math alive!

VISUAL MATHEMATICS COURSE III
BLACKLINE MASTERS

This packet contains one copy of each display master and student activity page.

Math Alive!
Visual Mathematics, Course III
by Linda Cooper Foreman and Albert B. Bennett Jr.
Blackline Masters

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Blackline Masters

		Copies / Transparencies
LESSON 1	Connector Master A	1 per group, 1 transp.
	Connector Master B	1 per group, 1 transp.
	Connector Master C	1 per group, 1 transp.
	Connector Master D	2 per student, 2 transp.
	Focus Master A	1 per student, 1 transp.
	Focus Student Activity 1.1	1 per student, 1 transp.
	Focus Student Activity 1.2	1 per student, 1 transp.
	Follow-up Student Activity 1.3	1 per student
LESSON 2	Focus Master A	1 per student, 1 transp.
	Focus Master B	1 per student, 1 transp.
	Focus Master C	2 per student, 1 transp.
	Focus Master D	1 transp.
	Focus Master E	1 transp.
	Focus Master F	1 per student, 2 transp.
	Focus Master G	1 per student, 2 transp.
	Focus Master H	1 per student, 2 transp.
	Focus Master I	1 transp.
	Focus Master J	1 per student, 1 transp.
	Focus Student Activity 2.1	1 per student, 1 transp.
	Focus Student Activity 2.2	1 per student, 1 transp.
	Focus Student Activity 2.3	1 per student, 1 transp.
	Follow-up Student Activity 2.4	1 per student
LESSON 3	Connector Master A	1 per group, 1 transp.
	Connector Master B	1 per student, 1 transp.
	Focus Master A	1 transp.
	Focus Master B	4 per two students, 1 transp.
	Focus Master C	1 per student
	Focus Master D	1 per two students, 1 transp.
	Focus Student Activity 3.1	1 per student, 1 transp.
	Focus Student Activity 3.2	1 per student, 1 transp.
	Focus Student Activity 3.3	1 per student, 1 transp.
	Focus Student Activity 3.4	1 per student, 1 transp.
	Follow-up Student Activity 3.5	1 per student
LESSON 4	Connector Student Activity 4.1	1 per student
	Focus Master A	1 transp.
	Focus Master B	1 transp.
	Focus Master C	1 per two students, 1 transp.
	Focus Student Activity 4.2 (optional)	1 per student
	Follow-up Student Activity 4.3	1 per student
LESSON 5	Connector Master A (optional)	1 per teacher
	Connector Master B	1 per two students, 1 transp.
	Connector Master C	1 per student, 1 transp.
	Connector Master D	1 transp.
	Focus Master A	1 transp.
	Focus Master B	1 transp.
	Focus Master C	1 transp.
	Focus Master D	1 per student, 1 transp.
	Focus Student Activity 5.1	1 per student, 1 transp.
	Follow-up Student Activity 5.2	1 per student

Blackline Masters (continued)

		Copies / Transparencies
LESSON 6	Connector Master A	1 per group, 1 transp.
	Focus Master A	2 per student, 1 transp.
	Focus Master B	1 per group, 1 transp.
	Focus Master C	1 per group, 1 transp.
	Focus Student Activity 6.1	1 per student, 1 transp.
	Focus Student Activity 6.2	1 per student, 1 transp.
	Focus Student Activity 6.3	1 per student, 1 transp.
	Focus Student Activity 6.4	1 per student, 1 transp.
	Focus Student Activity 6.5	1 per student, 1 transp.
	Follow-up Student Activity 6.6	1 per student
LESSON 7	Connector Master A	1 transp.
	Focus Master A	1 per student, 1 transp.
	Focus Master B	1 per group, 1 transp.
	Focus Master C	1 per group, 1 transp.
	Focus Master D	1 transp.
	Focus Master E	1 transp.
	Focus Student Activity 7.1	1 per student, 1 transp.
	Follow-up Student Activity 7.2	1 per student
LESSON 8	Connector Master A	2 per student, 1 per group, and 1 transp.
	Connector Master B	1 per group, 1 transp.
	Connector Master C	1 per group, 1 transp.
	Connector Master D	1 per group, 1 transp.
	Connector Student Activity 8.1	1 per group, 1 transp.
	Focus Master A	1 transp.
	Focus Master B	1 transp.
	Focus Master C	1 per two students, 1 transp.
	Focus Master D	1 transp.
	Focus Student Activity 8.2	1 per two students, 1 transp.
	Follow-up Student Activity 8.3	1 per student
LESSON 9	Connector Master A	1 per two students, 1 transp.
	Connector Master B	1 per two students, 1 transp.
	Connector Master C	1 per two students, 1 transp.
	Connector Student Activity 9.1	1 per student, 1 transp.
	Focus Master A	1 per group, 1 transp.
	Focus Master B	1 transp.
	Focus Master C	1 transp.
	Focus Master D	1 per group, 1 transp.
	Focus Master E	2 per student, 1 transp.
	Focus Student Activity 9.2	1 per student, 1 transp.
	Focus Student Activity 9.3	1 per student, 1 transp.
	Follow-up Student Activity 9.4	1 per student

Blackline Masters (continued)

		Copies / Transparencies
LESSON 10	Connector Master A	1 transp.
	Focus Master A	1 transp.
	Focus Master B	1 per two students, 1 transp.
	Focus Master C	1 transp.
	Focus Master D	1 per two students, 1 transp.
	Focus Master E	1 transp.
	Focus Student Activity 10.1	1 per student, 1 transp.
	Focus Student Activity 10.2	1 per student, 1 transp.
	Focus Student Activity 10.3	1 per student, 1 transp.
	Focus Student Activity 10.4	1 per student, 1 transp.
	Follow-up Student Activity 10.5	1 per student
LESSON 11	Connector Master A	1 transp.
	Connector Student Activity 11.1	1 per student, 1 transp.
	Connector Student Activity 11.2	1 per student, 1 transp.
	Focus Master A	1 transp.
	Focus Master B	1 transp.
	Focus Master C	1 transp.
	Focus Master D	1 per group, 1 transp.
	Focus Master E	1 transp.
	Focus Master F	1 transp.
	Focus Master G	1 transp.
	Focus Student Activity 11.3	1 per student, 1 transp.
	Focus Student Activity 11.4	1 per student, 1 transp.
	Focus Student Activity 11.5	1 per student, 1 transp.
	Focus Student Activity 11.6	1 per student, 1 transp.
	Focus Student Activity 11.7	1 per group, 1 transp.
	Follow-up Student Activity 11.8	1 per student
LESSON 12	Connector Student Activity 12.1	1 per two students, 1 transp.
	Focus Master A	1 per group, 1 transp.
	Focus Master B	1 transp.
	Focus Master C	1 per group, 1 transp.
	Focus Master D	1 per student, 1 transp.
	Focus Master E	1 per group, 1 transp.
	Focus Student Activity 12.2	1 per student, 1 transp.
	Focus Student Activity 12.3	1 per student, 1 transp.
	Focus Student Activity 12.4	1 per student, 1 transp.
	Follow-up Student Activity 12.5	1 per student
LESSON 13	Connector Master A	1 transp.
	Focus Master A	1 transp.
	Focus Master B	1 per two students, 1 transp.
	Focus Master C	1 per two students, 1 transp.
	Focus Master D	1 transp.
	Focus Master E	1 transp.
	Focus Master F	1 per two students, 1 transp.
	Follow-up Student Activity 13.1	1 per student

Blackline Masters (continued)

		Copies / Transparencies
LESSON 14	Focus Master A	1 transp.
	Focus Master B	1 per group, 1 transp.
	Focus Master C	1 per group, 1 transp.
	Focus Master D	1 per student, 1 transp.
	Focus Student Activity 14.1	1 per student, 1 transp.
	Focus Student Activity 14.2	1 per student, 1 transp.
	Follow-up Student Activity 14.3	1 per student
LESSON 15	Connector Student Activity 15.1	1 per student, 1 transp.
	Focus Master A	1 transp.
	Focus Master B	1 transp.
	Focus Master C	1 per student, 1 transp.
	Focus Master D	1 per student, 1 transp.
	Focus Student Activity 15.2	1 per group, 1 transp.
	Focus Student Activity 15.3	1 per student, 1 transp.
	Focus Student Activity 15.4	1 per student, 1 transp.
	Focus Student Activity 15.5	1 per student, 1 transp.
	Focus Student Activity 15.6	1 per student, 1 transp.
	Focus Student Activity 15.7	1 per student, 1 transp.
	Follow-up Student Activity 15.8	1 per student
LESSON 16	Connector Master A	1 transp.
	Connector Master B	1 transp.
	Connector Master C	1 transp.
	Connector Student Activity 16.1	1 per two students, 1 transp.
	Focus Student Activity 16.2	1 per student, 1 transp.
	Focus Student Activity 16.3	1 per student, 1 transp.
	Focus Student Activity 16.4	1 per student, 1 transp.
	Follow-up Student Activity 16.5	1 per student
LESSON 17	Connector Master A	1 per student, 1 transp.
	Connector Master B	1 per student, 1 transp.
	Connector Student Activity 17.1	1 per group, 1 transp.
	Focus Master A	1 transp.
	Focus Master B	1 per student, 1 transp.
	Focus Master C	1 transp.
	Focus Student Activity 17.2	1 per group, 1 transp.
	Focus Student Activity 17.3	1 per group, 1 transp.
	Follow-up Student Activity 17.4	1 per student

ROTATIONS/TURNS

a) Complete this procedure:

- Position your note card so that it fits in its frame with no gaps or overlaps.
- Mark a point anywhere on your card with a dot, and label this point P.
- Place a pencil point on your point P and hold the pencil firmly in a vertical position at P.
- Rotate the card about P until the card fits back into its frame with no gaps or overlaps.

b) How many *different* rotations of the card about your point P are possible so that the card fits back in its frame with no gaps or overlaps? Assume that rotations are different if they result in different placements of the card in its frame.

c) If only a 360° (or 0°) rotation about your point P brings the card back into its frame, find another position for P on the card so that *more than one different rotation* about this point is possible. What are the measures of the rotations and how did you determine them?

REFLECTIONS/FLIPS

Figure 1 below shows the frame for a rectangular card with a line *l* drawn across the frame. In Figure 2, the card has been placed in the frame. Figure 3 shows the result of *reflecting*, or *flipping*, the card over line *l*. Notice that after the reflection over line *l*, the card does not fit back in its frame.

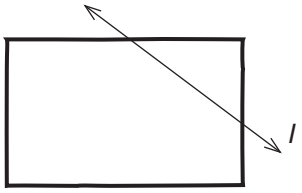


Figure 1

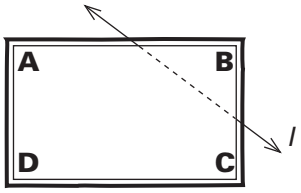


Figure 2

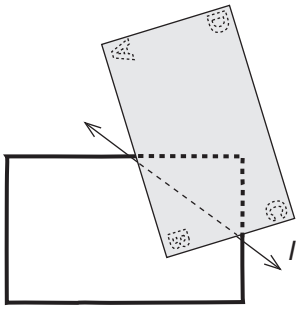
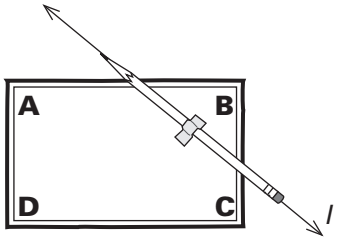


Figure 3

Determine all the *different* possible placements of line *l* so that when you flip your card once over *l*, the card fits back in its frame with no gaps or overlaps.

HINT: As a guide for flipping the card about a line, you could tape a pencil or coffee stirrer to the card along the path of line *l*, as shown below. Then keep the pencil or coffee stirrer aligned with line *l* as you flip the card.



Connector Master C

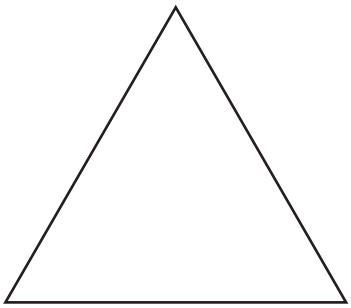
a) Discuss your group's ideas and questions about the meanings of the following terms. Talk about ways these terms relate to a nonsquare rectangle such as your note card. Record important ideas and questions to share with the class.

i) reflectional symmetry
ii) axis of reflection (also called *line of reflection*)
iii) rotational symmetry
iv) center of rotation
v) frame test for symmetry

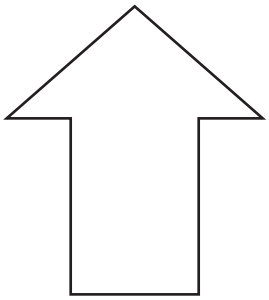
b) If a shape is symmetrical, its *order of symmetry* is the number of different positions for the shape in its frame, where different means the sides of the shape and the sides of the frame match in distinctly different ways. Develop a convincing argument that your rectangular note card has symmetry of order four.

Connector Master D

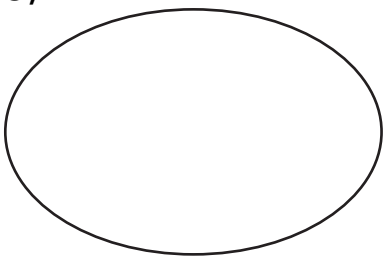
a)



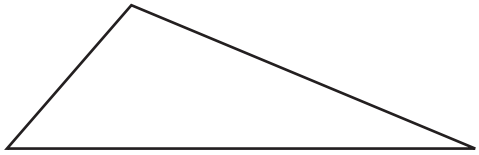
b)



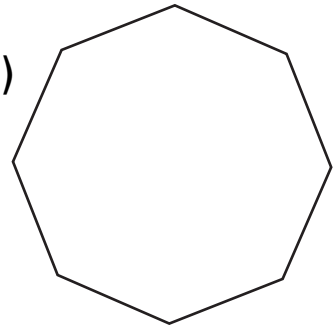
c)



d)

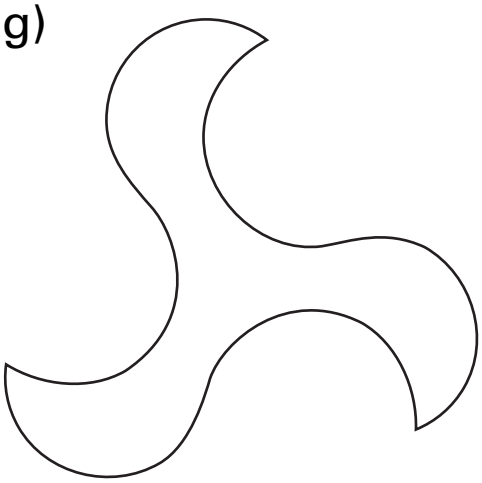


e)

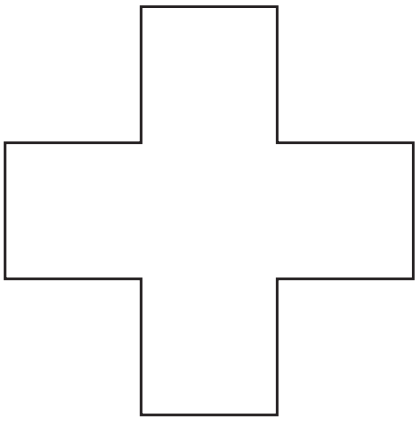


f)

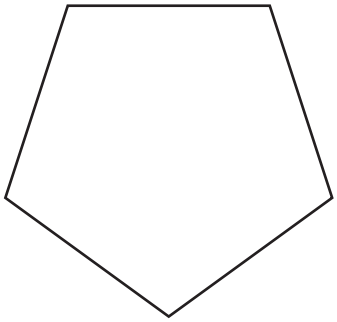
g)



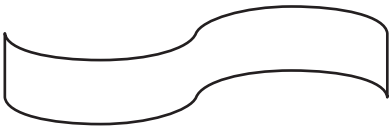
h)



i)



j)



Focus Master A

Our Goals as Mathematicians

We are a community of mathematicians working together to develop our:

a) visual thinking,

b) concept understanding,

c) reasoning and problem solving,

d) ability to invent procedures and make generalizations,

e) mathematical communication,

f) openness to new ideas and varied approaches,

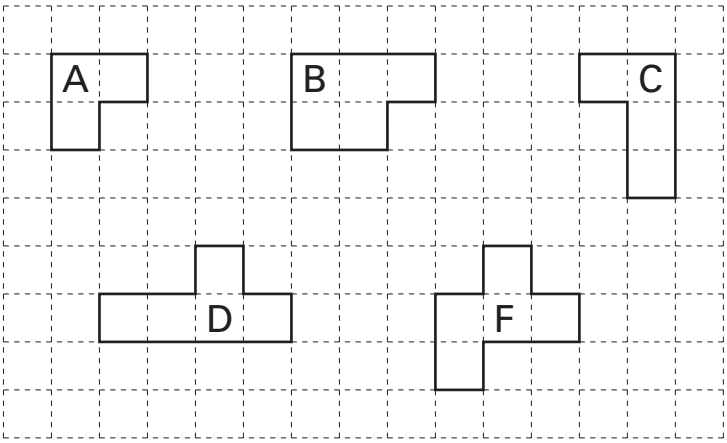
g) self-esteem and self-confidence,

h) joy in learning and doing mathematics.

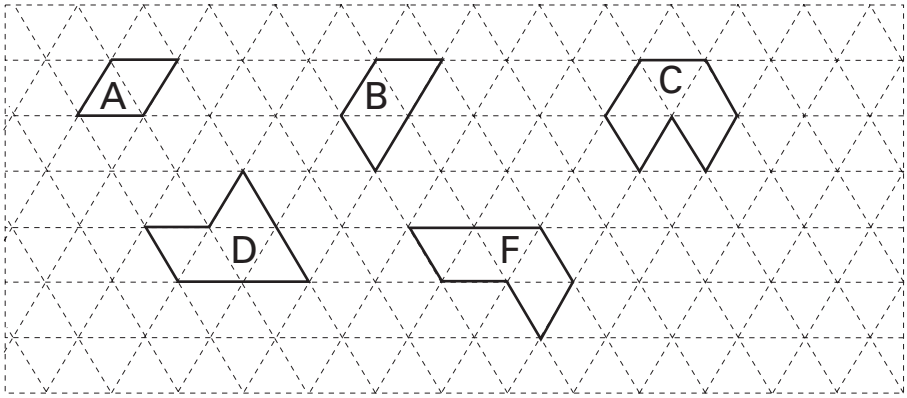
Focus Student Activity 1.1

NAME _____ **DATE** _____

1 For each shape below, determine mentally how many ways one square of the grid can be added to the shape to make it symmetrical. Assume no gaps or overlaps and that squares meet edge-to-edge.



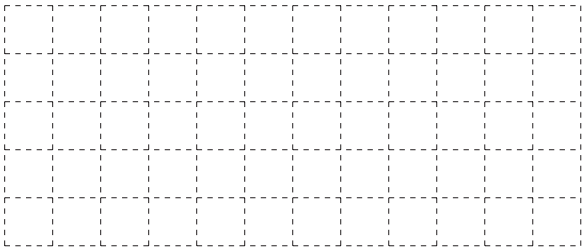
2 For each shape below, determine mentally how many ways one triangle of the grid can be added to the shape to make it symmetrical. Assume no gaps or overlaps and that triangles meet edge-to-edge.



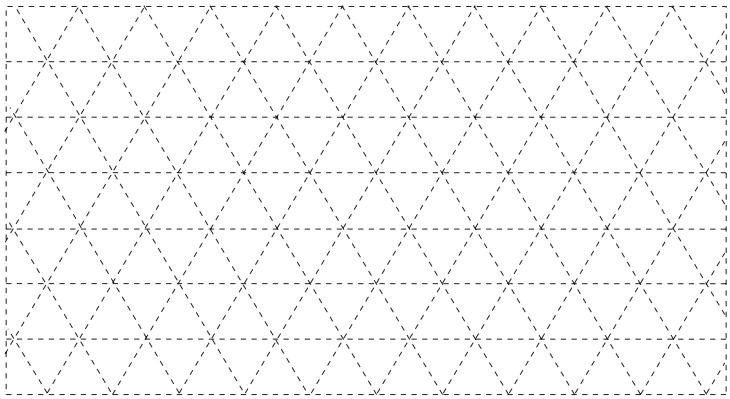
(Continued on back.)

Focus Student Activity (cont.)

3 Create a shape that is made of squares joined edge-to-edge (no overlaps) and has exactly 3 ways of adding one additional square to make the shape symmetrical.



4 Create a shape that is made of triangles joined edge-to-edge (no overlaps) and has exactly 4 ways of adding one additional triangle to make the shape symmetrical.



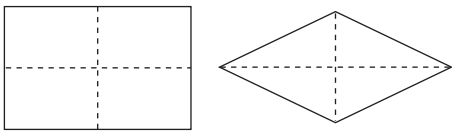
Focus Student Activity 1.2

NAME _____ **DATE** _____

Write a well-organized, sequential summary of your investigation of *one* of Problems 1 or 2. Include the following in your summary:

- a statement of the problem you investigate
- the steps of what you do, including any false starts and dead-ends
- relationships you notice (small details are important)
- questions that occur to you
- places you get stuck and things you do to get unstuck
- your AHA!s and important discoveries
- conjectures that you make—include what sparked and ways you tested each conjecture
- evidence to support your conclusions.

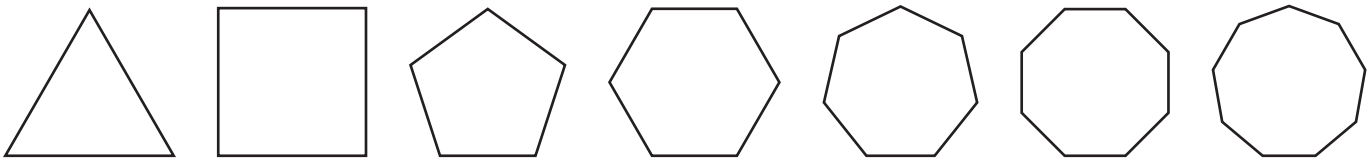
1 A nonsquare rectangle and a nonsquare rhombus each have 2 reflectional symmetries. However, the 2 lines of symmetry are of 2 different types—the lines of symmetry of a rectangle connect the midpoints of opposite sides and the lines of symmetry of a rhombus connect opposite vertices. Investigate other polygons with exactly 2 lines of symmetry of these 2 types. Generalize, if possible.



2 What, if any, is the minimum number of sides for a polygon with 3 rotational symmetries and no reflectional symmetry? What, if any, is the maximum number of sides? What, if any, is the minimum number of sides for polygons with 4 rotational symmetries and no reflectional symmetries? 5 rotational and no reflectional symmetries? *n* rotational and no reflectional symmetries? Investigate.

Follow-up Student Activity 1.3

NAME _____ **DATE** _____



1 Trace and cut out a copy of each of the above regular polygons. Use the copies and original polygons, but no measuring tools (no rulers, protractors, etc.), to help you complete the following chart:

								
No. of different positions in frame								
No. of reflectional symmetries								
No. of rotational symmetries								
Measures of *all* angles of rotation								
Measure of each interior angle*								

*Interior angles are the angles "inside" the polygon and are formed by intersections of the sides of the polygon.

Complete the following problems on separate paper. Be sure to write about any AHA!s, conjectures, or generalizations that you make.

2 Explain the methods that you used to determine the angles of rotation and the interior angle measures for the chart above. Remember, no protractors.

3 Label the last column of the chart in Problem 1 "Regular *n*-gon" and then complete that column. For each expression that you write in the last column, draw a diagram (on a separate sheet) to show "why" the expression is correct.

(Continued on back.)

Follow-up Student Activity (cont.)

4 Discuss the symmetries of a circle. Explain your reasoning.

5 Locate a resource that shows flags of the countries of the world. For each of the following, if possible, sketch and color a copy of a different flag (label each flag by its country's name) and cite your resource.

a) rotational symmetry but no reflectional symmetry,

b) reflectional symmetry across a horizontal axis only,

c) no symmetry,

d) both rotational and reflectional symmetry,

e) 180° rotational symmetry.

6 Sort and classify the capital letters of the alphabet according to their types of symmetry.

7 Attach pictures of 2 different company logos that have different types of symmetry. Describe the symmetry of each logo.

8 Create your personal logo so that it has symmetry. Record the order of symmetry for your logo, show the location of its line(s) of symmetry, and/or record the measures of its rotational symmetries.

9 Jamaal made conjectures a) and b) below. Determine whether you think each conjecture is always/sometimes/never true. Give evidence to show how you decided and to show why your conclusion is correct. If you think a conjecture is not true, edit it so that it is true.

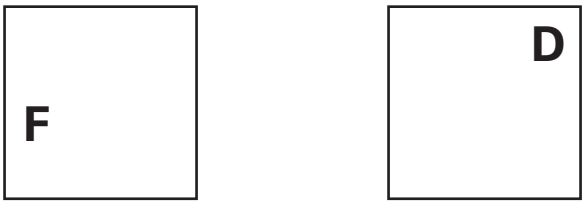
If a shape has exactly 2 axes of reflection, then

a) those axes must be at right angles to each other.

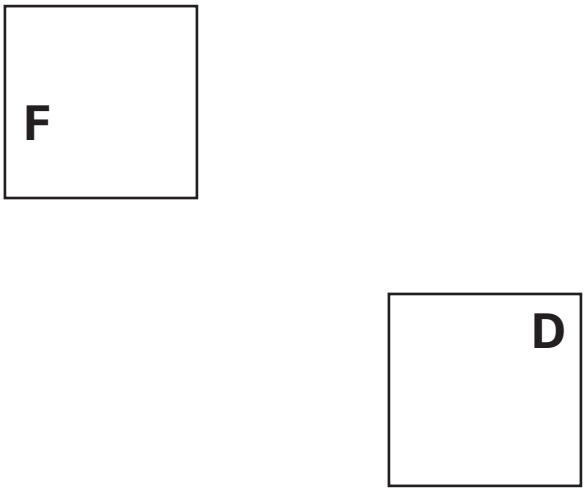
b) the shape also must have 2 rotational symmetries.

Focus Master A

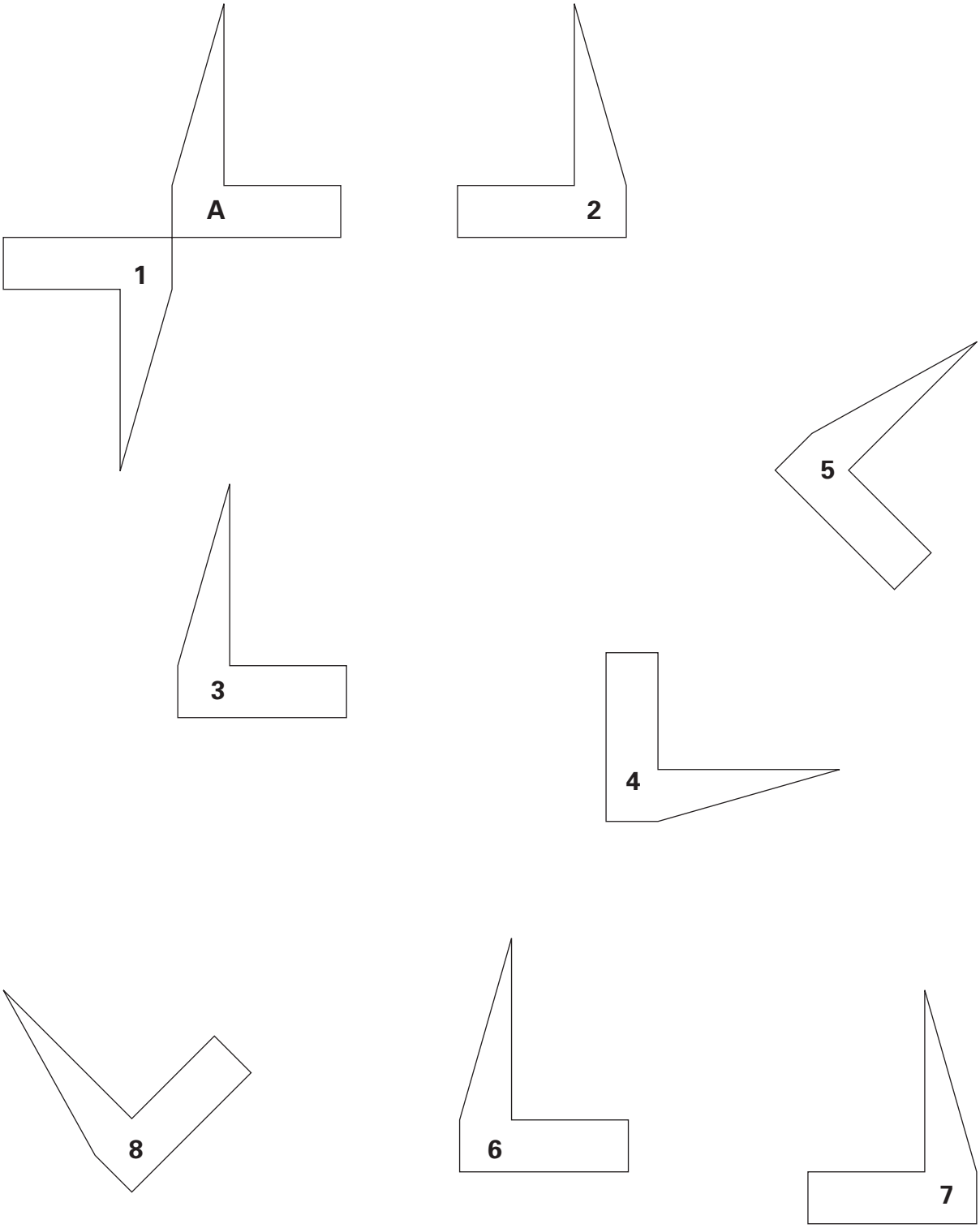
Investigate ways to use slides, flips, and/or turns to move Square F exactly onto Square D. Use words and/or mark diagrams to explain the movements that you use.



Focus Master B



Focus Master C



Focus Master D

Part I

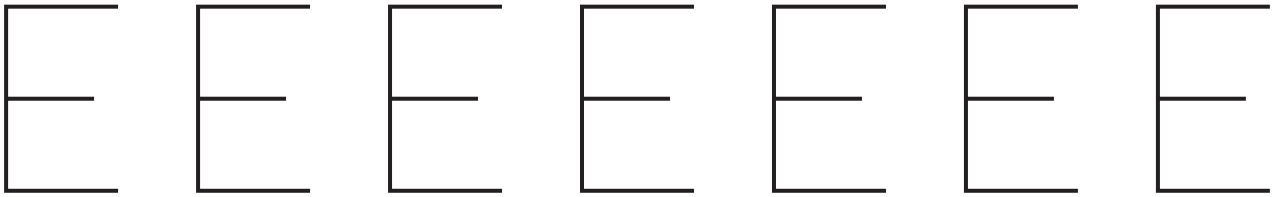
It is possible to move Shape A directly to several of the numbered positions using exactly one of these isometries only once: translation, reflection, or rotation. Find each position for which this is possible, and tell the single motion that moves Shape A to that position.

Part II

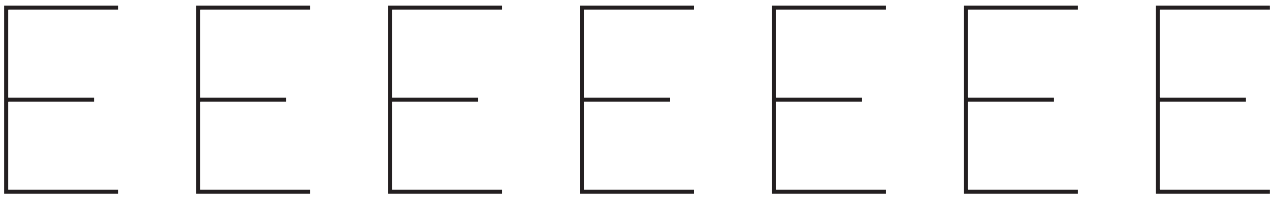
Describe ways to move Shape A from its starting position to each numbered position using a combination of exactly two reflections, rotations, and/or translations. Note: combinations of more than one type of motion are allowed as long as no more than two motions are used.

Focus Master E

Frieze A



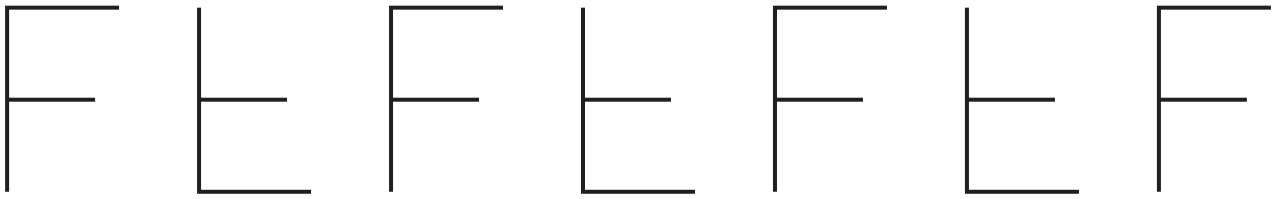
Frieze B



Focus Master F

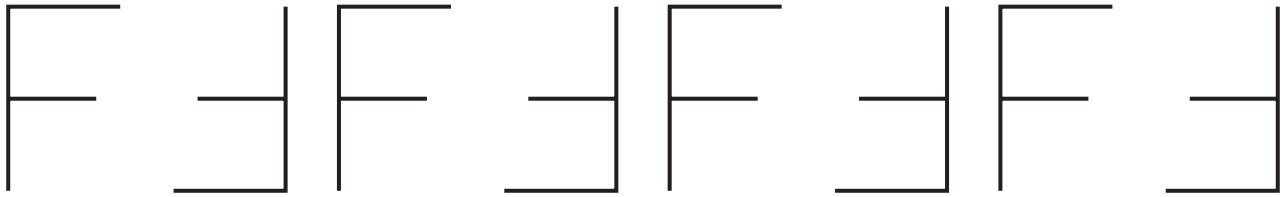
Frieze A

Frieze B

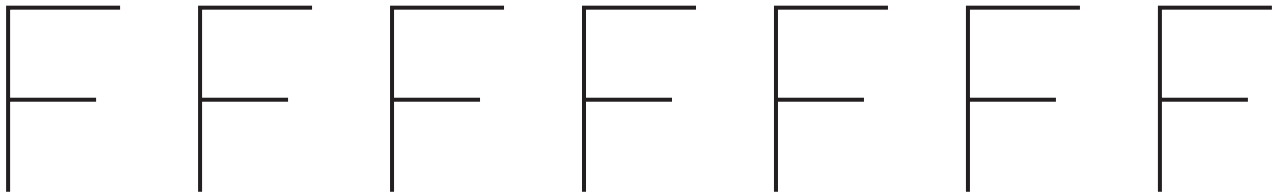


Focus Master G

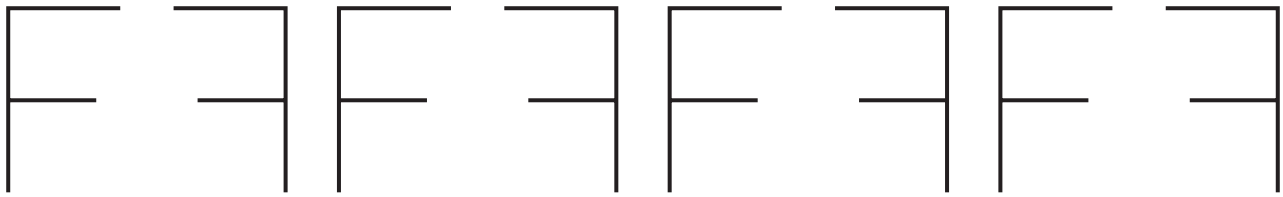
Frieze A



Frieze B

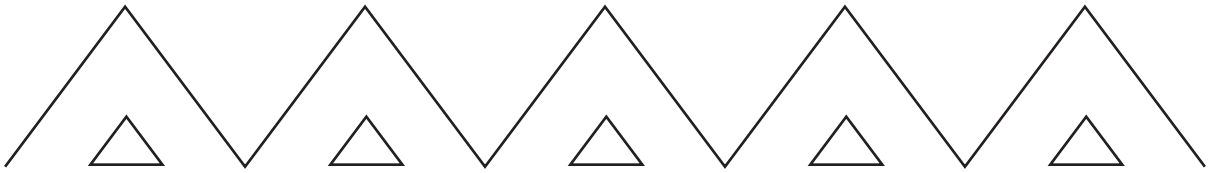


Frieze C

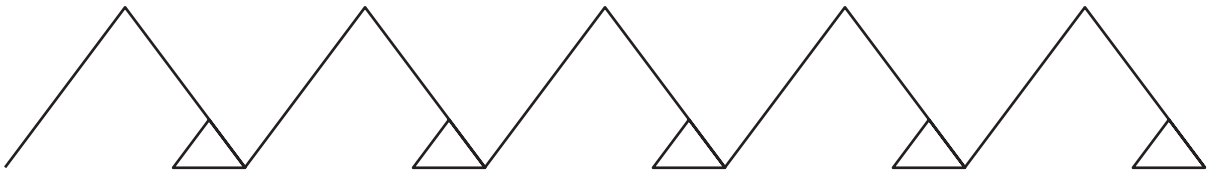


Focus Master H

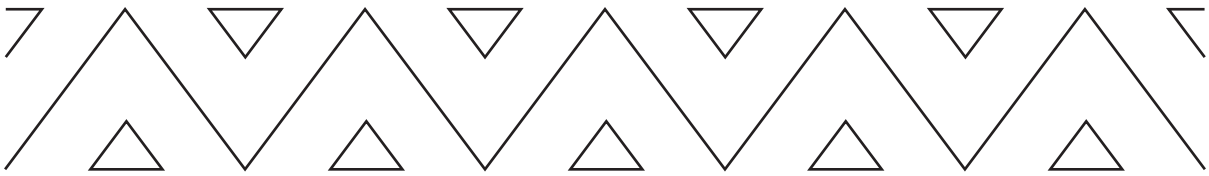
Frieze A



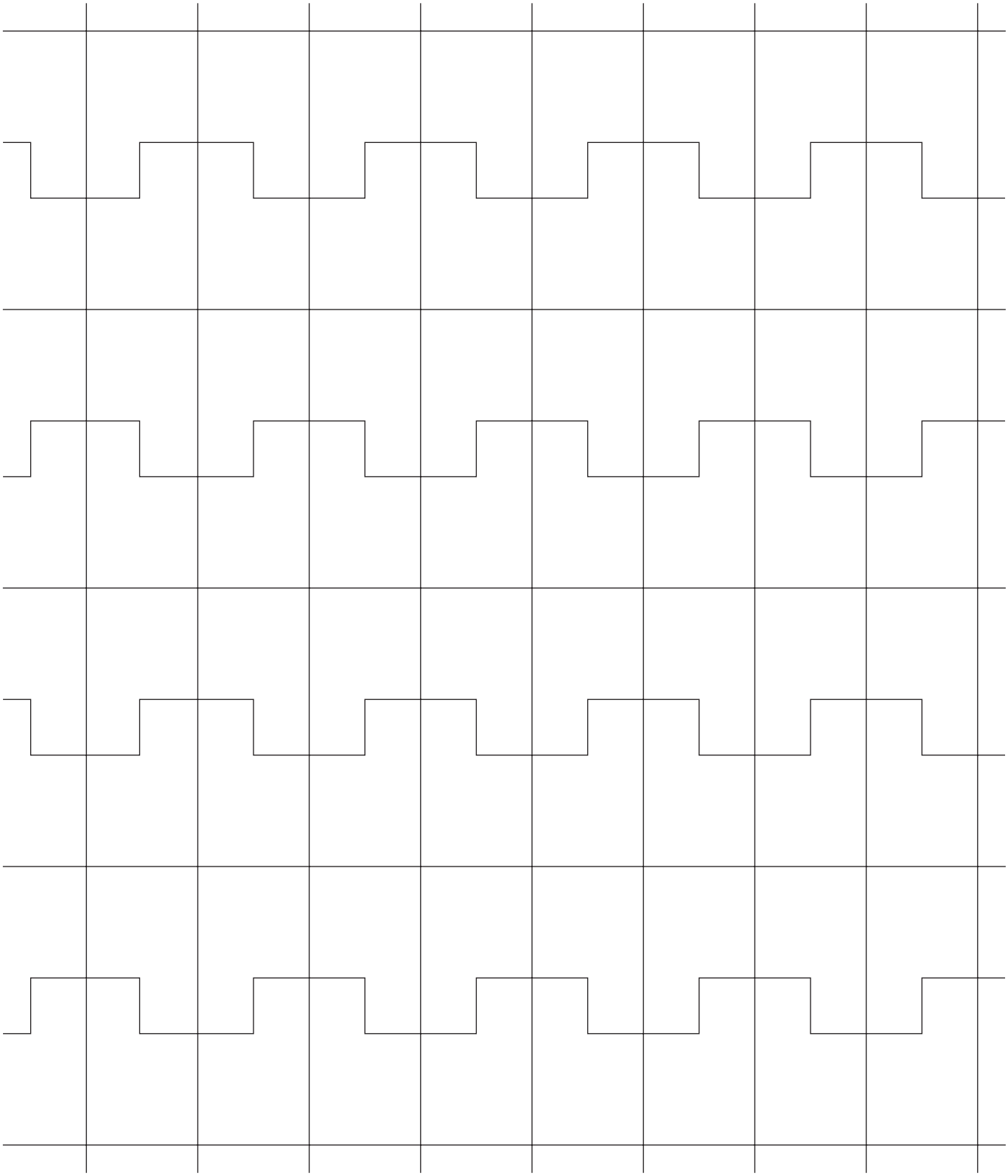
Frieze B



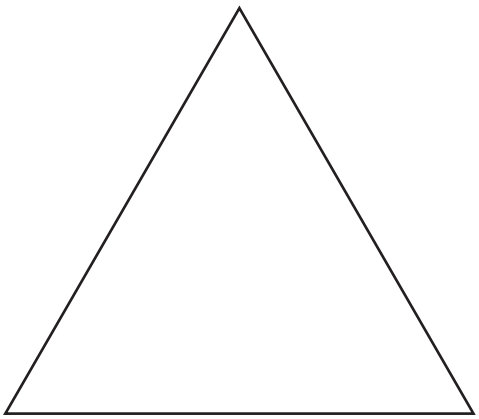
Frieze C

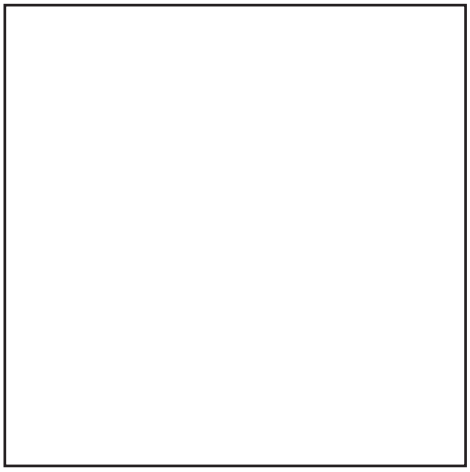


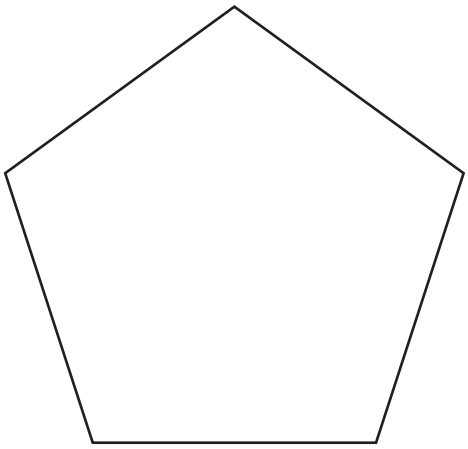
Focus Master I

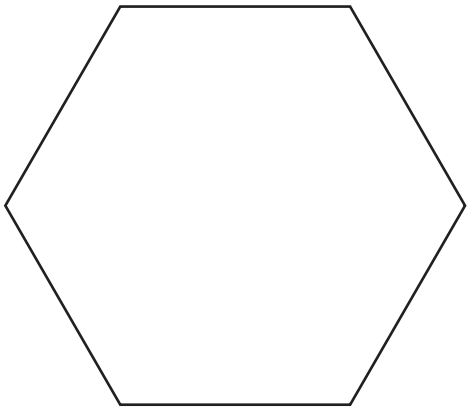


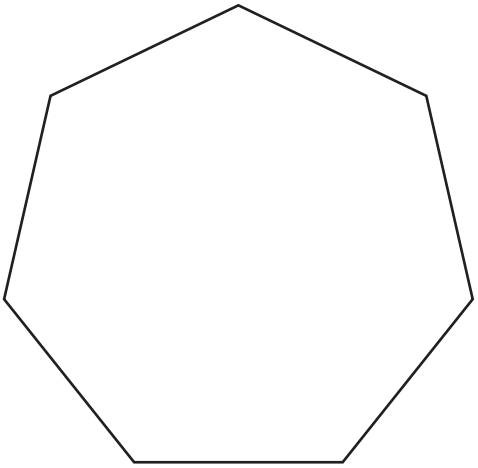
Focus Master J

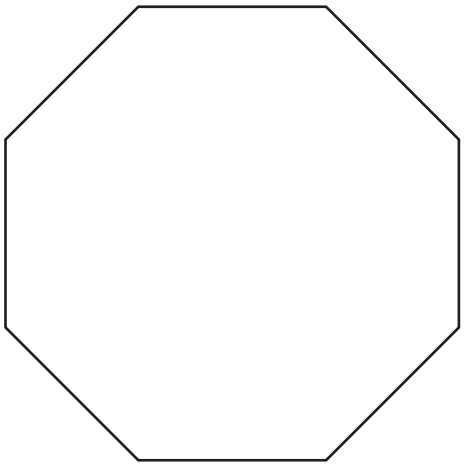










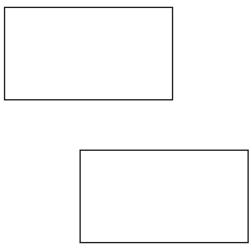


Focus Student Activity 2.1

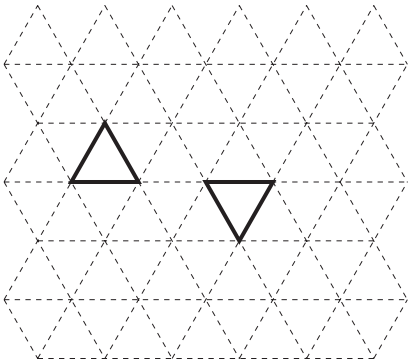
NAME _____ **DATE** _____

1 Shown below are several pairs of congruent shapes. Investigate ways to use one or more translations, reflections, rotations, or combinations of them, to move each first shape exactly onto the second. For each pair of shapes, write an explanation in words only of your "favorite" motion or combination of motions; explain in enough detail that a reader would be able to duplicate your motions without additional information.

