. Sampling procedures (including copies of all surveys, questionnaires, etc.).

0	1	2	3	4
not given	lacks clarity and/or details	clear and complete		very clear and detailed

3. Attempts to avoid bias in the survey and data collection procedures.

0	1	2	3	4
not given or extreme bias possible		shows evidence that sources of bias were considered		shows very solid understanding of sources of bias; detailed evidence of effort to avoid bias

4. An organized display and description of the data collected.

0	1	2	3	4
little or no organization	some organization lacking	organized display and explanation		exemplary organization and display; explanations throughout memorable

5. Statistical analysis of the data.

   a) Shows appropriate use of a variety of graphing techniques (e.g., line plot, stem and leaf plot, bar graph, circle graph, scatter plot).

0	1	2	3	4
graphs are all inaccurate, incomplete, or missing	some inaccuracy or inappropriate uses	accurate and appropriate		exemplary use of graphing throughout; multiple graphs shown for most data; explanations include rationale for choice of graphs

(Continued on back.)

Focus Master E (cont.)

b) Shows ability to interpret the shape, centers (mean, median, mode), and spread (range) of the data graphed, including any evidence of bias.

0	1	2	3	4
no interpretations or excessive inaccuracies		correct interpretations		exemplary and memorable interpretations; detailed analysis

6. Conclusions

a) What they learned about their research question, based on their survey results.

0	1	2	3	4
no conclusion given		conclusion is logical and adequate		memorable; thorough; very well supported by statistical and graphical evidence; communicated eloquently

b) What they learned about themselves as statisticians.

0	1	2	3	4
not given	some information given			extremely reflective; shows high level of understanding of data analysis process; used specific examples

7. Overall Rating

0	1	2	3	4
minimal effort reflected in presentation; missed the point of the project		adequate presentation; reveals solid grasp of concepts and procedures of data collection and analysis		exemplary and memorable; shows very high level of understanding of concepts and procedures of data collection and analysis; shows exceptional ability to communicate mathematically

8. Comments:

Focus Master E

Oral and Written Report Feedback Form

Feedback for _____ by _____

Please record an **X** on each line to show your ratings.

1. Purpose for the survey (i.e., what is their research question?).

0	1	2	3	4
not given		clear		very clear

2. Sampling procedures (including copies of all surveys, questionnaires, etc.).

0	1	2	3	4
not given	lacks clarity and/or details	clear and complete		very clear and detailed

3. Attempts to avoid bias in the survey and data collection procedures.

0	1	2	3	4
not given or extreme bias possible		shows evidence that sources of bias were considered		shows very solid understanding of sources of bias; detailed evidence of effort to avoid bias

4. An organized display and description of the data collected.

0	1	2	3	4
little or no organization	some organization lacking	organized display and explanation		exemplary organization and display; explanations throughout memorable

5. Statistical analysis of the data.

   a) Shows appropriate use of a variety of graphing techniques (e.g., line plot, stem and leaf plot, bar graph, circle graph, scatter plot).

0	1	2	3	4
graphs are all inaccurate, incomplete, or missing	some inaccuracy or inappropriate uses	accurate and appropriate		exemplary use of graphing throughout; multiple graphs shown for most data; explanations include rationale for choice of graphs

(Continued on back.)

Focus Master E (cont.)

b) Shows ability to interpret the shape, centers (mean, median, mode), and spread (range) of the data graphed, including any evidence of bias.

0	1	2	3	4
no interpretations or excessive inaccuracies		correct interpretations		exemplary and memorable interpretations; detailed analysis

6. Conclusions

a) What they learned about their research question, based on their survey results.

0	1	2	3	4
no conclusion given		conclusion is logical and adequate		memorable; thorough; very well supported by statistical and graphical evidence; communicated eloquently

b) What they learned about themselves as statisticians.

0	1	2	3	4
not given	some information given			extremely reflective; shows high level of understanding of data analysis process; used specific examples

7. Overall Rating

0	1	2	3	4
minimal effort reflected in presentation; missed the point of the project		adequate presentation; reveals solid grasp of concepts and procedures of data collection and analysis		exemplary and memorable; shows very high level of understanding of concepts and procedures of data collection and analysis; shows exceptional ability to communicate mathematically

8. Comments:

Focus Master E

Oral and Written Report Feedback Form

Feedback for _____ by _____

Please record an **X** on each line to show your ratings.

1. Purpose for the survey (i.e., what is their research question?).

0	1	2	3	4
not given		clear		very clear

2. Sampling procedures (including copies of all surveys, questionnaires, etc.).

0	1	2	3	4
not given	lacks clarity and/or details	clear and complete		very clear and detailed

3. Attempts to avoid bias in the survey and data collection procedures.

0	1	2	3	4
not given or extreme bias possible		shows evidence that sources of bias were considered		shows very solid understanding of sources of bias; detailed evidence of effort to avoid bias

4. An organized display and description of the data collected.

0	1	2	3	4
little or no organization	some organization lacking	organized display and explanation		exemplary organization and display; explanations throughout memorable

5. Statistical analysis of the data.

   a) Shows appropriate use of a variety of graphing techniques (e.g., line plot, stem and leaf plot, bar graph, circle graph, scatter plot).

0	1	2	3	4
graphs are all inaccurate, incomplete, or missing	some inaccuracy or inappropriate uses	accurate and appropriate		exemplary use of graphing throughout; multiple graphs shown for most data; explanations include rationale for choice of graphs

(Continued on back.)

Focus Master E (cont.)

b) Shows ability to interpret the shape, centers (mean, median, mode), and spread (range) of the data graphed, including any evidence of bias.

0	1	2	3	4
no interpretations or excessive inaccuracies		correct interpretations		exemplary and memorable interpretations; detailed analysis

6. Conclusions

a) What they learned about their research question, based on their survey results.

0	1	2	3	4
no conclusion given		conclusion is logical and adequate		memorable; thorough; very well supported by statistical and graphical evidence; communicated eloquently

b) What they learned about themselves as statisticians.

0	1	2	3	4
not given	some information given			extremely reflective; shows high level of understanding of data analysis process; used specific examples

7. Overall Rating

0	1	2	3	4
minimal effort reflected in presentation; missed the point of the project		adequate presentation; reveals solid grasp of concepts and procedures of data collection and analysis		exemplary and memorable; shows very high level of understanding of concepts and procedures of data collection and analysis; shows exceptional ability to communicate mathematically

8. Comments:

Focus Master E

Oral and Written Report Feedback Form

Feedback for _____ by _____

Please record an **X** on each line to show your ratings.

1. Purpose for the survey (i.e., what is their research question?).

0	1	2	3	4
not given		clear		very clear

2. Sampling procedures (including copies of all surveys, questionnaires, etc.).

0	1	2	3	4
not given	lacks clarity and/or details	clear and complete		very clear and detailed

3. Attempts to avoid bias in the survey and data collection procedures.

0	1	2	3	4
not given or extreme bias possible		shows evidence that sources of bias were considered		shows very solid understanding of sources of bias; detailed evidence of effort to avoid bias

4. An organized display and description of the data collected.

0	1	2	3	4
little or no organization	some organization lacking	organized display and explanation		exemplary organization and display; explanations throughout memorable

5. Statistical analysis of the data.

   a) Shows appropriate use of a variety of graphing techniques (e.g., line plot, stem and leaf plot, bar graph, circle graph, scatter plot).

0	1	2	3	4
graphs are all inaccurate, incomplete, or missing	some inaccuracy or inappropriate uses	accurate and appropriate		exemplary use of graphing throughout; multiple graphs shown for most data; explanations include rationale for choice of graphs

(Continued on back.)

Focus Master E (cont.)

b) Shows ability to interpret the shape, centers (mean, median, mode), and spread (range) of the data graphed, including any evidence of bias.

0	1	2	3	4
no interpretations or excessive inaccuracies		correct interpretations		exemplary and memorable interpretations; detailed analysis

6. Conclusions

a) What they learned about their research question, based on their survey results.

0	1	2	3	4
no conclusion given		conclusion is logical and adequate		memorable; thorough; very well supported by statistical and graphical evidence; communicated eloquently

b) What they learned about themselves as statisticians.

0	1	2	3	4
not given	some information given			extremely reflective; shows high level of understanding of data analysis process; used specific examples

7. Overall Rating

0	1	2	3	4
minimal effort reflected in presentation; missed the point of the project		adequate presentation; reveals solid grasp of concepts and procedures of data collection and analysis		exemplary and memorable; shows very high level of understanding of concepts and procedures of data collection and analysis; shows exceptional ability to communicate mathematically

8. Comments:

Focus Student Activity 25.1

NAME _____ **DATE** _____

Defining My Question (What, Why, When, Where, How)

1 Here are 2 questions I think would be interesting to investigate.

Question A:

Question B:

2 Here are some reasons *why* I am interested in these questions.

Question A:

Question B:

(Continued on back.)

Focus Student Activity 25.1 (cont.)

3 Here are some ideas I have about using a survey to answer these questions.

Question A

When: Where: How:

Question B

When: Where: How:

4 Here are some obstacles/problems I think I could run into trying to collect data about Questions A and B.

Question A:

Question B:

5 Here is the question I would most like to research:

Focus Student Activity 25.2

NAME _____ **DATE** _____

Research question(s) the survey is designed to help answer: _____

_____

_____

1 Survey draft:

(Continued on back.)

Focus Student Activity 25.2 (cont.)

2 Data collection plans

a) Describe *who* will be in your sample and *what* population they represent.

b) Explain *how* you will select each person in your sample.

c) Tell the *size* of your sample.

d) Explain *where* you will conduct your survey.

e) Tell *when* you plan to conduct the survey and *when* you expect to have all data collection completed.

Follow-up Student Activity 25.3

NAME _____ DATE _____

The Wesell Advertising Agency is looking for a team of student researchers to assist in the design, implementation, and analysis of surveys. The purpose of these surveys will be to help Wesell create high quality advertisements for their clients.

1 Write a 2 (or more) page letter to Ms. Ida Noyu, Director of Personnel, applying for a job on the Wesell Student Researchers Team. This letter should clearly communicate the details of your understanding of how to design, carry out, analyze, and report the results of a survey. Give her specific information and examples to show:

- that you understand what are the most important ideas to include and to avoid in the survey process,
- the important statistical techniques you use when you analyze surveys,
- other information that you feel will demonstrate your expertise as a surveyor and data analyst,
- why you would make a good team member.

By the way, the rumor is that Ida is really picky—she will hire only high quality communicators and she wants lots of *specific details, with examples*, about an applicant's knowledge.

2 Once you complete your letter to Ida Noyu, give this sheet to an adult to read. Then have that adult read your letter and give you specific feedback regarding the effectiveness of your communication and whether it is clear in your letter that you thoroughly understand the survey design/data collection/data analysis/reporting process. Ask for their feedback regarding ways to improve your letter to assure that you will be offered a job by Ida. Have the adult write two or more "I appreciate..." statements about your letter and two or more "I wish..." statements about your letter. Turn in the letter and the adult's "I appreciate" and "I wish" statements.

(Continued on back.)

Follow-up Student Activity (cont.)

3 (Optional) Based on the adult feedback you received, make adjustments in your letter. Turn in the edited version and label it "Edited letter."

Focus Student Activity 26.1

NAME _____ DATE _____

Answer the following on another sheet of paper.

a) Form the first 3 arrangements in a sequence for which $v(n) = 2(n + 1) + 3$. Sketch these 3 arrangements.

b) Use Algebra Pieces to form a representation of the nth arrangement in the above sequence. Sketch your representation.

c) Based on what you can "see" in your nth arrangement, write some equivalent expressions for $v(n)$.

d) Show how you can reason from the Algebra Pieces (without having to "guess and check") to determine which arrangement has value 225. Add a few words or equations to help explain your methods.

e) Which is the largest arrangement that has value 500 or less? Explain how you decided this.

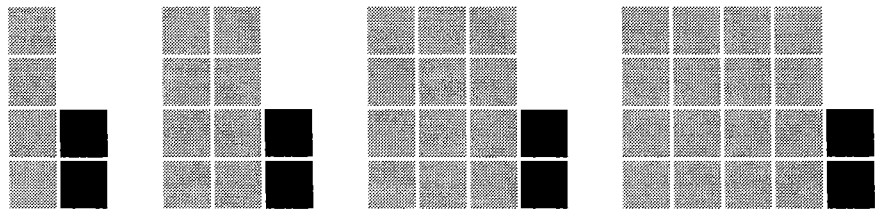
f) On a sheet of grid paper, make a coordinate graph showing the value of at least 5 different arrangements in the sequence. Label the horizontal and vertical axes of your graph, and label the coordinates of each ordered pair that you plot. Record some relationships you notice in your graph.

g) Challenge. Two successive arrangements have a total value of 400. Find a method of using the Algebra Pieces to determine which arrangements these are. Then make a sketch to show how your method works. (Note: successive means one arrangement follows right after the other.)

Focus Student Activity 26.2

NAME _____ **DATE** _____

1



a) Shown above are the first 4 arrangements in a sequence of counting piece arrangements. Sketch an Algebra Piece model of the nth arrangement and list several equivalent expressions for $v(n)$.

b) Find $v(15)$ = _____. Explain your methods.

c) If $v(n) = -250$, then n = _____. Explain how it is possible to reason from your Algebra Piece model to determine this.

d) On a sheet of grid paper, graph the values of at least 5 arrangements. Next to your graph, describe the location of the coordinate associated with the 75th arrangement.

e) Challenge. If $v(n) = -98$, then $v(n - 2)$ = _____.Explain your methods.

Focus Student Activity 26.3

NAME _____ DATE _____

1 Form the first 3 counting piece arrangements in a sequence for which the value of the nth arrangement is $(n + 1)(2n + 1)$. Sketch these 3 arrangements below.

b) Use Algebra Pieces to form a representation of the nth arrangement. Sketch your representation here:

c) Which arrangement has value 2145? Explain how you decide this.