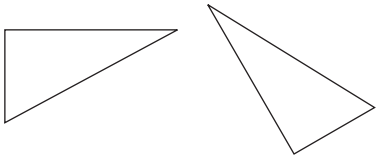
a)



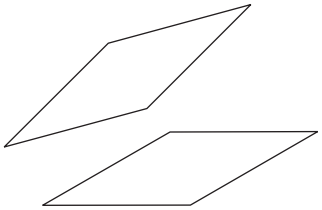
b)



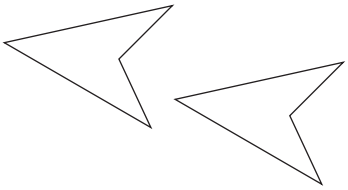
c)



d)



e)



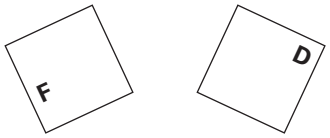
2 Challenge. Each motion or combination of motions that you determined for Problem 1 produces a *mapping* of the first shape (the *pre-image*) exactly onto the second (the *image*). How many different mappings are there for each of a)-e), if different means the sides of the pre-image and the sides of the image match in distinctly different ways.

3 Record your "I wonder…" statements, conjectures, or conclusions.

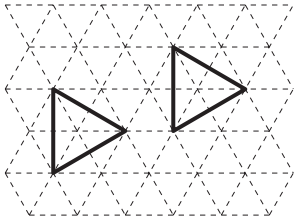
Focus Student Activity 2.2

NAME _____ **DATE** _____

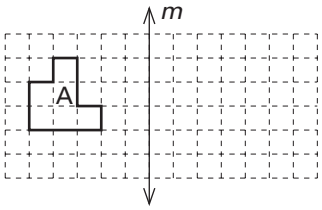
1 Shown at the right are 2 congruent squares. Determine ways to use exactly one isometry (translation, reflection, rotation, or glide reflection) to move Square F exactly onto Square D.



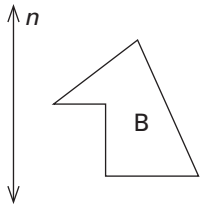
2 Repeat Problem 1 for the 2 equilateral triangles shown here:



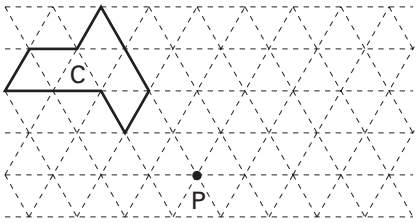
3 Sketch the reflected image of Shape A across line *m*. Next to your sketch write several mathematical observations about relationships you notice. Then explain how you verified that the image is a reflection of Shape A across line *m*.



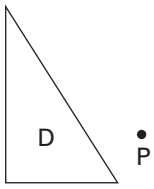
4 Challenge. Develop a method of accurately reflecting Shape B across line *n*. Show and describe your method of locating the reflected image of Shape B and tell how you verified that your method was correct. Can you generalize?



5 Sketch the image of Shape C after a 120° clockwise rotation about point P. Next to your sketch write several mathematical observations about relationships that you notice. Then explain how you verified that the image is a 120° rotation about point P.



6 Challenge. Invent a method of rotating Shape D 170° clockwise about point P, without using a grid. Show and describe your method of locating the rotated image of Shape D and tell how you verified that your method was correct.



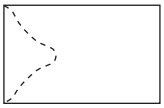
Focus Student Activity 2.3

NAME _____ **DATE** _____

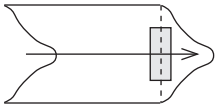
The following procedures create tessellations similar to the type created by the Dutch artist, Maurice C. Escher, who is famous for his tessellations of birds and animals. Escher's first inspirations came from the Alhambra, which was built in the 13th century in Granada, Spain, and is famous for its variety of tessellations. Procedure A below gives a method of creating a tessellation based on rectangles.

Procedure A—a translation tessellation based on rectangles.

Step 1: Beginning at one corner and ending at an adjacent corner, cut out a portion of the rectangle.

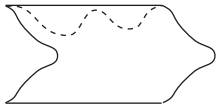


Step 2: Slide the cutout portion across the rectangle to the opposite side, matching the straight edges and the corners, as shown at the right. Then tape the pieces in place.

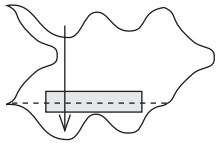


Hint: be careful not to flip the pieces over and don't let the tape extend beyond the edge of the figure.

Step 3: Beginning at an endpoint of one of the remaining two unaltered sides and ending at the other endpoint, cut another portion from the rectangle. An example is shown here.

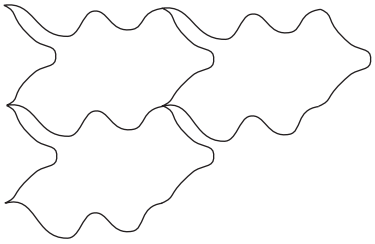


Step 4: Slide this cutout portion to the opposite side, matching edges and endpoints, as illustrated at the right. Tape the pieces in place.



Hint: It is important that nothing be trimmed or altered to fit!

Step 5: Tile a page with this new shape by repeatedly tracing the shape so the tracings fit together, with no gaps or overlaps (a portion of a tiled page is shown at the right). Describe the symmetries of the tessellation.

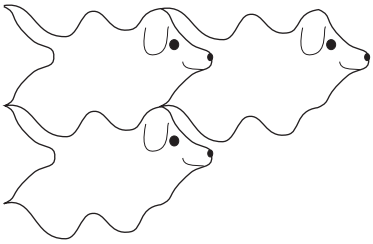


Hint: Remember not to flip the piece over when you tile with it.

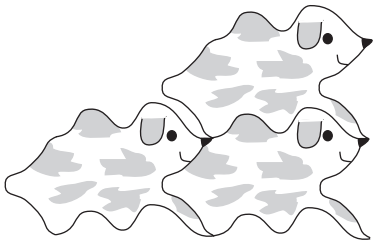
(Continued on back.)

Focus Student Activity 2.3 (page 2)

Step 6. Add color details to the figure to produce an interesting and creative work of art.



It could be puppy dogs on the run…



…or, puppy dogs at rest!

Project A

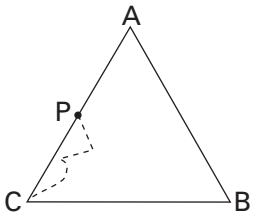
Part I Using a rectangle from page 7 of this activity and Procedure A, create an *original* tessellating shape. On another sheet of paper, trace enough copies of your figure to show that it forms a translation tessellation.

Part II Challenge. Using a shape that is not a rectangle, adapt Procedure A and create another figure that can form a translation tessellation. On another sheet of paper, trace enough copies of the figure to verify it forms a translation tessellation.

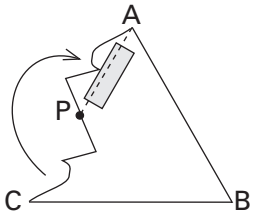
(Continued.)

Focus Student Activity 2.3 (page 3)

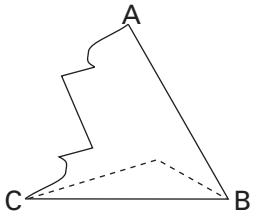
Procedure B—a rotation tessellation that is based on equilateral triangles.



Step 1. Mark the midpoint, P, of side AC on equilateral triangle ABC, as shown at the right. Beginning at vertex C and ending at P, cut a portion from the triangle.

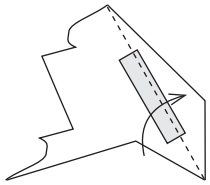


Step 2. Place a finger on P and rotate the cutout portion 180° about P. Then tape the rotated portion in place.



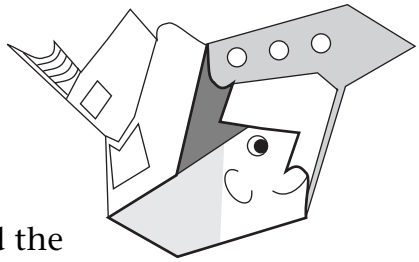
Step 3. Beginning at vertex C and ending at vertex B, cut out a portion of the triangle.

Step 4. Place a finger on vertex B and rotate the new cutout portion clockwise 60°. Tape the rotated portion in place.



Step 5. Tessellate the page by repeatedly tracing and fitting together (with no gaps or overlaps) the shape. Add detail and color to produce a creative piece of art that fills the page.

Hint: To tessellate with this shape it is necessary to rotate the figure.



Project B

Part I Use an equilateral triangle from page 7 of this activity and the method of Procedure B, or another procedure that you invent, to create an *original* shape that can be copied to form a rotation tessellation. On another sheet of paper, trace enough copies of your shape to verify that it forms a rotation tessellation.

Part II Challenge. Invent a way to alter a regular hexagon from page 7 to create a shape that forms a rotation tessellation of the plane. Verify. Show a tracing of several copies to illustrate the beginning of a tessellation.

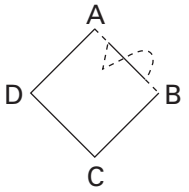
(Continued on back.)

Focus Student Activity 2.3 (page 4)

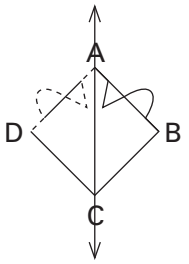
Procedure C—a reflection tessellation that is based on rhombuses.

Before you read on, trace a rhombus from page 7 and explore your ideas regarding ways to create a reflection tessellation based on nonsquare rhombuses. Then compare your ideas to the following procedure.

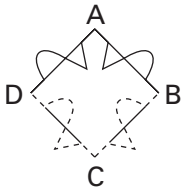
Step 1. Lightly trace the rhombus from page 7 on a blank sheet and label its vertices A, B, C, and D. Draw a curve from A to B (notice the curve can extend outside the rhombus). Note: in the remainder of this activity, the notation c (A-B) means *the curve from point A to point B*.



Step 2. Reflect c (A-B) about line AC. Hint: use tracing paper to increase accuracy of copied curves.

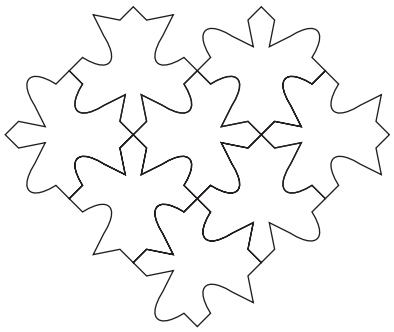


Step 3. Rotate c (A-D) 90° about point D to form c (D-C) so that A maps to C.



Step 4. Reflect c (D-C) about line AC so that D maps to B.

Step 5. Erase the lines of the original rhombus that are not part of the curve, and tessellate! Notice the lines of reflection in this tessellation.



Project C

Use a rhombus from page 7 of this activity and the method of Procedure C, or another method that you invent, to create an original shape that can be used to form a reflection tessellation. On another sheet of paper, trace enough copies of your shape to verify that it forms a reflection tessellation.

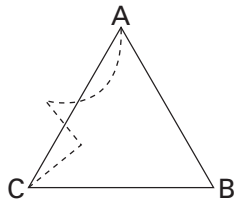
(Continued.)

Focus Student Activity 2.3 (page 5)

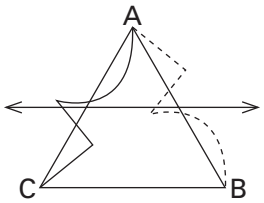
Procedure D—a glide reflection tessellation that is based on equilateral triangles.

Before you read on, trace an equilateral triangle from page 7 and explore your ideas regarding ways to create a glide reflection tessellation based on equilateral triangles. Then compare your ideas to the following procedure.

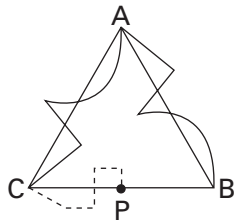
Step 1. Lightly trace the equilateral triangle from page 7 on a blank sheet and label its vertices A, B, and C. Draw a curve from A to C (as in Procedure C, the curve can extend outside the triangle).



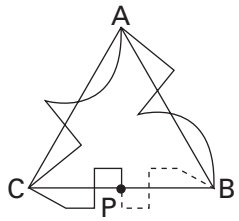
Step 2. Reflect c (A-C) about the line parallel to the base BC and passing through the midpoint of AC. Then translate this reflected curve to connect from A to B.



Step 3. Locate point P, the midpoint of side BC. Draw c (C-P).



Step 4. Rotate c (C-P) 180° about point P to form c (P-B).



Step 5. Tessellate!

Project D

Use Procedure D, or another method that you invented, to create an original shape that can be used to form a glide reflection tessellation. Verify by showing a tracing of several copies of your shape.

(Continued on back.)

Focus Student Activity 2.3 (page 6)

Project E

Part I

Pick your favorite tessellating shape from those you created for Projects A-D, or create a new tessellating shape based on Procedures A-D, or combinations of them, and:

a) *completely* tessellate a sheet of paper with the shape,

b) be creative in ways that you *color* and fill in the *details* of this tessellating figure,

c) mount the completed tessellation on a sheet of colored paper, or frame it creatively.

Note: to make a poster-sized tessellation you could enlarge the polygon that is the basis for your tessellation (e.g., double each edge of the hexagon pattern from page 7), create a new tessellating shape based on that enlarged polygon, and then tessellate a sheet of poster paper.

Part II

On the back of your tessellation (after you have mounted it):

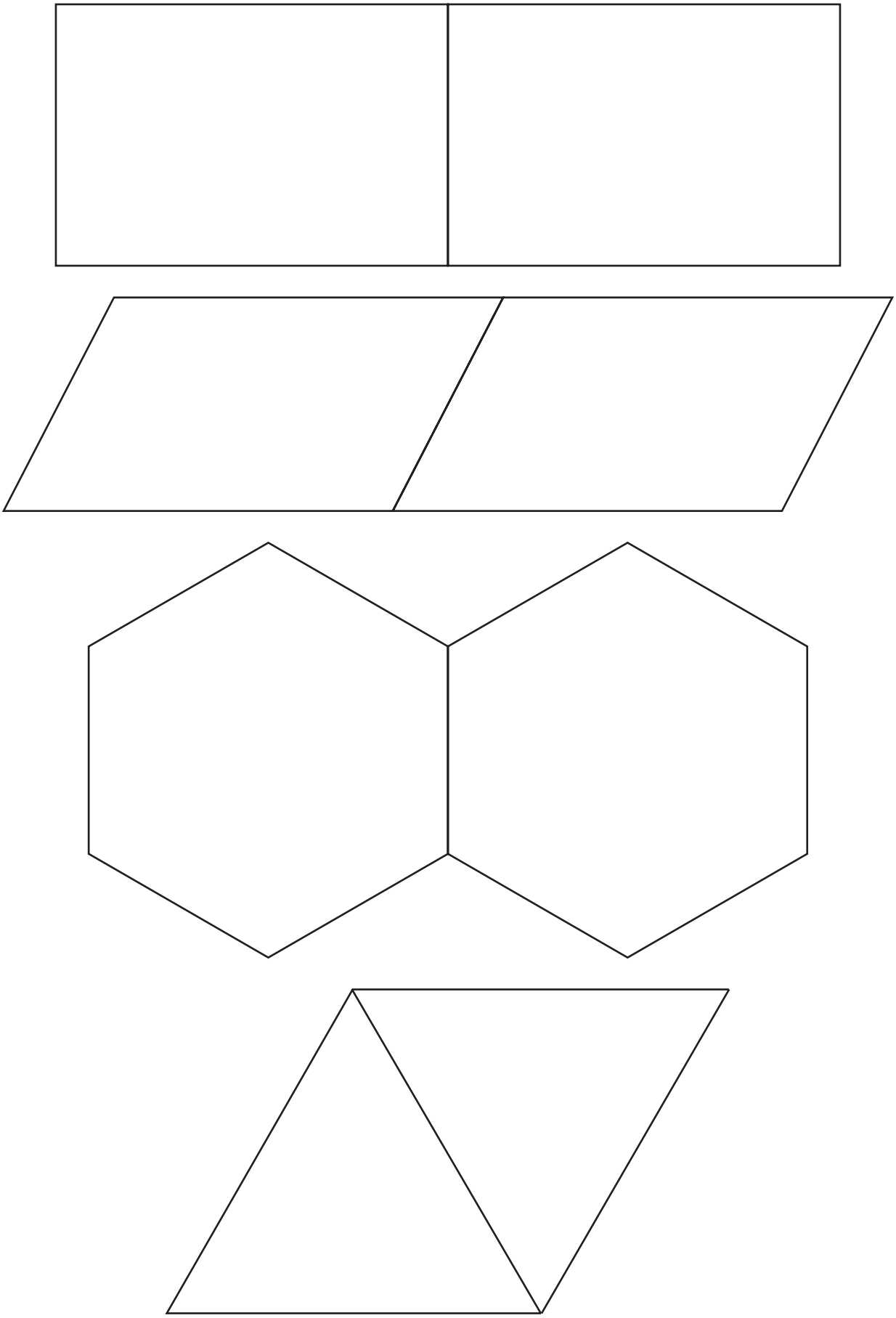
a) Trace the original polygon on which you based your tessellation.

b) Trace the tessellating shape.

c) Describe all symmetries, if any, of your tessellating shape, and all symmetries of your tessellation. Describe translation vectors and the locations of lines of reflection and centers of rotation for your tessellation.

(Continued.)

Focus Student Activity 2.3 (page 7)

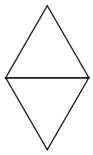


Follow-up Student Activity 2.4

NAME _____ **DATE** _____

Complete all of your work for this assignment on separate paper. Include a statement of each problem next to your work about the problem.

1 A shape made from 2 equilateral triangles joined edge-to-edge is called a *diamond*. In general, a shape made of 2 or more equilateral triangles joined edge-to-edge with no gaps or overlaps is called a *polyamond*. Use triangular grid paper to determine and make a chart that shows all the different diamonds, triamonds, tetramonds, pentamonds, and hexamonds and their symmetry types.



Diamond

2 Draw the polyamond with the least number of equilateral triangles that has exactly 3 rotational symmetries but no reflective symmetry. Label its center and angles of rotation.

3 Completely fill a ½-sheet of 2-cm triangular grid paper with a tessellation of your polyamond from Problem 2. Describe the symmetries of your tessellation. Label centers of rotations, lines of reflection, and/or translation vectors.

4 Explain in your own words the meanings of the term *isometry*. Describe the important features of each isometry explored in class.

5 Sherrill made conjectures a)-g) below. Examine several examples for each conjecture and decide whether you agree or disagree. For each conjecture, state your conclusion, show the examples you examine, and give solid mathematical evidence to support your conclusion. If you disagree, give a counter-example and tell how you would change her conjecture so it is true.

a) *If I draw 2 intersecting lines and reflect a shape, first over one of the lines and then over the other, I think the end result is the same as the result of a single rotation. The center and the measure of the rotation have a special relationship to the angle of intersection of the lines.*

b) *An enlargement is a transformation, but it is not an isometry.*

c) *For any translation, I can always locate 2 parallel lines so that the result of the translation is the same as the result of a reflection first across one of the parallel lines and then across the other.*

(Continued on back.)

Follow-up Student Activity (cont.)

d) *Every rotation about a point can be replaced by a reflection across first one line and then another.*

e) *It is not possible that a shape and its reflection can overlap. The same is true for a shape and its rotation, translation, or glide reflection.*

f) *A line of reflection is the perpendicular bisector of the line segments that connect each point on the pre-image to its corresponding point on the image.*

g) *There is no isometry that maps a shape back onto itself.*

6 Use a protractor and ruler to complete a)-c).

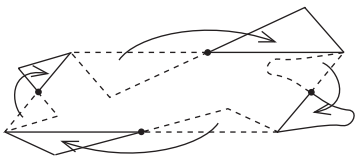
a) Draw a nonregular hexagon. Label one vertex of the hexagon H.
Draw a line M that does not intersect the hexagon.
Draw the reflection of the hexagon across line M.
Label the image of point H as H′.
Tell how you verify that your method is correct.

b) Draw a nonregular pentagon. Label one vertex P.
Mark a point Q that is not on the pentagon.
Draw a 45° rotation of the pentagon about point Q.
Label the image of point P as P′. Explain how you verify that point P rotated 45°.

c) Draw a scalene triangle. Label a point on the triangle C.
Label a point R that is not on the triangle. Draw a ray so that point R is the endpoint of the ray and so the ray does not intersect the triangle. Label a point S on your ray.
Sketch a translation of the triangle, using your ray with endpoint R as a translation vector with magnitude RS.
Label the image of point C as C′.

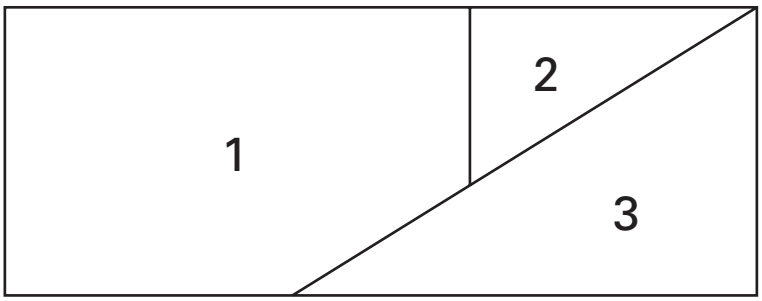
7 Create an *original* frieze pattern and describe its symmetries. Then adapt your design to create 6 new friezes that each illustrate a different possible symmetry type for friezes. Record the symmetries of each of your 7 friezes.

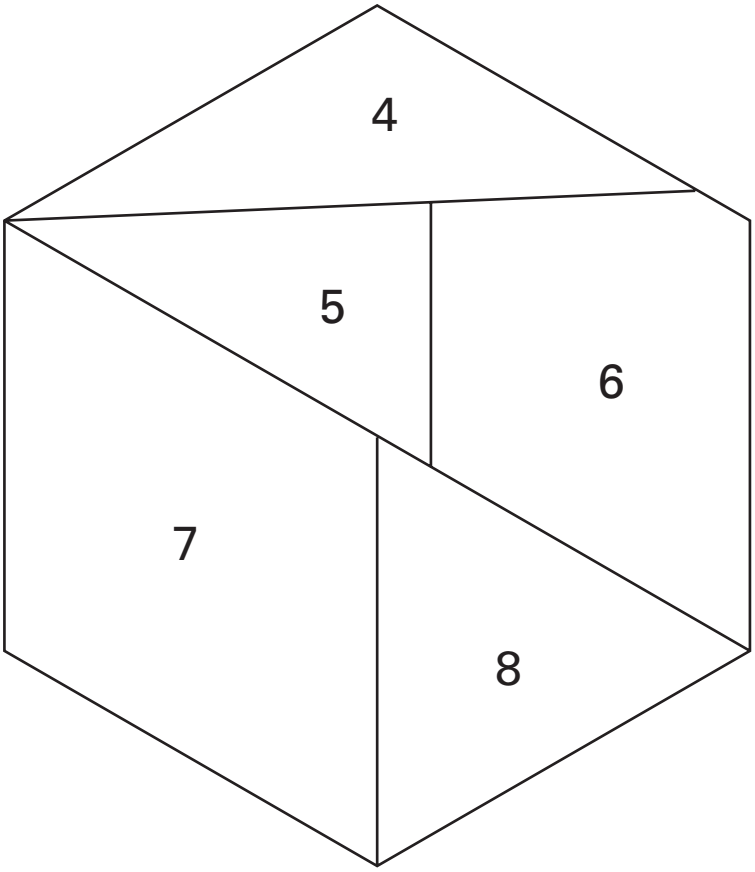
8 Study the following method of forming a tessellating figure that is based on a parallelogram. Investigate to see whether the method works for a trapezoid. If it works, complete such a tessellation of a trapezoid. If the method does not work, explain why not.



Connector Master A

The rectangle below has area 4 square units. When cut along the given lines, the rectangle can be reassembled to form a square. Similarly, the hexagon can be cut along the given lines and reassembled to form another square. Use this information to help you find a close approximation of the area of the hexagon. Then write a brief explanation of your methods. Note: use the same area and linear units for the rectangle and the hexagon.

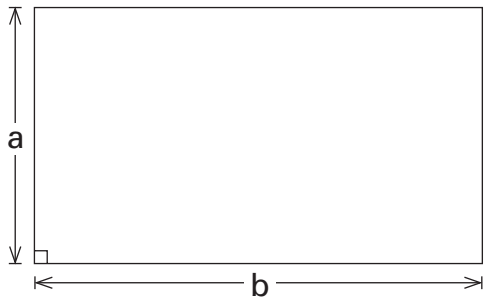




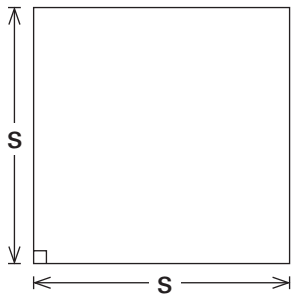
Connector Master B

Focus Master A

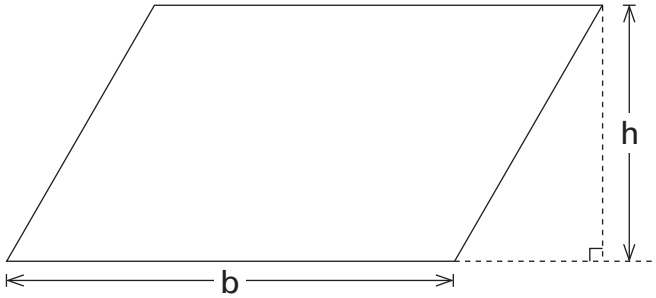
A Rectangle



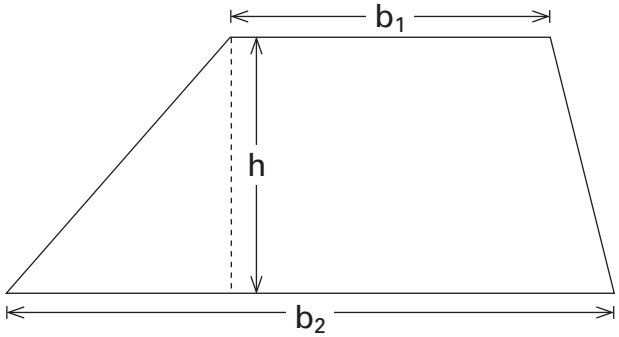
B Square



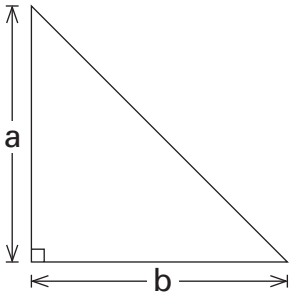
C Parallelogram



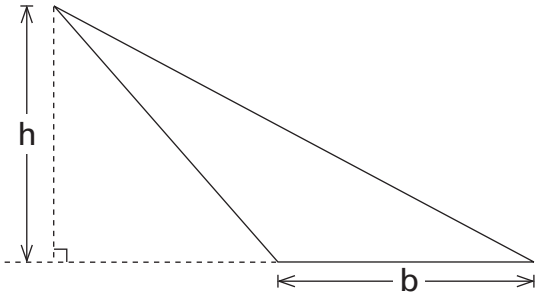
D Trapezoid



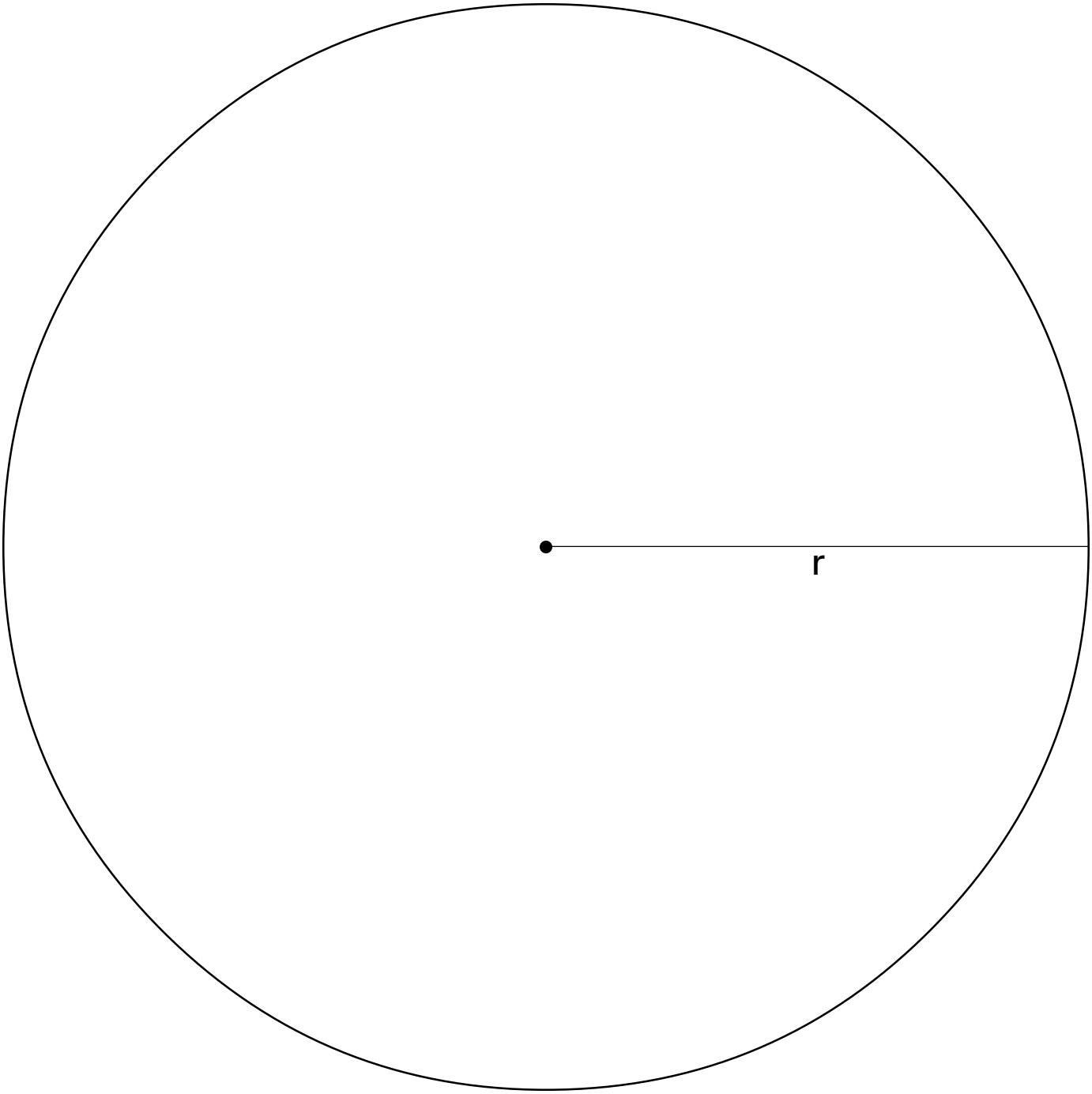
E Right Triangle



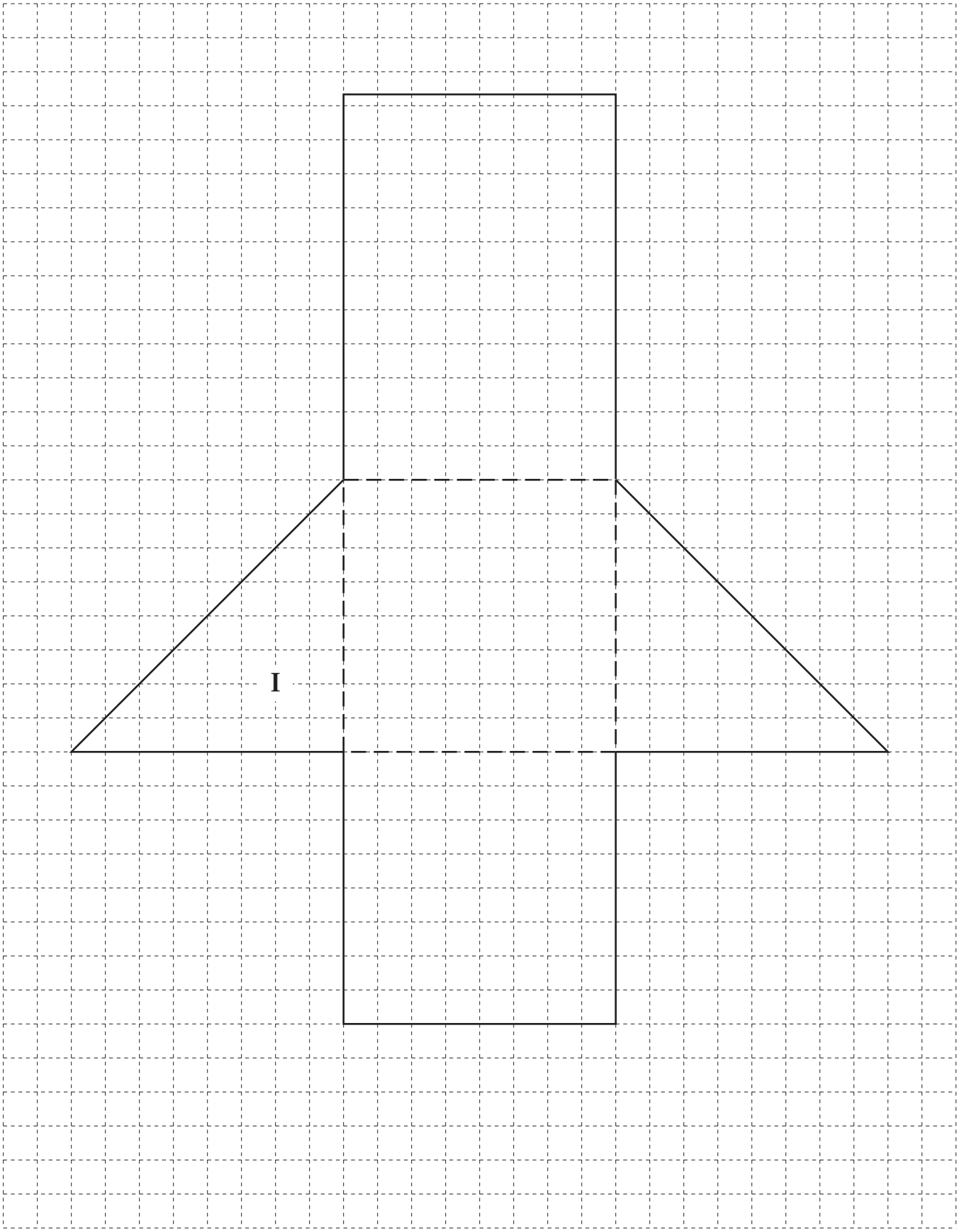
F Nonright Triangle



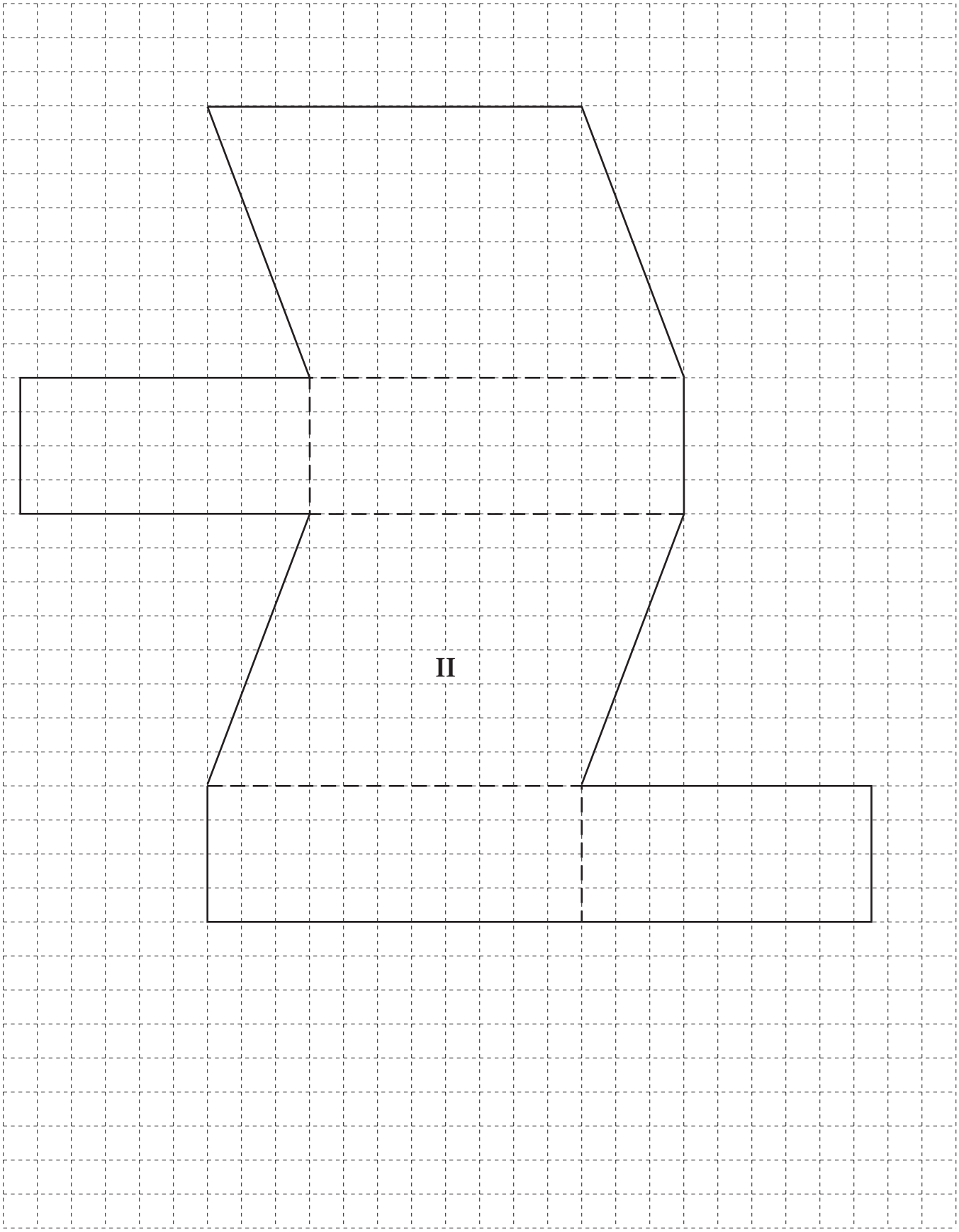
Focus Master B



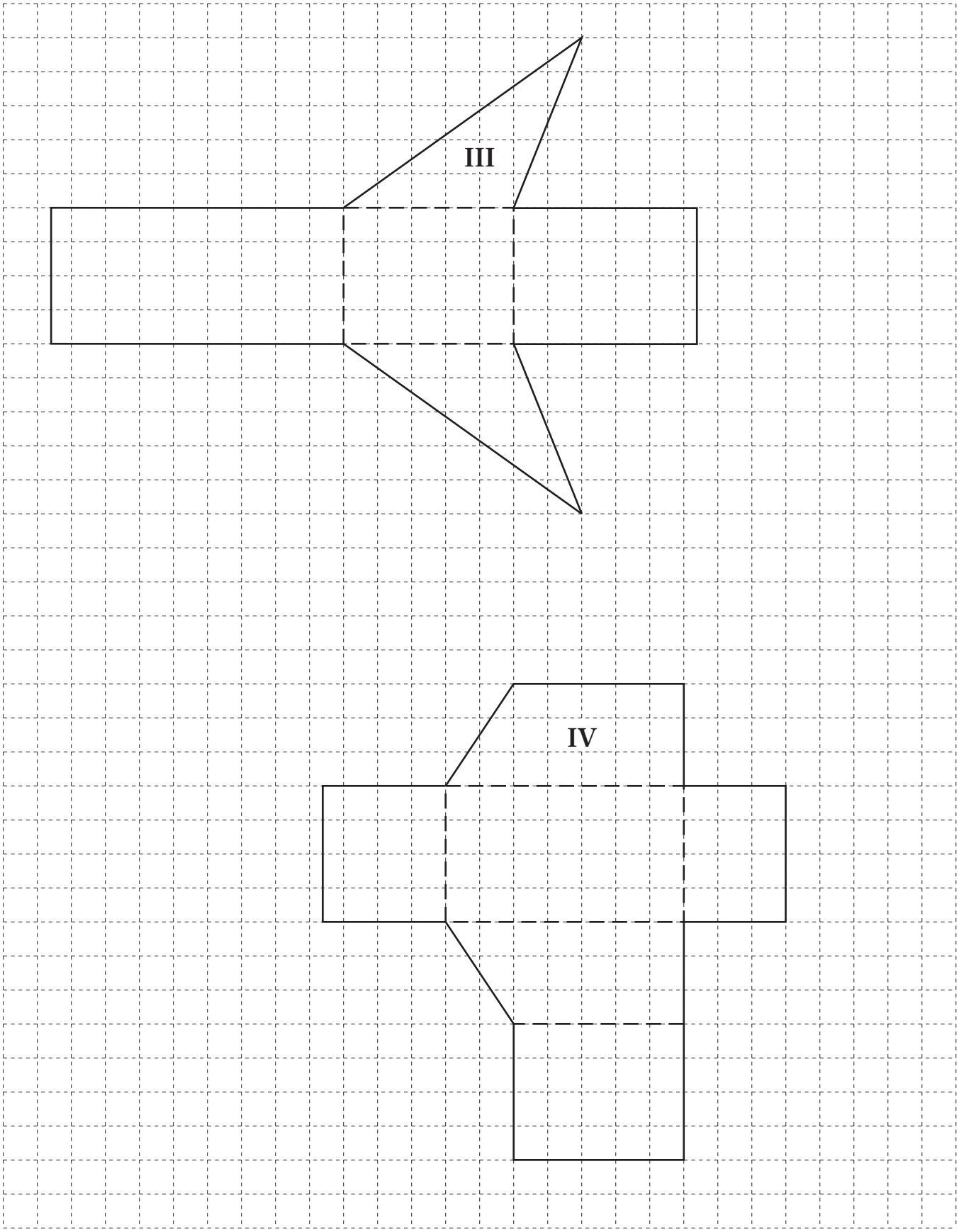
Focus Master C



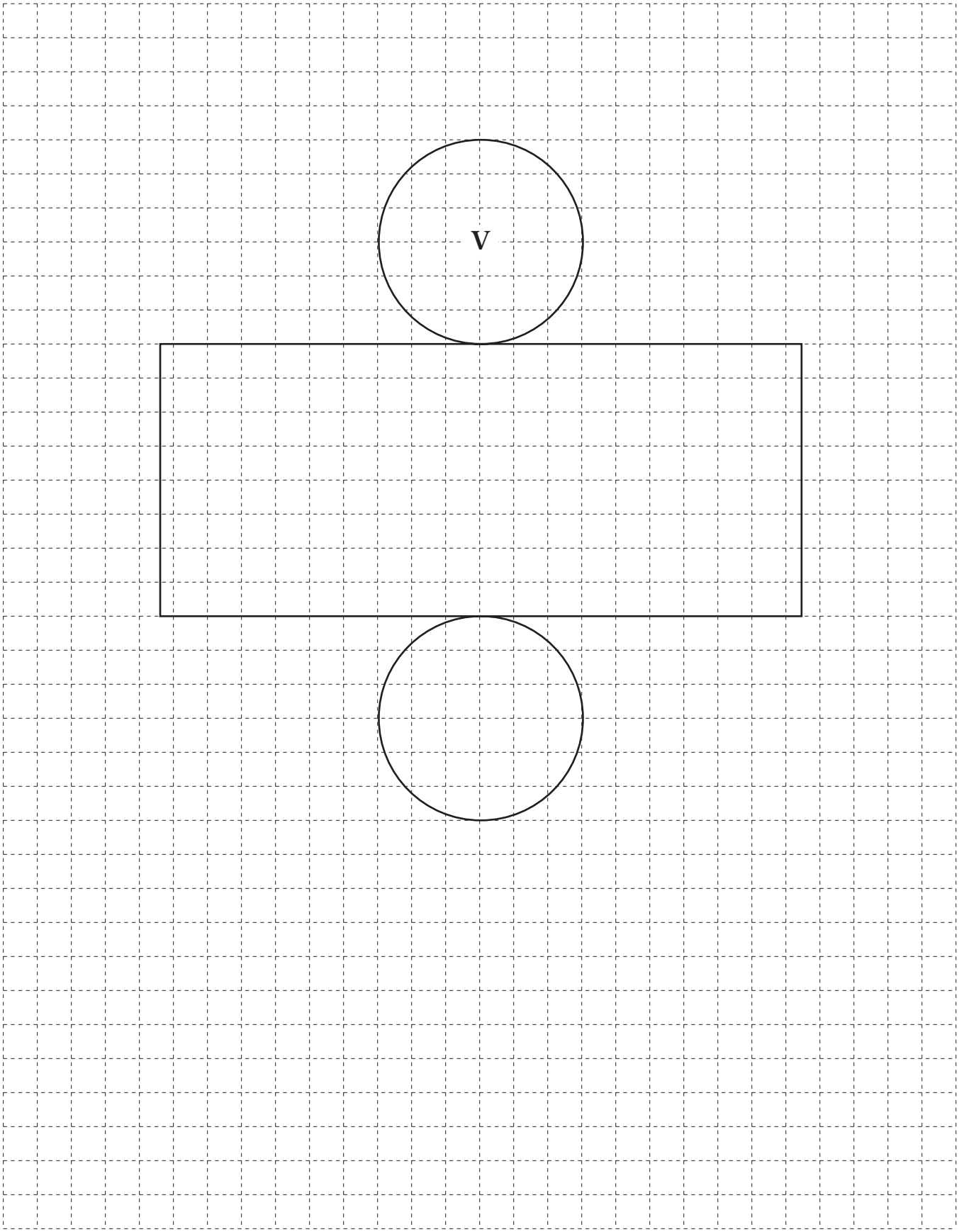
Focus Master C (page 2)



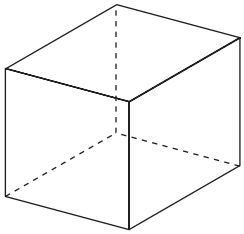
Focus Master C (page 3)



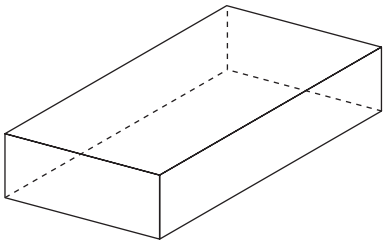
Focus Master C (page 4)



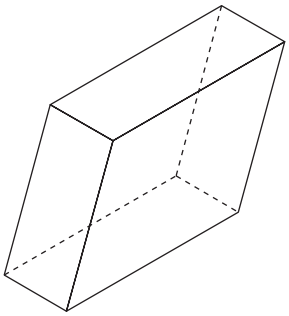
Focus Master D



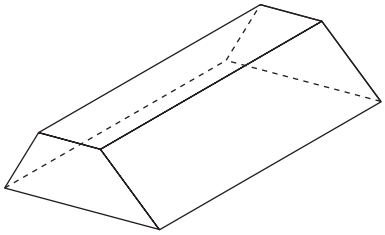
Cube



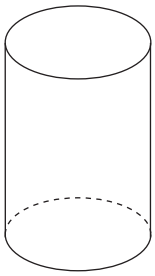
Noncubic Rectangular Prism



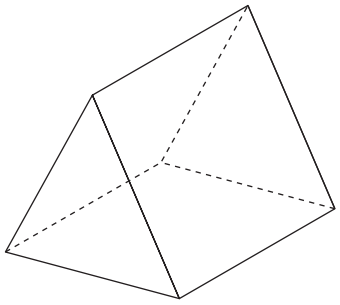
Nonrectangular Parallelepiped



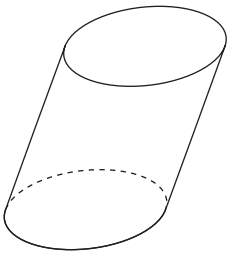
Trapezoidal Right Prism



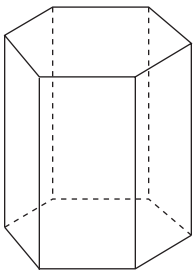
RIght Circular Cylinder



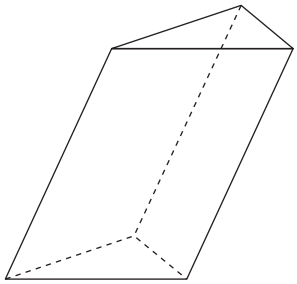
Triangular Prism



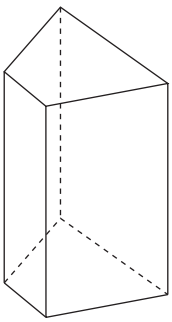
Oblique Circular Cylinder



Hexagonal Right Prism



Oblique Triangular Prism



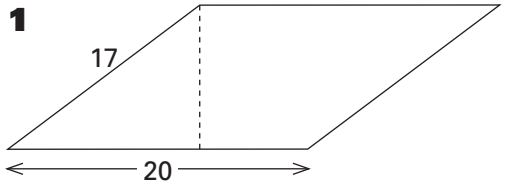
Quadrilateral Right Prism

Focus Student Activity 3.1

NAME _____ **DATE** _____

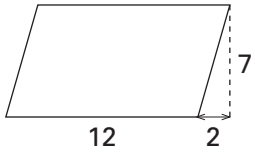
For each of the following polygons, use the given information to help you find the missing information. Mark each diagram or write equations or brief comments that communicate the steps of your thought processes. If it is not possible to find some of the missing information write NP and explain why. Note: diagrams are not drawn to scale; the dotted lines in the diagrams are altitudes; *a* represents area; *p,* perimeter; *h*, height (the length of the altitude); and *x*, *s*, and *b* are lengths that are marked on the diagrams.

1



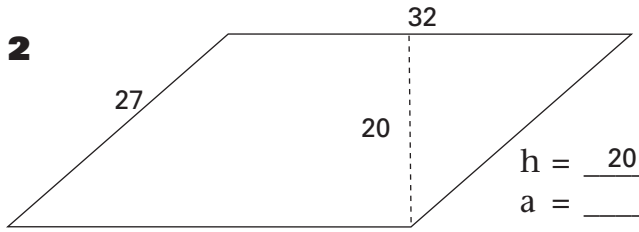
h = _____
a = 300
p = _____

4



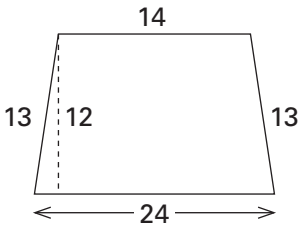
h = 7
a = _____
p = _____

2



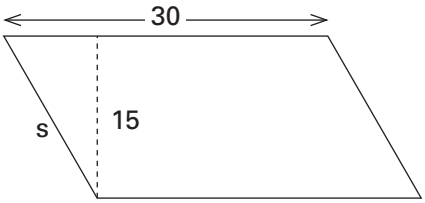
h = 20
a = _____
p = _____

5



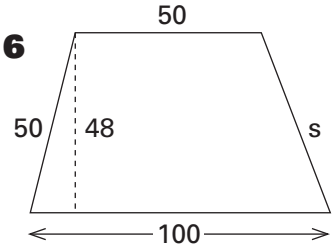
h = 12
a = _____
p = _____

3



h = _____
a = _____
p = 100
s = _____

6

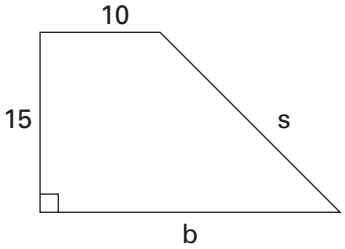


h = _____
a = _____
p = 260
s = _____

(Continued on back.)

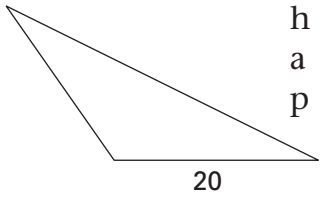
Focus Student Activity 3.1 (cont.)

7



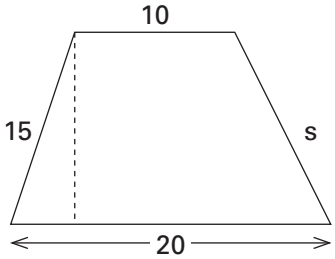
h = _____
a = 300
p = _____
s = _____
b = _____

10



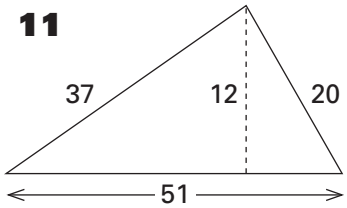
h = _____
a = 150
p = _____

8



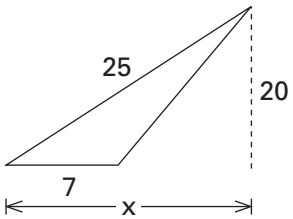
h = _____
a = 204
p = 62
s = _____

11



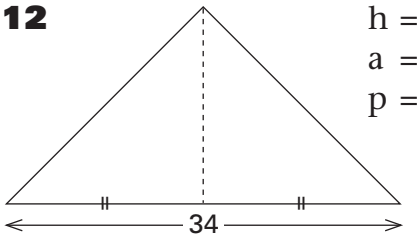
h = 12
a = _____
p = _____

9



h = _____
a = _____
p = _____
x = _____

12



h = _____
a = 216.75
p = _____

Focus Student Activity 3.2

NAME _____ **DATE** _____

Draw a diagram or sketch to solve each of the following problems. Use your calculator to carry out computations, and record all of the calculations that you do. Express each answer to the nearest tenth.

1 A circular lid has a diameter of 7 cm. What is its circumference, to the nearest tenth of a centimeter? Remember to show a sketch, the keys you press to compute the answer, and your answer.

2 A circular fish pond has a circumference of 5.8 meters. What is its diameter?

3 The diameter of one circle is twice as long as the diameter of another circle. How do the circumferences of the circles compare?

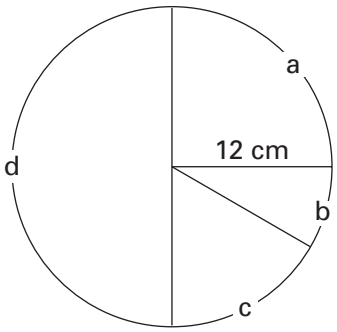
(Continued on back.)

Focus Student Activity 3.2 (cont.)

4 A circular patio has a circumference of 18.7 meters. What is its radius?

5 For a pony ride at the fair, Pete Pony is tied to the end of a 10 foot pole. The other end of the pole is connected to a "hub" that rotates 360°. One day at the fair, Pete completely circled the hub 220 times. How many miles did Pete travel on that day? 5,280 feet = 1 mile.

6 For the circle shown at the right, find a close approximation of the length of each of the indicated arcs (a, b, c, and d). Use math symbols to communicate your method of determining each arc length.



a = _____ Method:

b = _____ Method:

c = _____ Method:

d = _____ Method:

Focus Student Activity 3.3

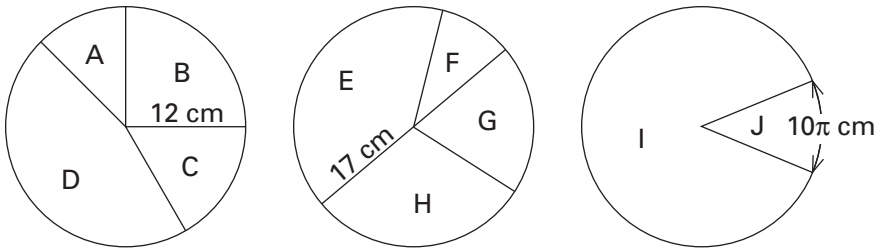
NAME _____ **DATE** _____

Write your responses to problems 1-7 on another sheet of paper. Be sure to put a copy of the problem next to your responses.

1 Write and illustrate two methods that you could use to show a younger student how to approximate the area of a circle with a radius of 7 cm.

2 Explain and sketch a diagram to show how a parallelogram can be used to find the area of a circle with a radius of 17 cm.

3 For each of sectors A-J shown below, without using a protractor, predict the area of the sector and the arc length along the edge of the sector. Record your predictions and reasoning. Then use a protractor to verify your predictions. Record your measurements and calculations.



4 Papa Paulo's Pizza Crust Company sells a crust with a 13 inch diameter for $2.29. They have decided to add a square crust to their product line. Assuming Papa Paulo wants to price all of his crusts at the same rate, what should he charge for a square crust with side length 13 inches?

5 On Carmen's clock, the second hand is 15 cm long and the hour hand is 10 cm long. Explain your methods of solving each of the following problems about her clock. Give answers to the nearest tenth of a centimeter.

a) How far does the tip of the second hand travel in 45 minutes?

b) How far does the tip of the hour hand travel in 45 minutes?

(Continued on back.)

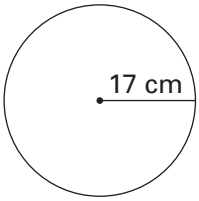
Focus Student Activity 3.3 (cont.)

6 In order to get a sign post that rises 8 feet above the ground to stand at right angles, Zane fastened one end of a wire to the top of the post and the other end to the ground 3 feet from the base of the post. If she needs 6 extra inches of wire on each end for fastening, how long (to the nearest inch) must she cut the wire? Make a diagram and show your reasoning.

7 What is the area (to the nearest square centimeter) of the largest circle that can be cut from a square piece of sheet metal 73 cm on a side? Explain your reasoning.

8 The following figures show the radius, diameter, circumference, or area of a circle. Use the information given to *mentally approximate* the requested measures. Use math symbols to communicate your *mental* calculations.

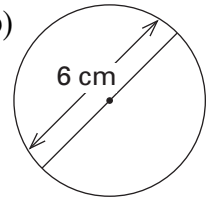
a)



Circumference ≈_____
Mental calculations:

_____

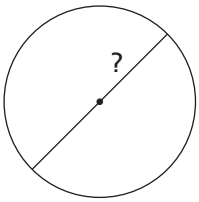
b)



Area ≈_____
Mental calculations:

_____

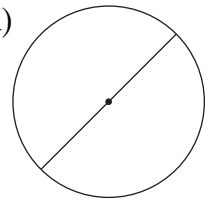
c)



Area = 75 cm^2
Radius ≈_____
Mental calculations:

_____

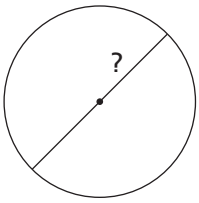
d)



Circumference = 21 cm
Area ≈_____
Mental calculations:

_____

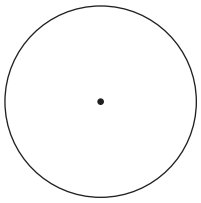
e)



Circumference = 15 cm
Radius ≈_____
Mental calculations:

_____

f)



Area = 48 cm^2
Circumference ≈_____
Mental calculations:

_____

Focus Student Activity 3.4

NAME _____ **DATE** _____

1 Suppose you need to fill ½ of a right cylindrical can with water. Assuming that you have no measuring tools and no other containers, how can you determine when the can is half full?

2 When an object is placed in a container of water it changes the level of the water. The amount of water that is *displaced* is equal to the volume of the object. This fact is useful for finding the volume of irregularly shaped objects. Suppose that:

a) You drop a rock into a 5" × 5" × 12" square prism that is filled with water to a height of 7". The rock displaces the water level by 4". What is the volume of the rock?

b) You drop some coins in a right circular cylinder with radius 7 cm and altitude 15 cm. The coins displace the water level from 6 cm to 13 cm. What is the volume of the coins to the nearest cubic centimeter?

c) Gilly Goldfish and his sisters all jump out of their "filled-to-the-brim" rectangular-prism-shaped fish tank. The water level in this 12" by 14" by 18" tank drops ½" below the 18" height. What is the total volume of Gilly and his sisters?

3 The perimeter of a face of Cube X is $4(\sqrt[3]{125})$ inches. Determine the surface area and volume of Cube X. Show your methods and reasoning.

4 The following questions relate to a rectangular parallelepiped with volume 36 cubic units:

a) What are all the possible whole number dimensions this parallelepiped could have?

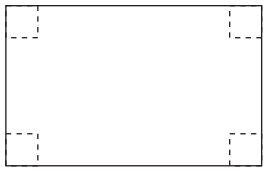
b) What is the minimum possible surface area for this parallelepiped if the dimensions are only whole numbers? if the dimensions are not whole numbers?

(Continued on back.)

Focus Student Activity 3.4 (cont.)

c) What is the maximum possible surface area for this parallelepiped? Explain.

5 A box (without a lid) can be formed by cutting squares out of corners of a rectangular sheet of paper, folding the "flaps" that remain after the cuts, and taping . Assume that you can cut out squares whose sides have only whole number lengths.



a) Determine the dimensions of the squares to cut from the corners of a 40" by 50" sheet in order to to maximize the volume of the box.

b) Repeat a) for a 36" by 36" square sheet.

c) What are the dimensions of the box with minimum surface area that can be formed from a 40" × 50" sheet? the box with maximum surface area?

6 Generalize about the dimensions, surface area, and volume of all pairs of trapezoidal prisms for which:

a) one prism is a reduction of the other by a scale factor of $\frac{1}{2}$.

b) one prism is a reduction of the other by a scale factor of $\frac{2}{3}$.

c) one prism is an enlargement of the other by a factor of $\frac{7}{3}$.

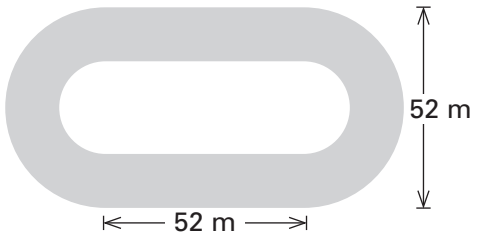
d) one prism is an enlargement of the other by a factor of n.

Follow-up Student Activity 3.5

NAME _____ **DATE** _____

For this activity use $\pi \approx 3.14$. Record your responses on separate paper, and include a statement of each problem next to your work.

1 The outer edge of the racetrack shown below encloses a square region and 2 half circles.



a) If the racetrack is 6 meters wide, what is the area, to the nearest tenth of a square meter, of *only* the racetrack (shown by the shaded part of the diagram)? Explain your reasoning.

b) How much farther (to the nearest tenth of a meter) would a person running along the *outside* edge of the racetrack run than a person running along the *inside* edge? Explain.

2 For a square, a nonsquare rectangle, and a circle:

a) Suppose each encloses a region with area 100 cm^2. Order these figures from least to greatest perimeter. Explain your reasoning.

b) Suppose each has perimeter 100 cm. Compare their areas. Explain.

3 Given: a nonsquare rectangle with dimensions l and w; a square with side length s; a circle with radius r; a rectangular prism with dimension l, w, and h; and a right circular cylinder with radius r and height h. Support your conclusions about the following:

a) Suppose the sides of the square, both dimensions of the rectangle, and the radius of the circle are all doubled; tripled; multiplied by 5; by ¼; by n. What is the effect of each factor on the perimeter and area of each figure?

(Continued on back.)

Follow-up Student Activity (cont.)

b) What is the effect on surface area and volume if the 3 dimensions of the prism, and the radius and height of the cylinder are doubled? all tripled? multiplied by 5? by ¼? by *n*?

4 A guard dog is chained to a clothesline that is 15 feet long and 5 feet high. The dog's chain is 10 feet long and can slide from one end of the clothesline to the other. Determine the area and perimeter of the dog's territory. Show your reasoning and conclusions.

5 The Big Wheeler ferris wheel has a radius of 25 feet, and the Little Wheeler has a radius of 20 feet. Both ferris wheels makes 4 complete revolutions in one ride. A ride on Big Wheeler costs $1.00. What is a fair price for a ride on Little Wheeler? Explain your reasoning.

6 Outline a presentation about a) and b) below. Then give your presentation to an adult. Turn in your outline, an explanation of anything you did differently from your outline, and 3 "I appreciate…" and 2 "I wish…" statements from the adult.

a) The relationships between the circumference, diameter, and radius of a circle.

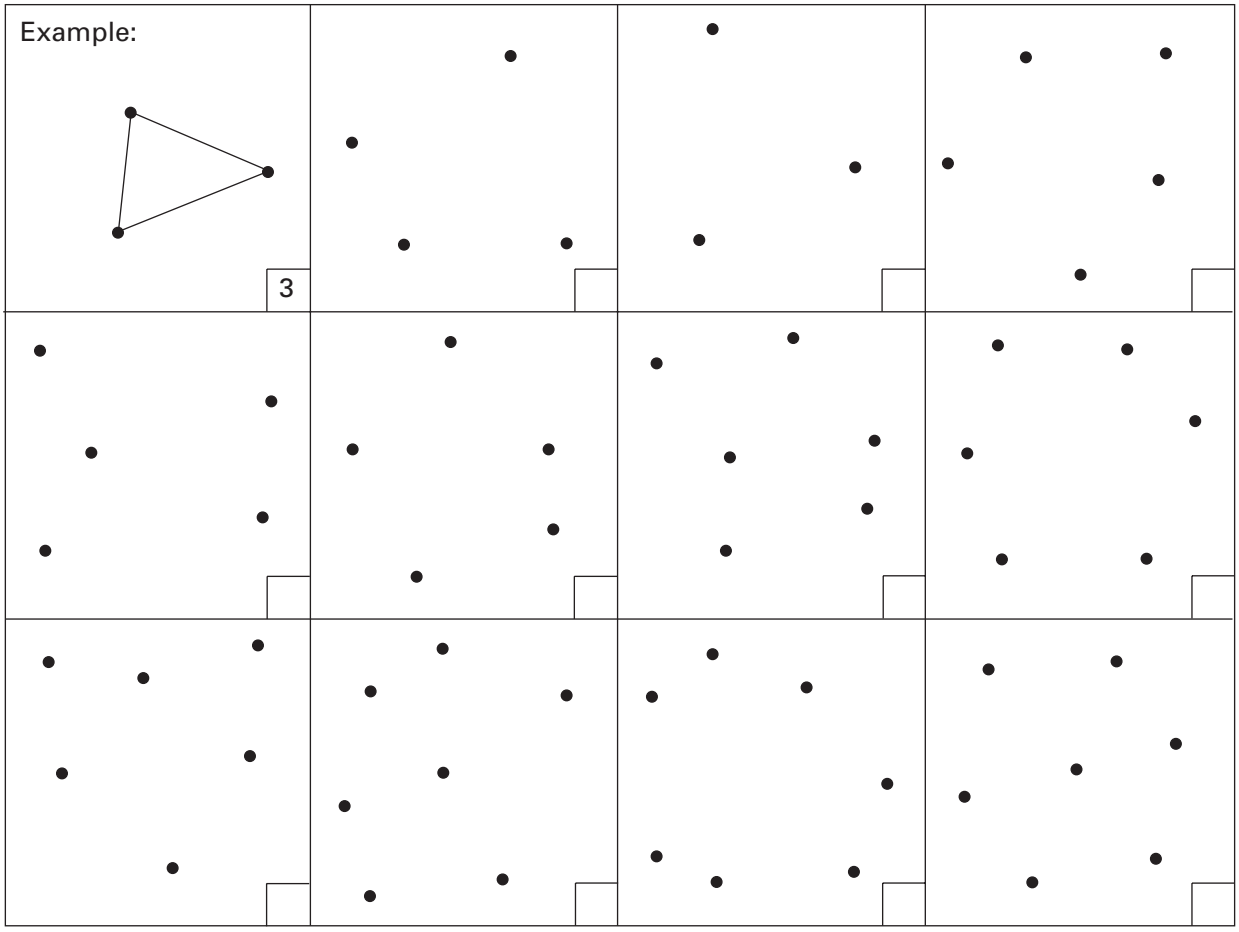
b) Visual "proofs" of formulas for: the area of a square, nonsquare rectangle, parallelogram, trapezoid, right triangle, nonright triangle, and circle; and the volume and surface area of a rectangular prism and a right circular cylinder. For each formula that you prove, give a specific example.

7 Challenge. Suppose the earth is a smooth sphere and a piece of string is wrapped tightly around it at the equator. At the point where the string meets end-to-end with no overlap, 8 feet of string is added. Then, at the equator, the earth is rewrapped by this longer piece so that the distance from the equator to the string is equal all around the earth. Which do you think is more likely to fit between the earth and the lengthened piece of string (predict before problem solving): a) a piece of paper, b) your fist, c) you crawling, or d) you walking? Finally, determine which of a)-d) is actually most likely to fit, and support your conclusion with mathematical reasoning. (Hint: you do not need to know the measure of the earth's radius.)

Connector Student Activity 4.1

NAME _____ **DATE** _____

1 In each square below use a straightedge to draw straight line segments that connect each dot to *all* other dots in the square. Then, in the corner of each square, write how many segments you drew.



2 Tell about any patterns you notice or conjectures you have about counting segments between dots.

(Continued on back.)

Connector Student Activity (cont.)

3 A student in another class said she thought that finding the number of segments was just like solving the "handshake problem." What do you think she meant?

4 Suppose you have to find the number of lines it takes to connect 125 dots. Describe a way to find this number without having to draw the segments and dots. (Just tell your method—you don't need to compute the answer!)

Focus Master A

a) Imagine, build, or sketch a staircase of cubes for the sum of the first 10 positive integers, 1 + 2 + 3 + … + 10.

i) What are the dimensions of a rectangle formed by combining 2 of these staircases?
ii) How many cubes does this rectangle contain?
iii) How many cubes are in each of the 2 staircases?

b) Now imagine or sketch a staircase of cubes representing the sum 1 + 2 + 3 + … + 100.

i) What are the dimensions of a rectangle formed by combining 2 of these staircases?
ii) How many cubes does the rectangle contain?
iii) How many cubes are in each staircase?

c) Imagine or sketch a staircase of cubes representing the sum 1 + 2 + 3 + … + n, where n is a whole number.

i) What are the dimensions of a rectangle formed by combining 2 of these staircases?
ii) How many cubes does the rectangle contain?
iii) How many cubes are in each staircase?

d) Suppose a staircase contains a total of 465 cubes. The first step contains 1 cube, and each step is 1 cube higher than the preceding step.

i) Reason from a sketch to determine how many steps it contains.

e) Challenge. Sketch a staircase of cubes to represent the sum 23 + 24 + 25 + … + 77.

i) Reason from the sketch to find the number of steps in the staircase.
ii) Use your sketch to find the number of cubes in the staircase.

Focus Master B

Part I

a) Use cubes to form and combine matching staircases to compute the sum 2 + 4 + 6 + 8 + 10.

b) Sketch a staircase representing the first 20 consecutive even numbers. Then determine S_{20}, where the symbol S_{20} represents the sum of the first 20 consecutive even numbers. Then find S_{100}, the sum of the first 100 even numbers. Finally, based on your diagram, invent a formula for S_n.

c) Reasoning from relationships you can "see" in a staircase and without counting, determine the number of even numbers from 28 through 104.

d) Find the sum of the even numbers from 28 through 104; from 8 through 818; from 152 through 346.

Part II

a) Use cubes to form and combine matching staircases to compute the sum of 1 + 3 + 5 + 7 + 9.

b) Sketch and label a staircase representing S_n, the sum of the first *n* consecutive odd numbers. By reasoning from your sketch, invent a formula for S_n, the sum of the first *n* odd numbers.

c) Reasoning from relationships you can "see" in a staircase and without counting, determine the number of odd numbers from 79 through 245.

d) Use staircases to find the sum of the odd numbers from 79 to 245; from 31 through 331.

Focus Master C

Any sequence of numbers that can be represented by a staircase whose steps increase by a constant amount is called an *arithmetic sequence.* That is, in an arithmetic sequence there is a constant difference between any term and the term that follows it. The constant is called the *common difference*. (Note: each number in a sequence of numbers is called a *term* of the sequence.)

For example, the sequence of even numbers and the sequence of odd numbers are each an arithmetic sequence whose common difference is 2.

Explore the following problems and display your responses on a poster. Next to each response write the corresponding problem.

a) Create 2 different arithmetic sequences that each contain 7 terms and with common difference 3. Show a staircase model of each sequence. Note: you don't need to show every cube in your diagram, but label the size of each step.

b) Create an arithmetic sequence that contains 10 terms. The 1st term is 18 and the 2nd term is 23. Show how to use staircase methods to find the sum of the numbers in this sequence.

c) Do you think the staircase method sometimes/always/never works for finding the sum of the first *n* terms in *any* arithmetic sequence? Use diagrams and/or formulas to support your conclusions. Be sure to label your diagrams carefully and tell what any variables you use represent.

d) Following are descriptions of 3 arithmetic sequences. Sketch a staircase model (you don't have to show every step) of each sequence and label the values of the first and last term, the common difference, and the number of terms for each sequence.

(Continued on back.)

Focus Master C (cont.)

Next to each model explain how you determine *d*, the common difference.

i) The first term of this sequence is 28 and the last term is 208. There are 61 terms.
ii) There are 37 terms in this sequence, the first term is 8, and the last term is 332.
iii) Challenge. This sequence has *n* terms, the first term is a_1, and the last term is a_n.

e) Write a formula for a_n, the value of the *n*th term of the arithmetic sequence: 11, 18, 25, 32, 39, … (Examples: $a_1 = 11$ and $a_2 = 18$.)

f) Challenge. Write a formula (using only the variables *l*, *n*, *a*, and *d*) for *l*, the last term of any arithmetic sequence with first term *a*, common difference *d*, and *n* terms. Use a diagram and a brief explanation to show how and why your formula works. Label your diagram to show what each variable represents.

g) Repeat Problem d) for these 3 sequences, but this time explain how you determined *n*, the number of terms in each sequence.

i) The first term of this sequence is 32, the common difference is 2, and the last term is 160.
ii) The first term is 7, the common difference is 10, and the last term is 97.
iii) Challenge. The first term is a_1, the common difference is *d*, and the last term is a_n.

Focus Student Activity 4.2

Peer Feedback Sheet

REVIEWEES _____ **REVIEWERS** _____

We appreciate…

We question…

We wish…

We learned…

Follow-up Student Activity 4.3

NAME _____ **DATE** _____

Record all of your responses on separate paper. Include a statement of each problem next to your work about the problem.

1 Describe your understanding of the meaning of an *arithmetic sequence*. Then give an example of a sequence that is an arithmetic sequence and one that is not.

2 Show how to use staircases to find the sum of the first 100 counting numbers.

3 For each of series a) and b) below do the following:

i) Sketch and label a staircase that represents the series.
ii) Explain how to reason from your sketch to determine the number of steps in the staircase.
iii) Show how to use staircase methods to add the numbers in the series.

a) $2 + 4 + 6 + \ldots + 164$

b) $7 + 13 + 19 + 25 + \ldots + 181 + 187$

4 Draw a diagram to show how the staircase method can be used to find the sum of the first n terms of any arithmetic sequence. Label your diagram to show what any values or variables you use represent. Write a brief step-by-step explanation of the staircase method for adding the terms in an arithmetic sequence.

5 Draw diagrams to show how staircase methods can be used to solve each of the following problems. Label your diagrams in detail, and make sure that each step of your solution process is clearly communicated. Show all calculations that you do, and explain what they represent. Then tell how you verified your solution.

a) Suppose that your class takes a field trip to the fair, and every student goes on exactly one ride with each of the other students in the class. If only 2 students can ride together and each ride costs $1.50, how much will the class spend?

(Continued on back.)

Follow-up Student Activity (cont.)

b) Holly asked for the following allowance: 1¢ on the first day, 3¢ on the second day, 5¢ on the 3rd day, 7¢ on the 4th day, 9¢ on the 5th day, and so on, continuing for the 365 days in the year. How much would Holly earn on the 365th day? Altogether how much would she earn for the year?

6 For each of the following arithmetic series, create an interesting problem about a situation outside of school, so that the solution to the problem would involve finding the sum of the series. Then record the answer to your problem and show your solution methods.

a) 14 + 15 + 16 + … + 176

b) 49 + 51 + 53 + … + 97

7 Explain or draw diagrams to show your methods of determining each of the following Mystery Arithmetic Sequences. List the first 5 and last 5 terms in each arithmetic sequence.

a) The first term of Mystery Arithmetic Sequence A is 1 and the last term is 241. There are 61 terms in the sequence.

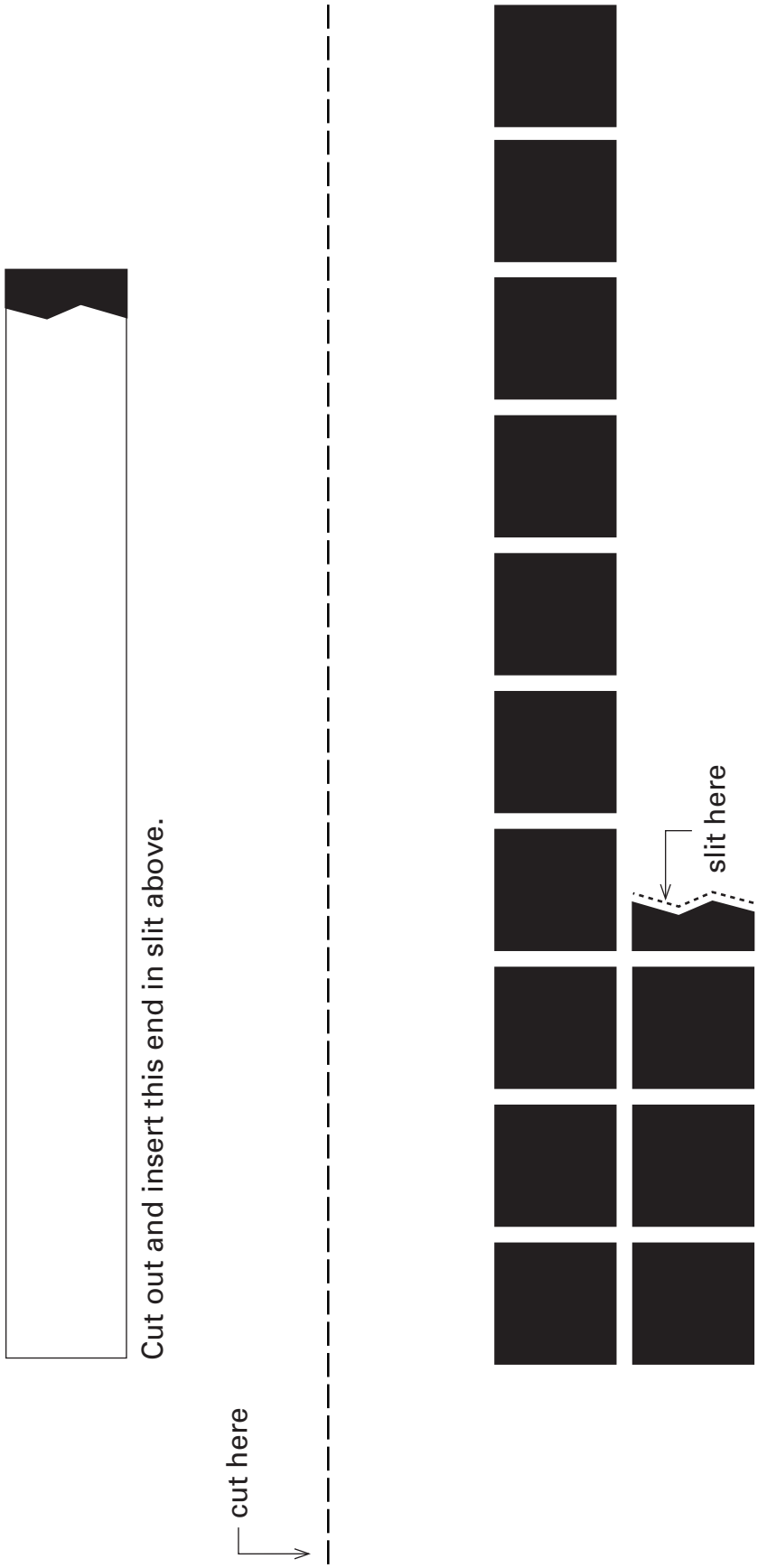
b) The common difference in Mystery Arithmetic Sequence B is 8. There are 17 terms and the last term is 157.

c) The sum of the 25th through the 29th terms of Mystery Arithmetic Sequence C is 165. The first term of the sequence is 7 and there are 40 terms in the sequence.

8 Challenge: Several high school mathematics texts list the following formulas regarding arithmetic series, where a_n = the nth term of the sequence, d = the common difference, and S_n = the sum of the first n terms of the series. For each of the following formulas: i) sketch and label a diagram of staircases to show what you think the formula means and what you think each part of the formula represents; ii) write a sentence or two to explain your thinking about what the formula means.

a) $a_n = a_1 + (n - 1)d$ b) $a_n = a_{n-1} + d$ c) $S_n = \frac{n(a_1 + a_n)}{2}$

Connector Master A

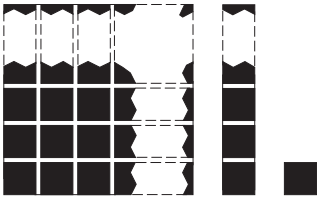


Connector Master B

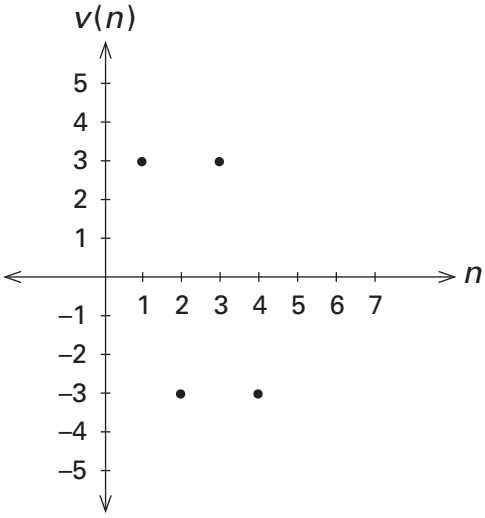
Following are representations of 5 different sequences.

1. 

2. $v(n) = -13 + 5(n - 1)$

3. 

4. 3, –3, –9, –15, –21, –27 …

5. 

Connector Master B (cont.)

Part I

For each of Sequences 1-5 represented on the previous page, do the following (if the information is already given, write "given" and then skip that part):

a) Form and then sketch counting piece arrangements for the first 6 terms of the sequence.

b) Form and then sketch an Algebra Piece representation of the *n*th arrangement of the sequence.

c) Write a formula for $v(n)$, the value of the *n*th arrangement of the sequence. "Loop" a diagram to show how your formula works.

d) If a_n represents the *n*th term of the sequence, record the values of the following: a_1, a_2, a_3, a_4, a_5, a_6, a_{20}, a_{73}, and a_{200}.

e) Make a coordinate graph, plotting and labeling the first 5 or more terms in the sequence.

Part II

1. Which of Sequences 1-5 are arithmetic sequences? Explain your reasoning.

2. *Challenge*: For any of Sequences 1-5 that are arithmetic sequences, show how to use the method of combining two matching staircases to find S_{50}, the sum of the first 50 terms of those sequences.

Connector Master C

1. Use Algebra Pieces to do the following:

a) Form a collection that is ½ of a collection of 2 *n*-strips.

b) Now, add 4 units to the collection formed in a).

2. Write an algebraic expression that describes your actions in Problem 1.

3. One way to think about algebraic expressions is as representations of actions with Algebra Pieces. For the expression $\frac{4+2n}{2}$ write the steps of Algebra Piece actions, in order, that the expression could represent. At the end of each step, sketch the resulting collection of Algebra Pieces.

4. In the final collection that you formed for Problem 3, can you "see" an algebraic expression that is equivalent to the original expression? If so, record that expression.