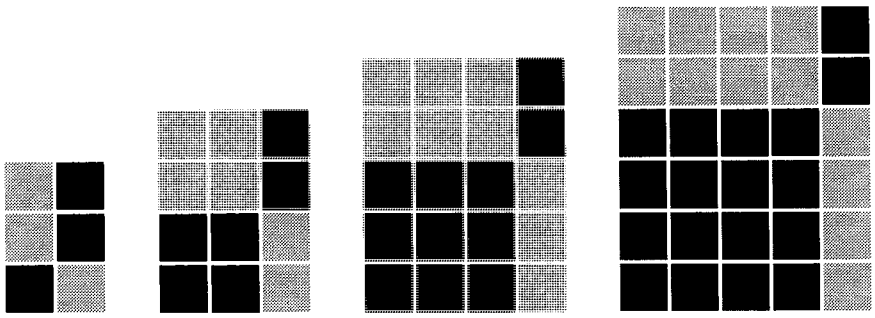
d) On a sheet of grid paper, make a coordinate graph that shows the net value of 5 or more arrangements in this sequence. Next to your graph describe the location of the coordinate associated with the 100th arrangement.

e) Challenge. If the value of a certain arrangement in the sequence is doubled, 50 more black counting pieces are needed to form a 50×50 square. Use your Algebra Pieces to help you determine which arrangement this is. On another sheet, explain and show diagrams of your methods.

f) Challenge. The larger of 2 successive arrangements in the sequence contains 125 more tile than the smaller. Which 2 arrangements are these? On another sheet, explain and show diagrams of your methods. (Remember that successive arrangements are next to each other.)

Focus Student Activity 26.3 (cont.)

2



a) Shown above are the first 4 arrangements in a sequence of counting piece arrangements. Sketch the nth arrangement and list several equivalent expressions for $v(n)$.

b) Find the following values, and explain the methods that you use:

$v(20) =$ _____ $v(40) =$ _____ $v(100) =$ _____

c) On a sheet of grid paper, graph the values of at least 5 arrangements. Next to your graph, describe the location of the coordinate associated with the 75th arrangement and record your observations about the graph.

d) Challenge. Explain how one can reason from the Algebra Pieces to solve each of these problems:

i) If $v(n) = 90$, then $n =$ _____.

ii) If $v(n+1) - v(n) = 50$, then $n =$ _____.

Follow-up Student Activity 26.4

NAME _____ **DATE** _____

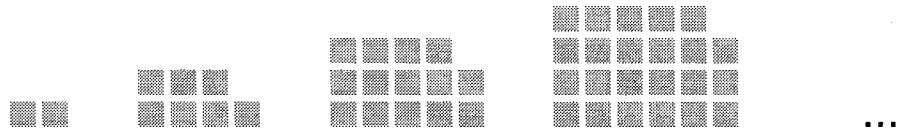
Answer all problems on separate paper.

1 For Sequences a) and b) shown below, record the following:

- several different formulas for $v(n)$, the value of the nth arrangement in the sequence, "loop" sketches of the nth arrangement to show how you "see" each formula;
- a graph showing the value of the first 5 or more arrangements in the sequence, label the axes of your graph and label the coordinates of each point you plot.

a)

b)



2 Each of a)-j) below is a formula for the value of the nth arrangement in a sequence of counting piece arrangements. For *each* sequence:

- sketch the first 3 arrangements and the nth arrangement;
- make a coordinate graph showing the net values of at least 5 arrangements in the sequence (label each point you plot);
- next to the graph record the following values: $v(13)$, $v(72)$, $v(100)$.

a) $v(n) = 2n + 3$

b) $v(n) = -2n + 3$

c) $v(n) = -2n - 3$

d) $v(n) = -(2n + 3)$

e) $v(n) = -(-2n + 3)$

f) $v(n) = 2n^2 + 3$

g) $v(n) = -(2n^2 + 3)$

h) $v(n) = -2n^2 + 3$

i) $v(n) = -2(n + 3)^2$

j) $v(n) = (-2n + 3)^2$

(Continued on back.)

Follow-up Student Activity (cont.)

3 Look back at your nth arrangements, formulas, and graphs for Problems 1 and 2.

a) Describe conjectures or generalizations that you have about relationships you notice among the nth arrangement of a sequence, its formula, and the shape of its graph.

b) List any other conjectures that you have based on your observations of formulas and graphs.

4 In a formula such as $v(n) = 4n + 5$, n is called a *variable*. Explain in your own words the meaning of the term variable.

5 Explain the meanings of each of the following algebraic expressions: $n + 2$, $2n$, n^2, and $(2n)^2$. Are they sometimes/always/never equal in value? Explain, using diagrams of Algebra Pieces and examples to support your explanations.

6 The following situation can be modeled as a sequence of counting piece arrangements. Sketch the first 5 arrangements and the nth arrangement of the sequence. Briefly explain how your nth arrangement relates to the situation.

The bicycle rental shop charges a $5 flat fee plus $4 for every hour the bike is out of the shop.

7 Draw diagrams of Algebra Pieces and write explanations of the meanings of the following: odd numbers, even numbers, square numbers. Include several observations and generalizations about these numbers (e.g., their sums, differences, formulas, graphs, ways they are related, etc.)

Follow-up Student Activity 27.1

NAME _____ **DATE** _____

Write your responses to all of these problems on separate paper.

1 The value of the nth arrangement of Sequence X is $v(n) = -2n - 3$, and the value of the nth arrangement of Sequence Y is $v(n) = -3n + 10$.

a) Sketch the first 3 and the nth arrangements of both sequences.

b) Show how Algebra Pieces can be used to find the value of n for which the nth arrangements of Sequence X and Sequence Y have the same net value.

c) Verify that the value of n you determined in b) is correct.

2 Make sketches that show how to use Algebra Pieces to determine positive values of n for which the following equations are true (add a few words or phrases to clearly communicate your methods):

a) $2n - 8 = 4$

b) $7 - 3n = 5n - 9$

c) $n^2 + 3n - 2 = 23 + 3n$

d) $4n^2 - 6n + 3 = 3(n^2 + n + 1) - 18 + n^2$

e) $4(n - 1) = (5n - 5) - (n - 1)$

f) $n^2 + 7 = n^2 - 7$

3 Explain your answers to the following questions. Use diagrams of Algebra Pieces to support your answers.

a) For positive values of n, which is greater $2n$ or $n + 2$?

b) If it is true that $3n = 27$, is it possible that $2n + 5 = 13$?

c) Suppose that $4n - 5 = 35$. Which of the following, if any, are true:

i) $3n = 21$

ii) $n + 4 = 12$

iii) $2n - 8 = 6$

(Continued on back.)

Follow-up Student Activity (cont.)

4 Make sketches to show *how* to *use Algebra Pieces to solve* the following puzzle problems:

a) The sum of three consecutive *odd* numbers is 129. What are the numbers?

b) The length of a rectangle is 3 meters longer than its width. The perimeter of the rectangle is 114 meters. What are the dimensions of the rectangle?

c) Nifty Tractor Rental charges a $15 fee plus $2 for every mile a tractor is driven. If Jon paid $101 to rent a tractor, how far did he drive it?

d) Dylan is n years old. Jessica is 3 years older than Dylan. Jaime is twice as old as Jessica. When their ages are added together, the total is 81. How old is each person?

e) Two consecutive square numbers have a difference of 39. What are the numbers?

5 Use diagrams of Algebra Pieces to explain why the following number trick works:

Step 1: Pick a number.
Step 2: Triple your number.
Step 3: Subtract 3 from your answer in Step 2.
Step 4: Add 9 to your answer in Step 3.
Step 5: Divide your answer in Step 4 by 3.
Step 6: Add 2 less than your original number.
The answer will always be double your original number!

6 Write a detailed report about the uses of Algebra Pieces. Assume the person who will read your report knows how to use red and black counting pieces to add, subtract, multiply, and divide integers, and they have some experience with generalizing patterns. However, they have never seen Algebra Pieces. Give plenty of examples, clear explanations, and draw diagrams so they can "see" with understanding what the pieces represent and how to use them to solve equations.

Focus Master B

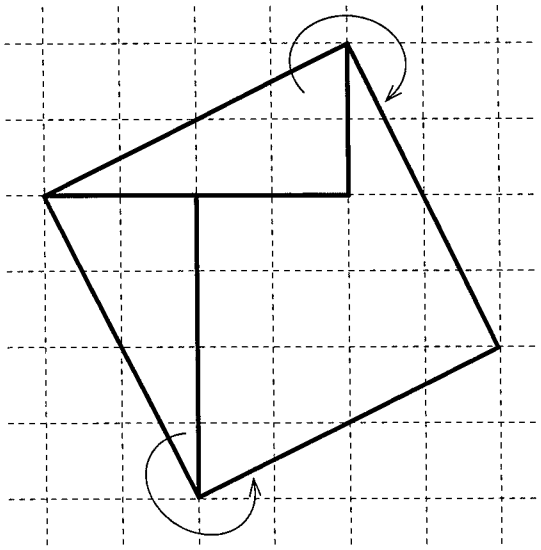


Figure 1

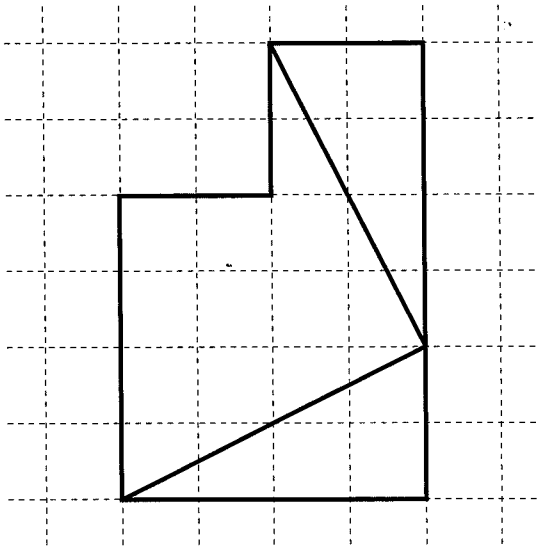
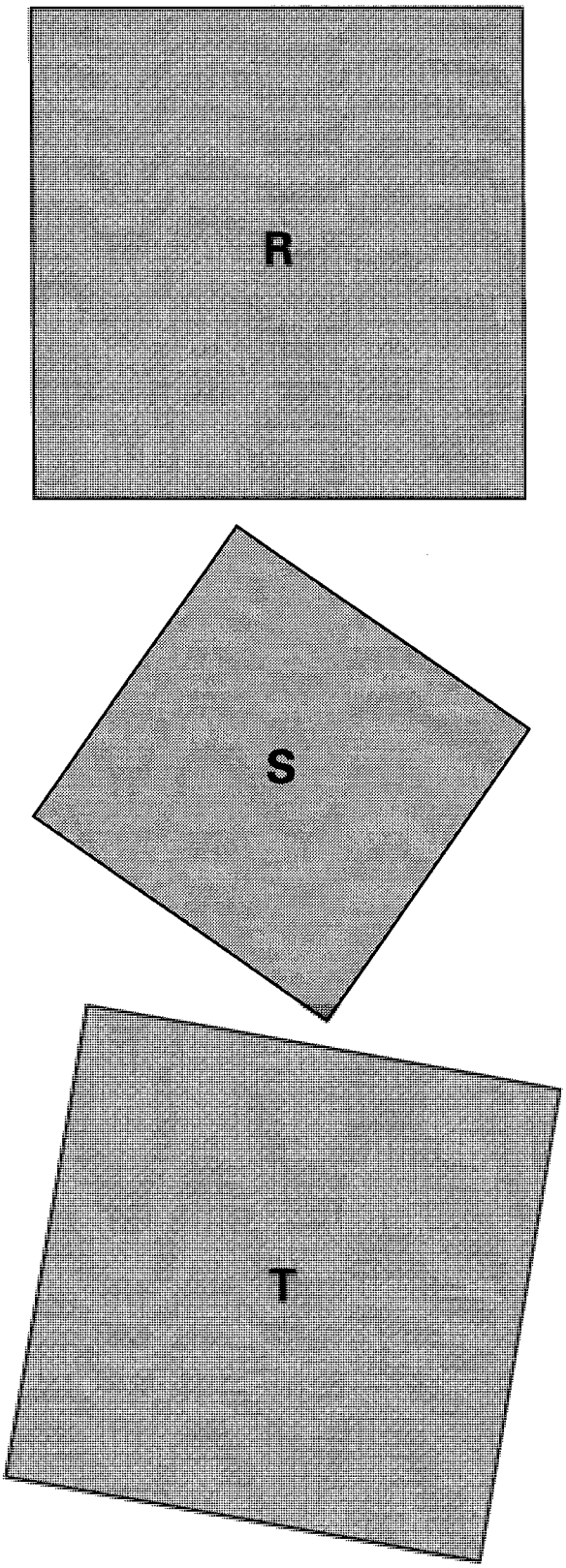


Figure 2

Focus Master C

a) Dissect and cut apart each of these squares:



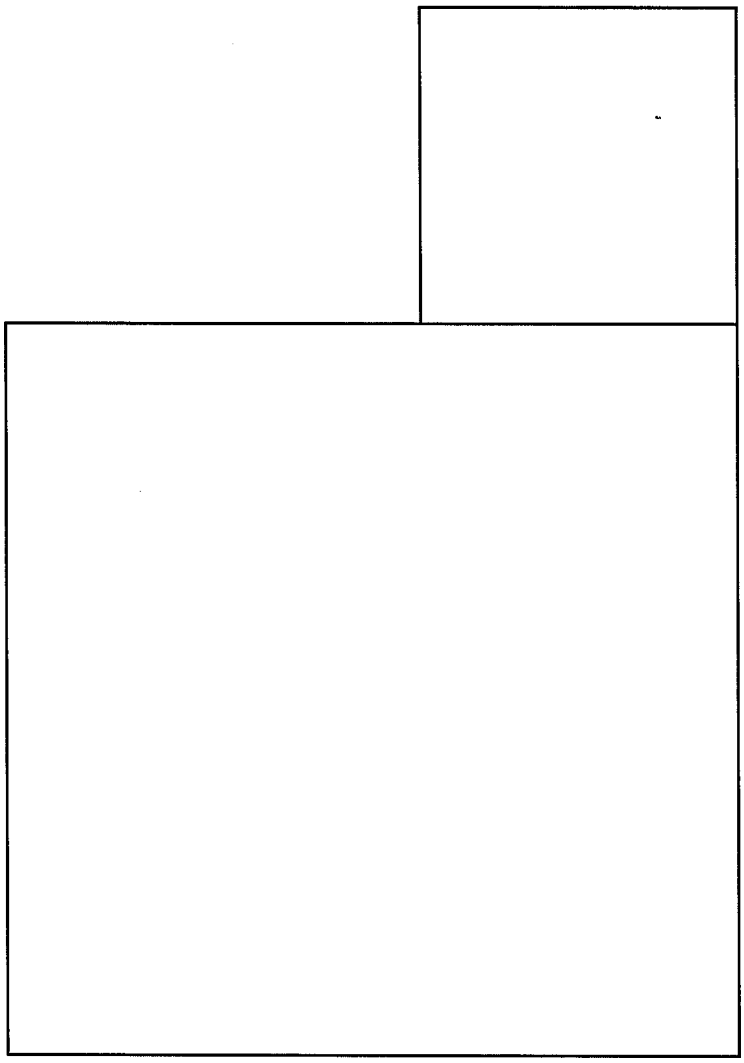
b) Reassemble the pieces of each square from a) to form 2 adjacent squares. Tape results here:

R:

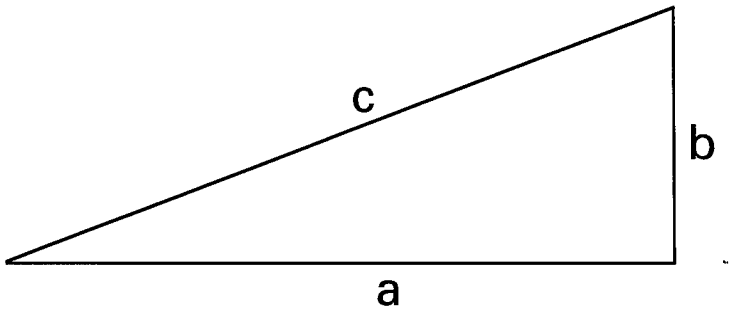
S:

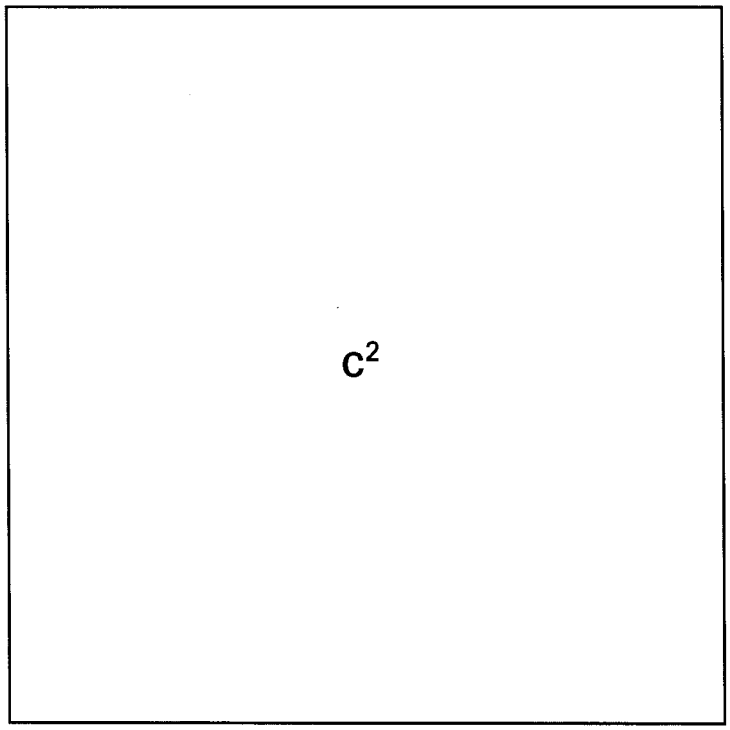
T:

Focus Master D



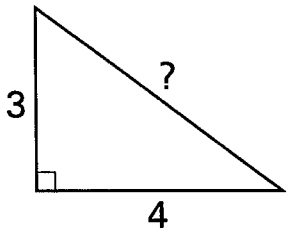
Focus Master E



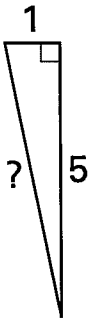


Focus Master F

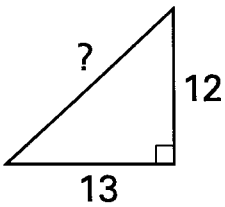
a)



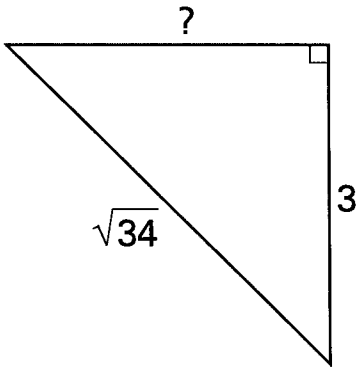
b)



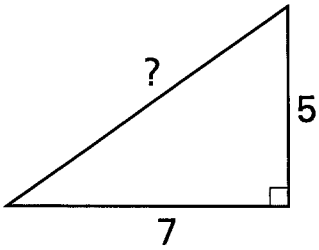
c)



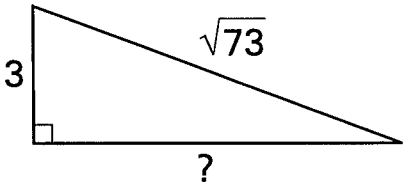
d)



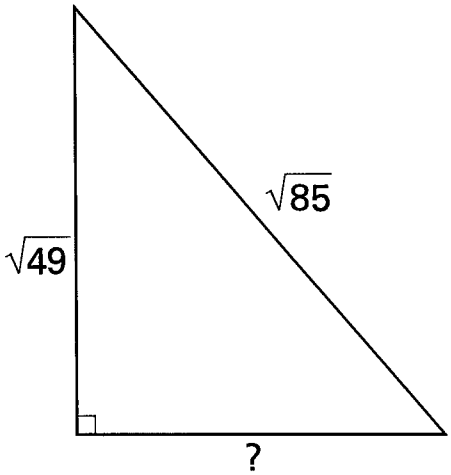
e)



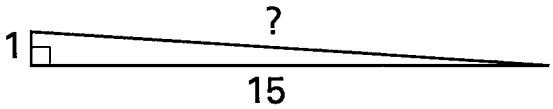
f)



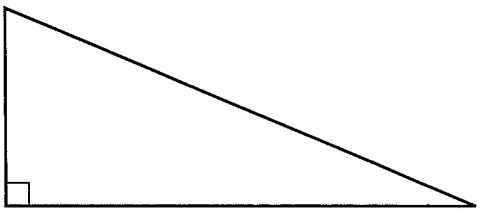
g)



h)



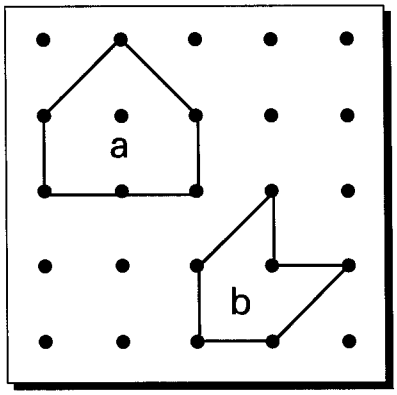
i)



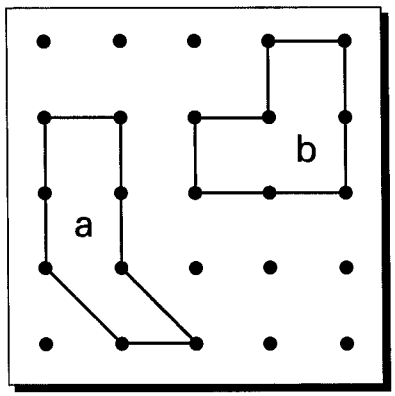
Given a "generic" right triangle, what are several possibilities for the lengths of its 3 sides (different from those given in a)-h)?

Focus Master G

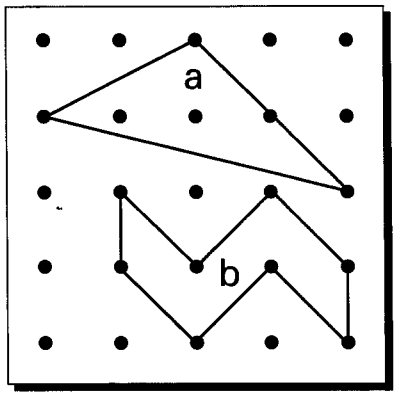
Pair 1



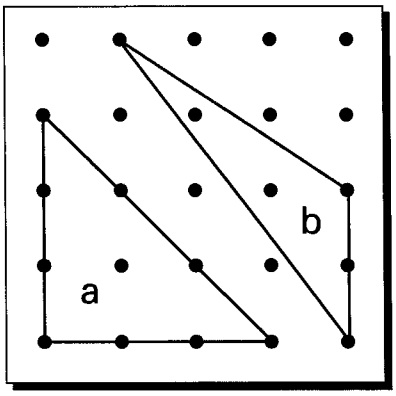
Pair 2



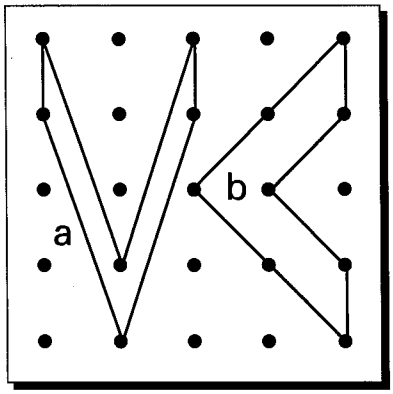
Pair 3

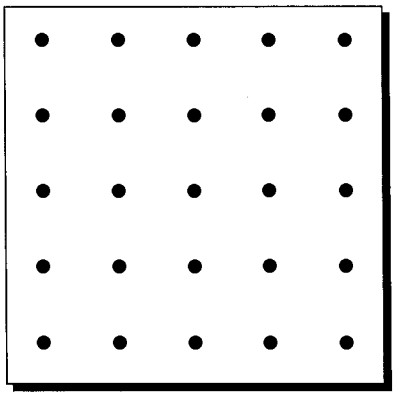


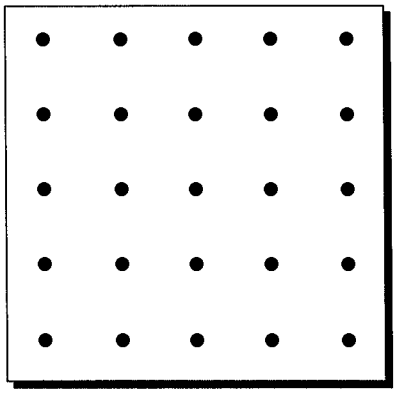
Pair 4

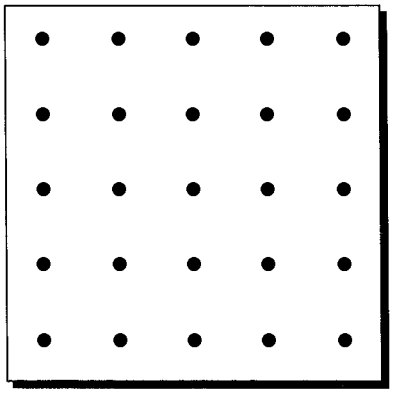


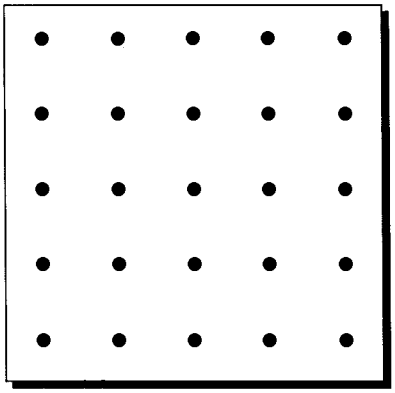
Pair 5











Follow-up Student Activity 28.1

NAME _____ **DATE** _____

Complete these problems on separate paper.

1 On geoboard recording paper, draw all the segments of *different lengths* that can be constructed on a 5-pin by 5-pin geoboard. Record the actual (i.e., not approximate) length of each segment. Explain the methods you used to find the length of one of the "slanted" segments.

2 For each length in Problem 1 that can't be written as a whole number, write a decimal approximation for the length.

3 Explain in your own words the meaning of square root.

4 On dot paper, make sketches to verify that the lengths $\sqrt{8}$ and $2\sqrt{2}$ are equal. Then make sketches to show that $\sqrt{20} = 2\sqrt{5}$.

5 What are some different ways of naming the length $\sqrt{32}$? Make dot paper diagrams to show that your conclusions are correct.

6 Suppose a square has sides of length $2\sqrt{7}$ units. What is the area of the square? Explain your reasoning.

7 According to Pythagorean theorem, for any right triangle with legs of length a and b and hypotenuse of length c, $a^2 + b^2 = c^2$. Use the method of dissecting and reassembling squares to show this is true for a right triangle with legs of length 3 and 7.

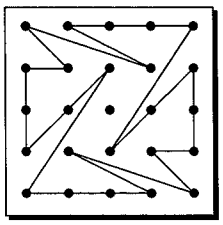
8 Determine whether each set of numbers in a)-h) represents a right triangle. Explain or draw diagrams to show how you decided for each set of lengths.

a) 3, 4, 5
b) 4, 4, 4
c) 7, 3, 5
d) 6, 8, 10
e) 3, 2.5, 3.5
f) 7.5, 10, 12.5
g) 20, 21, 29
h) 30, 40, 50

(Continued on back.)