5. Repeat Problems 3 and 4 for the following expressions:

a) $\frac{4n-2n}{2}$

b) $\frac{4-2n}{2}$

c) $4^2 - 2n^2$

d) $4^2 - (2n)^2$

e) $(4 - 2n)^2$

f) $4 + 4n - 8 \div 4$

g) $\frac{4+4n-8}{4}$

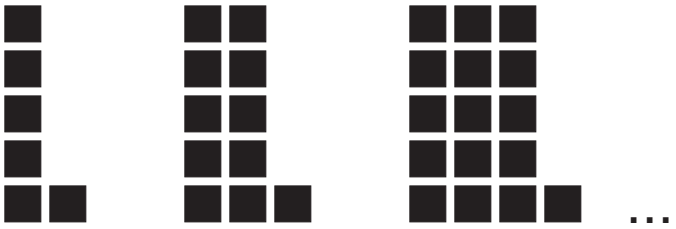
h) $6 - 12n \div 3 + 3$

i) $6 - 12n \div (3 + 3)$

6. Record any general observations, AHA!s, conjectures, or important ideas you noticed.

Connector Master D

A



B

C

D

E $v(n) = -4n + 7$

F $v(n) = 2n + {}^{-}17$

Focus Master A

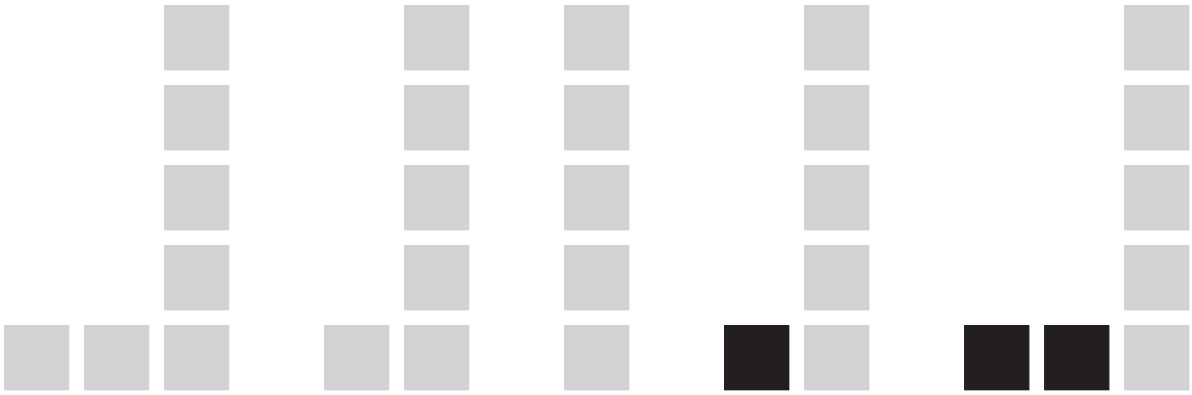
DIAGRAM A

DIAGRAM B

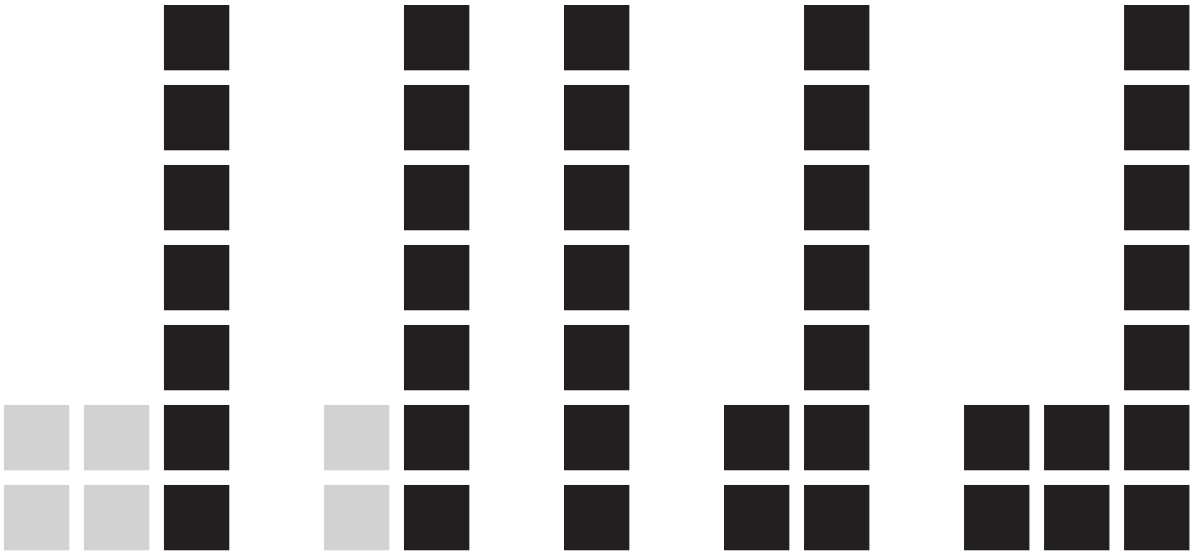
Focus Master B

Arrangement number, *n*: –2 –1 0 1 2

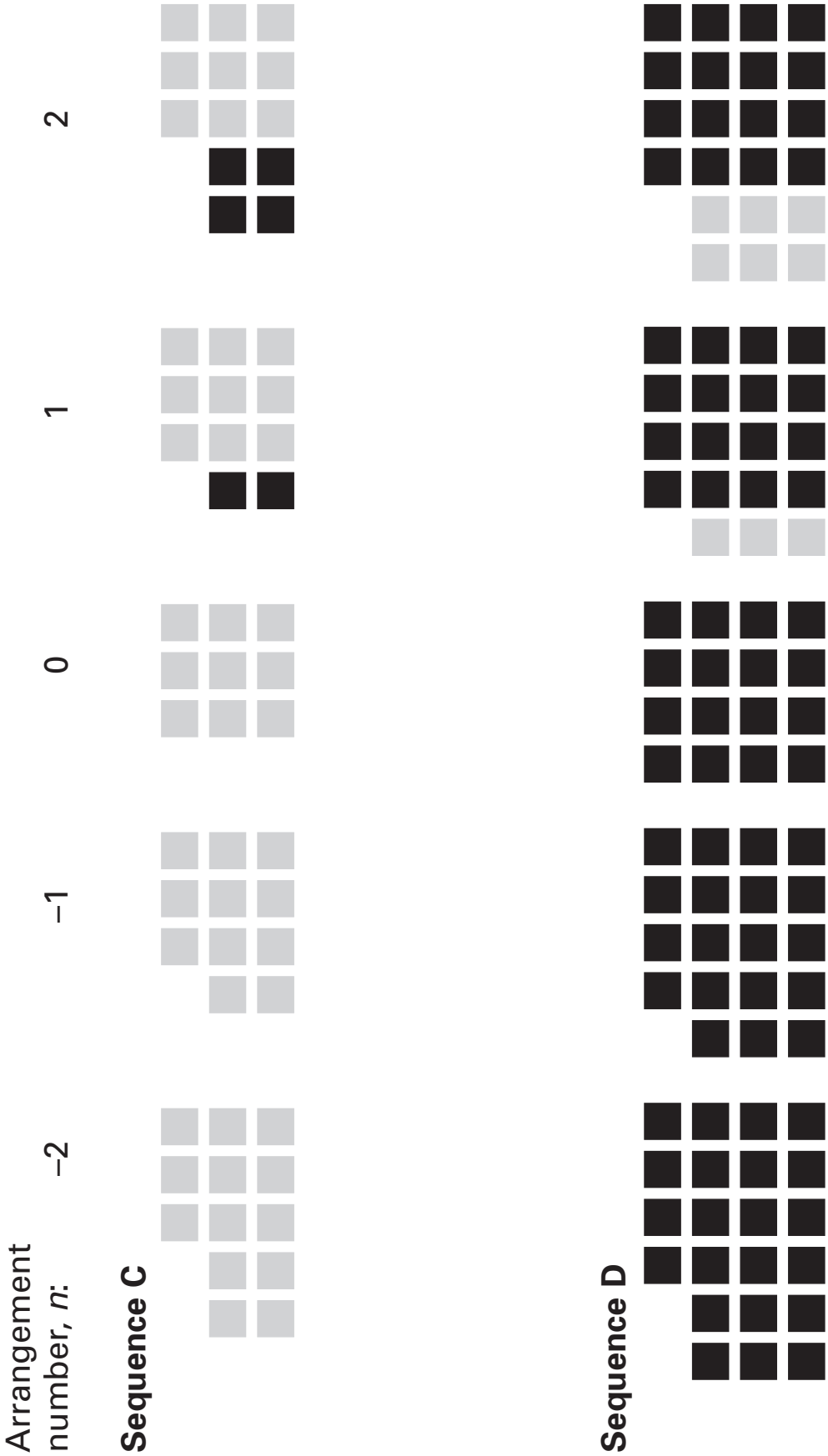
Sequence A



Sequence B



Focus Master C



Focus Master D

1. When you see the "–" symbol, what do you think of?

2. When you are done solving an equation, how do you test your answer to make sure that it is correct?

3. What are common things that you do when solving an equation?

4. Define n and $-n$.

5. Two important ideas in solving an equation are addition and subtraction of negative and positive pieces. Generalize and explain these processes and give an example of each.

6. The n-frame is an important tool to understand. Explain what they are used for, how they are used, and why.

7. What is an algebra equation you can solve in your mind, by simply imagining the pieces?

8. Explain when, if ever, $-n$ is less than, more than, and/or equal to zero. Give examples and go into detail.

9. Why are there no n^2-frames? Build a convincing argument and explain your thinking on this question.

10. Tell what is meant by an "extended" sequence of counting piece arrangements.

(Continued on back.)

Focus Master D (cont.)

11. Figure out what is wrong with the way Jane solved the following equation. Then solve it the way you think would use the correct procedures and least amount of steps.

Jane's Steps:

1. $2n + 3 + 3\,(n - 4) = 8n + 3$
2. $5n + {}^{-}9 = 8n + 3$
3. $5n + {}^{-}9 = 8n + 3 - 5n$
4. $5n + {}^{-}9 - 3n = 3n + 3 - 3n$
5. $2n + {}^{-}9 = 3$
6. $2n + {}^{-}9 + 9 = 3 + 9$
7. ${}^{2n}/_{2} = {}^{12}/_{2}$
8. $n = 6$

12. Explain how the equation $-3n + 2 = 4n - 12$ relates to sequences of counting piece arrangements.

13. Explain how edge pieces and/or edge frames are used to represent products of whole numbers, integers, and algebraic expressions.

14. Can you think of other thoughtful questions about the ideas in this lesson? Try to think of ones that require *understanding* of important ideas.

Focus Student Activity 5.1

NAME _____ **DATE** _____

Luise, Bob, and Patty used Algebra Pieces to solve the equation $3(n - 4) = 5(n + 4)$. Then they wrote the following to represent each step of their thoughts and actions with the Algebra Pieces. Next to each line of the three methods, write a brief explanation of what thoughts or actions you think are represented by the algebra statement on that line.

Luise's Method

$3(n - 4) = 5(n + 4)$
$3n - 12 = 5n + 20$
$3n - 12 - 3n = 5n + 20 - 3n$
$-12 = 2n + 20$
$-12 + {}^{-}20 = 2n + 20 + {}^{-}20$
$-32 = 2n$
$-16 = n$

Bob's Method

$3(n - 4) = 5(n + 4)$
$3n - 12 = 5n + 20$
$3n - 12 + 5n + {}^{-}5n = 5n + 20$
$3n - 12 - 5n = 20$
${}^{-}2n - 12 = 20$
${}^{-}2n - 12 = 20 + 12 + {}^{-}12$
${}^{-}2n = 32$
${}^{-}2n \div 2 = 32 \div 2$
${}^{-}n = 16$
$-({}^{-}n) = -16$
$n = -16$

Patty's Method

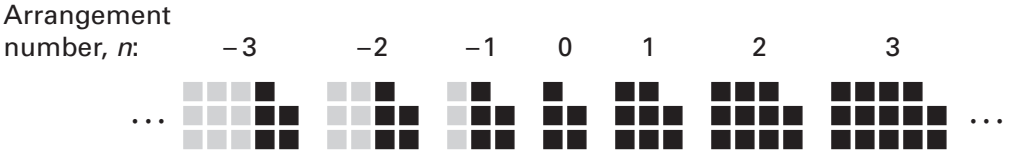
$3(n - 4) = 5(n + 4)$
$3n - 12 = 5n + 20$
$3n - 12 + {}^{-}3n + {}^{-}20 = 5n + 20 + {}^{-}3n + {}^{-}20$
$-32 = 2n$
$\frac{1}{2}(-32) = \frac{1}{2}(2n)$
$-16 = n$

Follow-up Student Activity 5.2

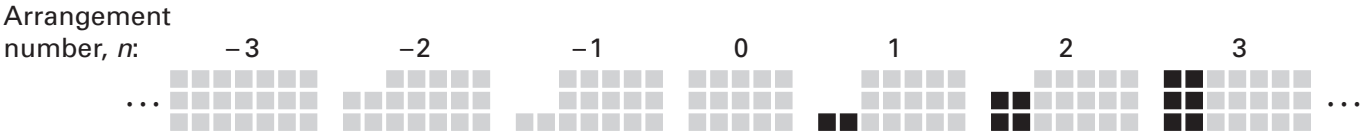
NAME _____ **DATE** _____

Complete the problems on this Follow-up on separate paper.

Sequence A



Sequence B



1 a) Sketch the Algebra Piece representations of the nth arrangement of Sequence A and the nth arrangement of Sequence B.

b) Draw diagrams to show each step of Algebra Piece procedures for finding the value of n for which Sequences A and B have the same net value. Write brief comments, as needed, to help communicate your methods.

c) Tell what equation you solved in b).

2 Sketch the –3rd through 3rd arrangements of a sequence of counting piece arrangements with net value $v(n) = 3n + 4$. Then show how to use Algebra Pieces to determine the value of n for which $v(n) = 190$.

3 Draw diagrams to show how to use Algebra Pieces to solve, if possible, the following equations. Write brief comments to explain what you do in each step. If there is no solution, explain why. If there is more than one solution, explain how many and why.

a) $7n + 2 = 8n - 4$

b) $4n^2 + 3n - 5 = (2n + 1)^2 + 8$

c) $3(2n - 3) = 9n + 6$

d) $-16 + 24n = 272$

e) $3 + n = -3 + n$

f) $7(n + 2) = 7n + 14$

(Continued on back.)

Follow-up Student Activity (cont.)

4 For each of the following conditions, write an equation (not already on this Follow-up) which meets the given conditions. Then make a diagram or write a brief explanation to show why your equation satisfies the conditions.

a) This equation has exactly one solution and that solution is negative.

b) This equation has no solutions.

c) This equation has an infinite number of solutions.

5 Use Algebra Pieces to solve the equation $8n + 36 = 4(n + 1)$. Then:

a) using algebraic symbols only, record each step of your thought processes and Algebra Piece methods;

b) write a brief explanation of the thoughts and actions represented by each step you wrote in a).

6 Solve the equation $7(n + 3) = 5(n - 3) + 6$ using whatever methods you choose. Explain or illustrate each step of your thought processes and actions. Then tell how you can be sure that your solution is correct.

7 Give one or more different equations (not already on this Follow-up) for each of the following. Show or explain your methods of solving each equation.

a) an algebra equation you think is most convenient to solve by simply imagining the Algebra Pieces in your mind's eye (i.e., without building or sketching models or writing equations);

b) an algebra equation you think is most convenient to solve by using algebraic symbols to represent the Algebra Pieces;

c) an algebraic equation that you can solve and you think is difficult.

8 Write several "tips" you recommend that others keep in mind when solving equations or representing expressions with Algebra Pieces.

Connector Master A

1. The county plans to evenly distribute 7 signs along the 5 miles of highway construction.

2. Tia and 4 friends are going to the movies. They have 3 candy bars to share equally.

3. The 4 members of the Wilkinson family plan to share a pizza equally.

4. The 32 members of Ms. Callahan's class plan to share 8 pizzas equally.

5. On a 10 kilometer relay, the 3 members of the Run-abouts each ran an equal distance.

6. The 7 members of the Greenspace Garden Project each were allocated an equal portion of the garden.

7. The Bergmans own a rectangular strip of land that runs 7 miles along the freeway. The area of the strip is 5 square miles. The Bergmans need to replace the fencing around the strip.

Focus Master A

Focus Master B

a) Given a length of $3/5$, here is a method of forming a length of 3, and this method relies on thinking about $3/5$ as $3/5$ of 1 unit (i.e., using the part-to-whole concept of $3/5$):

b) Given a length of $3/5$, here is a method of forming a length of 3, and this method relies on thinking about $3/5$ as $1/5$ of 3 or $3 \div 5$ (i.e., according to the division concept of $3/5$):

c) Given a length a/b, where a and b are positive integers, following is a set of instructions for forming the length a, and this method relies on the part-to-whole concept of a fraction:

d) Given a length a/b, where a and b are positive integers, following is a set of instructions for forming the length a, and this method relies on the division concept of a fraction:

e) Given a length a/b, where a and b are positive integers, here is a set of instructions for constructing the length 1 unit, and this method does not require subdividing the length a/b into a equal parts.

Focus Master C

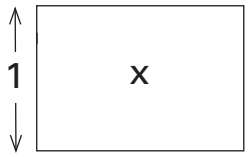
For each of the following diagrams:

i) Label dimensions and areas not shown in the diagrams. Then write several equations that represent mathematical relationships in the diagram. Be sure to include some equations that involve fractions.

ii) For each equation in i), give 2 different examples involving specific numbers in place of the variables. Relate each example to the diagram. If there are specific values that are not possible for some variables, list those values and tell why they are not possible.

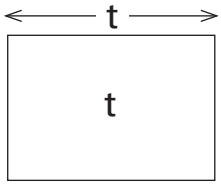
Diagrams are not necessarily to scale.

a)

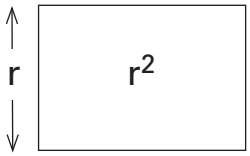


b)

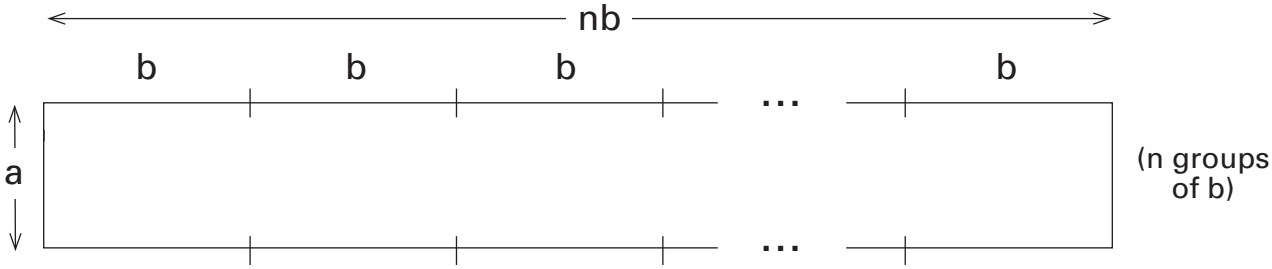
c)



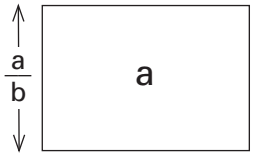
d)



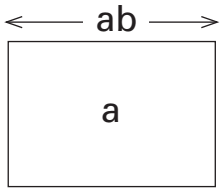
e)



f)



g)



Focus Student Activity 6.1

NAME _____ **DATE** _____

1 Use the parallel line sheet to divide each segment into the indicated number of parts.

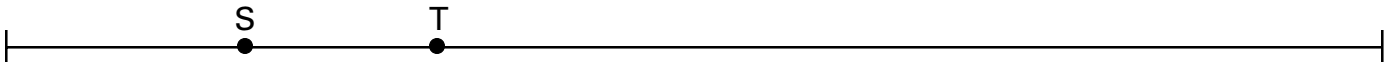
(5 parts)

(3 parts)

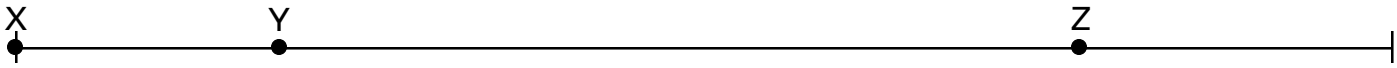
(8 parts)

(7 parts)

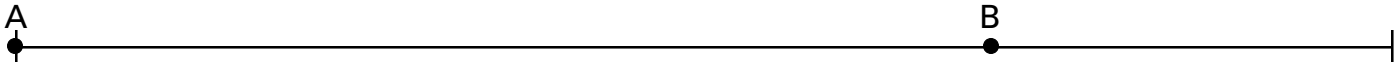
2 Locate points to the right of T and to the left of S so that the distance between adjacent points is the same as ST.



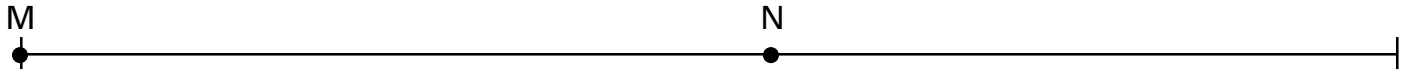
3 If the distance from X to Y is 1 unit, what is the distance from X to Z?



4 If the distance from A to B is 7 units, locate a point P which is 5 units from A.



5 If MN is 3 units, find point Q so that MQ is 5 units.



Focus Student Activity 6.2

NAME _____ **DATE** _____

1 Use the parallel line sheet to divide each segment into the indicated number of parts. Then write a fraction name for each part.

Length of One Part

3 units (4 parts) a. ☐

4 units (3 parts) b. ☐

5 units (2 parts) c. ☐

6 units (7 parts) d. ☐

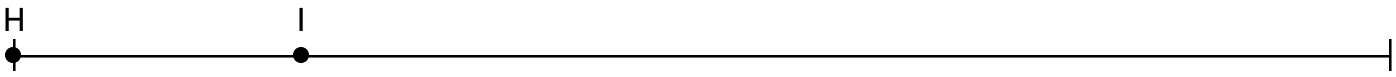
2 Use the parallel lines to locate the indicated fraction on the given number line.

3 units Locate $\frac{3}{5}$.

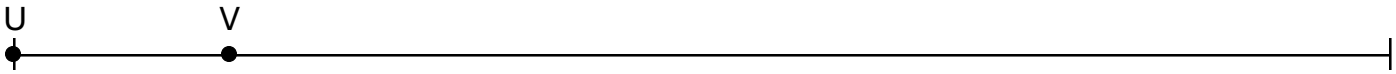
5 units Locate $\frac{5}{2}$.

7 units Locate $\frac{7}{10}$.

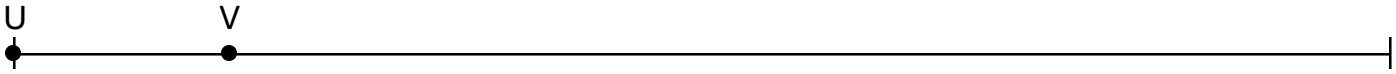
3 HI is $\frac{1}{4}$ of a unit. Find point J so that HJ is 1 unit.



4 UV is $\frac{3}{5}$ of a unit. Find point M so that UM is 3 units.



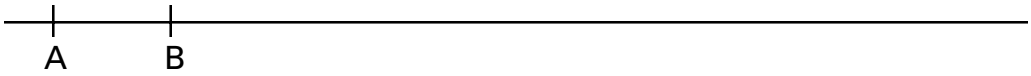
5 UV is $\frac{3}{5}$ units. Find point W so that UW is 1 unit.



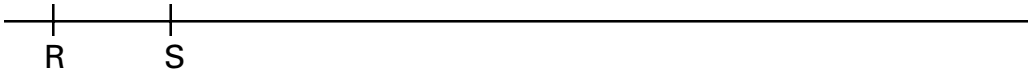
Focus Student Activity 6.3

NAME _____ **DATE** _____

1 If the length of $\overline{AB}$ is $1/7$ of a unit, find and label point C so that the length of $\overline{AC}$ is 1 unit.

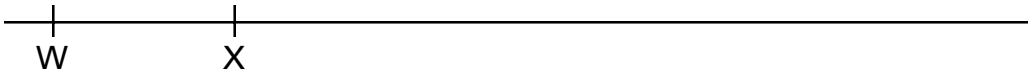


2 If the length of $\overline{RS}$ is $3/5$ of a unit, find and label point T so that the length of $\overline{RT}$ is 3 units.

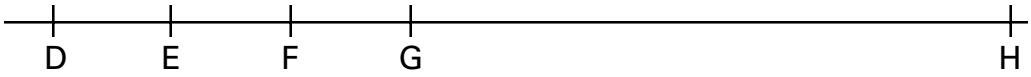


3 Find and label point U on the line in Problem 2 so that $\overline{RU}$ is 1 unit in length.

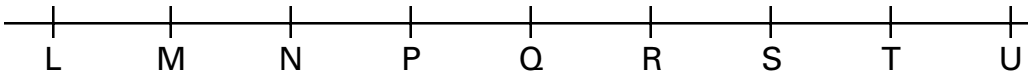
4 If the length of $\overline{WX}$ is 1 unit, find and label point Y so that the length of $\overline{WY}$ is $5/3$ units.



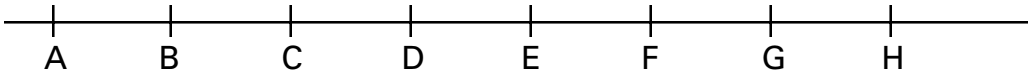
5 If $\overline{DH}$ is 9 units in length, how long is $\overline{DG}$? $\overline{EG}$? $\overline{FG}$?



6 The length of $\overline{PQ}$ is $3/8$ of a unit. How long is $\overline{LU}$? $\overline{LQ}$? $\overline{PR}$?



7 The length of $\overline{AH}$ is 3 units. How long is $\overline{AB}$? $\overline{DE}$? $\overline{AC}$? $\overline{DF}$?



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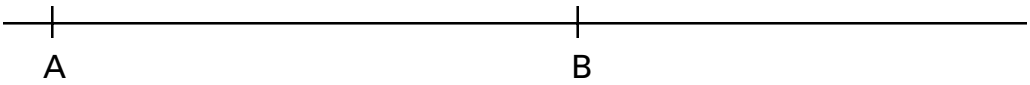
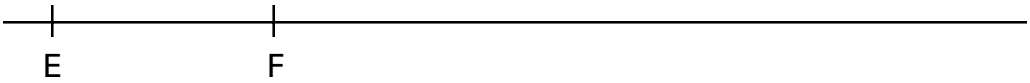
Focus Student Activity 6.3 (cont.)

8 If the length of $\overline{AB}$ is 3 units, find and label point D so that the length of $\overline{AD}$ is 5 units.

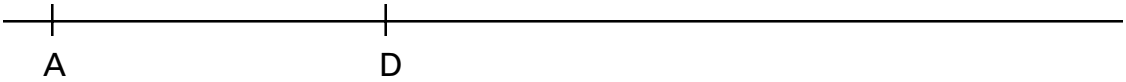


9 Locate point C on the line above so that the length of $\overline{AC}$ is $5/2$ units.

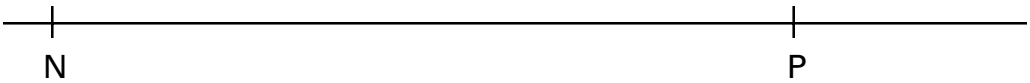
10 The length of $\overline{EF}$ is $1/4$ of a unit. Find and label point G so that the length of $\overline{EG}$ is 1 unit.



11 If the length of $\overline{AD}$ is $4/3$ units, locate and label point E so that $\overline{AE}$ is 4 units in length.



12 If the length of $\overline{NP}$ is $5/4$ units, find and label point Q so that $\overline{NQ}$ is 1 unit in length.



13 Locate and label point R on the line above so that the length of $\overline{NR}$ is $4/3$ units.

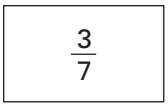
14 Sketch an equilateral triangle, a rectangle, a regular hexagon, and a rhombus. Label their areas 13, 23, 37, and 91 square units. Subdivide each into 4 congruent subregions and label the area of each subregion.

Focus Student Activity 6.4

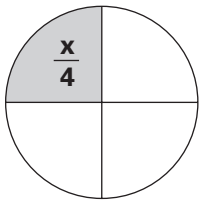
NAME _____ **DATE** _____

Complete the following. Use a straightedge, but no ruler. Express all fractions in improper form.

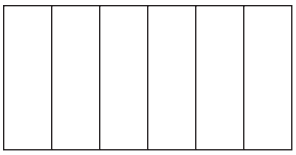
1 Assuming the area of the rectangle shown below is 3/7 square units, sketch a rectangle with area 3 units.



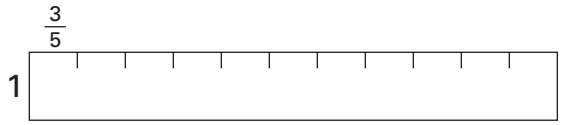
2 The area of the shaded region is x/4 square unit. What is the area of the whole circle?



3 The area of this field is 13 acres. What is the area of each part?



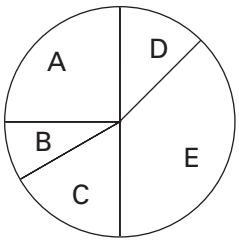
4



a) Shade a region with area 3/2 square units.

b) Draw a bold line around a rectangle with area 6 square units.

5 The area of this circle is X square units. Find the area of each region.



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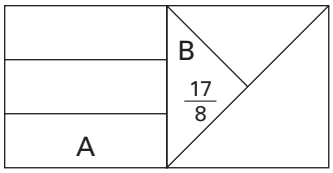
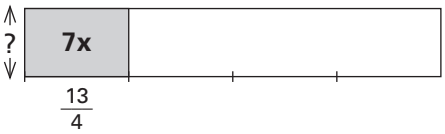
Focus Student Activity 6.4 (cont.)

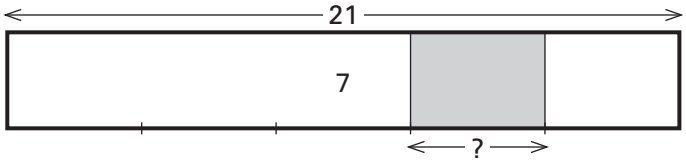
6 The area of region B is $17/8$. Find the area of region A.



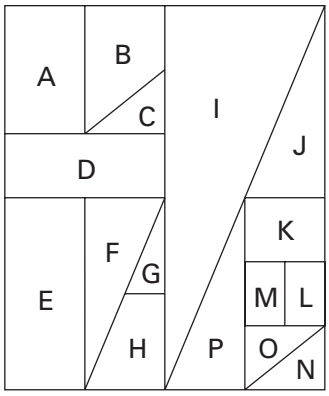
7 The shaded region has area $7x$ and length $13/4$. Find the area, length, and height of the large rectangle.



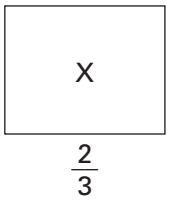
8 The area of the large outer rectangle is 7 and its length is 21. Find the area and length of the shaded region.



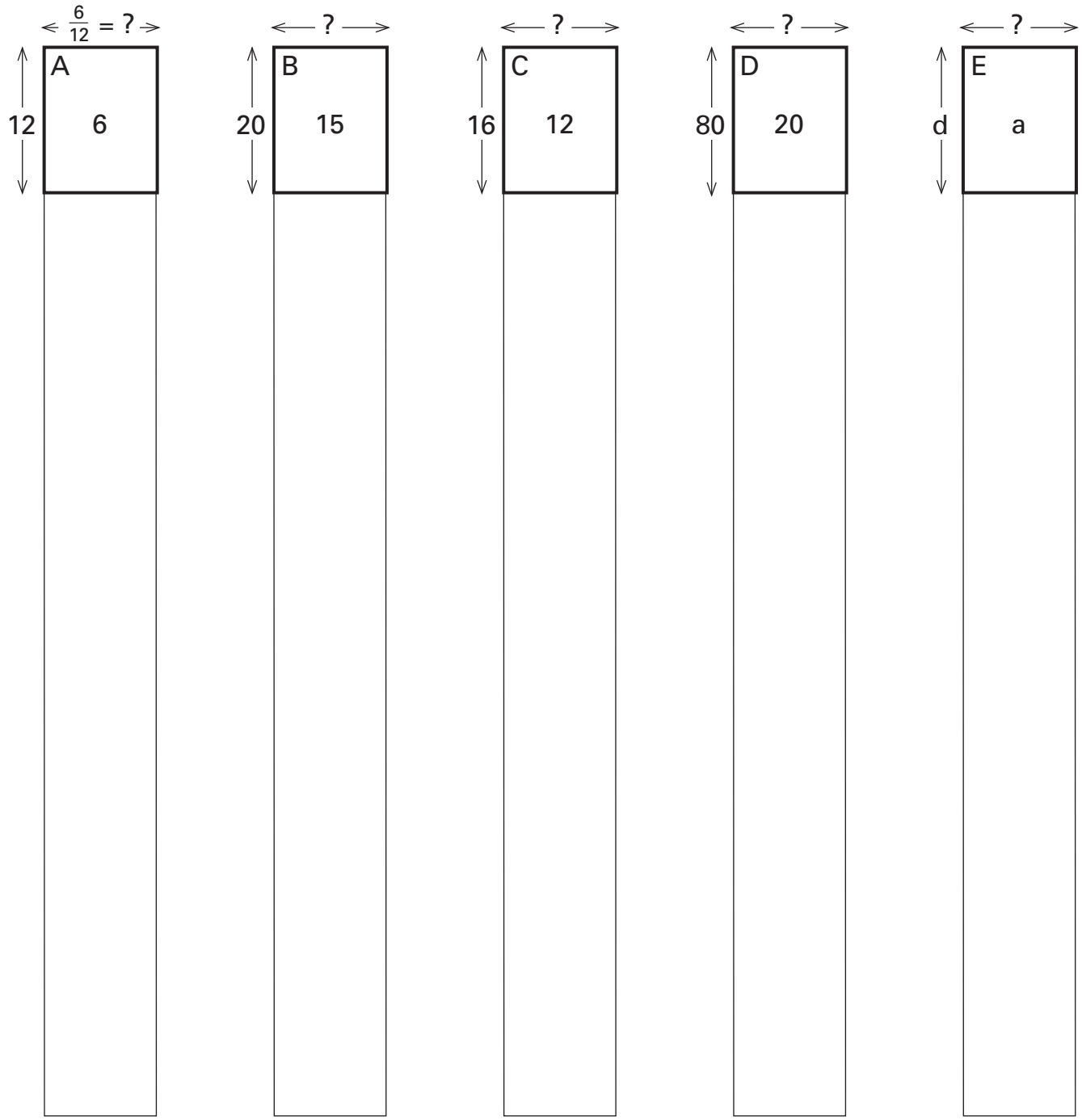
9 The area of the large outer rectangle below is 29. Find the area of each subregion.



10 The rectangle below has area X and length $2/3$. Extend the right edge of the rectangle to form a rectangle with length 2. Record its area and mark the rectangle to show your methods.



Focus Student Activity 6.5



Equivalent fractions:

Follow-up Student Activity 6.6

NAME _____ **DATE** _____

Write all of your responses on other sheets of paper. Be sure to write the problem next to your response.

1 For each of these concepts of a fraction—*part-to-whole*, *ratio*, *division*, and *area*—do the following:

a) explain the meaning of a fraction a/b when it is viewed according to that concept;

b) write a situation in which you "think" according to that meaning, and make a sketch that illustrates your thinking.

2 Explain the meaning of *equivalent fractions*. Show how to use each of the 4 concepts of a fraction to determine 3 pairs of equivalent fractions.

3 For each of the 4 concepts of a fraction, show how to determine an infinite set of fractions that are equivalent to a/b, for a and b not equal to zero.

4 Suppose $a \neq 0$. Demonstrate *why*:

a) $0/a = 0$

b) $0/0$ is undefined

c) $a/0$ is not possible

d) 1 is the multiplicative identity; 1 is the division identity; 0 is the additive identity; and 0 is the subtractive identity.

5 Suppose a and b are whole numbers not equal to zero. Create at least 6 "visual proofs" of mathematical relationships that you can show on rectangles whose dimensions and areas are a, b, 1, or fractions whose numerators and/or denominators are a, b, or 1.

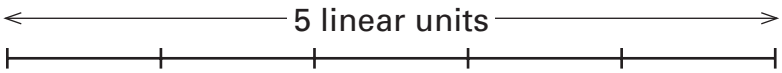
(Continued on back.)

Follow-up Student Activity (cont.)

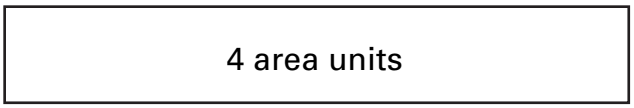
6 On the attached sheet of 1-cm grid paper, outline a rectangle that is 18 cm × 12 cm. Suppose it has area 17 square units for a certain area unit. Subdivide the rectangle to form rectangular subregions with the following areas and with no overlaps. Label the area of each subregion.

Region A, $\frac{17}{12}$ square units.
Region B, $\frac{17}{24}$ square units
Region C, $\frac{34}{12}$ square units
Region D, $\frac{34}{24}$ square units
Region E, $\frac{17}{6}$ square units
Region F, $\frac{34}{6}$ square units
Region G, $\frac{17}{8}$ square units

7 Use the parallel line sheet to mark off lengths or areas for the given fractions. Explain your reasoning.

a) 

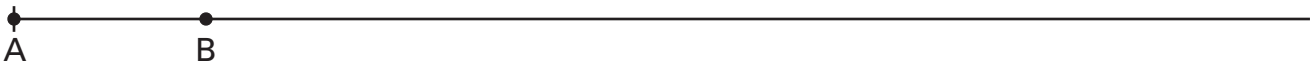
Show length $\frac{3}{5}$.

b) 

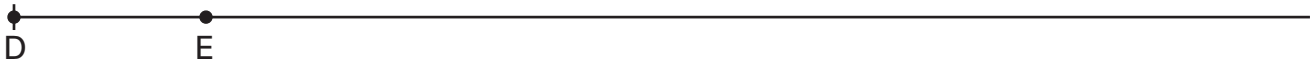
Show area $\frac{4}{7}$.

8 Use the parallel line sheet to locate and label points satisfying the given conditions. Mark the diagram to show your methods.

a) AB is $\frac{2}{3}$ units. Locate point C so that AC is 4 units.



b) DE is $\frac{4}{5}$ units. Locate point F so that DF is 3 units.



c) GH is 3 units. Locate point K so that GK is $2\frac{3}{5}$ units.

Connector Master A

Complete the following thought starter. If you think about a term from the list in more than one way, be sure to include all of your ideas.

Here is how I think about the meaning of each term below, together with an example or examples to illustrate my understanding of the term and ways it is related to other terms in the list…

a) addition
b) subtraction
c) multiplication
d) division
e) whole number
f) fraction
g) decimal
h) integer
i) percent

Focus Master A

Assume that a, b, and c are integers such that denominators are not zero.

a) $a\left(\frac{c}{a}\right) = c$

b) $\left(\frac{1}{b}\right)b = 1$

c) $ab = ba$

d) $\frac{ab}{a} = b$

e) $\frac{b}{b} = 1$

f) $b\left(\frac{a}{b}\right) = a$

g) $\frac{a}{a/b} = b$

h) $\frac{b}{b/a} = a$

i) $b\left(\frac{a}{c}\right) = \frac{ba}{c}$

j) $a(b + c) = ab + ac$

k) $\frac{a+b}{a} = 1 + \frac{b}{a}$

l) $a(bc) = ab(c)$

m) $\frac{1}{1/a} = a$

n) $\frac{a^2 + ab}{a} = a + b$

o) $\frac{a}{b} = \frac{ac}{bc}$

p) $\frac{abc}{b} = ac$

q) $\frac{abc}{c} = ab$

r) $\frac{abc}{bc} = a$

s) $\frac{1}{b/c} = \frac{c}{b}$

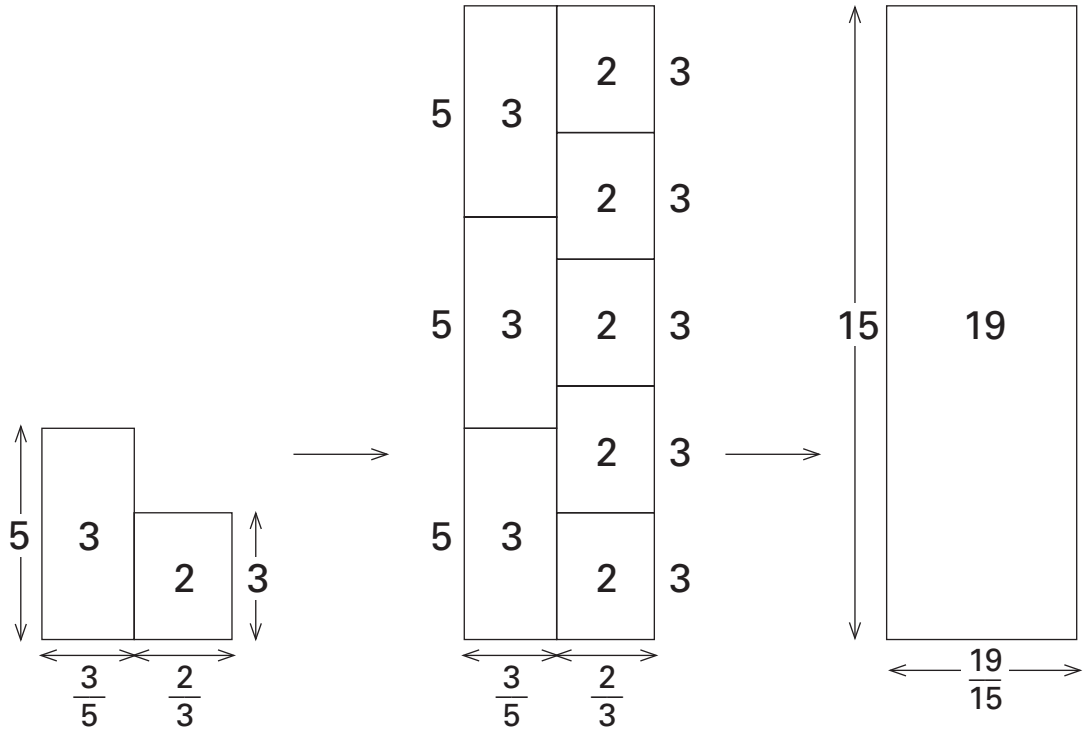
t) $\frac{a+b}{a+b} = 1$

u) $\frac{-ab}{b} = -a$

Focus Master B

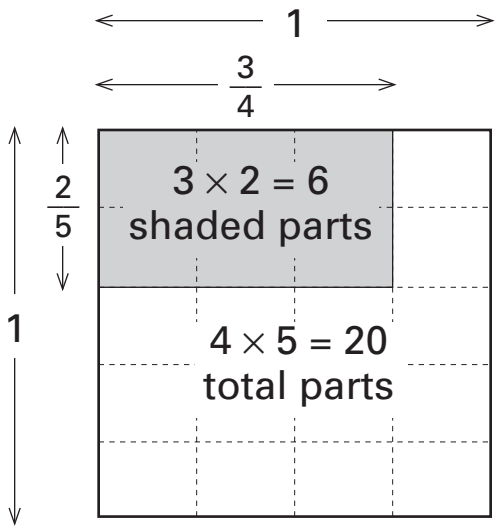
Julie's Method

$$\frac{3}{5} + \frac{2}{3}$$



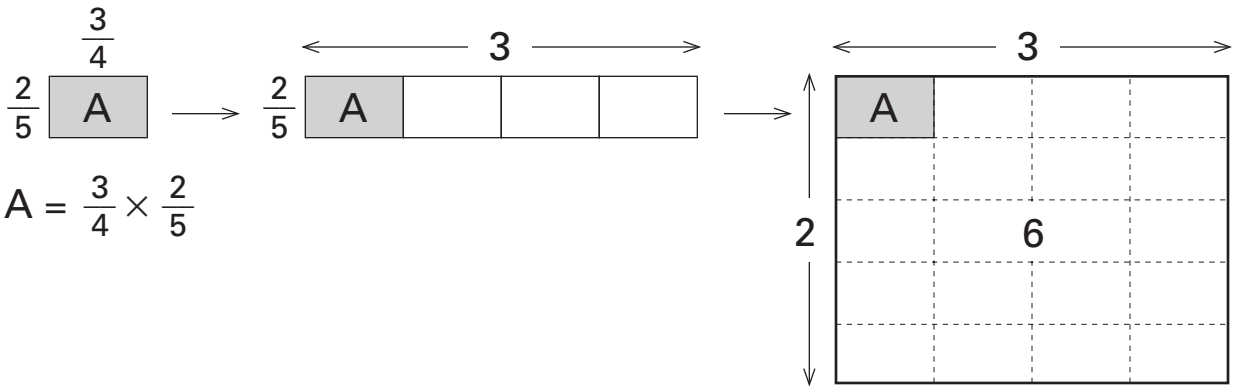
Focus Master C

Linden's Method: $\frac{3}{4} \times \frac{2}{5}$



Therefore, $\frac{3}{4} \times \frac{2}{5} = \frac{3 \times 2}{4 \times 5} = \frac{6}{20}$.