Follow-up Student Activity (cont.)

9 On a sheet of geoboard recording paper, sketch 4 or more geoboard polygons with perimeter $2 + 2\sqrt{2} + 2\sqrt{5}$ but different areas. Next to each polygon, record its area, its actual perimeter, and a decimal approximation of the perimeter.



10 Compute the actual perimeter of the polygon shown at the right.

11 Sketch 2 geoboard polygons with perimeters greater than the polygon shown in Problem 10. Label each polygon to show its exact perimeter and a decimal approximation of the perimeter.

12 Challenge. Determine the polygon with greatest perimeter that can be formed on a 5-pin by 5-pin geoboard.

13 Present the following to an adult (use lots of diagrams and models to help them understand):

a) your understanding of the meaning of square root;

b) methods of dissecting and rearranging a square into 2 adjacent squares, and vice versa;

c) a description of Pythagorean theorem and how your methods in b) show that Pythagorean theorem works;

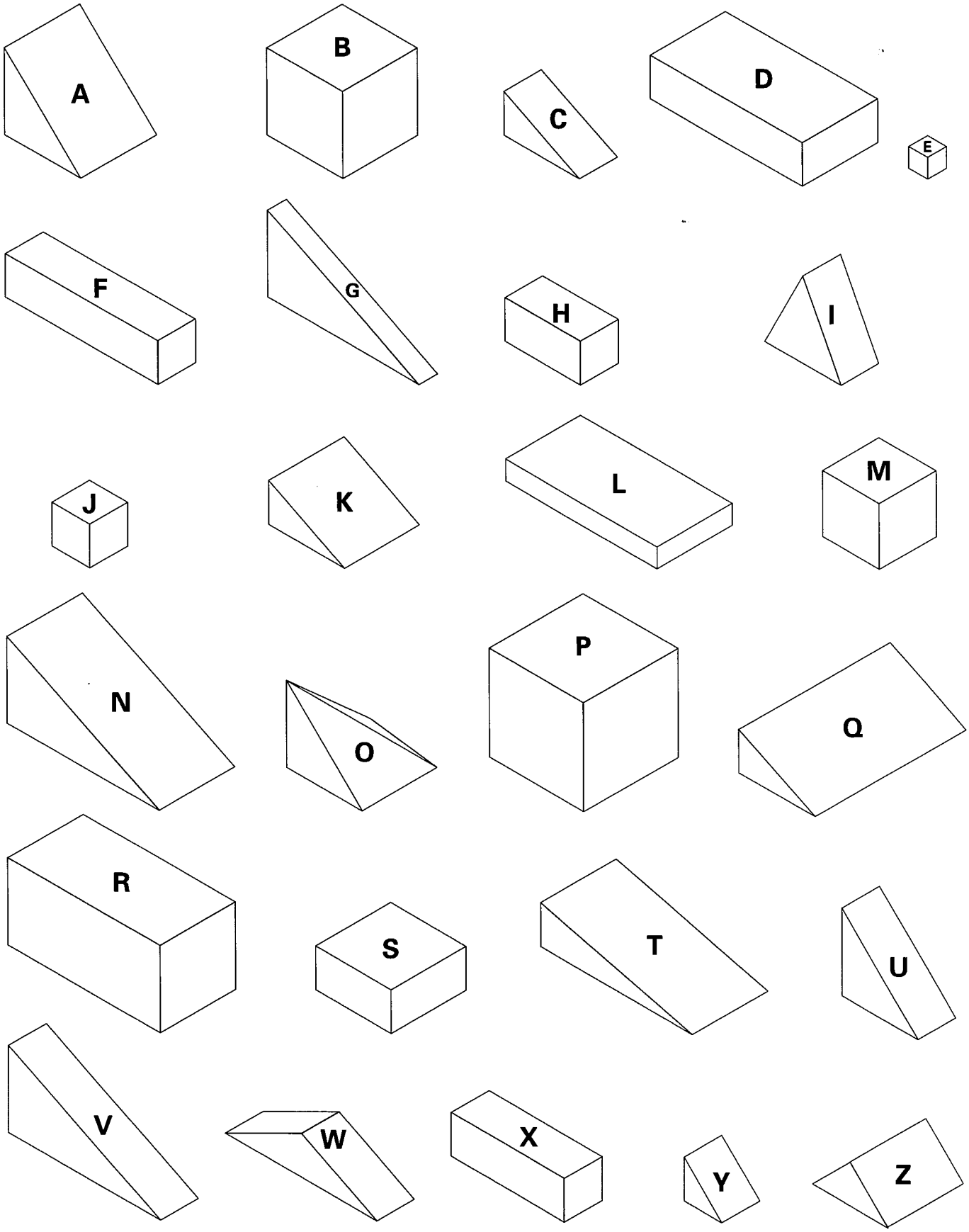
d) ways you can use Pythagorean theorem to compute lengths.

Before you teach the adult, prepare an outline of what you will do, including the examples you will use. After you finish presenting, write a brief summary that tells:

- to whom you presented,
- how (if) your presentation was different from your outline,
- whether the adult understood your explanations and how you could tell,
- how you felt after you finished,
- questions or new thoughts about the ideas you presented.

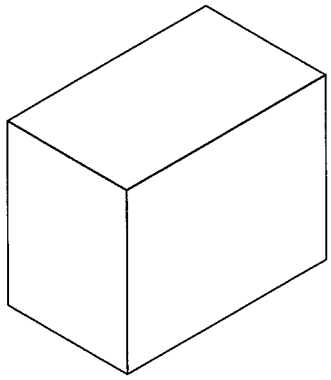
Have the adult write 2 "I appreciate..." statements about your presentation and attach those to your summary. Attach your outline and your summary to this assignment.

Focus Master B



Focus Master C

Suppose that geoblock J is the volume unit, and the length of its edge is the linear unit. The sketch below represents a rectangular solid whose dimensions are 15 linear units × 20 linear units × 25 linear units.



a) Determine a method of "seeing" and counting (other than one by one) the volume of the above solid. Explain how your method works and how you can be sure that it is correct.

b) Determine a method of seeing and counting (other than one by one) the surface area of the above solid. Explain how your method works and how you can be sure that it is correct.

c) Will your methods in a) and b) above work for any rectangular solid? If yes, explain how and why this is so. If not, explain why not, and develop a method that would work for any rectangular solid.

d) Suppose the volume unit is now geoblock E, and the length of its edge is the linear unit. Now what are the volume, surface area, and dimensions of the solid shown above? Explain your methods.

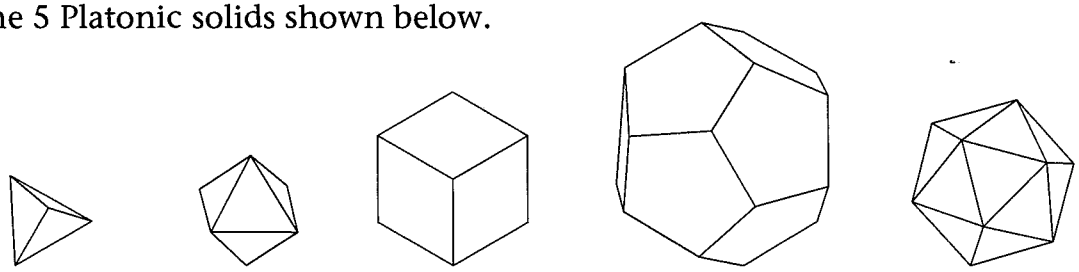
e) Challenge: Investigate general methods of finding the volume of solids that are not rectangular prisms.

Follow-up Student Activity 29.1

NAME _____ **DATE** _____

Write all of your responses on separate paper.

1 Using the patterns provided by your teacher, cut along the solid lines, fold along the dotted lines, and tape edges together to form the 5 Platonic solids shown below.



Tetrahedron Octahedron Cube Dodecahedron Icosahedron

a) These Platonic solids are also called regular polyhedra. Do some research to find out what the term *regular polyhedra* means, and why these are called the *Platonic solids*. Explain what you learned and where you obtained your information.

b) For each Platonic solid, trace its diagram above and mark the locations of at least 2 axes of symmetry, and at least 2 planes of symmetry.

c) Use estimation to order the Platonic solids from least to greatest volume. Record your ordering and explain your estimation methods.

d) For each Platonic solid, describe at least 2 different cross sections that you think could be formed by slicing the solid. Trace the above diagrams, mark the position of your cuts, and sketch the resulting cross section.

e) In the chart below, record the numbers of vertices, faces, and edges for each of the Platonic solids:

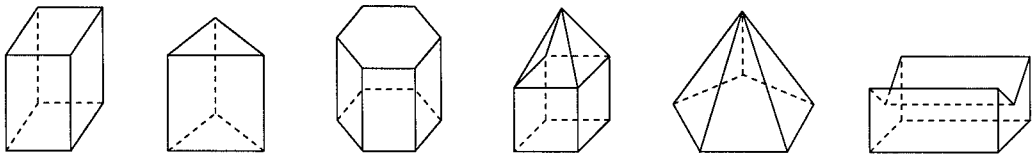
	Vertices (V)	Faces (F)	Edges (E)
Tetrahedron			
Cube			
Octahedron			
Dodecahedron			
Icosahedron			

(Continued on back.)

Follow-up Student Activity (cont.)

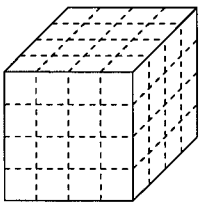
f) There is a special relationship among the numbers of vertices, faces, and edges in the solids you formed. This relationship is called Euler's formula, and is named for the Swiss mathematician, Leonhard Euler, who discovered it in 1752 (it was actually first stated by Rene Descartes about 1635). Study your chart and then record what you think Euler's formula might be (Euler used the variables V, F, and E).

g) Does your formula work on other solids, such as the ones shown below? Explain.



i) ii) iii) iv) v) vi)

2 Suppose a large cube is formed using 64 small cubes, and then all 6 faces of the large cube are painted. If the large cube is disassembled, how many small cubes will have one face painted? 2 faces painted? 3 faces painted? 4 faces? 5 faces? 6 faces? 0 faces? Write an explanation of the methods you used to determine this.

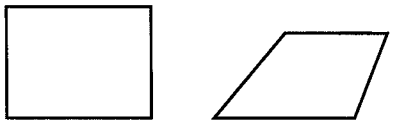


3 Now imagine that a large cube is formed using 1000 small cubes, and then the 6 faces are painted. If this large cube is disassembled, how many small cubes will have 1 face painted? 2 faces? 3? 4? 5? 6? 0? Explain how you determined this.

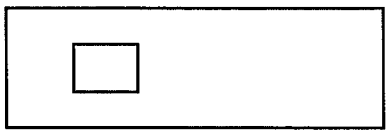
4 Generalize the above painted cube problems to explain the number of small cubes that would have 1, 2, 3, 4, 5, 6, and 0 faces painted for an n by n by n cube.

Connector Master B

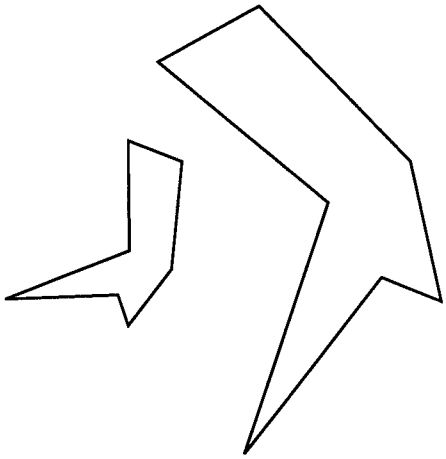
a)



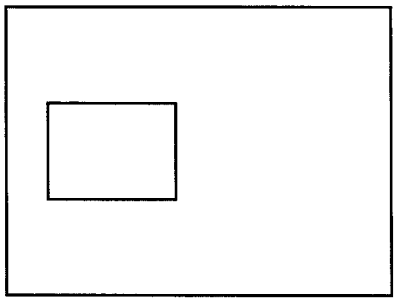
b)



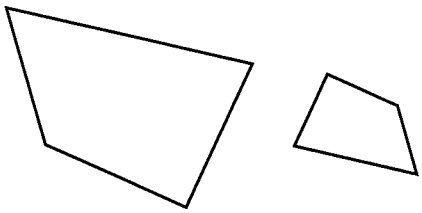
c)



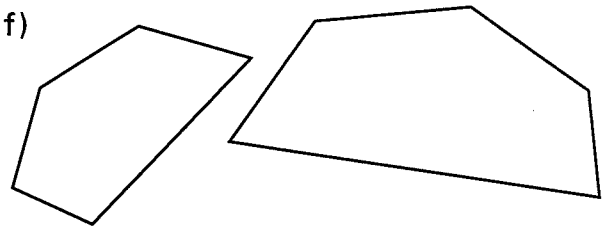
d)



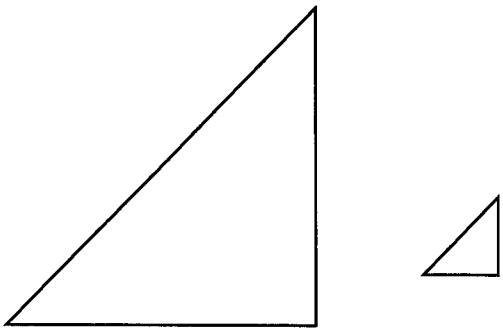
e)



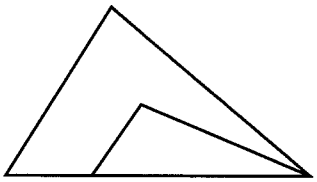
f)



g)



h)



Connector Student Activity 30.1

NAME _____ **DATE** _____

1 Using figures A-F on the following page, sketch figures A′-F′ so they are similar to A-F according to the following criteria (note the symbols A′ and B′ are read, "A prime" and "B prime"):

A′ is an enlargement of A by a scale factor of 2.