Erica's Method: $\frac{3}{4} \times \frac{2}{5}$



$A = \frac{3}{4} \times \frac{2}{5}$

So, $A = \frac{3}{4} \times \frac{2}{5} = (3 \times 2) \div (4 \times 5) = \frac{3 \times 2}{4 \times 5} = \frac{6}{20}$.

Focus Master D

For real numbers a, b, and c.

Equal sums: $a + b = (a - c) + (b + c)$

Equal differences: $a - b = (a + c) - (b + c)$, and
$a - b = (a - c) - (b - c)$

Equal products: $a \times b = (a \times n) \times (b \div n)$
That is, $ab = (an)(\frac{b}{n})$

Equal quotients: $a \div b = (a \times n) \div (b \times n)$, and
$a \div b = (a \div n) \div (b \div n)$
That is, $\frac{a}{b} = \frac{an}{bn}$, and $\frac{a}{b} = \frac{a/n}{b/n}$

Focus Master E

For each of the following conditions, create an interesting and challenging math problem based on a situation from everyday life outside of school. You could write a separate problem for each condition listed, or you could combine more than one condition in a problem. On a separate sheet, show how to solve your problems.

1. Fractions
   a) addition of 3 mixed numbers with unlike denominators
   b) subtraction of 2 mixed numbers with unlike denominators
   c) multiplication of 2 mixed numbers with unlike denominators
   d) division of 2 fractions with unlike denominators

2. Decimals
   a) addition, the sum is greater than 150.713
   b) subtraction, the difference is less than .001
   c) multiplication, the product is a mixed decimal
   d) division, 14.452 is a multiple of the quotient

3. Integers
   a) addition, the sum is negative
   b) subtraction, the difference is greater than $^{-}50$
   c) multiplication, the product is a negative square number
   d) division, the quotient is less than $^{-}250$

4. Percents, r% of s equals t
   a) r is unknown
   b) s is unknown
   c) t is unknown

(Continued on back.)

Focus Master E (cont.)

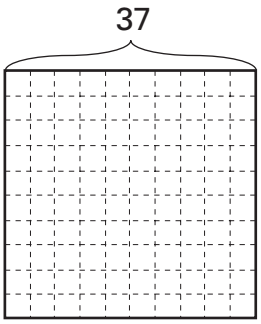
5. Proportions, $m/n = p/q$
   a) m is unknown; n, p, and q are positive integers
   b) n is unknown; m, p, and q are positive integers

6. Algebraic expressions
   a) addition
   b) subtraction
   c) multiplication
   d) division

Focus Student Activity 7.1

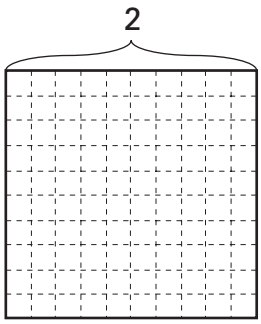
NAME _____ **DATE** _____

1 a)



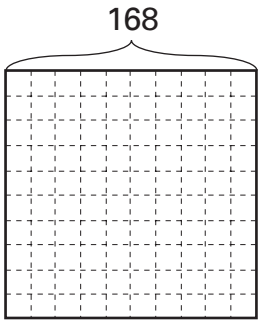
12% of 37 = _____

b)



35% of 2 = _____

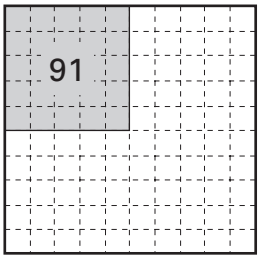
c)



59% of 168 = _____

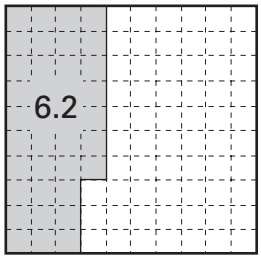
d) Generalizations and algorithms:

2 a)



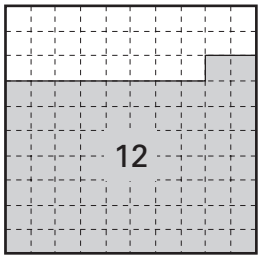
25% of _____ = 91

b)



_____% of _____ = 6.2

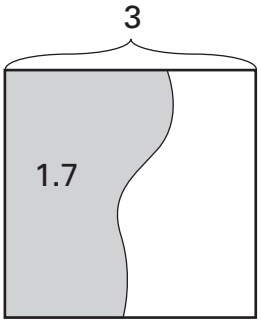
c)



_____% of _____ = 12

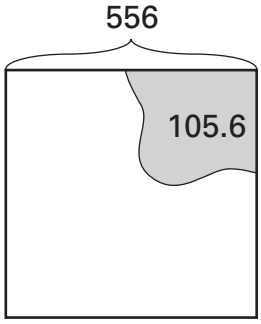
d) Generalizations and algorithms:

3 a)



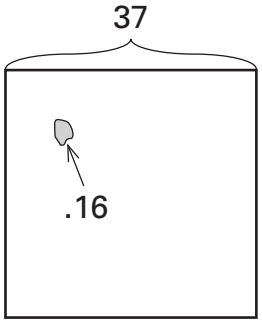
_____ % of 3 = 1.7

b)



____ % of ____ = 105.6

c)



____% of 37 = .16

d) Generalizations and algorithms:

Follow-up Student Activity 7.2

NAME _____ **DATE** _____

1 Determine if each equation below is true, assuming a, b, and c are integers such that denominators are not equal to zero. If an equation is true create a visual proof, and if it is false show a counter example.

a) $\frac{a-b}{a-b} = 1$

b) $\frac{a+b}{a} = 1 + b$

c) $\frac{a}{1/2} = 2a$

d) $a + (b \times c) = (a + b) \times (a + c)$

e) $\frac{a}{a+b} = \frac{1}{b}$

f) $\frac{a^2 - ab}{a} = a - b$

g) $a \div (b + c) = (a \div b) + (a \div c)$

h) $\frac{a}{b} = \frac{ac}{bc}$

i) $\frac{-abc}{c} = -ab$

j) $\frac{a}{1/b} = \frac{a}{b}$

2 Show how to solve each of the following computations, using whatever visual or symbolic method you wish. Be sure to show the steps of your thinking.

a) $\frac{3}{7} \times \frac{2}{5}$

b) $\frac{-1}{3} + \frac{5}{8}$

c) $\frac{3}{4} \div \frac{2}{9}$

d) $\frac{3}{8} - \frac{5}{6}$

e) $.2 + .43$

f) $.7 \times 32$

g) $.8 \div .15$

h) $.46 - .008$

i) $\frac{3x}{5} - \frac{2x}{3}$

j) $14\frac{5}{12} \div 2\frac{1}{3}$

k) $\frac{x+3}{2} + \frac{2x-1}{5}$

l) $\frac{6}{x} + \frac{1}{3x}$

m) $7.23 - 4.8$

n) $\frac{x}{3} \div \frac{2}{7}$

o) $3\frac{1}{8} - 1\frac{4}{7}$

3 Show one or more visual methods for computing each of the following, where a, b, c, and d are integers with $b \neq 0$ and $d \neq 0$. For each visual method, write a symbolic algorithm (using only numbers and math symbols) that is based on the method.

a) $\frac{a}{b} + \frac{c}{d}$ b) $\frac{a}{b} - \frac{c}{d}$ c) $\frac{a}{b} \times \frac{c}{d}$ d) $\frac{a}{b} \div \frac{c}{d}$

(Continued on back.)

Follow-up Student Activity (cont.)

4 Show how to solve each of the following computations using the equal sums, equal differences, equal products, or equal quotients strategy. Use numbers and math symbols to record your thought processes.

a) $17\frac{8}{9} + 6\frac{4}{9}$ c) $36.98 - 4.28$ e) 12×1155 g) $288 \div 60$

b) $14\frac{1}{9} - 8\frac{5}{9}$ d) $9.85 + 14.9$ f) $16 \div \frac{7}{3}$ h) $2\frac{2}{3} \times 27$

5 Show how to reason from diagrams to determine the missing value for each of the following percent problems. Next to each diagram show the steps in your reasoning and all of your calculations.

a) _____% of 350 = 63 b) 84% of _____ = 504 c) 28% of 92.5 = _____

6 An excerpt from Linden's journal is shown below. Write what you think she said in place of "…" and give examples to illustrate each statement.

I notice that the following aspects of each operation never change, regardless the type of number or variable that I use:

addition… subtraction… multiplication… division…

7 Suppose that a and b are integers. Tell whether each of the following is *always*, *sometimes*, or *never* true. If a statement is always true or never true, explain why. If a statement is sometimes true, explain the conditions necessary for it to be true. Give examples to support your reasoning.

$a + b$ is greater than both a and b

$a - b$ is greater than both a and b

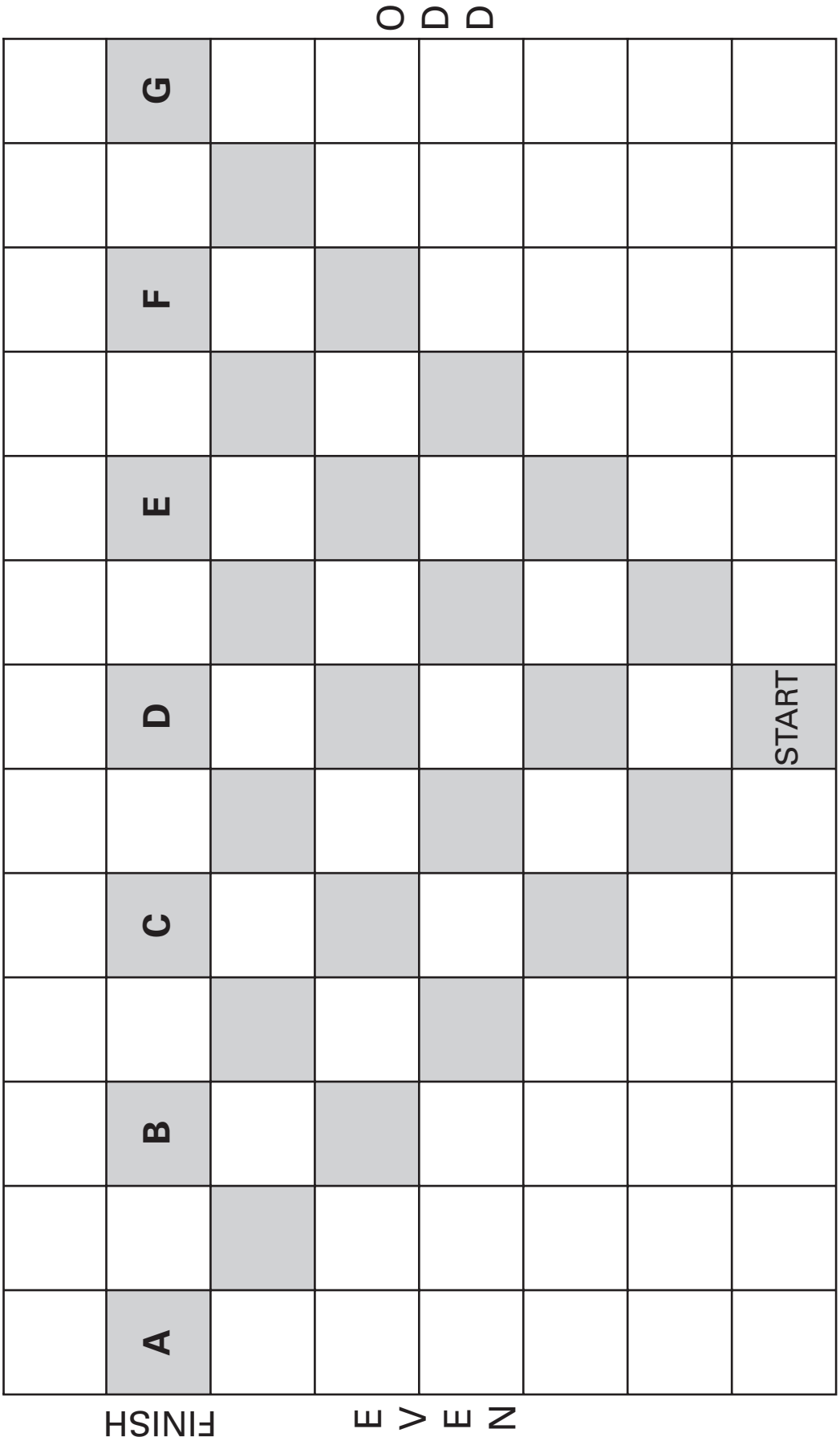
$a \times b$ is greater than both a and b

$a \div b$ is greater than both a and b

8 Repeat Problem 7, but suppose that a and b are fractions between 0 and 1.

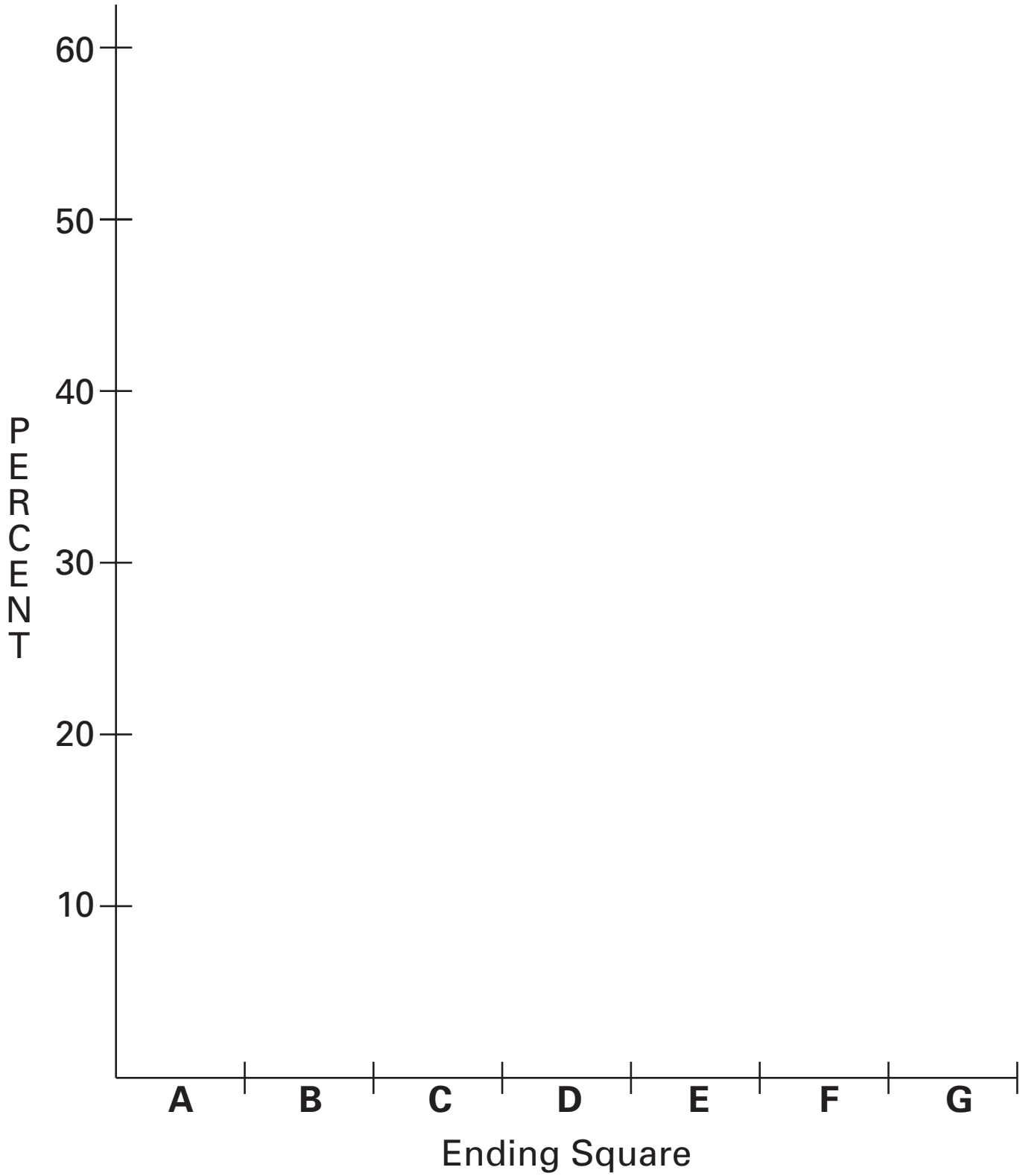
Connector Master A

Checker Game Board



Connector Master B

Relative Frequencies—Theoretical Results



Connector Master C

Checker-B Game

Replace each of the letters A-G on your group's Checker Game Board with one of the numbers 1-7 (one number per square and no numbers can be repeated) so that your group will be likely to win the following game.

Each group places a marker on the START square of their game board. The teacher tosses a pair of dice, then computes and announces the product of the numbers on the upturned sides of the dice. If the product is an EVEN number, each group moves their marker forward 1 square diagonally to the LEFT. If the product is an ODD number, each group moves their marker 1 square diagonally to the RIGHT.

The winning group is the group (or groups) whose marker ends in the square with the highest number.

Your Group's Task

Record your responses to a)-c) on a poster. Plan to spend 2-3 minutes presenting your results to the class.

a) Without any data analysis or data collection, where do you *predict* you should place the numbers 1-7?

b) Now reason mathematically to verify or adjust the numbering system you predicted in a). Write a sound mathematical argument showing why this numbering is most likely to be a winning one. Your arguments should include the following evidence: experimental *and* theoretical probabilities, graphs, diagrams, and concise mathematical language. If your theoretical and experimental probabilities suggest different numberings, explain how you deal with that.

c) Suppose your teacher were to pick at random a shaded square on the game board. On your poster, explain how to determine the probability that a marker would land on that square during a Checker-B game.

Connector Master D

Checker-C Game Procedures

Each player replaces the letters A-G on her/his Checker Game Board with the numbers 1-7 (one number per square, and no numbers can be repeated), positioning the numbers with the intent of winning the following game.

All players place a marker in the START square of their game boards. The teacher rolls a standard die, and announces the number obtained. If the number showing on the die is 1, 2, 3, or 4, all players move their markers forward 1 space diagonally to the LEFT. If the number showing on the die is 5 or 6, all players move their markers 1 space forward diagonally to the RIGHT. The winning player is the one whose marker ends in the square with the highest number.

Connector Student Activity 8.1

NAME _____ **DATE** _____

1 Each group member is to play the Checker-A game several times and keep a record of the number of times the marker lands in each of squares A-G. Your group needs to play a total of 50 games. When everyone has finished, record your data and totals on the following chart:

Players' Names	Ending Square on Game Board							Total Games
	A	B	C	D	E	F	G	
1								
2								
3								
4								
5								
Group Totals								50

Based on your group's experimental results, determine the experimental probability of landing in each of squares A-G on the game board. Express each answer as a fraction, decimal, and percent. Note: P(A) stands for "the probability of landing in square A."

a) P(A) =

b) P(B) =

c) P(C) =

d) P(D) =

e) P(E) =

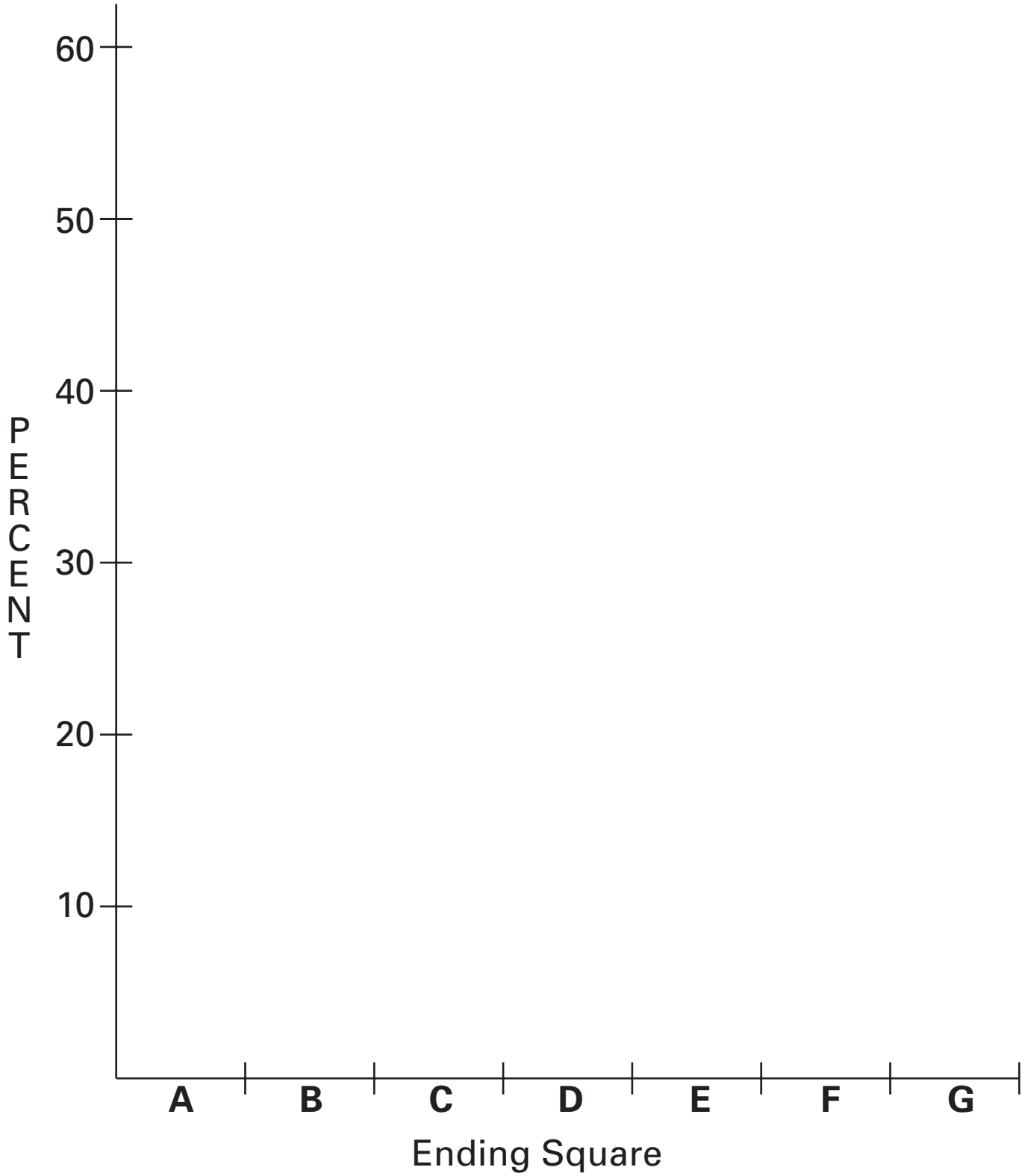
f) P(F) =

g) P(G) =

(Continued on back.)

Connector Student Activity 8.1 (cont.)

2 Make a bar graph showing the relative frequency of the marker ending in each column, based on your group's experimental results:



Focus Master A

Monty's Dilemma

On Monty's TV game show, there are 3 doors. Behind 1 of the doors is a valuable prize and behind the other 2 doors are gag prizes. To play Monty's game, a contestant is invited to choose (but not open) 1 of 3 doors.

After the contestant chooses a door, Monty reveals what is behind 1 of the other 2 doors, always showing a gag prize. Then Monty presents the contestant with the following dilemma:

"Would you like to STICK with the door you chose, or SWITCH to the other unopened door?"

Focus Master B

Monty's Strategies

After 1 of the doors with a gag prize has been opened, which of these 3 strategies is most likely to lead the contestant to the winning door?

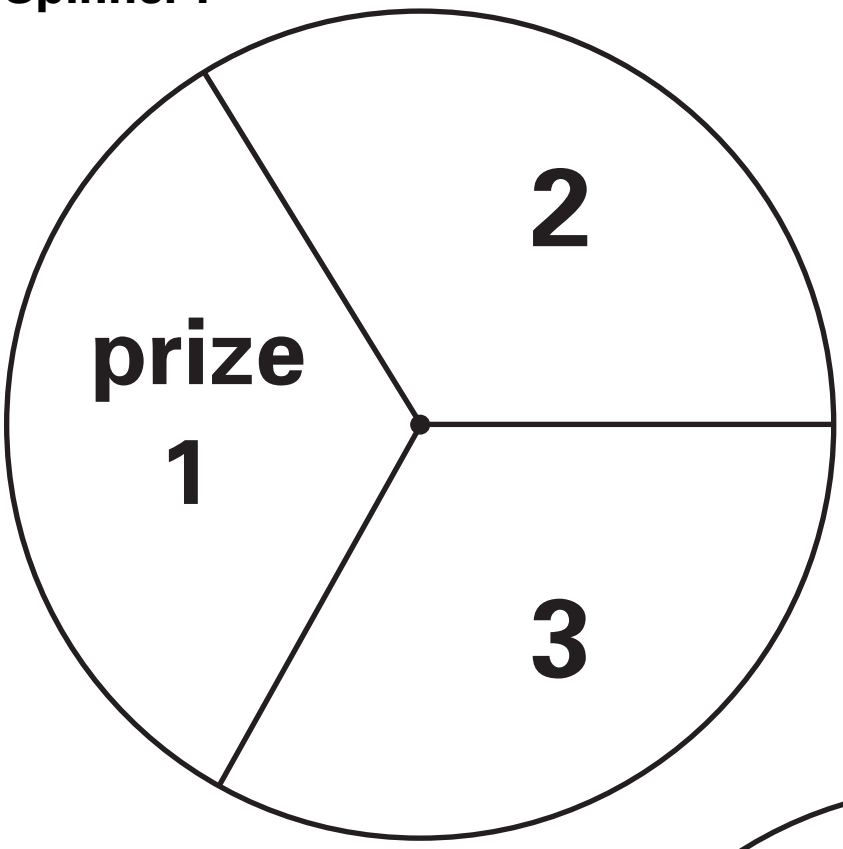
STICK strategy. Keep the door that was originally selected.

FLIP strategy. Choose again by randomly selecting a door from the remaining 2 closed doors.

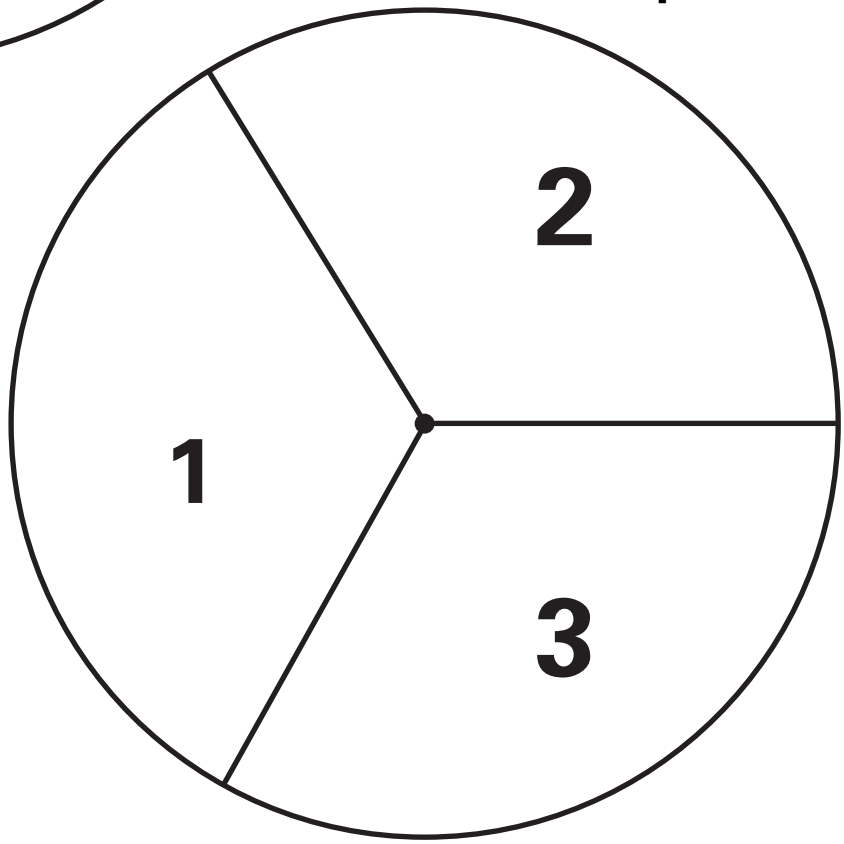
SWITCH strategy. Switch from the original door to the other closed door.

Focus Master C

Spinner I



Spinner II



Focus Master D

Kelsey's Dilemma

The school tennis team is holding a special drawing for a new tennis racket. A total of 10 tickets were given out—1 to each of the 10 tennis team members. This morning Coach Ward said the following to Kelsey, "I have 2 ticket stubs in my hand, yours and another one. One of these 2 ticket stubs has the winning number. Would you like to STICK with your ticket number, or SWITCH for the other number I am holding?" What should Kelsey do?

1. Design and carry out a simulation to help you solve Kelsey's dilemma.

   a) Describe the step-by-step procedures of your simulation.
   b) Make an organized listing of all the data that you collect.
   c) Use experimental probabilities as the basis for solving Kelsey's dilemma.

2. Use theoretical probabilities to solve Kelsey's dilemma. Explain how you determine these probabilities.

Focus Student Activity 8.2

NAME _____ **DATE** _____

Tallies and Totals for Each Strategy

	WINS	LOSSES
STICK		
	Total:	**Total:**
FLIP		
	Total:	**Total:**
SWITCH		
	Total:	**Total:**

Follow-up Student Activity 8.3

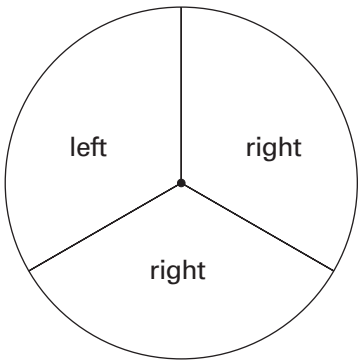
NAME _____ **DATE** _____

1 Explain in your own words the meaning of each of the following terms. Include an example to clarify each explanation.

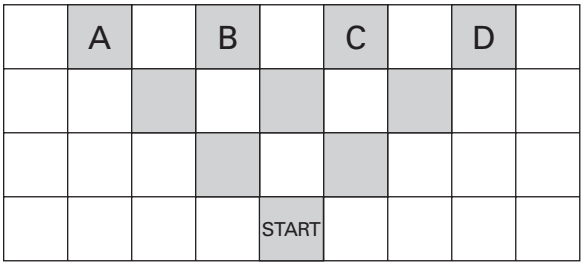
- Probability
- Experimental probability
- Theoretical probability
- Simulation
- Sample size
- Binomial experiment
- Equally likely outcomes
- Unequally likely outcomes

2 Replace the letters A-D on the Checker 1-4 Game Board shown below with the numbers 1-4 so that you are most likely to win the Checker 1-4 Game.

Checker 1-4 Game
Each player places a marker on the START square of their game board. One player spins a bobby pin or paper clip on the spinner shown below to determine the direction (left or right) of a forward diagonal move of every player's marker. Repeat until the markers each reach a numbered square. Each player records the number of the ending square for their marker. The winner is the player with the highest total after 20 games.



Checker 1-4 Game Board



Explain the mathematical reasoning behind your numbering system, using theoretical *and* experimental probabilities as supporting evidence. Show all data that you collect to determine experimental probabilities, and show all information that you use to determine theoretical probabilities.

(Continued on back.)

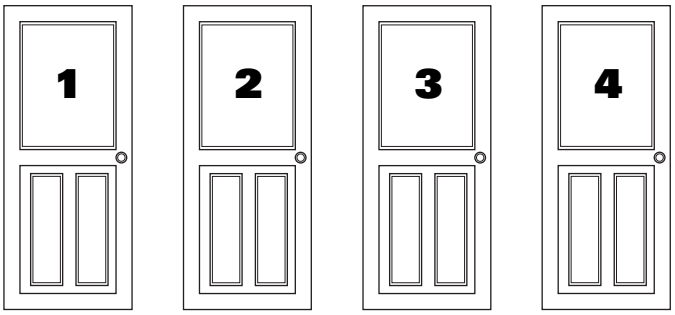
Follow-up Student Activity (cont.)

3 This version of the game, Monty's Dilemma, involves 4 doors—one with a valuable prize and 3 with gag prizes. After the contestant selects a door (but doesn't open it), Monty opens 2 of the remaining doors, both with gag prizes. A contestant may choose 1 of the following strategies:

i) STICK with the original choice

ii) FLIP a coin to determine which door to choose

iii) SWITCH from the original choice to the remaining door



a) Design a simulation of the game. Write a description of the step-by-step procedures of your simulation. Give several examples to show how each strategy works in your simulation.

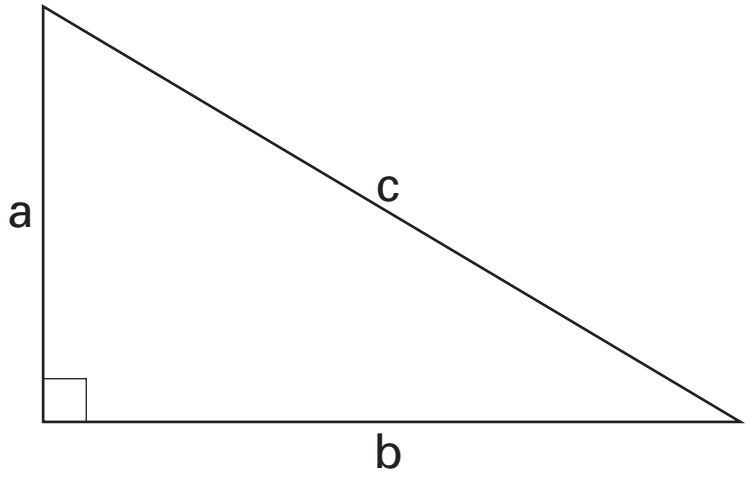
b) Carry out your simulation at least 10 times for each of the 3 given strategies. Show an organized listing of all the experimental data that you collect.

c) Based on an analysis of your experimental data, write a convincing argument telling which strategy is most likely to be a winning strategy for a contestant. Use experimental probabilities to support your position.

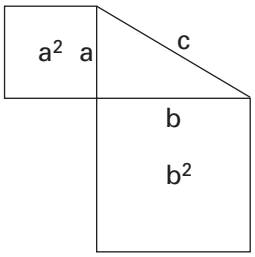
d) Based on theoretical probabilities, which strategy is most likely to be a winning strategy? Show how you determine each theoretical probability.

e) Determine the probabilities of winning and why with the stick, flip, or switch strategies for each of the following versions of Monty's game: there are 5 doors, 4 gag prizes, and Monty opens 3 doors; there are 6 doors, 5 gag prizes, and Monty opens 4 doors; there are n doors, $n - 1$ gag prizes, and Monty opens $n - 2$ doors.

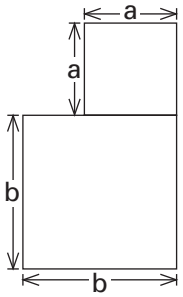
Connector Master A



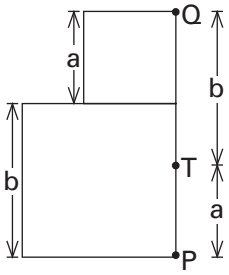
Connector Master B



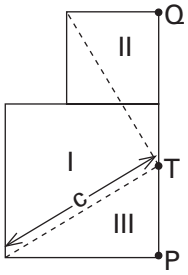
Step A Draw squares with edge lengths *a* and *b* on the legs of the right triangle on Connector Master A. Use a note card as a straightedge and guide for drawing right angles.



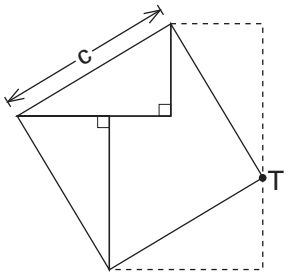
Step B Trace the squares from Step A to form 2 adjacent squares with right edges collinear.



Step C Label points P and Q as shown at the right. Locate a point T on edge $\overline{PQ}$ so that PT = *a*. Notice that TQ = *b*, since…



Step D Draw the dotted lines shown to form the regions labeled I, II, and III. Notice that regions II and III are congruent. Notice also that the angle formed at the intersection of the dotted lines is a right angle since…



Step E Cut out regions I, II, and III. Then reassemble the regions to obtain a square.

Step F Tape the square formed in Step E on the hypotenuse of the triangle on Connector Master A. Notice that…

Connector Master C

Use a note card as a straightedge and as a guide for drawing square corners.

a) From a geometric perspective, $\sqrt{n}$ is the length of the edge of a square whose area is n. On 1-cm grid paper and using a 1-cm square as 1 area unit, form all the actual lengths $\sqrt{n}$, for every integer n such that $1 \leq n \leq 12$. Tape your results, in order, on another sheet of paper, showing the square associated with each length. Label actual lengths.

b) On plain (no grid) paper, draw any 2 noncongruent squares so they are adjacent and their right edges are collinear. Then, dissect and rearrange these 2 adjacent squares to form a 3rd square whose area is equal to the sum of the areas of the 2 squares.

i) How many different 3rd squares are possible for a pair of adjacent squares?
ii) What if the 2 adjacent squares are congruent?

c) Challenge. On plain paper, draw a large square. Dissect and rearrange this large square to form 2 noncongruent adjacent squares whose total area equals the area of the large square.

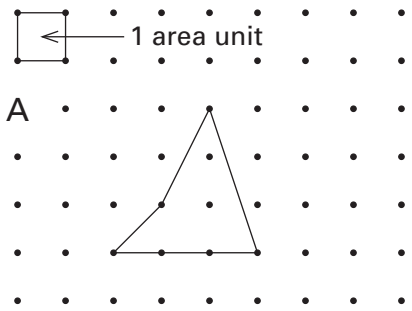
i) How many different pairs of adjacent squares can be formed so the sum of their areas equals the area of the large square?
ii) What if the 2 adjacent squares must be congruent?
iii) What if the area of the large square and the area of 1 of the 2 adjacent squares are given?

d) List your conjectures, questions, and generalizations.

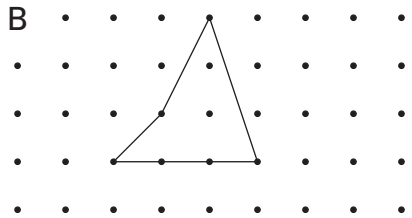
Connector Student Activity 9.1

NAME _____ **DATE** _____

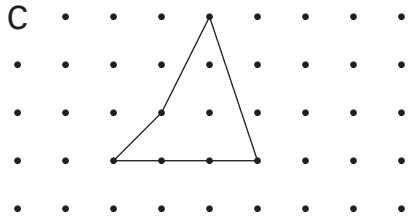
1 *Without* using any measuring tools, *without* using the Pythagorean Theorem, and *without* using area formulas, find the area and perimeter of Polygon A shown at the right. Mark the diagram to show your methods. Give *actual* measures rather than decimal approximations.



2 Use the Pythagorean Theorem where necessary to find all actual side lengths of Polygon B. Label each length on the diagram and show all of your calculations.



3 Use area formulas to compute the area of Polygon C. Label the diagram to show your calculations and formulas.

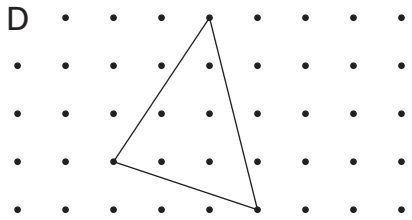


4 Complete a)-c) below for Polygon D. Show your methods.

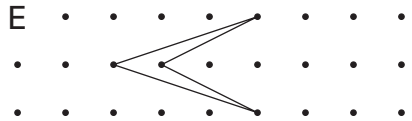
a) Determine its actual area.

b) Determine its actual perimeter.

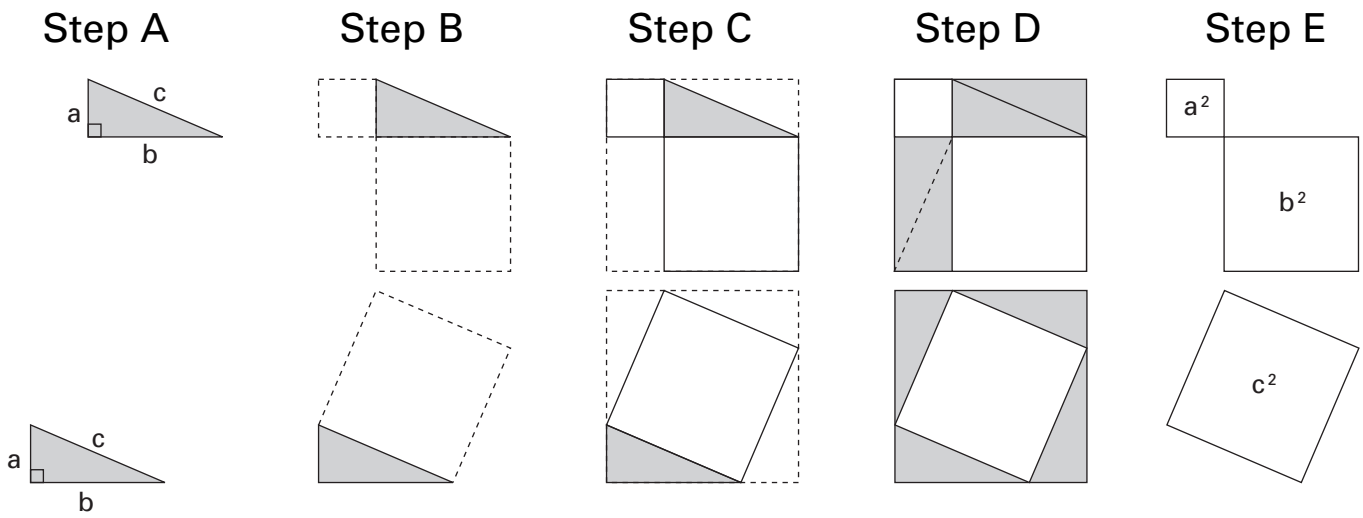
c) Determine the actual and approximate lengths of its 3 altitudes.



5 Show the methods that you use to find the area and perimeter of Polygon E.



Focus Master A



One visual proof of the Pythagorean Theorem is based on the diagrams shown above. Write statements that describe each step of this "proof." If needed, use the statements below as "thought starters."

Step A Draw 2 congruent right triangles, one triangle at the top of a sheet of paper, and the other at the bottom of the sheet. On each triangle, label the length of the short leg *a*, the long leg *b*, and the hypotenuse *c*.

Step B Draw a square on each leg of the top triangle, and draw a…

Step C Enclose the top figure and the bottom figure each in the smallest square possible. Notice that the areas of the enclosing squares are equal because…

Step D Draw a diagonal of the rectangle in the lower left corner of the top figure. Notice the 8 triangles (4 in the top figure and 4 in the bottom) are congruent because…

Step E If the 8 triangles are cut away, then… because…

In summary, if a right triangle has legs of length *a* and *b* and hypotenuse of length *c*, then…

Focus Master B

a) Reason visually to determine which, if any, of the following statements are true about *all* triangles with side lengths a, b, and c:

$a + b = c$
$a + b < c$
$a + b > c$

b) Use a straightedge to draw 4 noncongruent acute triangles.

i) Use a protractor and ruler to measure all side lengths and angles. Label your drawings to show lengths to the nearest tenth of a centimeter and angles to the nearest degree.

ii) Draw squares on each side of each triangle and record the areas of the squares.

iii) Using information from i)-ii) as evidence, reason inductively to complete Conjecture 1 below. Then give deductive arguments to show why your conjecture must always be true.