The ratio of the area of B to the area of B′ is 1 to 9.

The ratio of the perimeter of C to the perimeter of C′ is 2 to 1.

D′ is a dilation of D by a scale factor of 2.

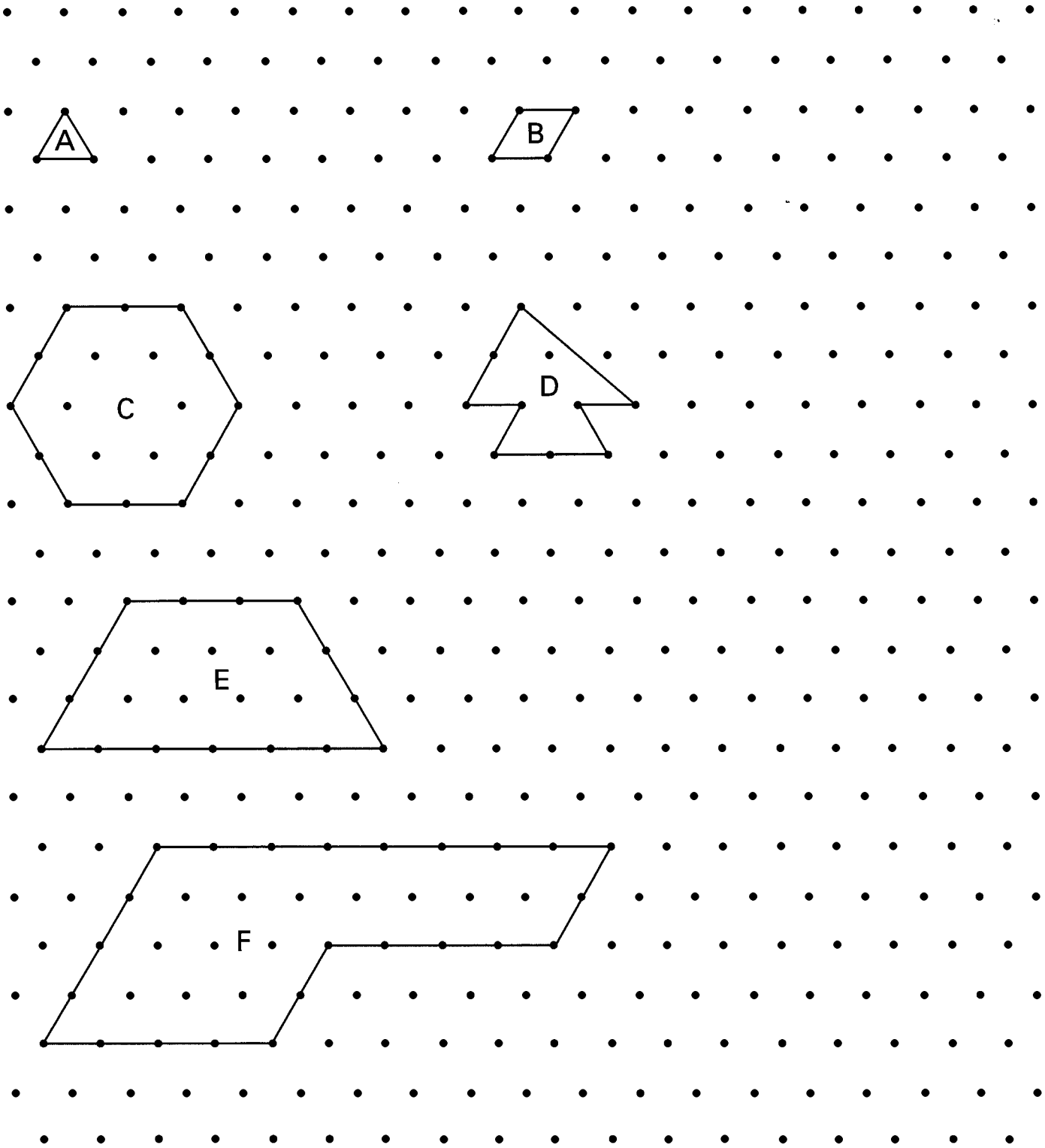
E′ is a dilation of E by a scale factor of $\frac{1}{3}$.

The ratio of the area of F to the area of F′ is 16 to 1.

2 Make a chart that shows the ratios of the areas of A-A′, B-B′, C-C′, D-D′, E-E′, and F-F′. Then repeat for the ratios of their perimeters.

3 Write your observations or generalizations, based on your results for Problems 1 and 2.

Connector Student Activity 30.1 (cont.)

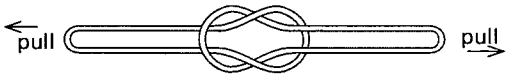


Focus Master A

1. The following method can be used to enlarge the curvilinear figure shown on the next page by a scale factor of 2 and with projection point P.

a) Trace point P and the figure on a blank sheet of paper.

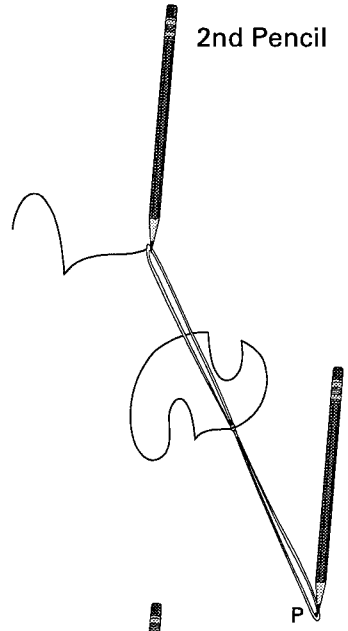
b) Knot 2 identical rubber bands together as shown here:



c) Using 2 pencils, place the pencil tips through the 2 unknotted ends of the bands.

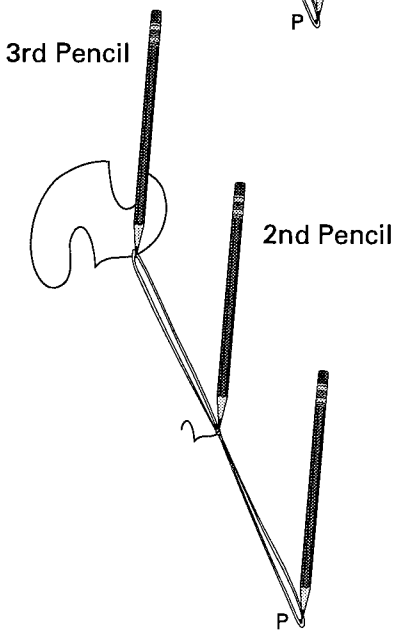
d) Hold one pencil point in place at point P.

e) Using the 2nd pencil point, stretch the bands until the knot is directly over a point on the curvilinear figure. (This procedure won't work if the bands aren't stretched tightly.)



f) Moving the 2nd pencil point, trace the knot along the perimeter of the shape (keep the rubber bands stretched tightly). As the knot traces over the shape, draw the path that the 2nd pencil point follows.

2. To form a reduction by a scale factor of ½, get a partner and use 3 pencils and 2 knotted bands as follows: with one pencil point hold the unknotted end of one band at point P; place a 2nd pencil point at the knot, and a 3rd pencil point through the other unknotted end of the bands; trace over the curvilinear figure with the 3rd pencil, steadying the middle pencil as it records the path of the knot.



(Continued on back.)

Focus Master A (cont.)

3. Investigate the procedures on the previous page for a variety of shapes, using a variety of scale factors that are greater and less than 1. Find some fun pictures to enlarge and reduce. Make a poster that includes the following:

a) several examples (show both the original pictures and your enlargements and reductions, and record the scale factors);

b) your mathematical observations, conclusions, conjectures, and/or generalizations.

Following are some questions to consider in your investigation:

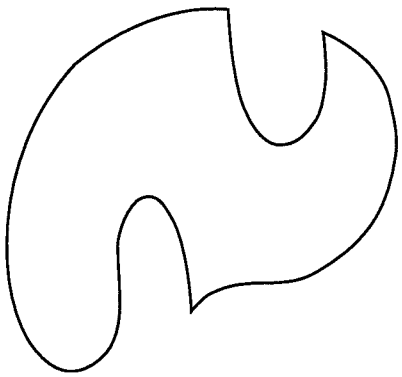
- Is there more than one way to form an enlargement/reduction by a given scale factor?

- What happens if you move the projection point to the inside of the shape? closer to the shape? farther way? on the shape?

- Does the size of the rubber bands affect the size of the enlargement/reduction?

- How can you make a reduction by a scale factor of $2/3$, $4/5$, $3/4$, $5/8$, $8/5$, etc.?

- How do distances on a figure enlarged by a factor of $3/2$ compare to the corresponding distances on a figure reduced by a factor of $2/3$?

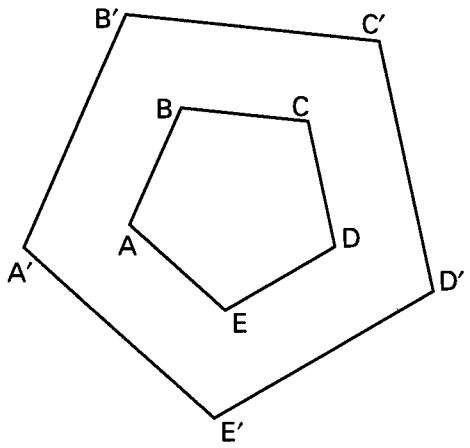


P •

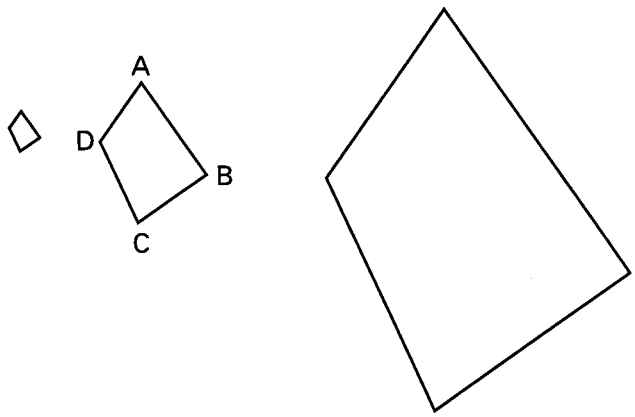
Focus Student Activity 30.2

NAME _____ **DATE** _____

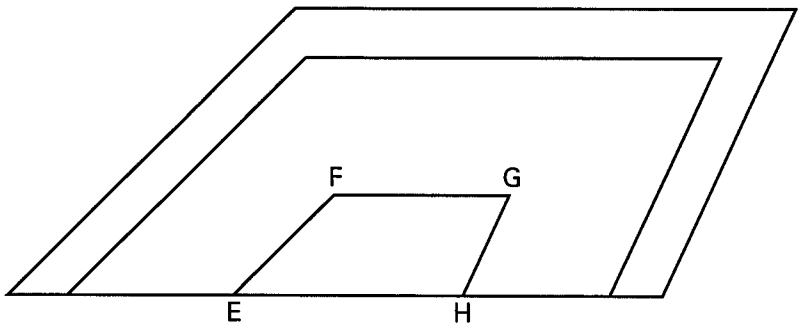
a)



b)



c)



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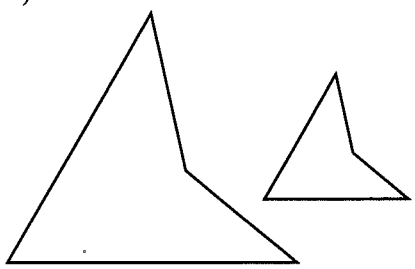
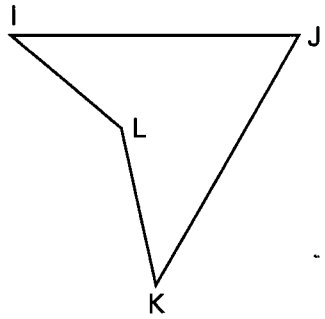
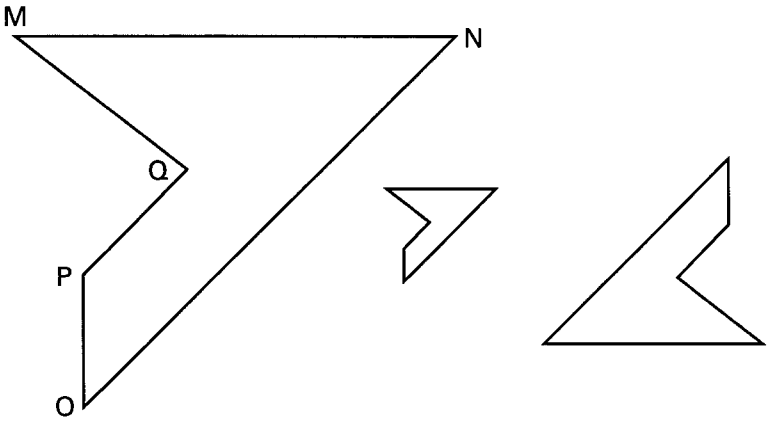
Focus Student Activity 30.2 (cont.)

d)





e)

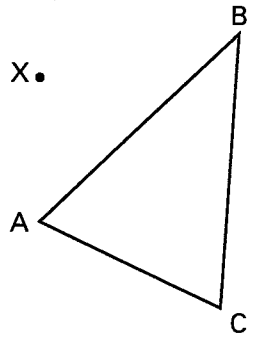


Focus Student Activity 30.3

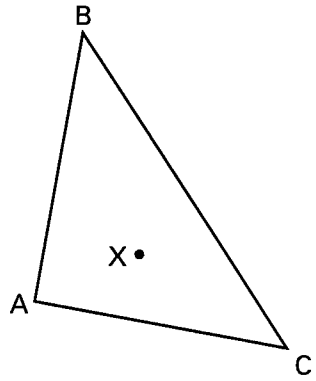
NAME _____ **DATE** _____

1 Trace each of the following diagrams on separate paper. Enlarge or reduce the polygons by the indicated scale factors with the center (X) of each dilation positioned as shown in the diagrams.

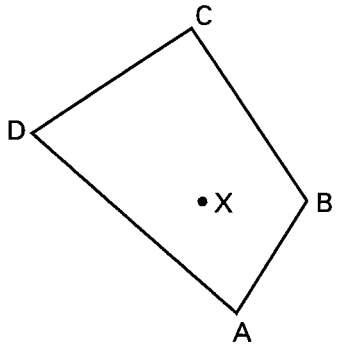
a) Scale Factor 2



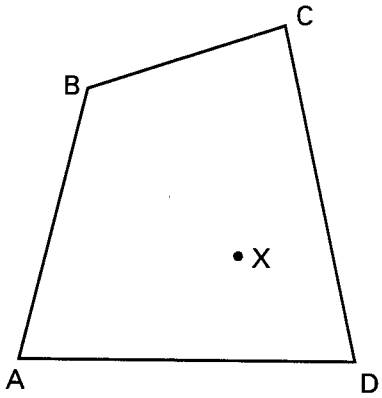
b) Scale Factor 2



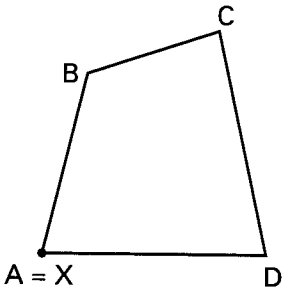
c) Scale Factor –2



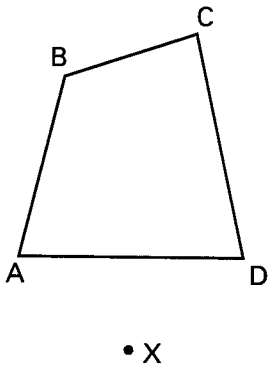
d) Scale Factor ½



e) Scale Factor 3



f) Scale Factor –3



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Focus Student Activity 30.3 (cont.)

2 How do the areas and perimeters of the polygon given in 1e) and the enlargement you formed compare? Give a visual demonstration of your answer.

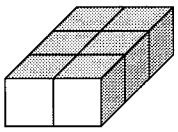
3 Draw a shape of your own and reduce it using a center of your choice and a scale factor of $-\frac{1}{3}$.

Follow-up Student Activity 30.4

NAME _____ **DATE** _____

Write your responses to all of these problems on separate paper.

1 The 1st solid in a sequence of solids is a $1 \times 2 \times 3$ rectangular solid. The 2nd solid is formed by doubling each dimension of the 1st solid. The 3rd solid is formed by tripling each dimension of the 1st solid. The 4th solid is formed by multiplying each dimension of the 1st solid by 4. And so on. Record the following:



a) the perimeter of the largest face of the first 3 or more solids from the sequence,

b) the surface area of the first 3 or more solids from the sequence,

c) the volume of the first 3 or more solids from the sequence,

d) the dimensions, surface area, and volume of the 100th and *n*th solids in the sequence. Explain how you determined these.

2 Draw a large triangle. Locate the midpoint of each side. Draw lines that connect the midpoints, forming 4 smaller triangles.

a) Describe mathematical relationships among the triangles.

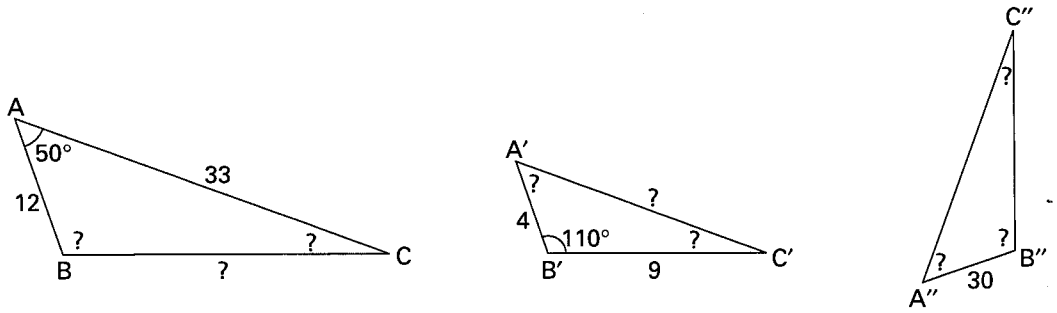
b) What happens if you repeat the above process on the small triangles? What if you continue this process? How are the new shapes related?

c) See if this process works on other triangles and what, if any, generalizations you can make.

(Continued on back.)

Follow-up Student Activity (cont.)

3 The triangles shown below are similar (they are not drawn to scale). Find the value of each "?". Explain your methods.



4 Sketch and label 3 more triangles that are similar to those above.

5 Draw the following using a picture of your choice (you can use the same or different pictures for each part):

a) an enlargement by a scale factor of 4 with the center of dilation outside the original picture;

b) a reduction by a scale factor of ⅓, with the center inside the original picture;

c) an enlargement by a scale factor of –2, with the center on the edge of the original picture.

6 Solve each of the following. Explain your reasoning and show diagrams to support your conclusions.

a) On a Blinkie Copy Machine, a 75% dilation of an image decreases each dimension of the image by 25%. What is the effect on the area of the image?

b) The ratio of the sides of 2 squares is 3 to 7. One square has a side length of 84. What are the possibilities for the side length of the other square.

c) Matthew built a rectangular solid with dimensions $2 \times 4 \times 6$. Then he formed a 2nd solid by increasing each dimension of the first solid by 100%. By what percentage did this increase the surface area of the 1st solid? the volume?

visual mathematics

COURSE II STUDENT ACTIVITIES

Grids & Tools

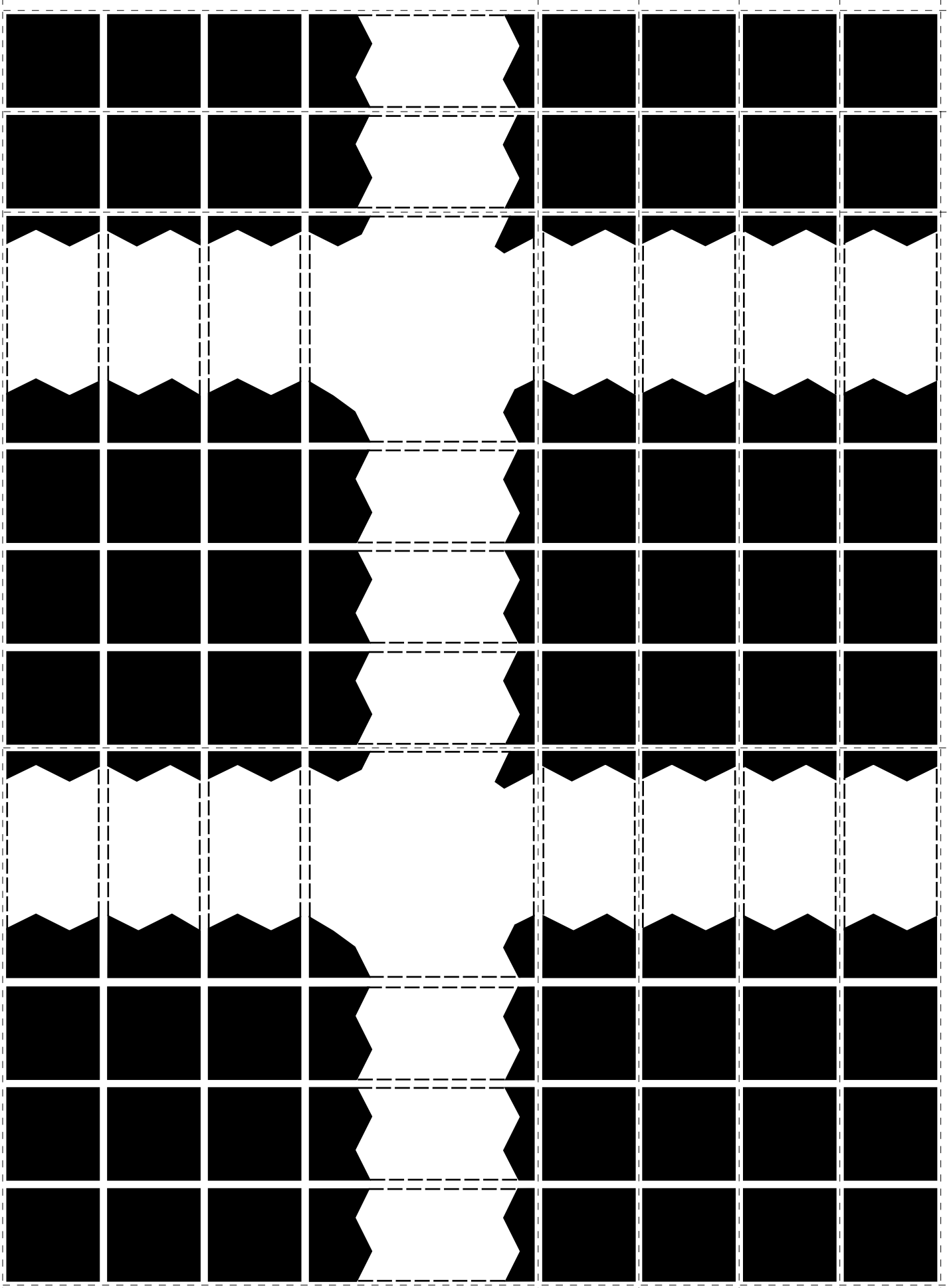
2 cm Grid Paper

2 cm Grid Paper

1-inch Grid Paper

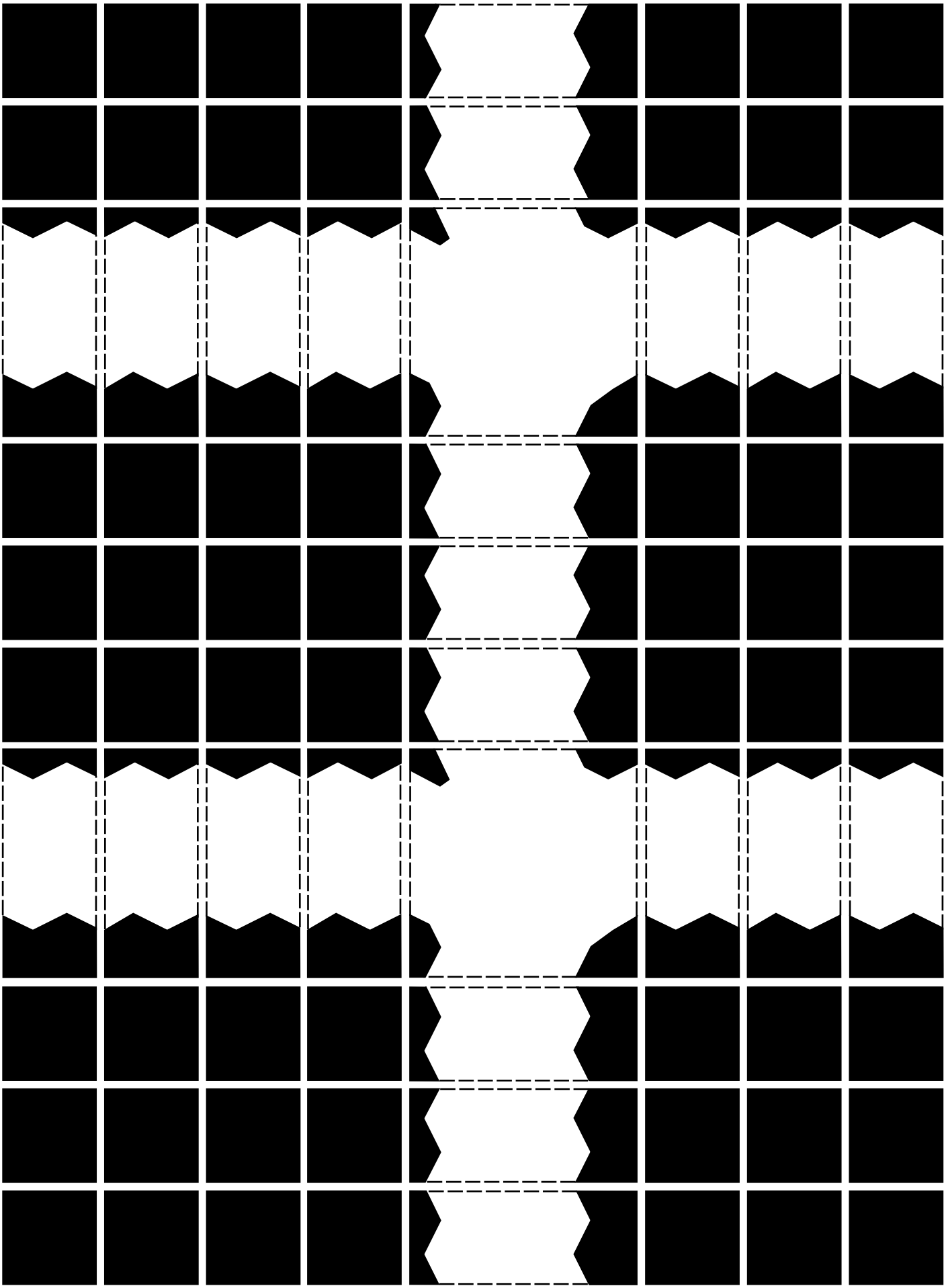
Algebra Pieces—front

(cut on thin dotted lines at arrowheads)