Conjecture 1 If a triangle is acute with side lengths a, b, and c, where c is the longest side length, then $a^2 + b^2$ ...

c) Complete Conjecture 2 below and give inductive and deductive arguments to support your conjecture:

Conjecture 2 If a triangle is obtuse with side lengths a, b, and c, where c is the longest length, then $a^2 + b^2$ ...

Focus Master C

a) Paintings in ancient Egyptian tombs from the 15th century B.C. show Egyptian geometers—called "rope stretchers," or surveyors—using long ropes with equally spaced knots. Historians believe these ropes were used to aid in the construction of right angles and right triangles. How do you think the rope stretchers did this?

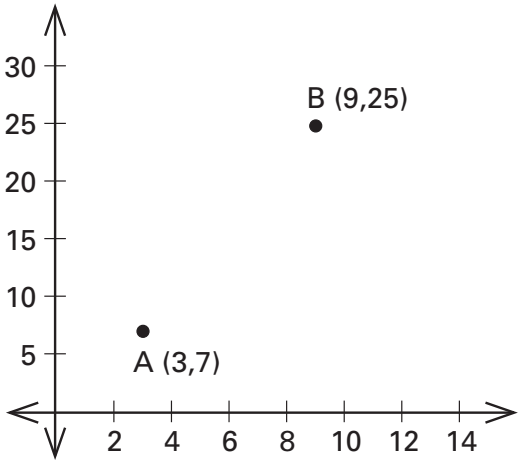
b) What are all the different right triangles with integral side lengths that can be formed from all or part of a 35-knot rope?

c) Three positive integers that work in the Pythagorean Theorem are called a *Pythagorean triple*. A Pythagorean triple is called a *primitive* if the 3 integers have no common factors other than 1—that is, a triple is a primitive if the 3 integers are *relatively prime*. Enlarging a right triangle by a scale factor that is a whole number creates a new Pythagorean triple that is called a *multiple* of the original triple.

What are all the Pythagorean triples such that each number in the triple is less than 35? Which of these are primitives? Which are multiples of primitives?

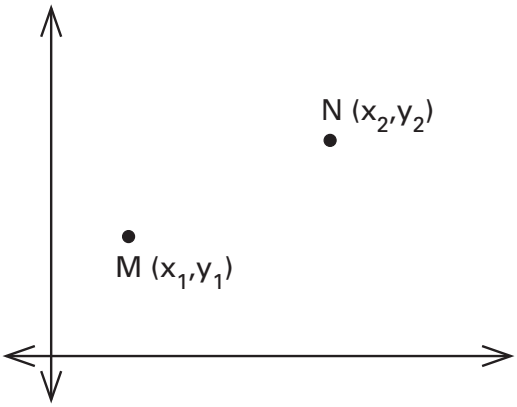
Focus Master D

1. Given 2 points A and B with coordinates (3,7) and (9,25):



i) Find the distance between A and B.
ii) Write the coordinates of the midpoint of $\overline{AB}$.

2. Given 2 points M and N with coordinates (x_1,y_1) and (x_2,y_2), respectively:



i) Write a formula for the distance between the 2 points. ___
ii) Write a formula for the coordinates of the midpoint of $\overline{MN}$.
iii) Do you think your results for i) and ii) work for any 2 points on the coordinate system? Explain your reasoning.

Focus Master E

<- _____ linear units ->

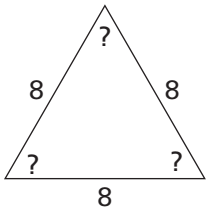
Area = _____ square units

Focus Student Activity 9.2

NAME _____ **DATE** _____

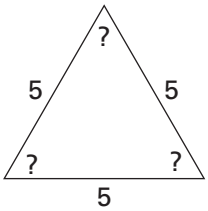
Fill in the blanks below. Be prepared to provide sound mathematical arguments to support your answers. Write all radical expressions in simplified form. For each problem, use only the given variables without adding other variables. A = area, P = perimeter, h = altitude, and s = side length.

1 a)



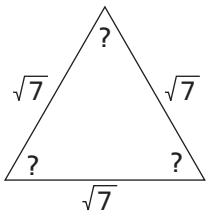
h = _____
A = _____
P = _____

b)



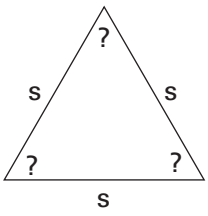
h = _____
A = _____
P = _____

c)



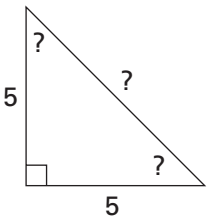
h = _____
A = _____
P = _____

d)



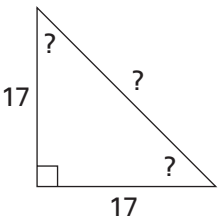
h = _____
A = _____
P = _____

2 a)



A = _____
P = _____

b)

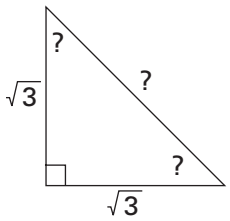


A = _____
P = _____

(Continued on back.)

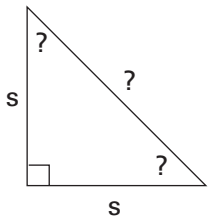
Focus Student Activity 9.2 (cont.)

c)



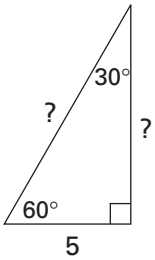
A = _____
P = _____

d)



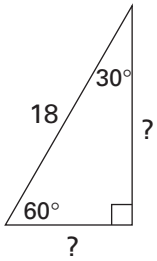
A = _____
P = _____

3 a)



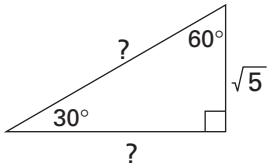
A = _____
P = _____

b)



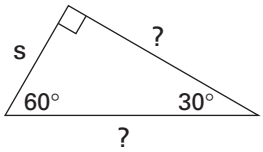
A = _____
P = _____

c)



A = _____
P = _____

d)

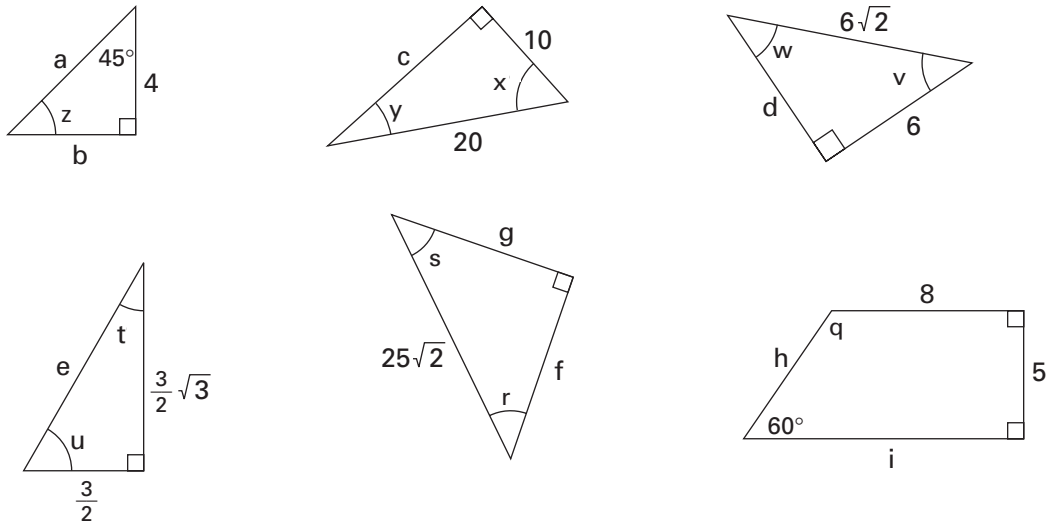


A = _____
P = _____

Focus Student Activity 9.3

NAME _____ **DATE** _____

1 Without measuring, find the value of each missing angle or length. Drawings are not necessarily to scale.



2 Draw diagrams to help you find the following values. Briefly explain and/or mark your diagrams to show your reasoning. Give actual measures rather than approximations.

a) The length of the diagonal of a square with area 225 cm^2.

b) The area of a regular hexagon with side length 12 inches.

c) The length of the diagonals of each face of a rectangular prism with dimensions 5 inches by 7 inches by 9 inches.

d) The length of the diagonals of the prism from Problem c). (Note: such diagonals extend corner to corner through the center of the prism.)

e) The area of an equilateral triangle with sides of length 17 feet.

f) The perimeter of an equilateral triangle whose area is $12\sqrt{3}$ square centimeters.

(Continued on back.)

Focus Student Activity 9.3 (cont.)

3 For each of the following, give an example to show evidence *that* the statement is true for positive values of x and y. Then demonstrate visually *why* each statement is true.

a) $\sqrt{x} \times \sqrt{y} = \sqrt{xy}$ b) $\sqrt{x} + \sqrt{y} \neq \sqrt{x+y}$ c) $\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}$

4 Invent a formula for the area, A, of a regular hexagon with side length s. Be sure that A and s are the only variables in your formula.

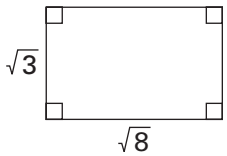
5 Simplify each of the following radical expressions.

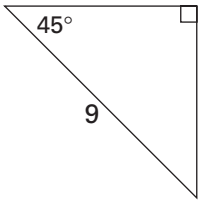
a) $\sqrt{24}$ d) $\sqrt{\frac{4}{5}}$ g) $\sqrt{32} \times \sqrt{5}$

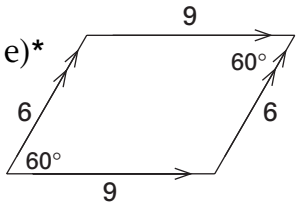
b) $\sqrt{\frac{32}{9}}$ e) $\sqrt{20} + \sqrt{60}$ h) $\sqrt{12} - \sqrt{36}$

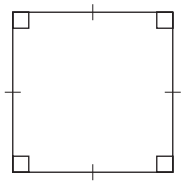
c) $\frac{1}{\sqrt{3}}$ f) $\sqrt{45} + \sqrt{75}$

6 Sketch each of the following polygons on another sheet. Show the reasoning you use to find the area and perimeter of each (in some cases the area or perimeter is given). Drawings are not to scale.

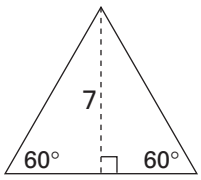
a)


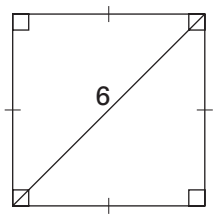
c)


e)*


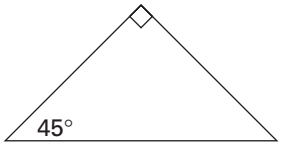
g)


perimeter = $4\sqrt{6}$

b)


d)


f)


h)


perimeter = $9 + \frac{9\sqrt{2}}{2}$

*Note: > and >> markings on sides indicate pairs of parallel sides.

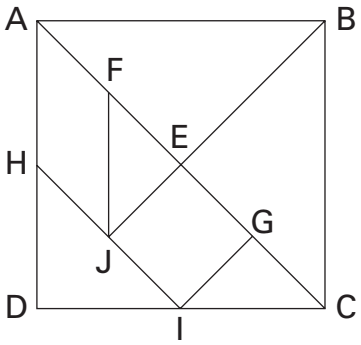
Follow-up Student Activity 9.4

NAME _____ **DATE** _____

1 One way to form the length $\sqrt{5}$ is to form a 1 by 2 rectangle and draw its diagonal. One way to form the length $\sqrt{20}$ is to double the length $\sqrt{5}$. The points $\sqrt{5}$, $-\sqrt{5}$, $\sqrt{20}$, and $-\sqrt{20}$ can be located on a number line by copying the length of a diagonal of a 1 by 2 rectangle. Determine how to locate points on the number line for the positive and negative square roots of all whole numbers less than 50, by constructing diagonals of the *minimum* number of rectangles. Make a chart to show the dimensions of the rectangles required, the length of the diagonals of each rectangle, and the points that can be located using each diagonal.

2 Tangrams, one of the oldest and most popular of the ancient Chinese puzzles, are made by constructing a geometric figure like the one below and then cutting it into the seven pieces shown.

In the square ABCD:
- Point E is the intersection of the diagonals.
- F and G are midpoints of $\overline{AE}$ and $\overline{EC}$, respectively.
- H and I are midpoints of $\overline{AD}$ and $\overline{DC}$, respectively.
- J is the midpoint of $\overline{HI}$.



If AB is 1 linear unit, find the side lengths and area of each different tangram piece. Show the calculations that you use to determine each measure.

3 Suppose that AB from Problem 2 is 5 linear units. Now find the side lengths and area of each different tangram piece. Show your reasoning.

4 Given the coordinates of any 3 points, without plotting the points on grid paper, how can you tell whether they form a right triangle? an obtuse triangle? an acute triangle? Show and explain your reasoning.

(Continued on back.)

Follow-up Student Activity (cont.)

5 If the lengths a, b, and c, form a right triangle, will $2a$, $2b$, and $2c$? $a/3$, $b/3$, and $c/3$? ka, kb, and kc, for k a positive integer? Explain.

6 Given a right rectangular prism with dimensions l, w, and h. What are the lengths of the diagonals of the prism. Justify your answer.

7 Given a cube with side length s. What are the lengths of its diagonals? Justify.

8 Draw each of the following quadrilaterals on 1-cm grid paper. Without using a calculator or a ruler to measure, determine and label the actual side lengths, diagonal lengths, and the area of each quadrilateral. Add comments as needed to communicate your methods and reasoning. Let 1 cm = 1 linear unit.

a) Squares with areas 1, 5, and 7 square units.

b) A rectangle with dimensions 1 by $\sqrt{2}$.

c) A square with diagonal length $\sqrt{6}$ linear units.

d) Two different nonsquare rectangles with diagonal lengths $\sqrt{6}$ linear units.

e) A square with side length $\sqrt{8}$ linear units and a nonsquare rectangle with diagonal length $\sqrt{8}$ linear units.

f) A rectangle with diagonal length $\sqrt{12}$ linear units.

9 Suppose that:

Nonsquare Rectangle R has sides of length a and b.
Square S has the same area as Rectangle R.
Square T has the same perimeter as Rectangle R.

Find the length of the sides of Square S and Square T. Show your reasoning.

Connector Master A

***Paperfold* to satisfy each of the following conditions. Do not use a protractor or ruler.**

a) Form 2 lines which are perpendicular to each other and not parallel to the edges of the paper.

b) Form 3 lines so that 2 of the lines are perpendicular to the 3rd line.

c) Form 2 parallel lines which are intersected by a 3rd line that is not perpendicular to the parallel lines.

d) Form a line segment and locate its midpoint. Then form a line perpendicular to the segment and passing through the midpoint of the segment.

e) Create an acute angle; then form a line that bisects the angle.

f) Form a line *m* and mark a point P not on the line. Then form a new line which is perpendicular to line *m* and passes through point P.

g) Form a line *r* and mark a point Q not on line *r*. Then form line *s* which passes through Q and is parallel to line *r*.

h) Form an isosceles triangle.

Construction 1

a) Use a straightedge to construct a line segment $\overline{AB}$.

b) Investigate ways to use a straightedge and compass only to construct a congruent copy of the line segment $\overline{AB}$ formed in a).

c) Devise a set of clear and concise, step-by-step instructions for *constructing* a copy of a line segment.

Construction 2

a) Use a straightedge to draw an angle with vertex X.

b) Investigate ways to use a straightedge and compass to construct $\angle V$ congruent to $\angle X$ formed in a).

c) Devise a clear set of instructions for *constructing* a copy of an angle.

Focus Master B

The 6 pieces of information shown on the next page make up the "maximal" set of information that you need to construct triangle ABC. What is a "minimal" set of information that you need to construct triangle ABC. How can you be sure you have all the information that you need? How can you be sure that you don't have more information than you need? Is there more than one minimal set of information?

Investigate. Test your ideas and build arguments to support your conclusions.

Make a poster of your conclusions. Show examples to support your ideas. List important observations, conjectures, and generalizations that you develop during the investigation.

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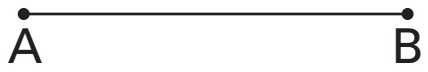
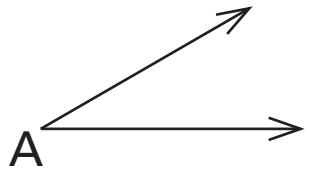
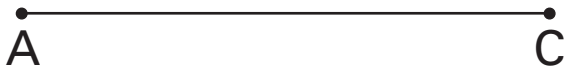
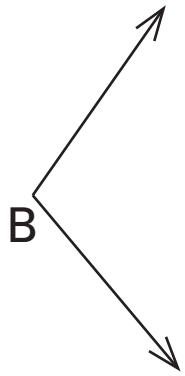
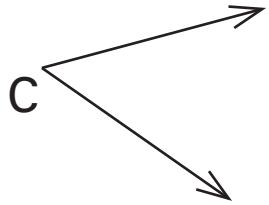
Focus Master B (cont.)











Focus Master C

According to the definition of congruence:

If there is a correspondence between the vertices of 2 triangles such that all corresponding segments are congruent and all corresponding angles are congruent, then the triangles are congruent.

The above statement gives the *maximal* set of conditions needed to establish that 2 triangles are congruent. For any 2 triangles ABC and DEF, what are all the possible *minimal* sets of conditions for determining whether ΔABC is congruent to ΔDEF? Give convincing evidence to support your conclusions.

Focus Master D

Which, if any, of the following conditions determine a unique triangle? an infinite collection of noncongruent triangles? a fixed number (greater than 1) of noncongruent triangles? no triangles? Show or explain your reasoning. Give examples to support your conclusions. Write conjectures and generalizations in "if *x*, then *y*" form.

a) the area of a scalene triangle

b) the perimeter of an equilateral triangle

c) one side and the area of a scalene triangle

d) the perimeter and the noncongruent side of an isosceles triangle

e) one leg and the area of a right triangle

f) 2 sides of an isosceles (nonequilateral) triangle

g) the area of an equilateral triangle

h) 2 sides of a right triangle

i) the perimeter of a right triangle

j) 2 sides of a triangle and the altitude to the vertex between the 2 sides.

k) Challenge: 2 sides and an angle of a scalene triangle.

l) 2 angles and a side of a scalene triangle

m) Challenge: the area and an acute angle of a right triangle

n) angles 110°, 47°, 28°

o) side lengths 4, 8, 10

p) side lengths 3, 9, 14

q) angles 57°, 63°, 60°

r) the area of an isosceles triangle that is not equilateral

Focus Master E

Sketch or construct, and describe, the locus of points in a plane which are:

a) equidistant from 2 intersecting lines.

b) equidistant from 2 parallel lines.

c) equidistant from 2 concentric circles.

d) equidistant from a given point A.

e) the midpoints of congruent chords of a circle with center A.

f) vertices of a right triangle with hypotenuse AB.

g) equidistant from the sides of a given angle B.

h) the midpoints of chords of a circle with center P such that all the chords have the same end point R on the circle.

i) equidistant from a line and a point Q not on the line.

j) such that the distance from a point A is 2 times the distance from a 2nd point B.

k) such that the sum of the distances to 2 fixed points A and B is the same number.

l) such that the difference of the distances from 2 fixed points C and D is the same number.

m) the centers of circles that are tangent to 2 intersecting lines.

Focus Student Activity 10.1

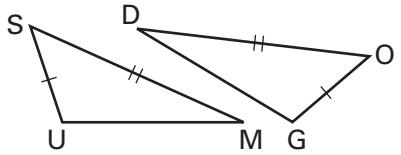
NAME _____ **DATE** _____

Using the information that is given (i.e., as indicated by the angle and hash marks) or that you can logically conclude from the given information, determine which of the following pairs of triangles are congruent. Note: the drawings may not be drawn to scale, so do not base your conclusions on measurements you make or on what appears to be true. For each of the following 12 pairs of triangles:

- If the 2 triangles are congruent tell which congruence property (SSS, SAS, or ASA) is the basis of your reasoning, and write a correct congruence statement (e.g., $\Delta ABC \cong \Delta MNO$).

- If the 2 triangles are not congruent, explain why.

1



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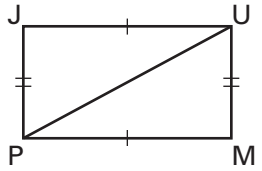

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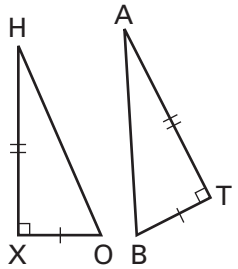

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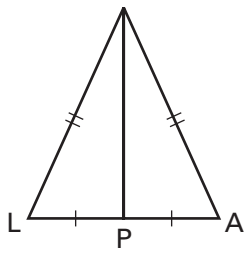

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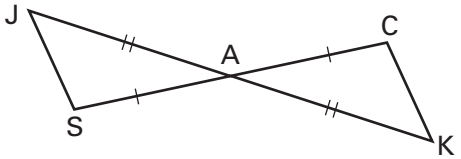

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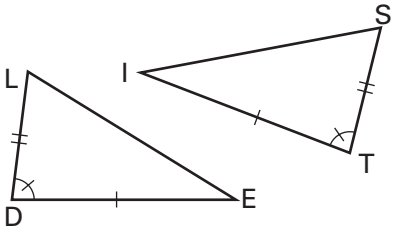

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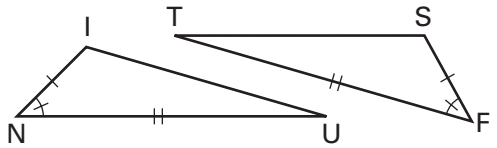

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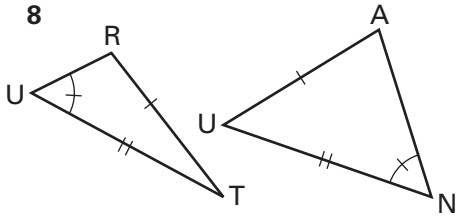

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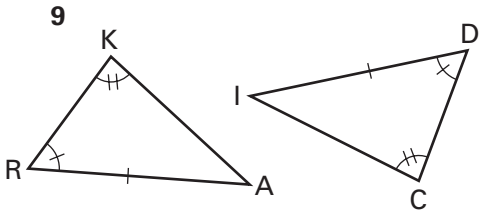
Focus Student Activity 10.1 (cont.)

9



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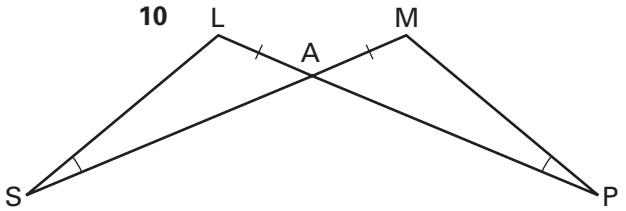

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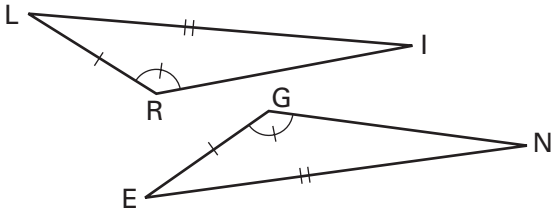

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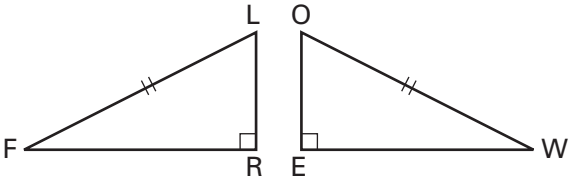


Focus Student Activity 10.2

NAME _____ **DATE** _____

On separate sheets of paper, carry out each of the following constructions using only a straightedge and compass. Next to each construction, list the steps of your construction, and write an explanation that tells how you can be certain (without measuring) that your constructed figure meets the given conditions.

1 Construct an equilateral triangle ΔABC.

2 Construct an isosceles triangle ΔABC which is not an equilateral triangle.

3 Construct the line that is the *perpendicular bisector* of a line segment AB. HINT: you must explain how you know the line is perpendicular to AB *and* you must explain how you know that the line divides AB into 2 congruent segments.

4 Construct a line that is the *angle bisector* of an angle A.

5 Construct a line perpendicular to a given line through a point P on the line.

6 Construct a line perpendicular to a given line through a point Q not on the line.

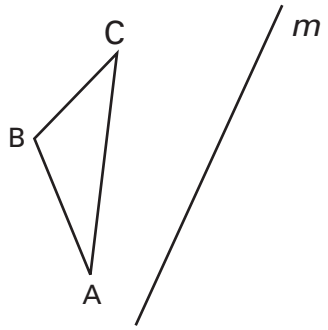
7 Construct a line parallel to a given line through a point not on the given line.

Focus Student Activity 10.3

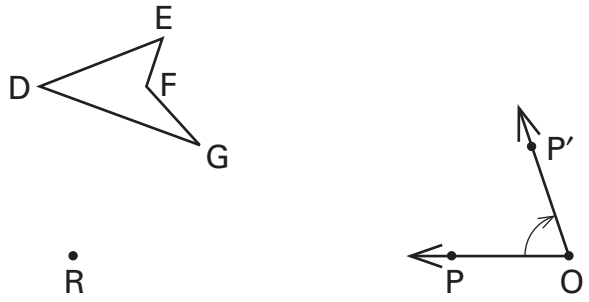
NAME _____ **DATE** _____

1 Use your straightedge and compass to construct the following transformations. Label the image of each point A as A′, the image of B as B′, etc. Don't remove your construction marks. Record your observations and conjectures.

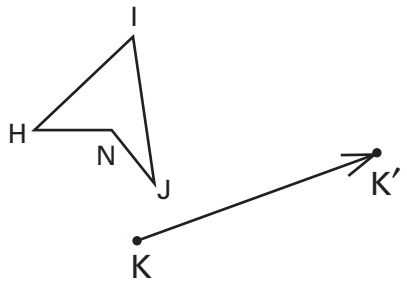
a) Reflect ΔABC across line *m*.



b) Rotate quadrilateral DEFG about point R, using ∠POP′ as the angle of rotation.



c) Translate quadrilateral HIJN, using translation vector $\overrightarrow{KK'}$.



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Focus Student Activity 10.3 (cont.)

2 Use your straightedge and compass for the following. Don't erase construction marks.

a) Draw a polygon with 4 or more sides. Construct the reflection of the polygon across the following lines. Then make observations and conjectures.

i) a line that intersects the shape at more than 1 point;

ii) a line that does not intersect the shape;

iii) a line that touches the shape at exactly 1 point.

b) Suppose you are given 2 shapes and 1 of the shapes is a reflection image of the other, but the line of reflection isn't given. Explain a method of constructing the line of reflection.

c) Suppose you are given 2 shapes and 1 is a translation image of the other, but the translation vector isn't given. Explain how to construct the translation vector.

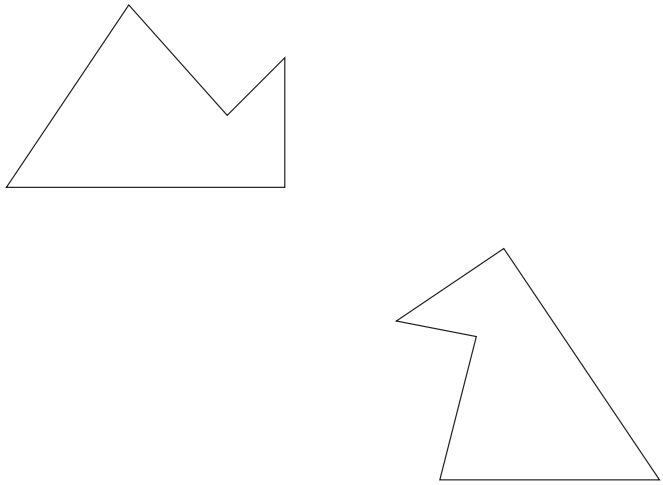
d) Draw a polygon with 4 or more sides. Elsewhere on the paper, draw an angle B. Construct the rotation of the polygon through $\angle B$ and about each of the following points. Then make conjectures and observations.

i) a point P outside the shape;

ii) a point Q inside the shape;

iii) a point R on the perimeter of the shape.

3 Challenge. For the pair of congruent figures at the right, label the vertices and write a congruence statement to show the corresponding vertices. Determine the type of mapping (translation, reflection, or rotation) which maps one figure onto the other. Then locate the line of reflection, translation vector, or center of rotation.



4 Challenge. Investigate general relationships between the location of the center of a rotation and the location of points and their rotation images.

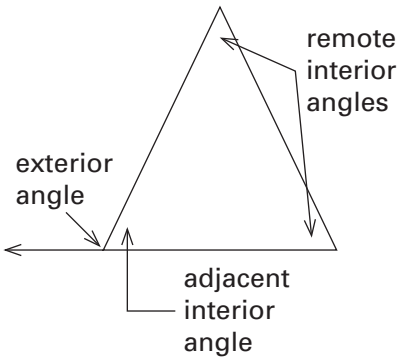
Focus Student Activity 10.4

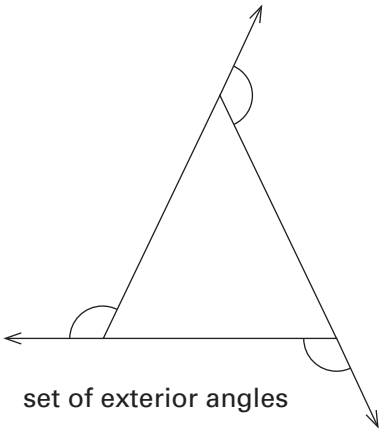
NAME _____ **DATE** _____

Find out all that you can about each idea below that you investigate. Whenever possible state your conjectures in "if… then…" format. Support your conjectures with evidence. Tell when your evidence is based on inductive reasoning from examples that suggest your conclusions seem to be true, and tell when your evidence is based on deductive reasoning that shows why your conclusions are correct.

Investigate…

a) The *exterior angles* of a triangle; a quadrilateral; a pentagon; a polygon with *n* sides. Note: Extending 1 side of a polygon forms 1 exterior angle; extending each side of a polygon to form 1 exterior angle at each vertex forms a set of exterior angles.





b) Minimal conditions to prove a quadrilateral is a parallelogram. Note: by definition a parallelogram is a quadrilateral with exactly 2 pairs of parallel sides.

c) The diagonals of kites, rhombuses, rectangles, squares, parallelograms, trapezoids (isosceles and nonisosceles).

d) Minimal conditions for congruence of 2 kites; 2 rhombuses; 2 rectangles; 2 squares; 2 parallelograms; 2 trapezoids (isosceles and nonisosceles); and 2 quadrilaterals.

(Continued on back.)

Focus Student Activity 10.4 (cont.)

e) Relationships between inscribed angles and the arcs and chords they intercept on a circle. Note: The measure of an arc in degrees is equal to the measure of the central angle that intercepts the arc.

f) Perpendicular bisectors of chords of circles.

g) Triangles that are inscribed in a circle (i.e., the circle circumscribes the triangle); and triangles that circumscribe a circle (i.e., the sides of the triangle are tangent to the circle).

h) One midsegment of a triangle and the set of all midsegments of a triangle; a midsegment of a rectangle and the set of all midsegments of a rectangle; of a rhombus; of a parallelogram; and of a trapezoid. Note: A midsegment of a triangle is a segment that connects midpoints of 2 sides of the triangle; a midsegment of a quadrilateral connects opposite midpoints.

i) Connecting midpoints of consecutive sides of quadrilaterals.

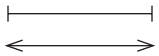
j) Minimal conditions to prove a quadrilateral is a rectangle. Note: by definition a rectangle is a quadrilateral with 4 right angles.

Follow-up Student Activity 10.5

NAME _____ **DATE** _____

Complete the following on separate paper; write each problem next to your work.

1 Given the length at the right is 1 linear unit. Use a straightedge and compass only to construct each of the following, if possible. Show all of your construction marks and briefly describe your step-by-step procedures. Then write a brief explanation why your method works. If the construction is not possible, explain why.



a) An equilateral triangle with perimeter $3\sqrt{2}$ units, and an enlargement of the triangle by a factor of 3.

b) A kite with diagonals $\sqrt{5}$ units and $2\sqrt{5}$ units.

c) A scalene right triangle with one side 2 units and one angle 45°.

d) A circle with area 6π square units.

e) A rectangle with diagonals of length 5 units and 4.5 units.

f) 3 noncongruent rectangles with diagonals of length 3 units.

g) An equilateral triangle with perimeter 6 and inscribed in a circle.

h) An isosceles right triangle with hypotenuse 5 and inscribed in a semicircle.

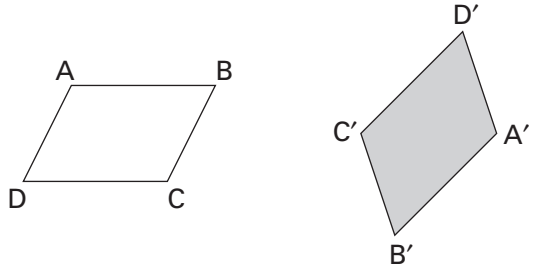
i) A regular hexagon whose perimeter is $12\sqrt{3}$ units and with an inscribed circle with radius 3 units.

2 Record, in "if… then…" format, 2 or more conjectures or generalizations that are based on your work for Problem 1. Give arguments to support each conjecture.

(Continued on back.)

Follow-up Student Activity (cont.)

3 A′B′C′D′ is the rotation image of ABCD. Trace these figures, construct the center of rotation, and briefly explain your steps.



4 A′B′C′D′ above is also the image of ABCD after 2 consecutive reflections across 2 different lines. Investigate and locate 2 such lines of reflection. Explain.

5 On a separate sheet of paper, draw line M and label point P not on M. Plot a few points whose distance from line M is twice the distance from point P. Form a conjecture about this locus of points.

6 For any 3 or more of the following statements, use a diagram and deductive arguments that are based on the diagram and properties that you know to argue why each of those statements is true.

a) The diagonals of a rhombus are perpendicular.

b) The area of a rhombus is ½ the product of its diagonals.

c) The altitude of an isosceles (nonequilateral) triangle bisects the vertex angle (i.e., the angle opposite the noncongruent side).

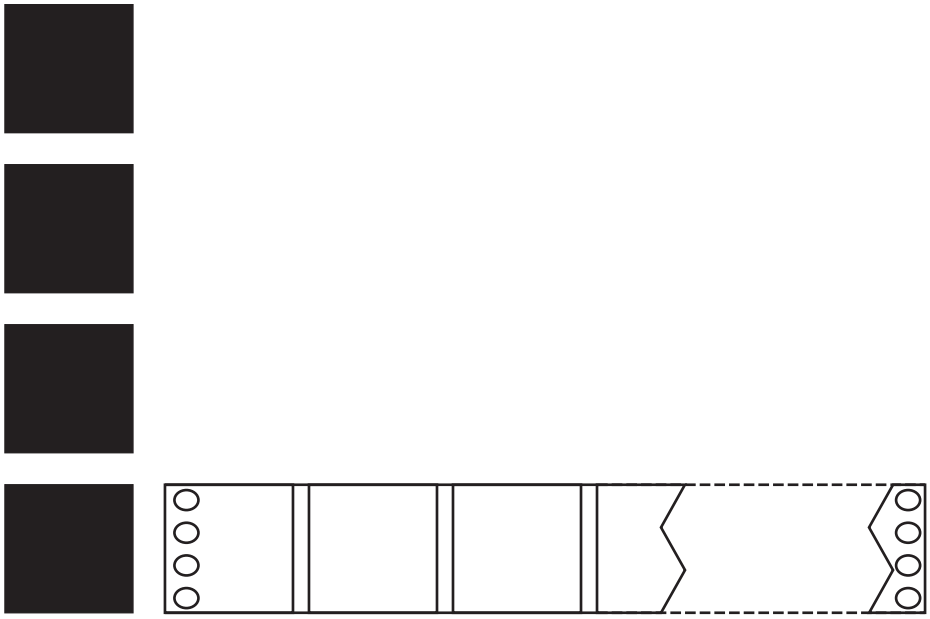
d) If a quadrilateral is a parallelogram, then a diagonal of the quadrilateral forms 2 congruent triangles.

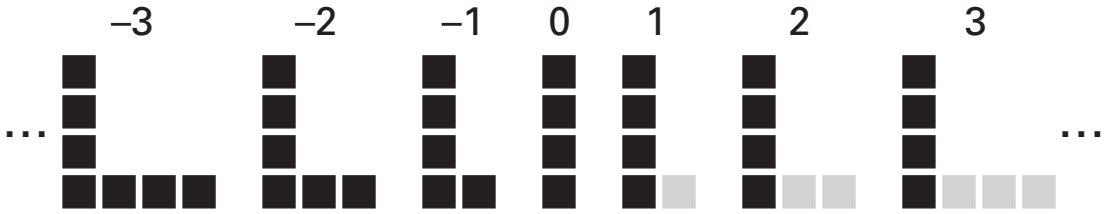
e) If the diagonals of a quadrilateral bisect each other, then the quadrilateral must be a parallelogram.

f) The sum of the measures of the exterior angles of a polygon is 360°.