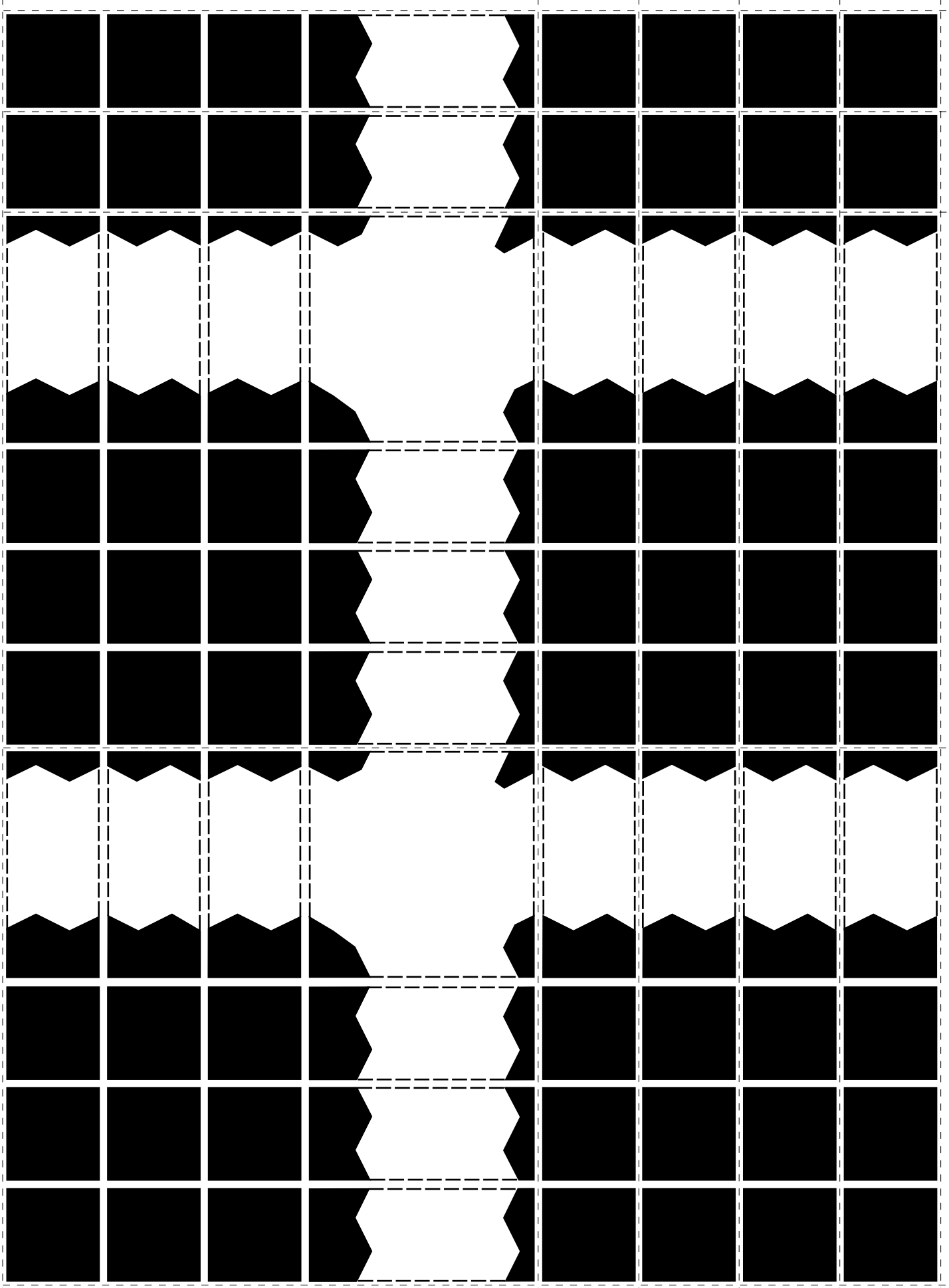
Algebra Pieces—back

(cut as indicated on front side)



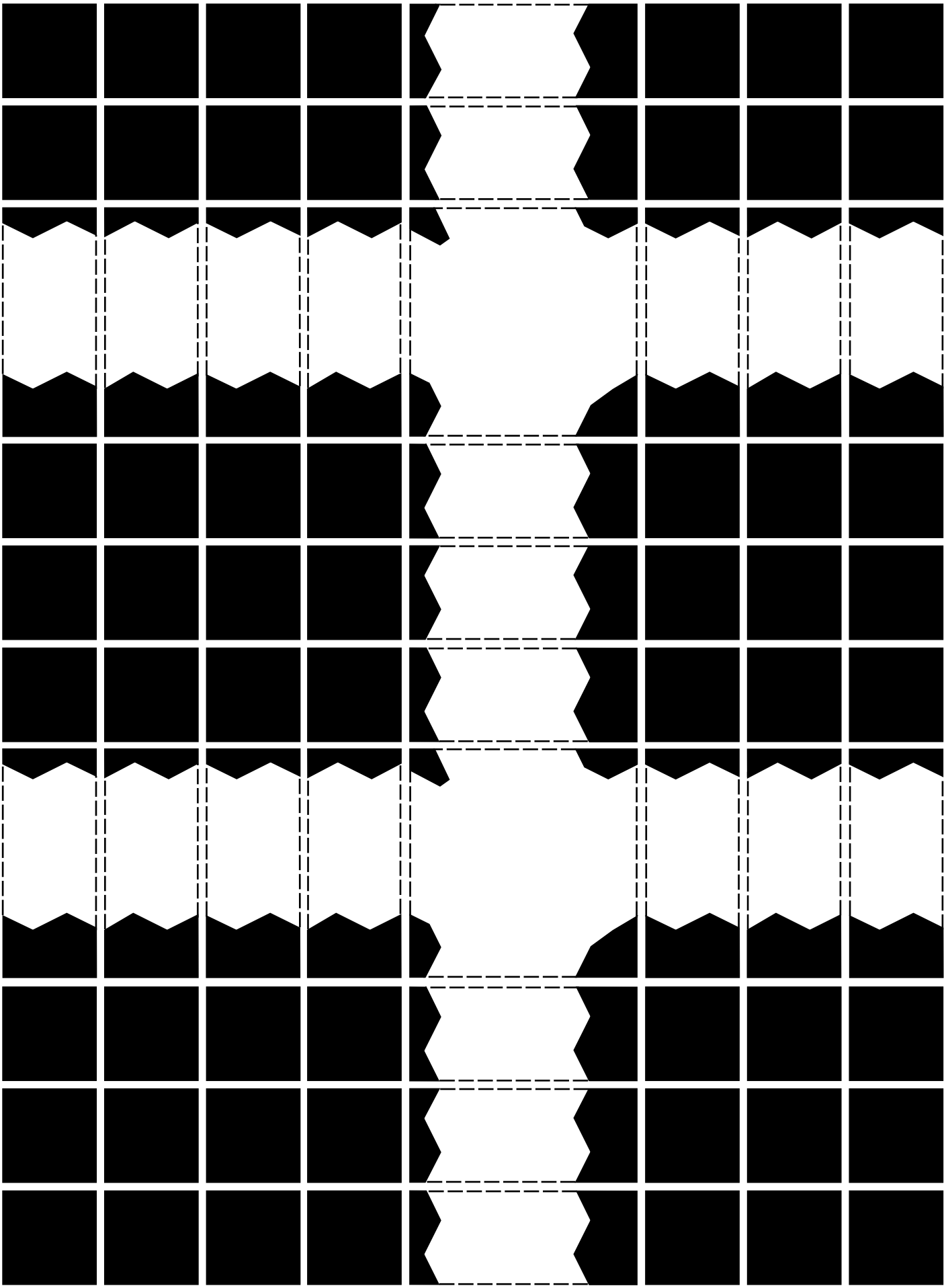
Algebra Pieces—front

(cut on thin dotted lines at arrowheads)



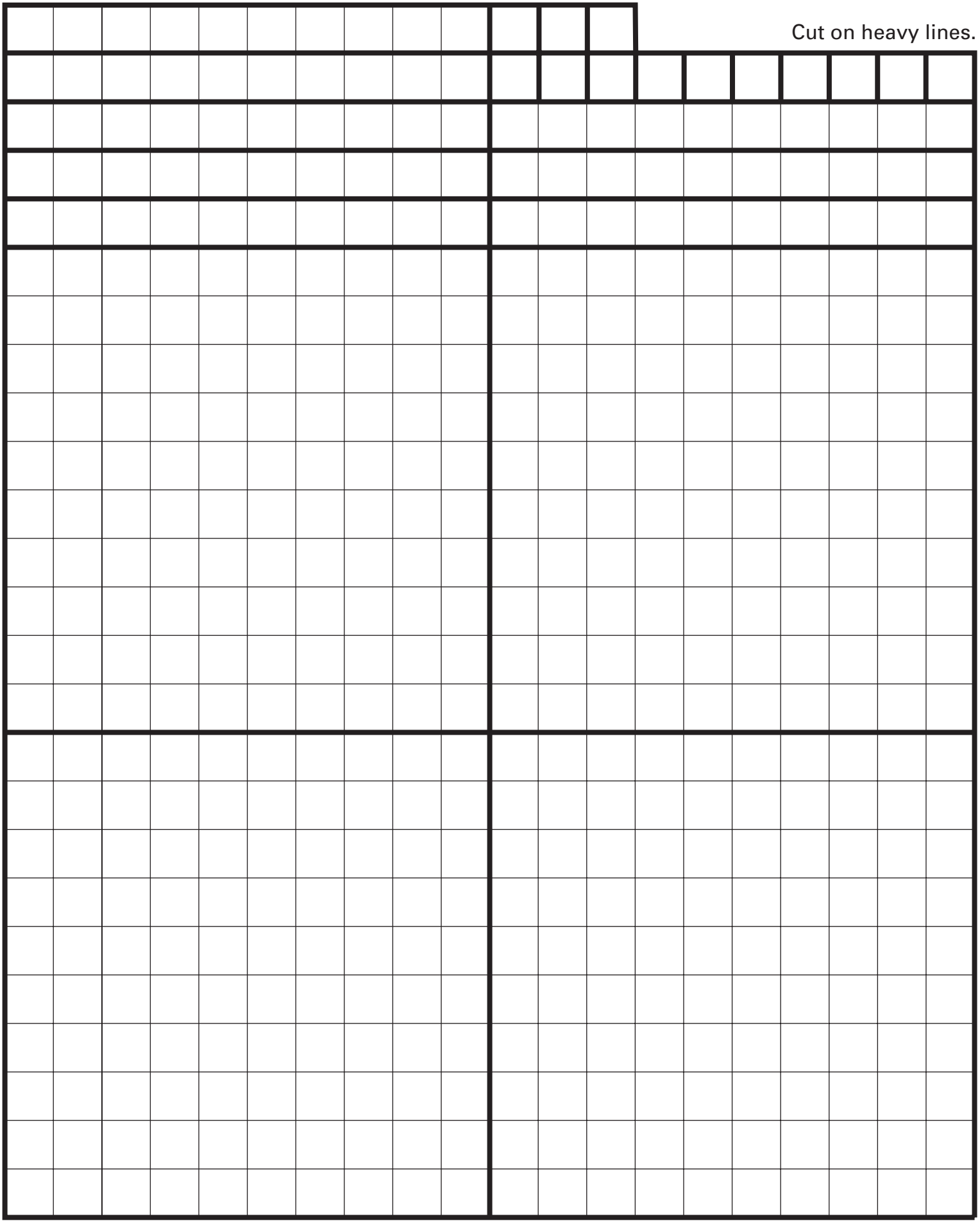
Algebra Pieces—back

(cut as indicated on front side)



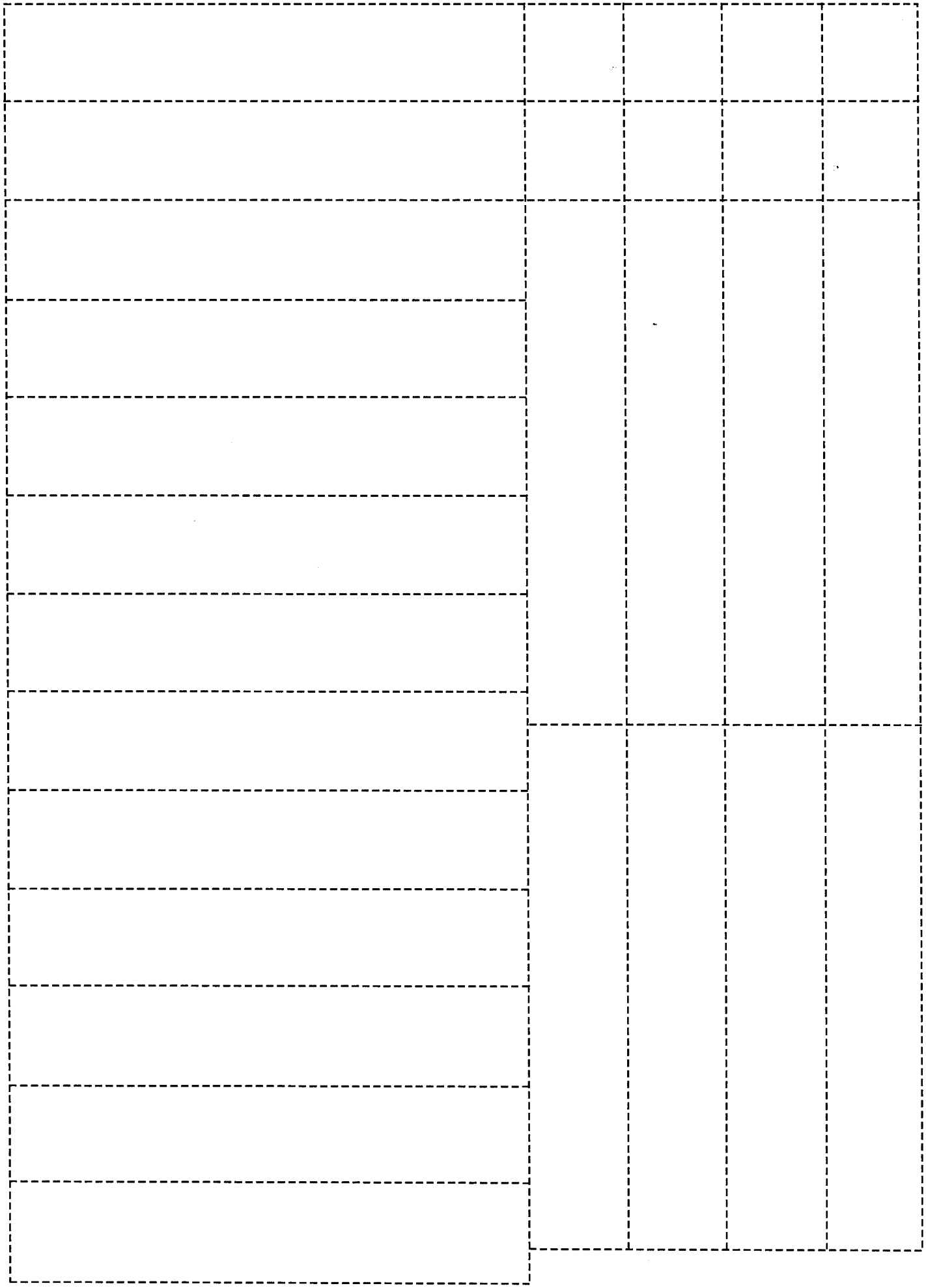
Base Ten Area Pieces

Cut on heavy lines.