7 Find several possible angle measures that can be constructed by straightedge and compass only. Explain your methods.

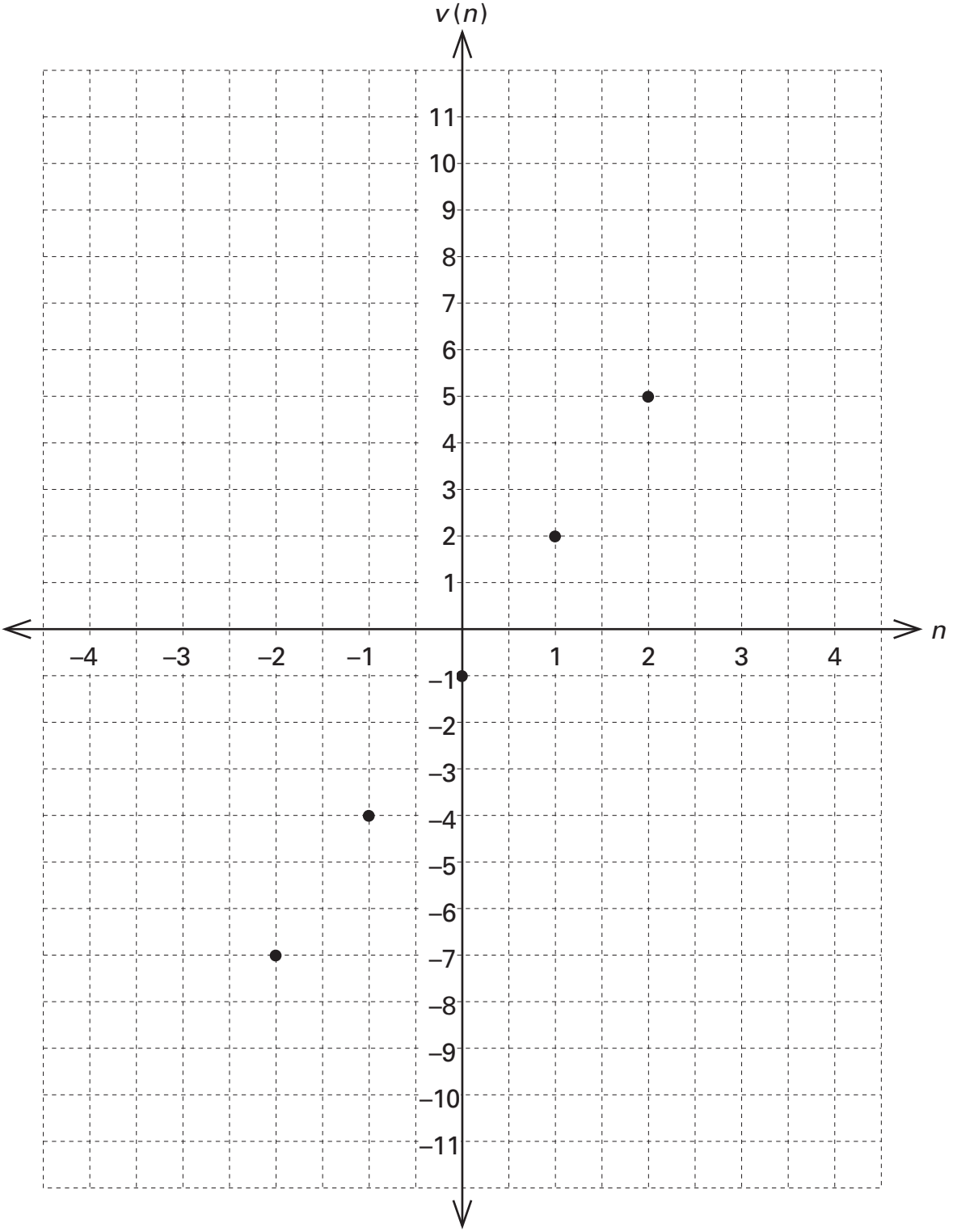
Connector Master A





Connector Student Activity 11.1

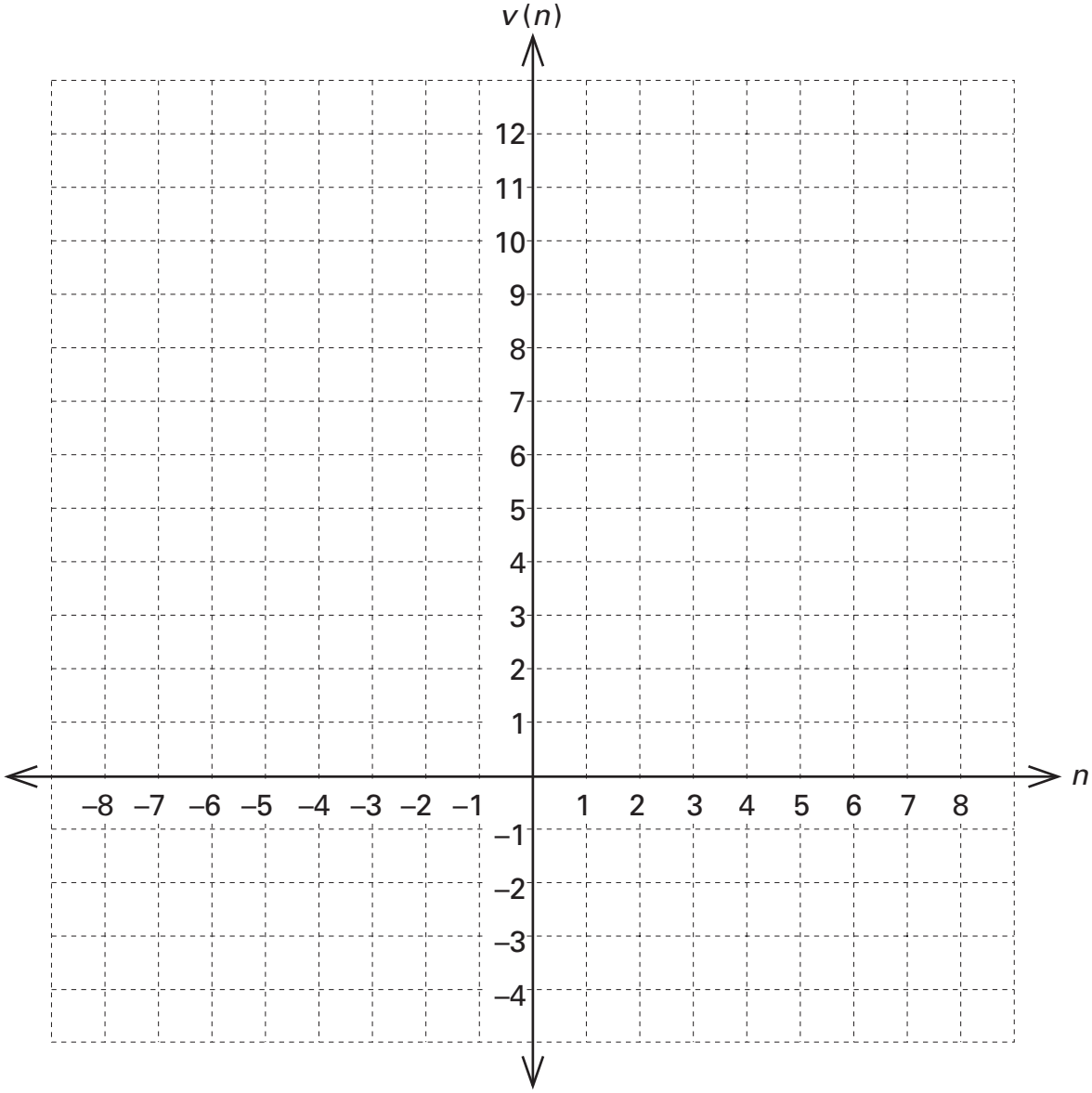
NAME _____ **DATE** _____



$v(n) =$ _____

Connector Student Activity 11.2

NAME _____ **DATE** _____

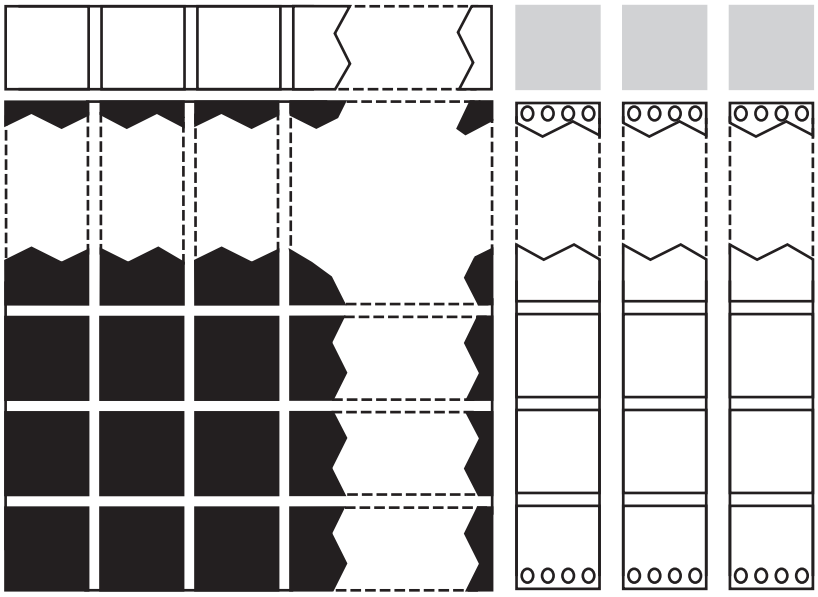


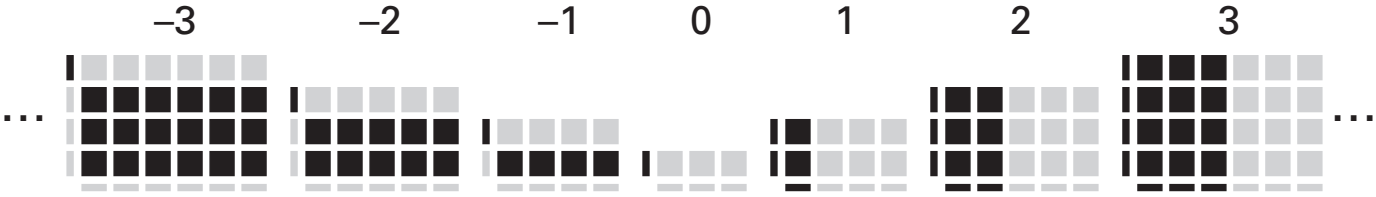
$v(n)$ = _____

Observations about the graph:

Focus Master A

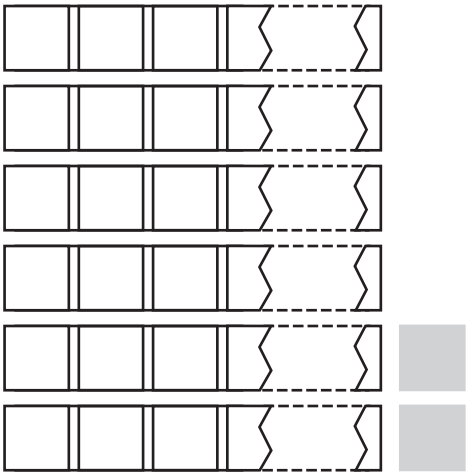
Focus Master B



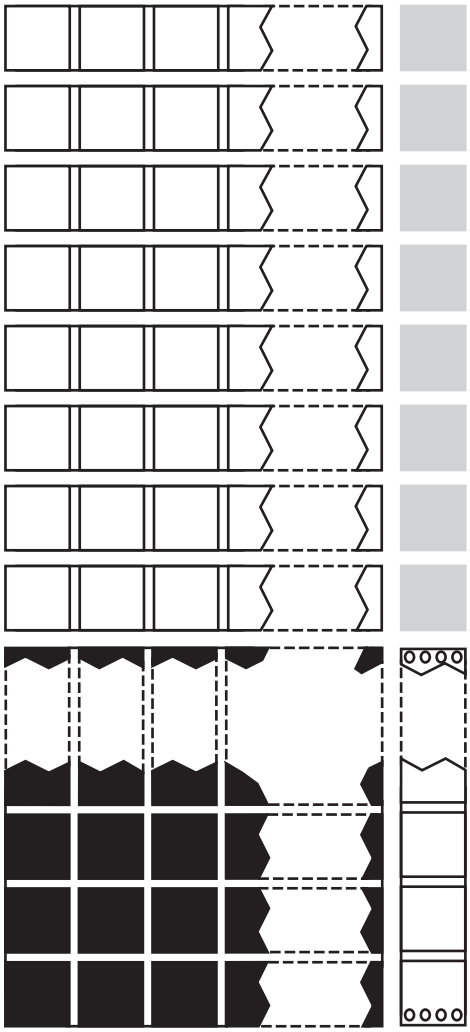


Focus Master C

I



II



Focus Master D

1. Formulas for the values of the *n*th arrangements of 3 pairs of extended sequences are given below. For each pair of sequences, please do the following:

a) Sketch the –3rd through 3rd arrangements and the *n*th arrangement of both sequences.

b) Make a table showing $v_1(n)$ and $v_2(n)$ for n from –3 to 3. Then graph $v_1(n)$ and $v_2(n)$ on the same coordinate axes.

c) Make mathematical observations about similarities, differences, and relationships you notice in the graphs of $v_1(n)$ and $v_2(n)$.

d) Use pictures or algebraic symbols to show your step-by-step methods of using Algebra Pieces to determine when $v_1(n) = v_2(n)$.

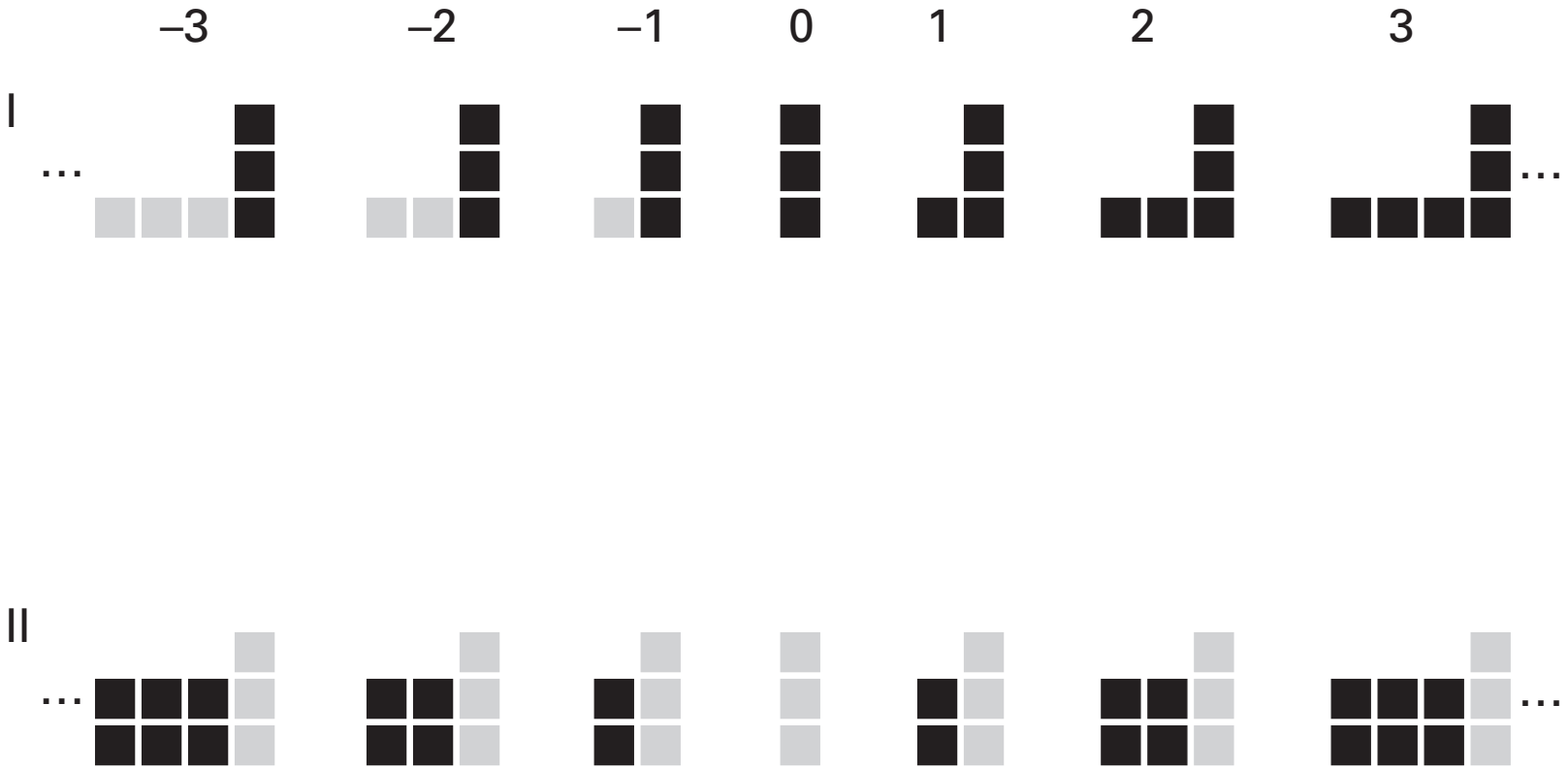
Pair 1
$v_1(n) = -3n + 2$ $\quad$ $v_2(n) = 4n - 12$

Pair 2
$v_1(n) = -7$ $\quad$ $v_2(n) = -n^2 - 2n + 8$

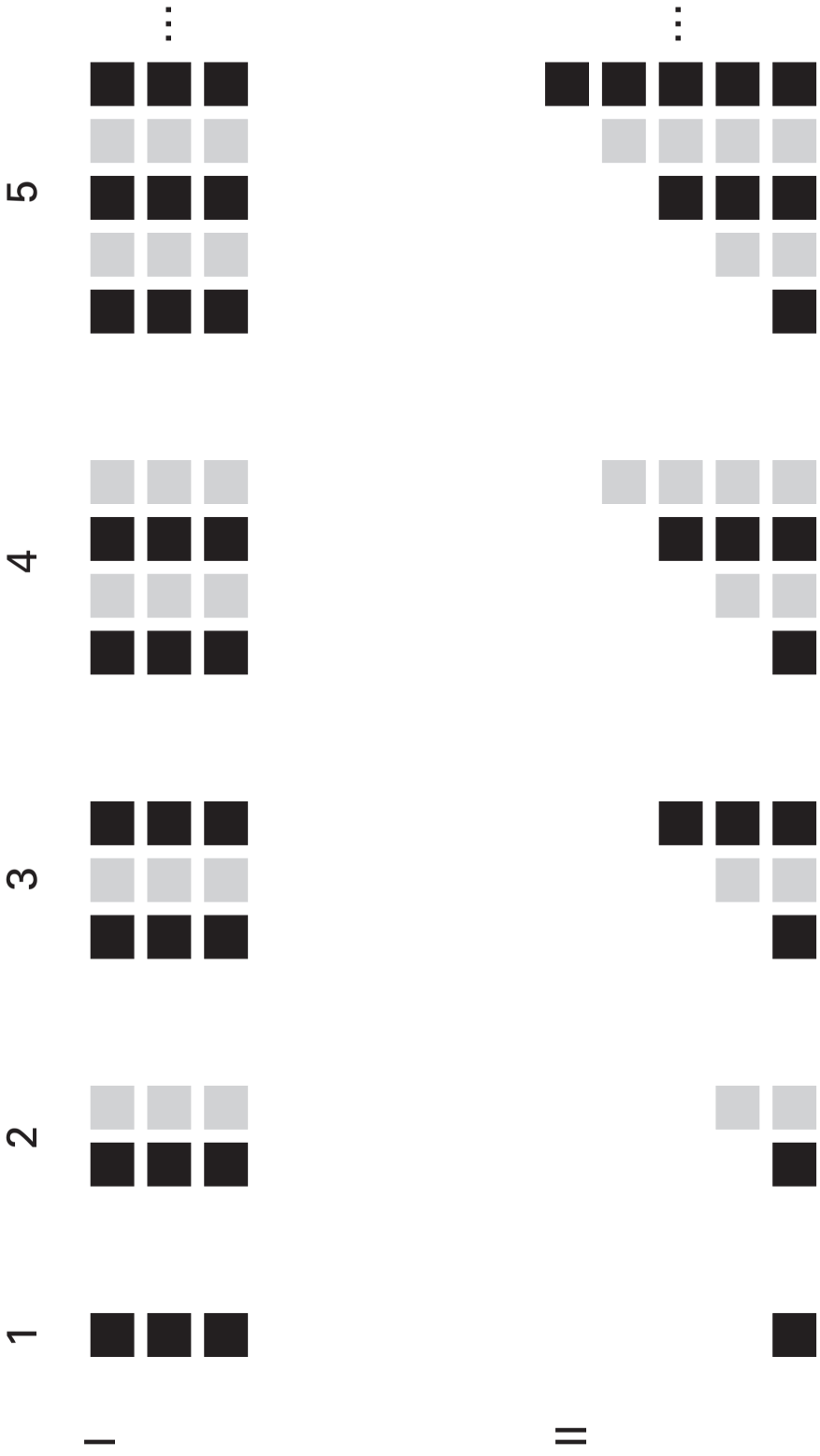
Pair 3
$v_1(n) = n^2 + 2$ $\quad$ $v_2(n) = -n^2 + 4$

2. Review your completed graphs for Problem 1 above and then record your general observations and conjectures about graphing equations.

Focus Master E

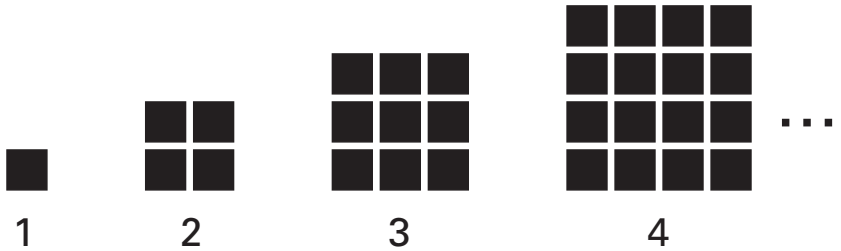


Focus Master F



Focus Master G

1. Situations a) and b) below refer to the following non-extended sequence of arrangements. For each situation, make an Algebra Piece model of the situation and then write several mathematical observations based on your model.



a) Suppose the values of 2 consecutive arrangements differ by 79 units.

b) Suppose the difference between 2 arrangement numbers is 3 and the difference between the values of the 2 arrangements is 111 units.

2. Suppose Mystery Sequence X is a nonextended sequence of arrangements of black counting pieces. To form the *n*th arrangement of Sequence X: form a rectangle that is *n* units wide and twice as long as it is wide; surround the rectangle with a 2-unit wide border (note: the outer perimeter of the border is a rectangle). Use Algebra Piece models as a basis for making mathematical observations about each of the following:

a) The 1st, 2nd, and *n*th arrangements of Sequence X.

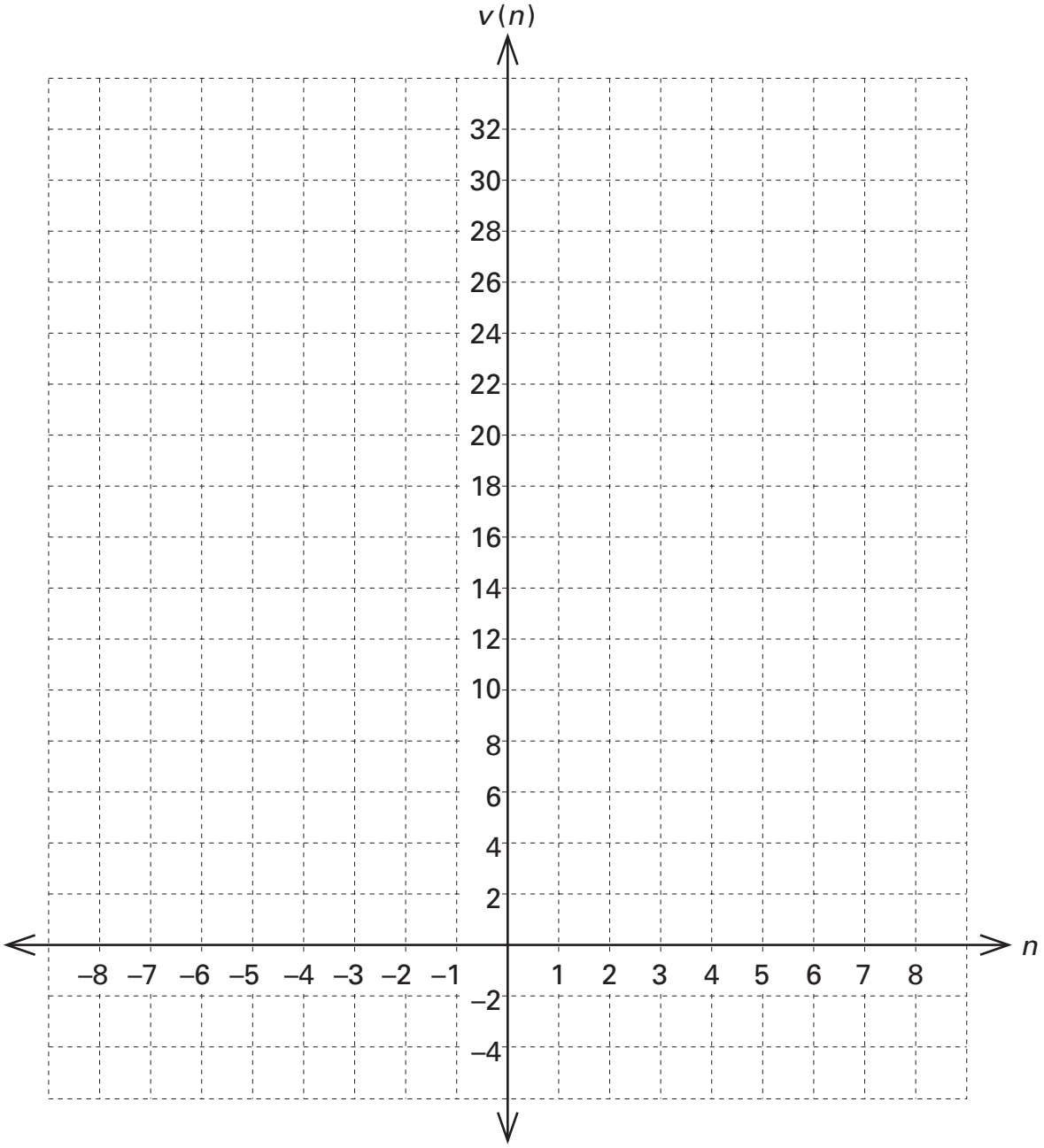
b) An arrangement whose border contains 160 units.

c) An arrangement that contains a total of 126 units.

d) Challenge. Two consecutive arrangements whose values differ by 62 units.

Focus Student Activity 11.3

NAME _____ **DATE** _____

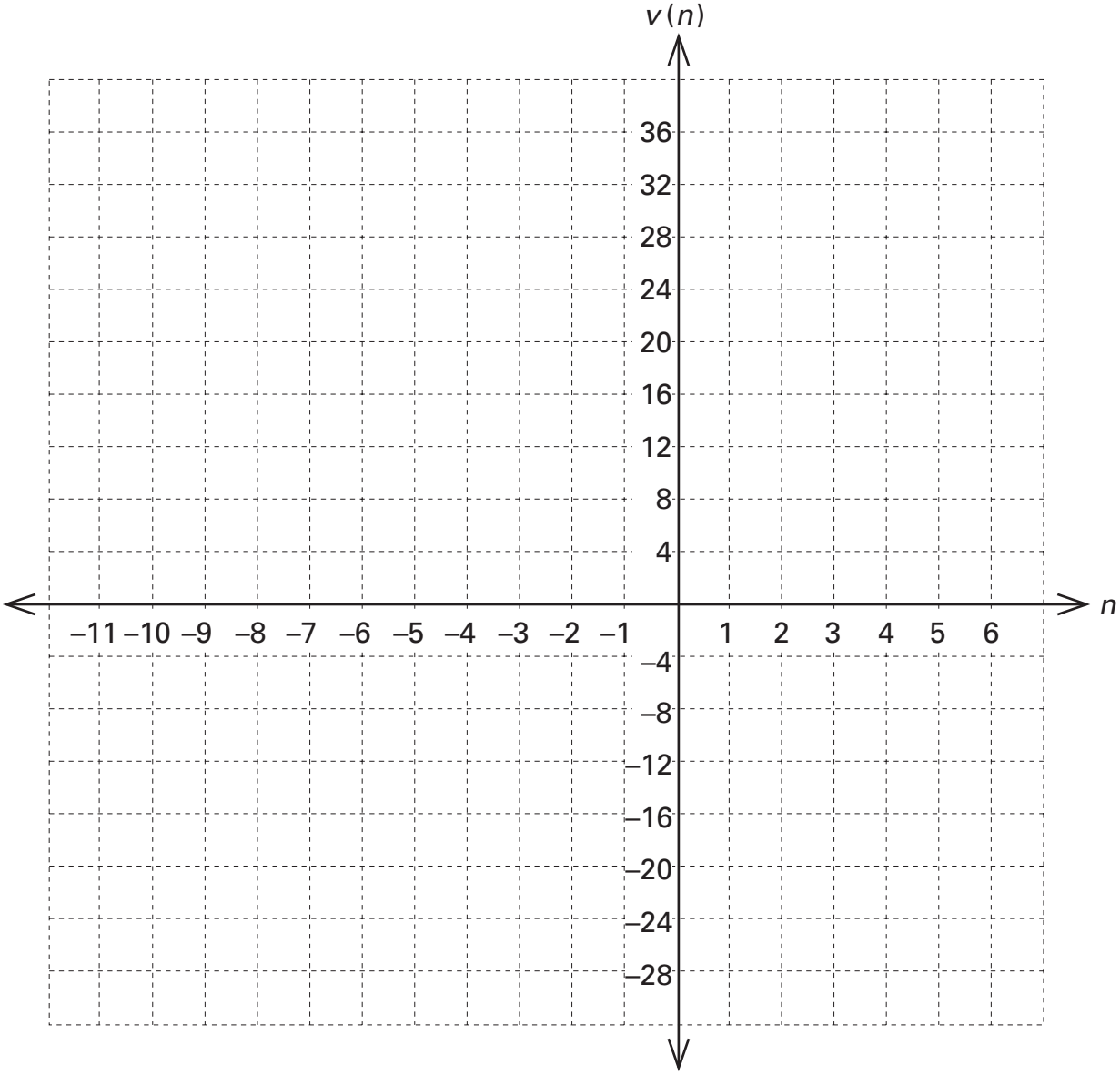


$v(n)$ = _____

Observations about the graph:

Focus Student Activity 11.4

NAME _____ **DATE** _____

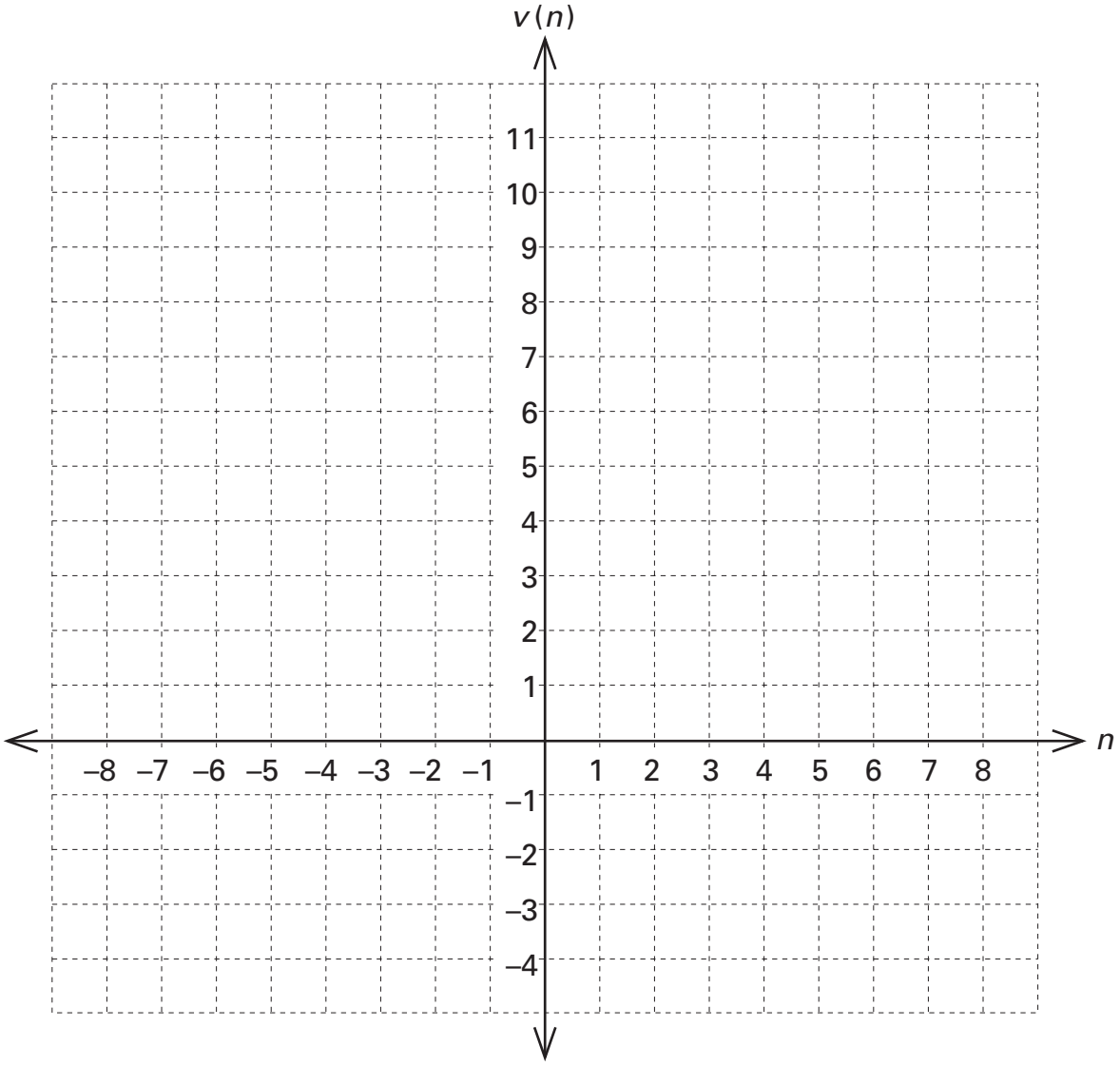


x $v_1(n)$ = _____ o $v_2(n)$ = _____

Observations about the graph:

Focus Student Activity 11.5

NAME _____ **DATE** _____

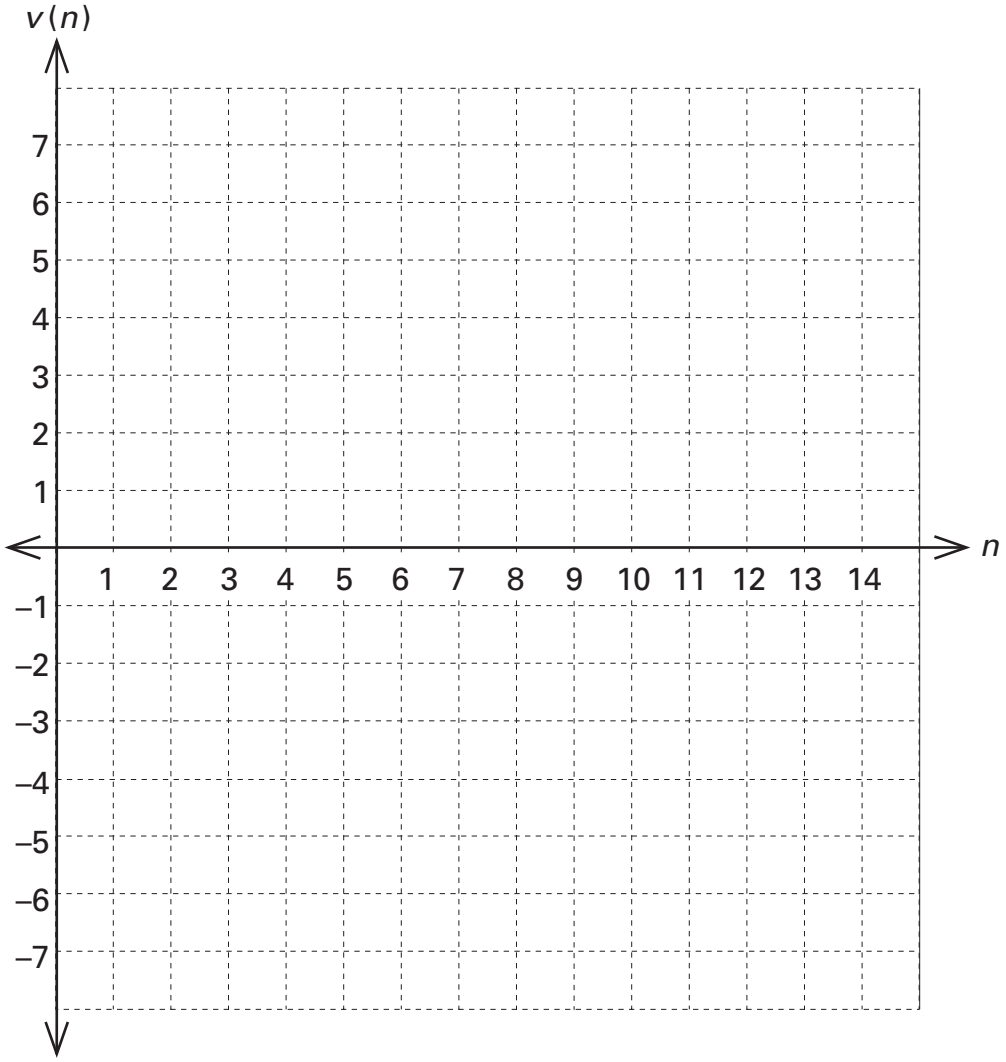


x $v_1(n)$ = _____ o $v_2(n)$ = _____

Observations about the graph:

Focus Student Activity 11.6

NAME _____ **DATE** _____



x $v_1(n)$ = _____ o $v_2(n)$ = _____

Observations about the graph:

Focus Student Activity 11.7

NAME _____ **DATE** _____

1 See the chart on page 3. Suppose that each Algebra Piece collection listed in a)-m) on the chart forms the *n*th arrangement of an extended sequence of counting piece arrangements. For each of a)-m):

i) Form the given collection of Algebra Pieces. In Column VII, write the standard quadratic form of the equation that represents the collection. Note: standard quadratic form is $v(n) = an^2 + bn + c$, where a, b, and c are integers and $a \neq 0$.

ii) If possible, form a rectangle without cutting any pieces and without changing the net value of the collection. In Column VIII, record the factored form of the quadratic equation. If no rectangles are possible, write NP (not possible) in Column VIII. Hint: it's okay to add zeros.

iii) In Column IX of the chart, record the values of n for which $v(n) = 0$.

iv) Graph $v(n)$. Be sure to plot enough points to show the shape of the graph. Label the coordinates of any zeroes of the graph.

v) Next to your graph and/or in the last column of the chart, record your observations, conjectures, and questions about factoring and graphing quadratic equations.

2 For each set of conditions a)-g) below, determine 3 different collections of Algebra Pieces which meet that set of conditions. For each collection, record the standard quadratic form and the factored form of the quadratic equation that represents the collection.

a) Each of these collections forms a square and contains both n-frames and $-n$-frames.

b) Each of these collections forms a square and contains no $-n$-frames.

c) Each of these collections forms a square and contains no n-frames.

(Continued on back.)

Focus Student Activity 11.7 (page 2)

d) These collections each form a rectangle that is twice as long as it is wide.

e) These collections each form a nonsquare rectangle with one edge whose value is n.

f) These collections each form a nonsquare rectangle that contains no n-frames or $-n$-frames.

g) These collections each form a nonsquare rectangle that contains both n-frames and $-n$-frames.

3 Record any new conjectures, questions, generalizations, and "I wonder…" statements that you have about quadratic equations and their factors.

4 Choose one idea that you recorded for Problem 1v) or for Problem 3 above. Investigate that idea further. Report your examples, reasoning, and results.

Focus Student Activity 11.7 (page 3)

	I	II	III	IV	V	VI	VII	VIII	IX	X
	n^2 mats	$-n^2$ mats	n frames	$-n$ frames	black units	red units	standard quadratic form	factored form/s	n for $v(n) = 0$	observations and conjectures
a)	1	0	8	0	7	0				
b)	1	0	0	8	7	0				
c)	1	0	8	0	0	0				
d)	1	0	5	0	6	0				
e)	0	1	0	5	0	6				
f)	1	0	7	0	6	0				
g)	1	0	0	5	0	6				
h)	1	0	5	0	0	8				
i)	2	0	0	0	0	18				
j)	1	0	0	0	0	25				
k)								$(-3n-3)(n+4)$ or $(3n+3)(-n-4)$		
l)								$(n+3)(n-3)$ or $(-n-3)(-n+3)$		
m)	1					15		$(n-3)(\quad)$ or $(\quad)(\quad)$		

Follow-up Student Activity 11.8

NAME _____ **DATE** _____

Do all work for this assignment on separate paper. Attach each problem to your work for the problem.

1 For each of a)-i) below, create an extended sequence of counting piece arrangements whose graph meets the given conditions. Then on coordinate grid paper, do the following:

i) sketch the –3rd to 3rd and *n*th arrangements of the sequence;
ii) write a formula for the value of the *n*th arrangement;
iii) graph the sequence (plot enough points that the shape of the graph is evident).

a) The graph of this sequence is linear (i.e., the points follow the path of a line) and it rises from left to right.

b) The graph of this sequence is linear, falls from left to right, and contains the point (0,2).

c) The points (1,5) and (2,8) lie on the graph of this sequence.

d) The graph of this sequence is U-shaped and the U "opens up."

e) The graph of this sequence is U-shaped and the U "opens down."

f) The graph of this sequence is a horizontal line.

g) The value of the 0th arrangement of this sequence is –3; the graph of the sequence is U-shaped, and (0,–3) is the turning point of the graph.

h) This sequence meets all of the criteria of g), but the graph of this sequence is not identical to the graph of the sequence you created for g).

i) The formula for values of negative numbered arrangements of this sequence differs from the formula for nonnegative numbered arrangements.

(Continued on back.)

Follow-up Student Activity (cont.)

2 Create 2 extended sequences whose graphs are both linear and have in common only the point (7,9). On coordinate grid paper, do the following:

a) sketch the –3rd through 3rd and *n*th arrangements of both sequences;

b) write formulas for the values of the *n*th arrangements of the sequences;

c) graph the sequences (on the same coordinate axes);

d) record pictures or algebra symbols that show your step-by-step Algebra Piece procedures for determining the value of *n* for which the *n*th arrangements of the sequences are equal.

3 Repeat Problem 2 a)-d) for 2 sequences whose graphs are parabolic (U-shaped); the turning point of each graph is (0,5) and they have no other points in common.

4 Show how to use the method "completing the square" to determine the values of *n* for which each of the following quadratic equations is true:

a) $n^2 + 2n = 35$

b) $(n - 8)(n + 2) = 0$

c) Challenge. $2n^2 - 10n = 48$

5 Use algebra symbols to record each step of your Algebra Piece methods in Problem 4b).

6 Using Algebra Pieces and using graphs are 2 methods of determining when the *n*th arrangements of 2 extended sequences of arrangements have the same value. Discuss your ideas about the advantages and disadvantages of each method.

Connector Student Activity 12.1

NAME _____ **DATE** _____

The 1st column of each table below lists, in order, the arrangement numbers of a sequence of counting piece arrangements, and the 2nd column lists the values of the arrangements. For each sequence:

a) Sketch 4 consecutive arrangements. Label the number and value of each arrangement.

b) Sketch the *n*th arrangement of the sequence.

c) On a coordinate grid, plot ordered pairs that represent several arrangements.

d) Fill in the blanks in the table.

e) Write a concise mathematical description of the set of all arrangement numbers in the sequence. Then describe mathematically the set of all numbers that are the values of the arrangements.

1

n	$v(n)$
1	4
2	9
3	14
4	19
5	24
⋮	⋮
13	__
14	__
⋮	⋮
__	154
⋮	⋮
n	$v(n)$ = _____
⋮	⋮

2

n	$v(n)$
⋮	⋮
–3	18
–2	8
–1	2
0	0
1	2
2	8
⋮	⋮
__	450
⋮	⋮
43	__
⋮	⋮
n	$v(n)$ = _____
⋮	⋮

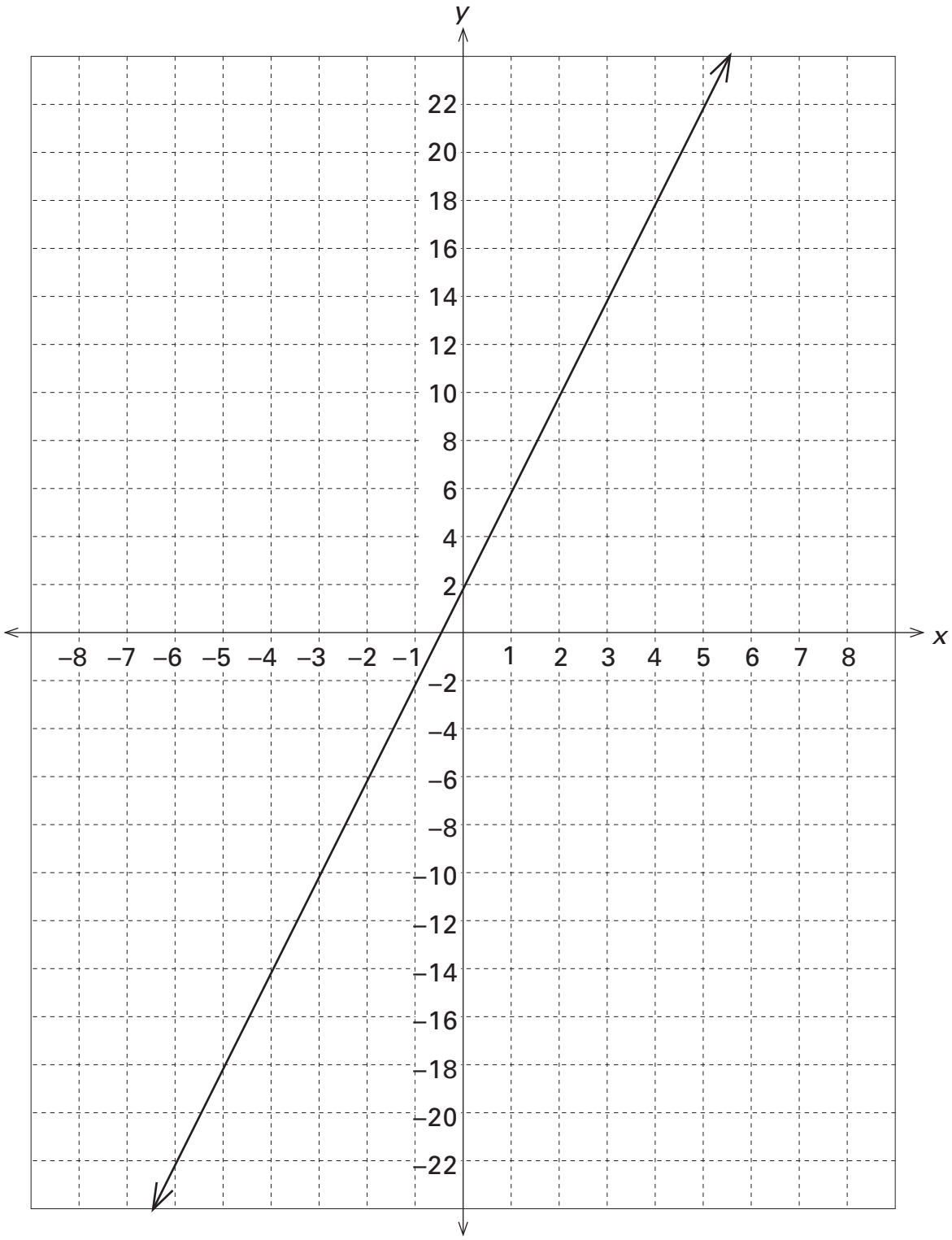
3

n	$v(n)$
–1	1
–2	2
–3	3
–4	4
⋮	⋮
n	$v(n)$ = _____
⋮	⋮

4

n	$v(n)$
1	3
2	–4
3	9
4	–8
5	15
6	–12
⋮	⋮
n	$v(n)$ = _____
⋮	⋮

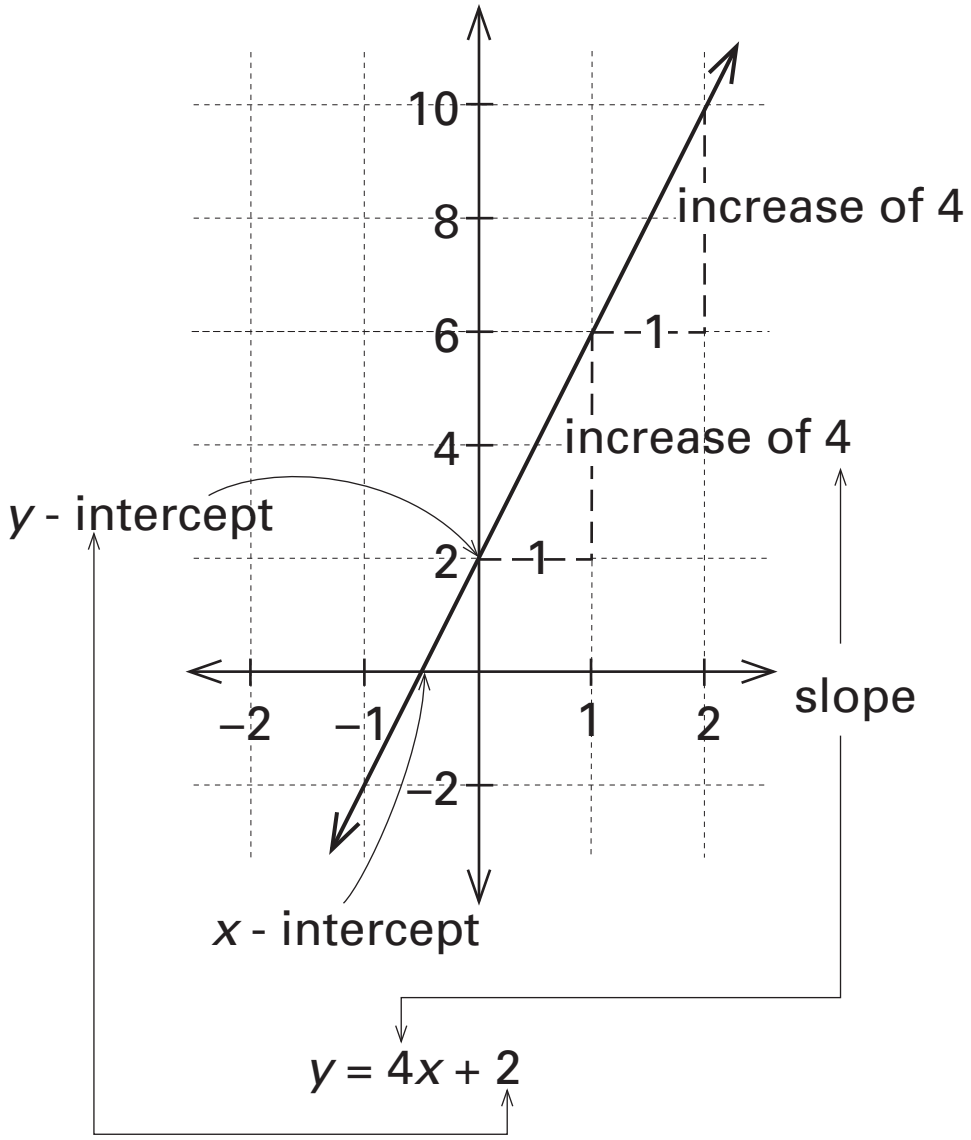
Focus Master A



$v(x)$ = _____

Sketch the xth arrangement:

Focus Master B



Focus Master C

a) Suppose that the value of the xth arrangement of a certain continuous sequence of arrangements is, $v_1(x) = \frac{1}{2}x^2 - x - 8$. Graph v_1 and determine the value of x if $v_1(x) = 4$; if $v_1(x) = -2.5$.

b) Suppose the xth arrangement of a 2nd continuous sequence has value $v_2(x) = 4$, and a 3rd has value $v_3(x) = -2.5$. Sketch the graphs of v_2 and v_3 on the same coordinate axes as v_1.

c) Circle and label the coordinates of the points where v_2 and v_3 intersect v_1. Explain how these points relate to the sequences of arrangements represented by the graphs.

d) Find all the values of x for which $\frac{1}{2}x^2 - x - 8 > 4$. Explain the relationship between these values and the sequences of arrangements for v_1 and v_2.

e) Find all the values of x for which $\frac{1}{2}x^2 - x - 8 < -2.5$. Relate these values to the sequences of arrangements for v_1 and v_3.

f) Draw a box around and label the coordinates of all x-intercepts of the graphs of v_1, v_2, and v_3. Relate your results to the sequences of arrangements.

g) Label the coordinates of the y-intercepts, if any, of the graphs of v_1, v_2, and v_3 and place a black dot at each y-intercept. Relate these points to the sequences of arrangements.

h) Place a V where you think v_1 stops decreasing and begins to increase. Explain how this turning point relates to the sequence of arrangements for v_1.

i) Challenge: give the coordinates of the turning point of v_1. Explain your methods.