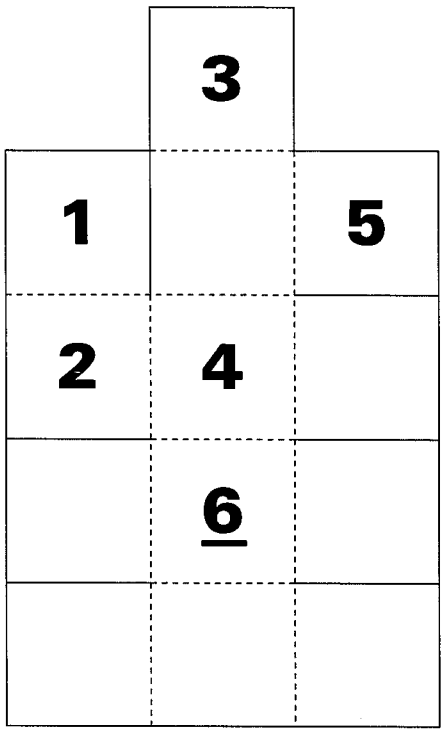
Blank Counting Pieces and N-Strips

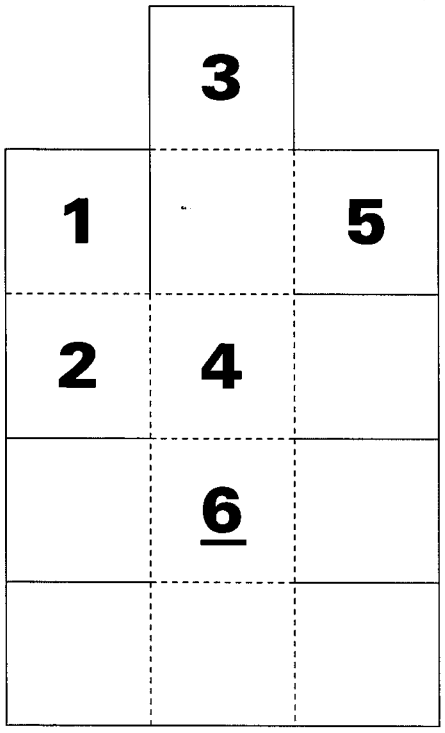
cut on dotted lines

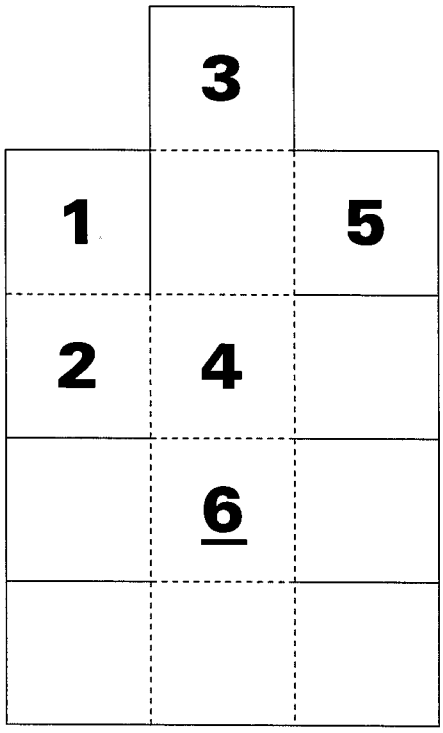


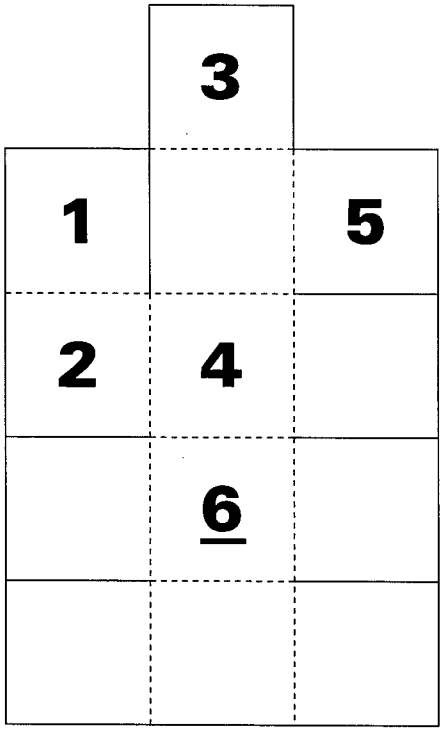
Cubical Dice Patterns

To make dice: Copy the patterns on cardstock or tagboard; cut on solid lines; score on dotted lines; fold so that the 6 numbered squares are on the outside; and use glue or rubber cement to secure.







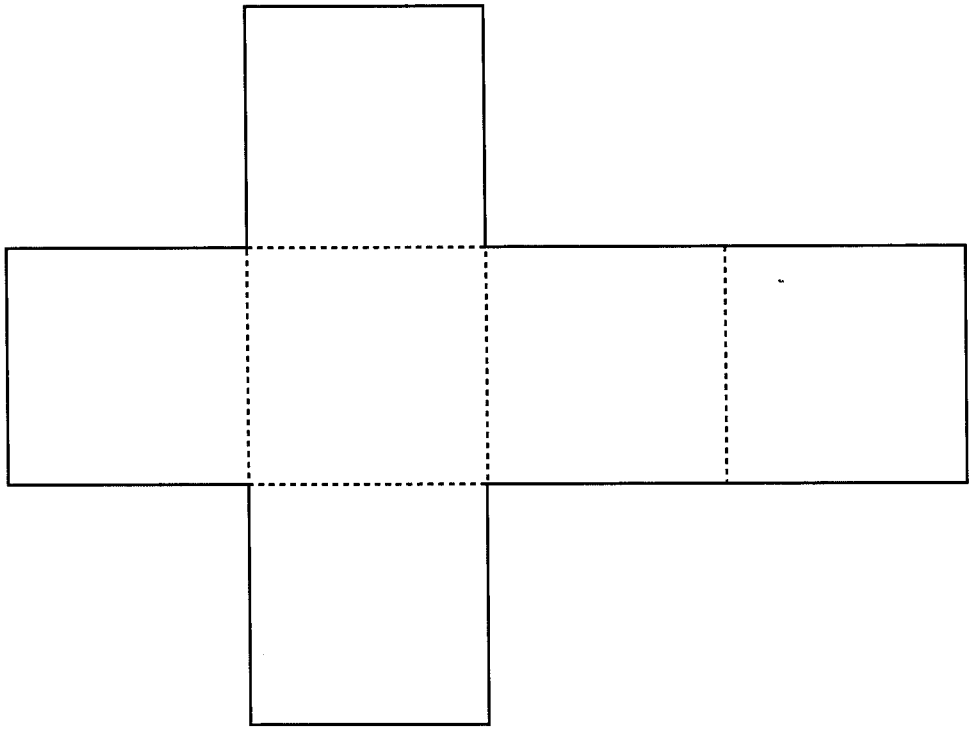


Red and Black Counting Piece Master — Front

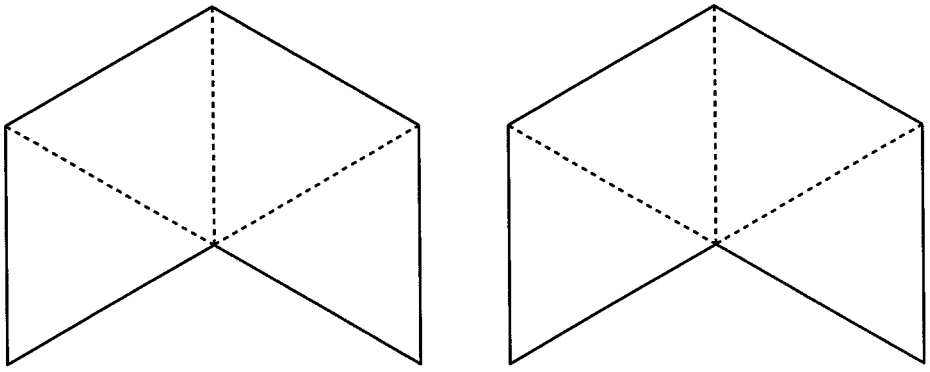
Red and Black Counting Piece Master — Back

Regular Polyhedra Patterns

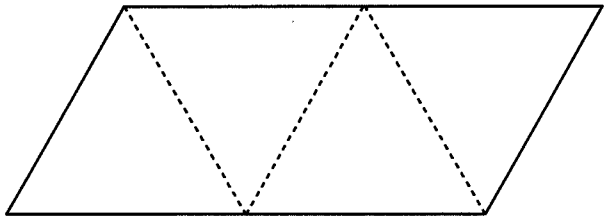
Cube (6 faces)



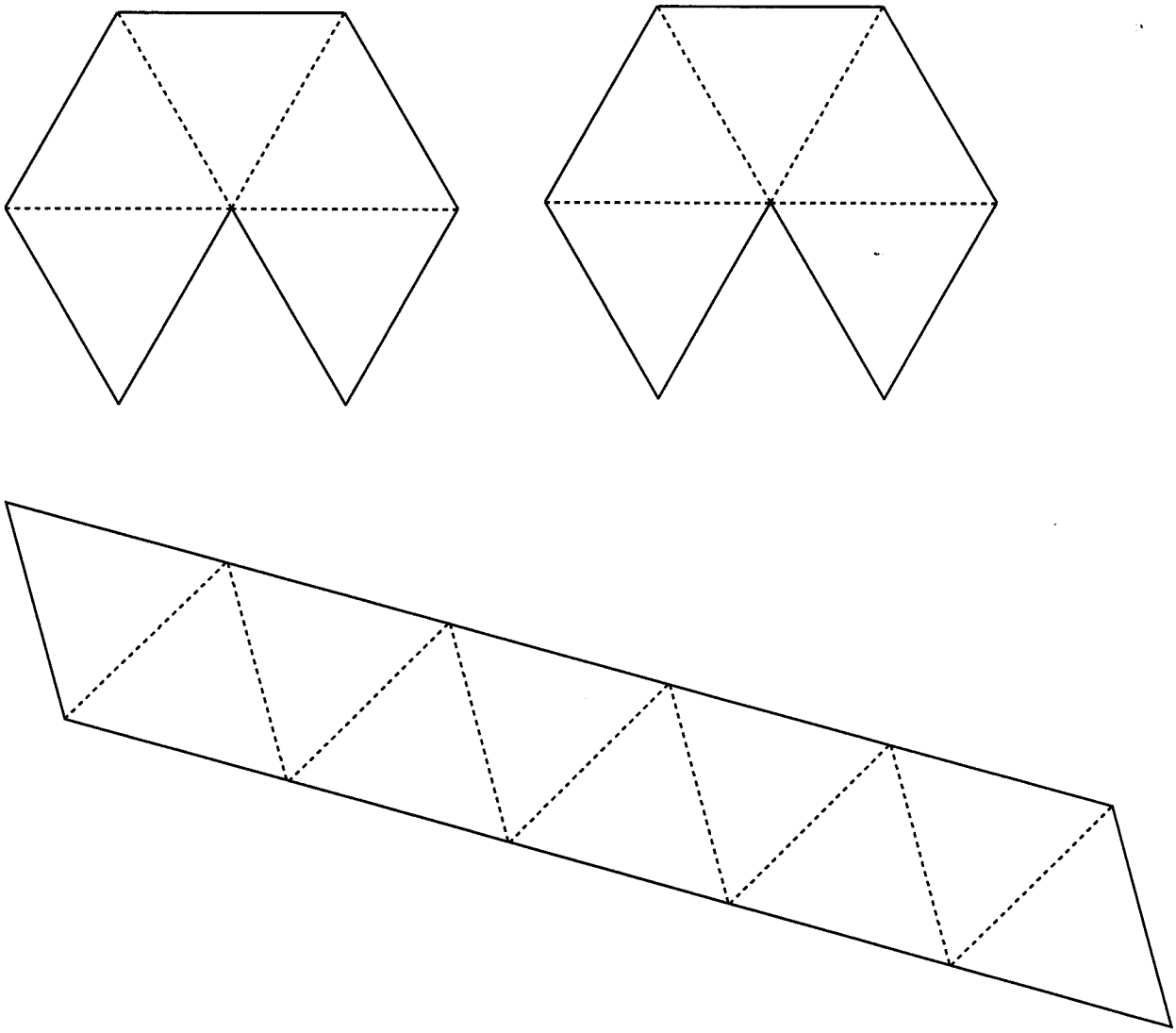
Octahedron (8 faces)



Tetrahedron (4 faces)



Icosahedron (20 faces)



Dodecahedron (12 faces)