Focus Master D

Each equation listed below describes the value of the xth arrangement of a continuous sequence of arrangements. For each pair of equations, complete a)-e). Record your responses to b)-e) next to each graph.

Pair 1: $v_1(x) = \frac{2}{3}x + 7$
$v_2(x) = 0$

Pair 2: $v_1(x) = 3x^2 - 9$
$v_2(x) = 0$

Pair 3: $v_1(x) = \frac{1}{2}x^2 + 3x - \frac{27}{2}$
$v_2(x) = -4x - 3$

Pair 4: $v_1(x) = (x - 4)(x - 2)$
$v_2(x) = -x^2 + 6x - 8$

a) Graph the pair on the same coordinate axes.

b) Sketch the xth arrangement of each sequence.

c) Find the value(s) of x, if any, for which $v_1(x) = v_2(x)$. Explain your methods and tell how you checked to be sure you are correct. Are the values exact or approximate?

d) Find the x-intercepts and y-intercepts, if any, of v_1 and v_2. Explain your methods and tell how you checked to be sure you are correct. Are your answers exact or approximate?

e) Write two inequality statements that describe relationships between the two graphs.

f) List your observations or conjectures.

Focus Master E

For each of the following situations, make (and label carefully) the indicated graph, and next to each graph write the following:

a) a mathematical formula that represents the relationships in the situation,

b) 2 mathematical conclusions that are based on information that you can "see" in your graph.

Situation 1 Jonathan earns $3 per hour baby-sitting. Make a graph that shows the amount Jonathan earns as related to the number of hours he works.

Situation 2 Erica's grandfather gives her $25 each year on her birthday. In addition, he gives her $1.50 for every year of her age. Make a graph that shows the amount of the birthday gift Erica receives from her grandfather as related to her age on her birthday.

Situation 3 The students in Ms. Cooper's math class are raising money for a field trip. The Whatsit Production Company has agreed to donate $5 for every hour a student works on roadside cleanup, plus an additional $10 if the student works more than 5 hours. Make a graph of the amount donated as related to the number of hours 1 student works.

Situation 4 When an object is dropped from an initial height of h_0 feet above the ground, it falls at a rate of 16 feet per second squared (Galileo observed this in 1604). Linden dropped a marble

(Continued on back.)

Focus Master E (cont.)

from the top of a 60-foot building into a pond at the base of the building. At the same time, Joel, who was standing next to Linden, dropped a brick. Make a graph that shows the height of the marble as related to the number of seconds after it is dropped. Do the same for the brick.

Situation 5 Katie and Malia designed the following game:

- The game board contains a number line. Both players start with a game marker at the origin of the number line (this is their first board position).

- Katie rolls a pair of standard dice (1 red die and 1 black) to determine where to move her marker. The number showing on the black die tells the number of spaces to move her game marker in the positive direction on the number line. The number showing on the red die tells the number of spaces to move the game marker in the negative direction.

- The number of points Katie earns is equal to the distance between the origin and her game marker after completing the moves indicated by the dice.

- Next Malia repeats the above procedures. Play continues until a player earns 25 points.

Graph all the possible ordered pairs (x,y) where x is a possible location of a player's marker after the first round of play and y is the number of points earned by landing in that position.

Focus Student Activity 12.2

NAME _____ **DATE** _____

A

B

C

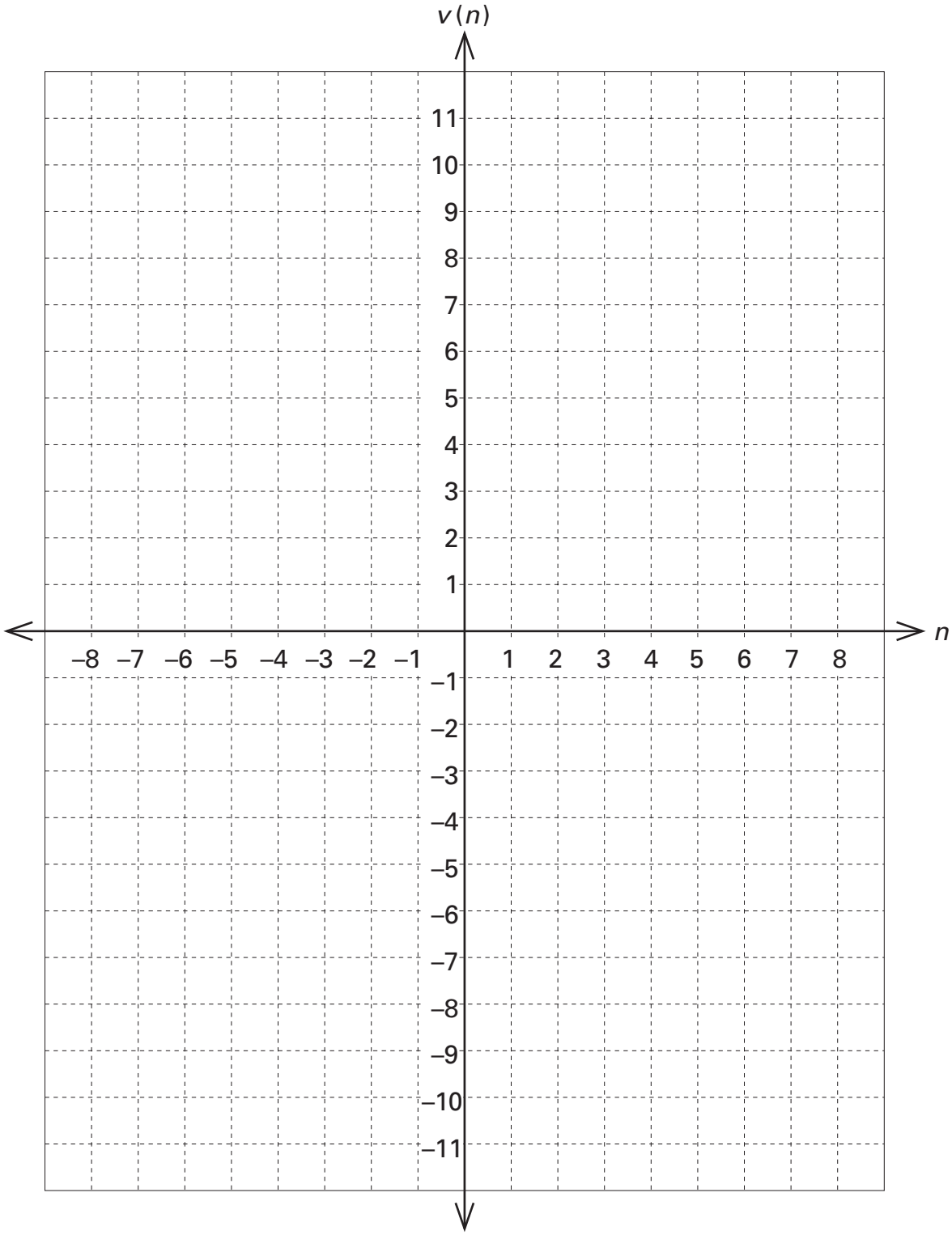
D

E

F

Focus Student Activity 12.3

NAME _____ **DATE** _____



$v(n) =$ _____

Focus Student Activity 12.4

NAME _____ **DATE** _____

Although you will have many opportunities during this course to become familiar with your graphing calculator, it will be helpful if you are comfortable with the functions listed below as soon as possible. Please investigate each function on your calculator and, if needed, in your calculator manual. A way to test yourself to see if you can comfortably use and recall a calculator function is to demonstrate its use to someone else (a family member, a classmate, a neighbor, etc.). Try to check off all functions in Part 1 below by the following date _____.

1 I am comfortable using the following calculator functions:

____ ON/OFF
____ CLEAR the screen
____ show blank coordinate axes in the calculator viewing screen
____ move the cursor around a blank coordinate axes
____ change the viewing WINDOW size
____ FORMAT the axes
____ determine the "standard" WINDOW size on my calculator (on many it is $-10 \le x \le 10$ and $-10 \le y \le 10$)
____ enter an equation y =
____ GRAPH an equation y =
____ TRACE a graph (What shows on the screen when you do this?)
____ ZOOM in on a graph
____ ZOOM in again—and again
____ ZOOM out on a graph
____ ZOOM back to the standard window
____ TRACE the graph of a function to determine the approximate value of the function at $x = 0$, $x = 19.75$, and $x = -37.5$
____ TRACE the graph of a function to determine the value of x when $y = 75$, when $y = -75$
____ GRAPH 2 equations on the same coordinate axes.
____ TRACE to approximate the intersection of 2 graphs
____ ZOOM and TRACE to improve your approximation
____ DRAW a horizontal line on coordinate axes and slide the line up and down
____ DRAW a vertical line on coordinate axes and slide the line left and right

(Continued on back.)

Focus Student Activity 12.4 (cont.)

____ view a TABLE of x- and y-coordinates of an equation
____ view a table of coordinates of 2 equations listed simultaneously
____ use a table to find when $0 = 5x + 1$
____ clear MEMory
____ reset defaults in MEMory
____ solve equations using the "solver" function from the MATH menu
____ use the "maximum" and minimum" functions from the CALC menu to find the turning point of a parabola
____ use the "intersect" function from the CALC menu to find the intersection of 2 graphs
____ use the "zero" function from the CALC menu to find the x-intercepts of a graph
____ use the "value" function from the CALC menu to find $v(x)$ for specific values of x
____ set the graphing style to shade the region above a graph; the region below a graph

2 Here are some other graphing calculator functions that I can use:

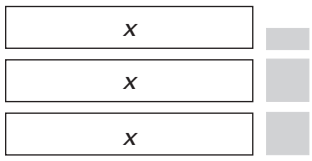
3 Here are some other functions I have tried but don't understand.

Follow-up Student Activity 12.5

NAME _____ **DATE** _____

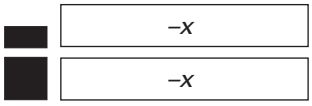
1 Using the given xth arrangement from a continuous sequence of arrangements, find the missing values in each table below. Explain how you determine the 3rd missing value in each table.

a)



x	$v(x)$
___	17
$-3\frac{1}{2}$	___
___	200
31	$90\frac{1}{2}$
x	___

b)



x	$v(x)$
55	___
___	–230
___	–121
___	178
x	___

2 For each of a)-d) below, sketch the graph of a straight line that satisfies the given conditions. Write an equation for the line.

a) slope of 3 and y-intercept of –2

b) passes through points (–2,–9) and (3,11)

c) passes through (2,1) and x-intercept is 3

d) slope is 0 and passes through (–3,–7)

3 Equations a) and b) below each represent the value of the xth arrangement of a continuous sequence of arrangements. Graph a) and b). Label the coordinates of the points for all x-intercepts and y-intercepts and the "turning points" of the graphs.

a) $v(x) = (x + 4)(x - 3)$ b) $v(x) = (2 - x)(5 + x)$

4 For each pair of equations given below: i) sketch the graphs of $v_1(x)$ and $v_2(x)$ on the same coordinate axes; ii) find a close approximation of the coordinates of the points where the graphs intersect; and iii) show the methods you use to approximate the intersection points.

a) $v_1(x) = {}^{-x}/_{2} + 8$ $v_2(x) = 3x - 5$

b) $v_1(x) = (x + 4)(x - 3)$ $v_2(x) = -x + 3$

c) $v_1(x) = x^2 - x - 12$ $v_2(x) = -x^2 + x$

(Continued on back.)

Follow-up Student Activity (cont.)

5 For each pair of graphs in Problem 4 write an inequality statement that tells when these conditions are satisfied:

i) The graph of $v_1(x)$ is above the graph of $v_2(x)$;
ii) The graph of $v_1(x)$ is below the graph of $v_2(x)$.

6 Sketch the xth arrangement of a sequence of arrangements for which $v(x) = x^2 + 6x - 7$. Arrange the pieces to illustrate the factored form of the quadratic expression $x^2 + 6x - 7$. Then sketch the graph of $v(x)$ and do the following, if possible:

a) Label the x-intercepts and y-intercepts of $v(x)$.

b) Determine the value of x if $v(x) = 14\frac{1}{4}$.

c) Find all values of x for which $v(x) < 9$.

d) Find all values of x for which $v(x) > 9$.

e) Place an M where you think $v(x)$ stops decreasing and starts increasing.

f) Explain how each of your answers for a)-e) relates to the sequence of arrangements for which $v(x) = x^2 + 6x - 7$.

7 For each of the following: i) graph the situation; ii) write 2 thoughtful mathematical questions whose answers can be determined from the graphs; iii) write the answers to your questions.

a) Maria can burn 4 calories per minute by using a treadmill. Sketch a graph that shows the amount of calories burned as related to the number of minutes she works out on the treadmill.

b) Bob's neighbor agreed to pay Bob \$8 for adjusting his lawn mower plus \$2 for every hour it runs without breaking down. Make a graph of the amount of money Bob will receive as related to the number of hours the lawn mower runs.

c) A seagull flying 80 feet above the ground drops a clam shell. The height of the shell can be represented by the equation $v(x) = 80 - 16x^2$, where x is the number of seconds since the seagull dropped the shell. Sketch a graph that shows the height as related to the number of seconds after release.

Connector Master A

For each of the following, suppose that whole square tile are used to form L-shapes.

a) Can 24 square tile be arranged in an L-shape? If so, in how many ways? What about 36 tile? 45 tile?

b) Determine ways to dissect an L-shape, using only straight cuts along edges of whole tile, so the pieces can be reassembled to form a rectangle. What is the minimum number of cuts necessary for any L-shape?

c) What rectangles can be dissected with exactly 1 straight cut (along edges of whole tile) and reassembled to form an L-shape?

d) What counting numbers can be written as the difference of 2 perfect squares? of 2 consecutive perfect squares? Why?

Focus Master A

Situations

a) The people at a meeting are separated into 2 groups.
The 1st group has 5 less people than 3 times the number in the 2nd group.
There are 43 people at the meeting.

b) There are 3 numbers.
The 1st number is twice the 2nd number.
The 3rd is twice the 1st.
The sum of the 3 numbers is 112.

c) The sum of 2 numbers is 40.
Their difference is 14.

d) The sides of square A are 2 inches longer than the sides of square B.
The area of square A is 48 square inches greater than the area of square B.

e) Melody has $2.75 in dimes and quarters.
She has 14 coins altogether.

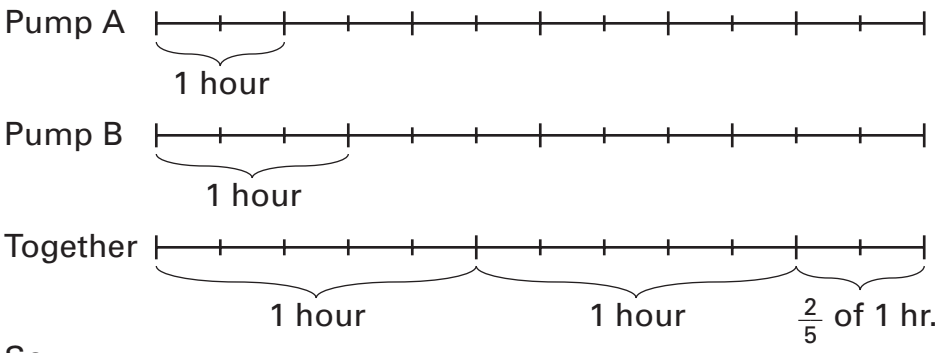
f) Three particular integers are consecutive.
The product of the 1st and 2nd integers is 40 less than the square of the 3rd integer.

g) Karen is 4 times as old as Lucille.
In 6 years, Karen will be 3 times as old as Lucille.

Focus Master B

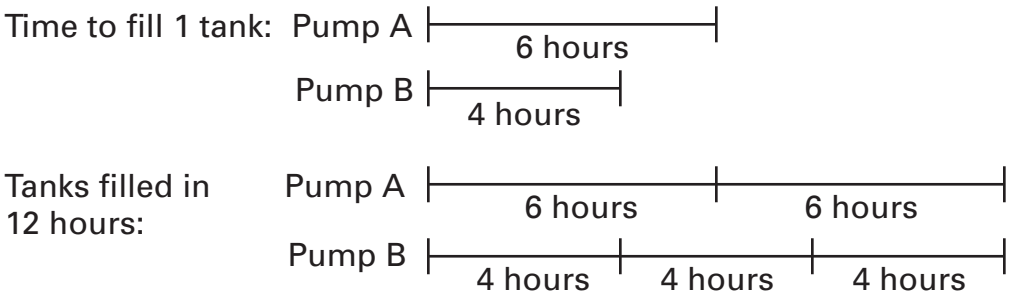
One pump can fill a tank in 6 hours. Another pump can fill it in 4 hours. If both pumps are used, how long will it take to fill the tank?

Solution 1



So…

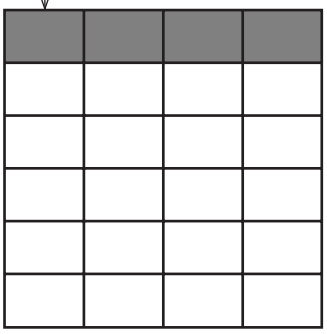
Solution 2



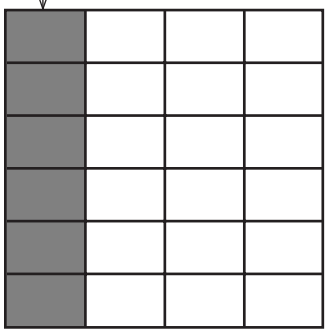
Pumps A and B fill 5 tanks in 12 hours; so…

Solution 3

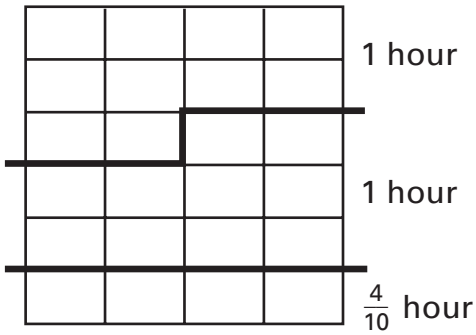
Pump A fills $\frac{1}{6}$ tank in 1 hour.



Pump B fills $\frac{1}{4}$ tank in 1 hour.



Pump A fills 4 subdivisions in 1 hour.
Pump B fills 6 subdivisions in 1 hour.
Together, they fill 10 subdivisions in 1 hour:



So…

Focus Master C

More Situations

a) A tank has 2 drains of different sizes.
If both drains are used, it takes 3 hours to empty the tank.
If only the first drain is used, it takes 7 hours to empty the tank.
On Tuesday only the 2nd drain is used to empty the tank.

b) Yesterday Maria and Lisa together had 20 library books.
Today Maria and Lisa visited the library; Lisa checked out new books and now has double the number of books that she had yesterday; Maria returned 3 of her books.
Now Maria and Lisa together have 30 books.

c) Of the students in Ms. Quan's class, $\frac{3}{5}$ are girls.
Ms. Nelson's class joined Ms. Quan's for a project; this doubled the number of boys and increased the number of girls by 6.
There are an equal number of boys and girls in the combined class.

d) On Moe's walk home from school, after 1 mile he stopped for a drink of water.
Next, Moe walked $\frac{1}{2}$ the remaining distance and stopped to rest at the park bench.
When Moe reached the park bench, he still needed to walk 1 mile more than $\frac{1}{3}$ the total distance from school to his home.

e) Jill has a gallon of paint that contains 20% red paint and 80% blue paint.
Jill adds more red paint until she has 50% red paint.

f) Standard quality coffee sells for $18.00 per kg.
Prime quality coffee sells for $24.00 per kg.
Every Saturday morning Moonman's Coffee Shop grinds a 40kg batch of a standard/prime blend to sell for $22.50/kg.

(Continued on back.)

Focus Master C (cont.)

g) Alex's collection of nickels, dimes, and quarters has 3 fewer nickels than dimes and 3 more quarters than dimes.
Alex's collection is worth $4.20.

h) For a school play, Kyle sold 6 adult tickets and 15 student tickets.
Kyle collected $48 for his ticket sales.
Matt sold 8 adult tickets and 7 student tickets for the same school play.
Matt collected $38 for his ticket sales.

i) The doctor mixed a 1200 ml of an 85% sugar solution (i.e., the container is 85% sugar and the rest is water).
The nurse added enough of a 40% sugar solution to create a 60% solution.

j) On Wednesday, Steve drove from Gillette to Spearfish in 1 hr. and 30 min.
On Thursday, driving 8 miles per hour faster, Steve made the return trip in 1 hr. and 20 min.

k) Michael averaged 78 points on 3 history tests.
His score on the 1st test was 86 points.
His average for the 1st 2 tests was 3 points more than his score on the 3rd test.

l) Traveling by train and then by bus, a 1200 mile trip took Wally 17 hours.
The train averaged 75 mph and the bus averaged 60 mph.

Focus Master D

a) The difference between 2 numbers is 6.
The sum of their squares is 1476.

b) The length of a rectangle is 6 units less than twice its width.
Its area is 836 square units.

c) The product of 2 consecutive even numbers is 2808.

d) The perimeter of a certain rectangle is 92 linear units.
The area of the rectangle is 493 square units.

e) The sum of 2 numbers is 32.
The sum of the squares of the numbers is 520.

f) A 40 foot by 60 foot rectangular garden is bordered by a sidewalk of uniform width.
The area of the sidewalk is 864 square feet.

Focus Master E

Situation A

There are several lifeboats on the USS Mathstar, and each lifeboat has enough rations to last 120 people for 8 days. The number of days that rations will last varies inversely with the number of people in a lifeboat. In a storm, the USS Mathstar sinks; 40 people climb aboard Lifeboat A.

Situation B

Ollie's wages vary directly with the time he works. Last week his wages for 20 hours of work were $350. This week he will work 44 hours.

Focus Master F

Puzzle Problems

1. For each of the following problems, make sketches that illustrate the type of variation given. Then show how to reason from your sketches to solve the problem. Mark your sketches and write equations to show your thinking and calculations.

a) The amount of profit for burger sales at Al's Burger Bar varies directly with the number of burgers sold. During the lunch rush yesterday Al sold 85 burgers and made $93.50 profit. During lunch today, Al sold 50 burgers. What was his profit today?

b) Each year the Math Club receives the same grant for students to attend the state math contest. The amount each student receives varies inversely with the number of Math Club students who attend the contest. Last year 12 students each received $25. If 16 students attend this year, what amount will each receive?

c) The amount raised during the school magazine subscription sale varies directly with the number of sales. This week the students raised $630 from sales of 450 subscriptions. If they sell 550 subscriptions this week, how much will they earn?

d) The distance an object falls in a given time varies directly as the square of the time. A certain object falls 64 feet in 2 seconds and hits the ground in 5 seconds. From what height did the object fall?

e) The current in a simple electrical circuit varies inversely with the resistance. If the current is 20 amps when the resistance is 5 ohms, what is the current if the resistance is 8 ohms?

(Continued on back.)

Focus Master F (cont.)

f) To predict the weight of his sculptures, Jim noticed that for his latest design, the weight of a statue seems to vary directly as the cube of its height. What will Jim predict a 5 foot statue will weigh, if a statue that is $1\frac{2}{3}$ feet high weighs 15 pounds?

g) Assuming that the temperature does not change, the pressure a gas exerts varies inversely with the volume of the gas. If a gas has a volume of 76 cubic inches when the pressure is 16 pounds per square inch, what is the volume when the pressure is 64 pounds per square inch?

h) The weight of an object above the surface of the earth varies inversely as the square of its distance from the center of the earth. How much does a person who weighs 100 pounds on earth weigh when 1000 miles above the earth's surface? Assume the radius of the earth to be 4000 miles.

i) Challenge. The length of a piece of wire varies directly as the weight of the wire and inversely as the square of the diameter of its cross section. If a 100 foot piece of wire weighs 6 pounds and has a $\frac{1}{8}$ inch diameter, how long is 9 pounds of wire of the same material but with a $\frac{1}{16}$ inch diameter?

2. Describe a situation (other than those given in Problem 1) from everyday life that involves direct variation and a situation that involves inverse variation.

Follow-up Student Activity 13.1

NAME _____ **DATE** _____

1 Investigate the following situation and write a detailed summary of your investigation, including: what you do and how you do it (don't forget to tell what doesn't work, as well as what does), what conclusions you make and how you know they are true, what you conjecture and why, what you wonder about when you finish, and how long you spend on the investigation.

If rectangles are formed on grids so the edges of the rectangles lie along grid lines, some rectangles can be cut along grid lines to form 2 congruent staircases and others cannot. Note: assume 1 stairstep is a 1×1 square of the grid.

2 For each of the following puzzle problems, make a sketch that illustrates the mathematical conditions of the problem. Then reason from your sketch to solve the problem. Mark each sketch to show your thinking and reasoning. If needed to fully communicate your thought processes and calculations, add brief comments next to each diagram.

a) If each side of Square X increases by 3 feet, the area increases by 63 square feet. What is the perimeter of Square X?

b) Two cars start from points 400 miles apart and travel toward each other. They meet after 4 hours. Find the average speed of each car if one travels 20 miles per hour faster than the other.

c) An ice-skating rink is 30 meters by 20 meters. Plans are made to double the rink's area by first adding a rectangular strip along one end of the rink, and then adding a strip of the same width along one side retaining a rectangular shape for the rink. What will be the width of these strips?

d) Four times the larger of 2 numbers exceeds their sum by 25; four times the smaller number exceeds their difference by 1. What are the numbers?

e) The length of a room is 3 feet more than its width. If the length increases by 3 feet and the width decreases by 2 feet, the area of the floor does not change. What are the dimensions of the room?

(Continued on back.)

Follow-up Student Activity (cont.)

f) Kay added a set of consecutive integers, $1 + 2 + 3 + \ldots + n$, to get the total 990. What is n?

g) If the length of each side of a square is decreased by 20%, the area is decreased by 72 square inches. What is the length of an edge of the original square.

h) What are 3 consecutive odd numbers whose sum is 213?

i) What is the length of the side of a square whose diagonal is 10 inches longer than the side?

j) At Henry High School, 1 less than $\frac{1}{5}$ of the students are seniors, 3 less than $\frac{1}{4}$ are juniors, $\frac{7}{20}$ are freshmen, and the remaining 28 students are sophomores. How many students attend Henry High?

k) If 40 cc of a 40% acid solution, 70 cc of a 50% acid solution, and 50 cc of pure acid are combined, what % acid solution results?

l) How many cubic centimeters of pure sulfuric acid must be added to 100 cc of a 40% solution to obtain a 60% solution?

m) If 8 shillings and 5 francs are worth \$2.14, and 9 shillings and 70 francs are worth \$15.54, what is the value of a shilling and the value of a franc?

n) A bag contains only white balls and black balls. Ten more than $\frac{1}{2}$ the total number of balls are black, and 6 more than $\frac{1}{2}$ the number of black balls are white. If 1 ball is randomly selected at random from the bag, what is the probability it will be white?

3 Select 3 or more of the puzzle problems from Problem 2 and write algebraic equations to represent important parts of your sketches and the steps of your thought processes. Be sure to tell what each variable and equation represents.

4 Use diagrams and brief explanations to show what each of the following means: a) direct variation and b) inverse variation. For each of a) and b), describe an everyday situation involving that type of variation and make a diagram that illustrates the mathematical relationships in the situation.

Focus Master A

I $y = -3x + 5$
II $y = -x + 5$
III $y = x + 5$
IV $y = 3x + 5$

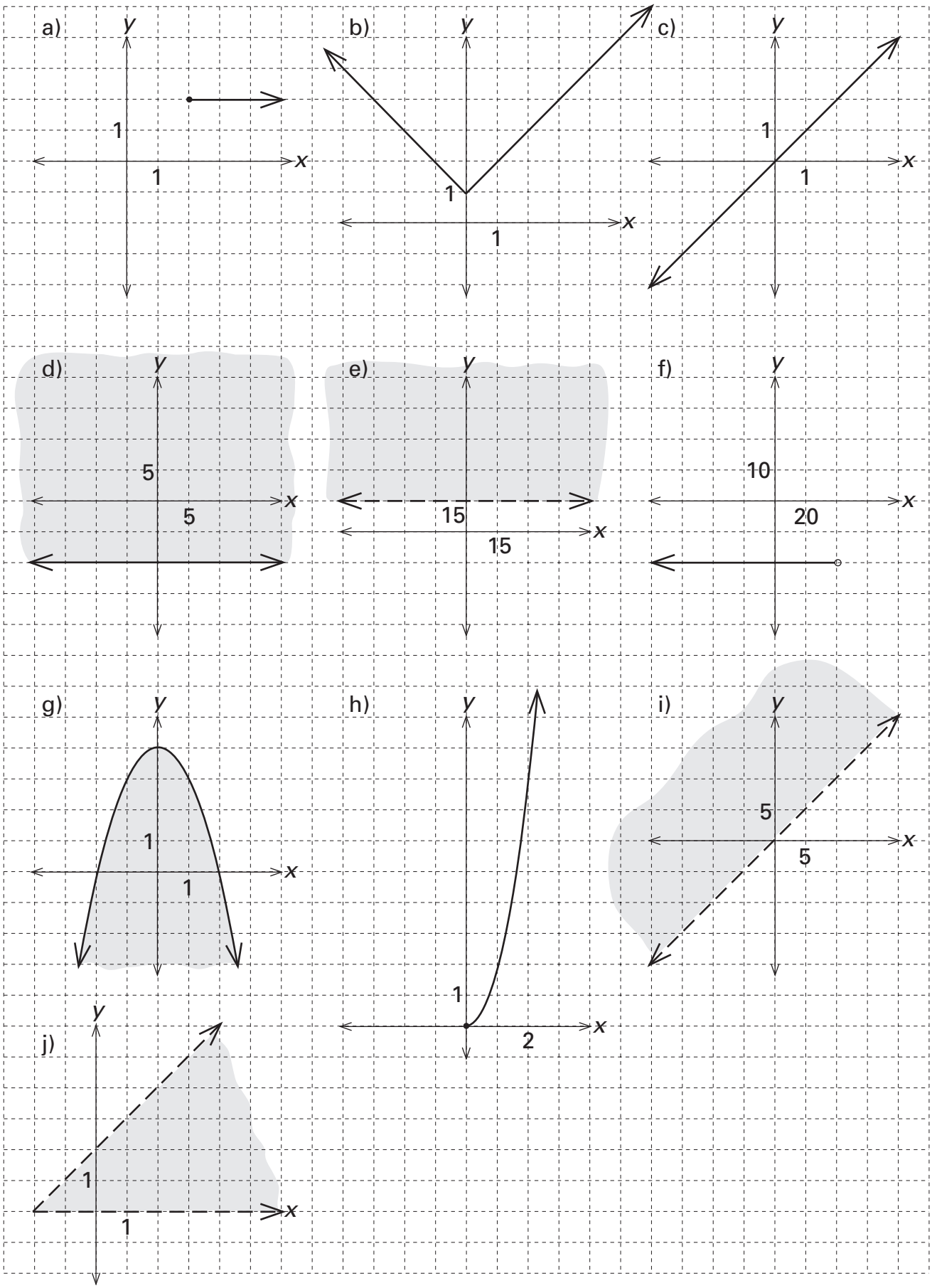
a) Imagine the graph of each of equations I-IV. What similarities and differences does your group *predict* about the graphs?

b) Now graph the 4 equations simultaneously on your graphing calculators. Do the results agree with your predictions? What else do you notice?

c) Equations I-IV are a "family" of equations. What characteristic(s) do you think make these equations a family? What are two other equations that could belong to this family?

d) What are similarities and differences among Algebra Piece representations of the xth arrangements of the sequences represented by equations I-IV?

Focus Master B



Focus Master C

Franko and his son, Marcus, plan to race one another on a track.

Marcus can run 20 meters in 5 seconds.

Franko can run 20 meters in 3 seconds.

They have agreed that Marcus will start 30 meters ahead of Franko.

Write several “We wonder…” statements about this situation.

Focus Master D

For each of the 3 Situations shown below, please do the following:

a) Make a diagram or sketch that illustrates the important mathematical relationships in the situation.

b) Write 3 or more worthwhile mathematical questions that a person might investigate about the situation.

c) Investigate one or more of your mathematical questions. While you investigate each question, keep a running account of your thought processes. Make note of your discoveries, stuck points, AHA's, important mathematical moments, changes in direction, etc.

d) After you complete your investigation of each question, write a summary that includes a restatement of the question, a clear and concise explanation of a solution process (this might be a refined version of your method or a different method you discovered during your work), your answer to the question, and verification that your answer works.

Situation 1
The Rent-A-Wreck and the We Hardly Try car rental companies charge the following prices:

We Hardly Try charges an initial fee of $10 and then charges $.10 per mile. Rent-A-Wreck does not charge an initial fee, but charges $.15 per mile.

Situation 2
The Saucey Pizza Company charges $7 for a pizza. The ingredients and labor for each pizza cost $2.50. The overhead costs (lights, water, heat, rent, etc.) are $100 per day.

Situation 3
Michael, the golf pro at U-Drive-It Golf Range, claims that when he hits the ball from the lower level tee, the height h of the ball after t seconds is:
$h = 80t - 16t^2$.

Michael also claims that when he hits the ball from the elevated tee, the ball reaches the following height in t seconds: $h = 20 + 80t - 16t^2$.

Focus Student Activity 14.1

NAME _____ **DATE** _____

For each equation family below, record the following on separate paper:

a) your predictions about the graphs of the 4 equations,

b) your observations about calculator graphs of the equations,

c) the characteristic(s) that you think make the equations a family,

d) two additional equations that would fit in the family,

e) similarities and differences among Algebra Piece representations of the 4 equations.

1
I $y = -3x + 5$
II $y = -3x - 5$
III $y = -3x + 2$
IV $y = -3x - 2$

2
I $y = x^2 - 6$
II $y = x^2 + 6$
III $y = -x^2 - 6$
IV $y = -(x^2 - 6)$

3
I $y = 4x^2$
II $y = (1/4)x^2$
III $y = -3x^2 - 6$
IV $y = (-3/4)x^2$

4
I $y = x(x - 3)$
II $y = x^2 - 2x$
III $y = x^2 + 2x$
IV $y = x(x + 3)$

5
I $y = (x - 3)(x - 4)$
II $y = x^2 - 7x + 12$
III $y = (x + 2)(x + 3)$
IV $y = (x + 1)(x - 2)$

6
I $y = (x - 2)(x - 5)$
II $y = 2(x - 2)(x - 5)$
III $y = -(x - 2)(x - 5)$
IV $y = -2(x^2 - 7x + 10)$

7
I $28x + 8y = 0$
II $7x + 2y = 6$
III $14x + 4y = 4$
IV $21x + 6y = -12$

8
I $y = -5x + 2/3$
II $3y = -15x + 2$
III $0 = -5x - y + 2/3$
IV $-2 = -15x - 3y$

Focus Student Activity 14.2

NAME _____ **DATE** _____

For each pair of equations given in a)-g):

i) Sketch and label counting piece arrangements to represent y_1 for $x = -3, -2, -1, 0, 1, 2$, and 3, and sketch an Algebra Piece representation of the xth arrangement of y_1. Repeat for y_2.

ii) Make a table that shows the corresponding values of x, y_1, and y_2 for the arrangements formed in i).

iii) Predict how you think the graphs of y_1 and y_2 will look in comparison to the graph of $y = |x|$. Then, on the same coordinate axes, sketch a graph of y_1, y_2, and $y = |x|$ over the domain of real numbers such that $-10 \le x \le 10$. Label the coordinates of the points from the table in ii).

iv) On the same coordinate axes of the graphing calculator, graph y_1, y_2, and $y = |x|$ over the domain given in iii).

v) Write 3 additional equations that form a family with y_1, and y_2, and explain the relationship that makes the 5 equations a family.

vi) Record conjectures and generalizations based on your observations from i)-v).

a) $y_1 = |x| - 3$ $\quad$ $y_2 = |x| + 4$

b) $y_1 = |x - 5|$ $\quad$ $y_2 = |x + \frac{1}{2}|$

c) $y_1 = |2x|$ $\quad$ $y_2 = |-2x|$

d) $y_1 = 5|x|$ $\quad$ $y_2 = -5|x|$

e) $y_1 = |x - 2x|$ $\quad$ $y_2 = |3x + 2|$

f) $y_1 = |x| - |2x|$ $\quad$ $y_2 = |3x| + |2|$

g) $y_1 = \frac{1}{x}$ $\quad$ $y_2 = \frac{3}{x}$

Follow-up Student Activity 14.3

NAME _____ **DATE** _____

1 For each of the following families of 3 equations, graph the equations on 1 coordinate axis, and list the characteristics that make the equations a family. Then create and graph 2 or more additional equations that have those characteristics. Label each graph with its equation.

a) $y = 4x - 1$
$y = 4x + 2$
$y = 4x - 5$

b) $y = 3x - 2$
$y = \frac{x}{5} - 2$
$y = -6x - 2$

c) $y = x^2 + 3$
$y = -2x^2 + 3$
$y = \frac{x^2}{4} + 3$

d) $y = |x| + 2$
$y = -3|x|$
$y + 2|x| - 3$

e) $y = \frac{1}{x} + 5$
$y = \frac{1}{x} - 4$
$y = \frac{1}{x} + 3$

f) $y = 4(x - 2)(x + 5)$
$y = -3(x - 2)(x + 5)$
$y = (x - 2)(x + 5)$

2 For each of the following systems of equations: i) show or describe how you solve the system; ii) sketch a graph of the system on coordinate grid paper; and iii) label the coordinates to the nearest tenth of all points of intersection.

a) $6x + 3y = 5$
$2y - 3x = 12$

b) $y = x^2 - 8x + 18$
$2x + y = 7$

c) $y = \frac{2}{x} + 3$
$y = x^2 + 4$

d) $y = \frac{x^2}{2} - \frac{x}{2} + 3$
$y = -x^2 - 3x + 5$

e) $y = x^2 - 3x - 2$
$x^2 + 3y - 18 = 0$

f) $y = -x^2 - x - 2$
$y = (x + 1)(x - 2)$

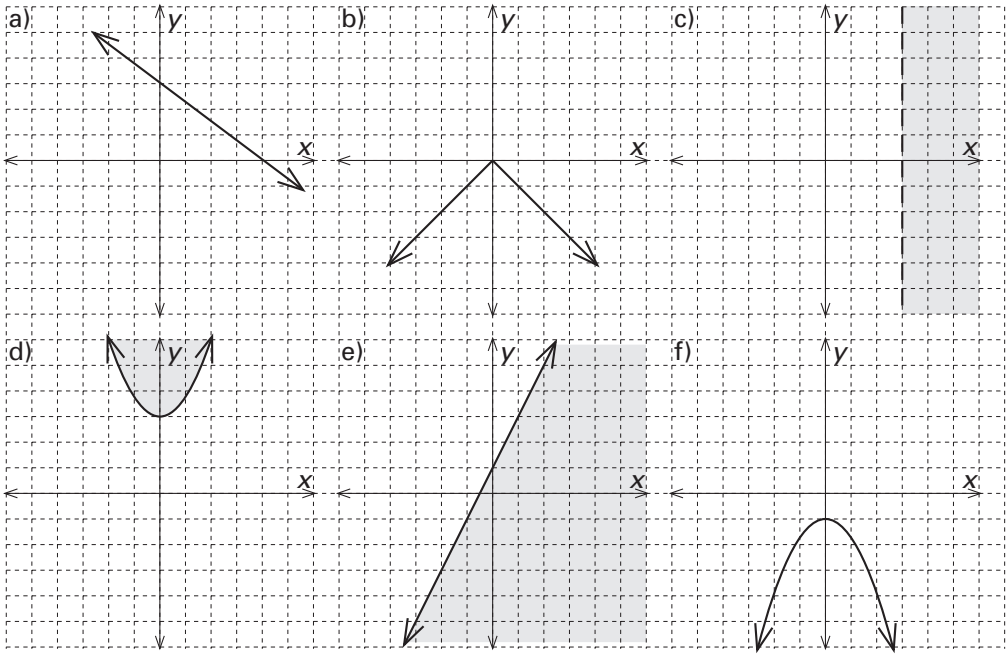
3 Verify your solutions to Problems 2a) and 2b) by solving each using another method. Show your thinking and reasoning.

4 Discuss the advantages and disadvantages of using the graphing calculator to solve equations. Give examples to illustrate your ideas.

(Continued on back.)

Follow-up Student Activity (cont.)

5 Write a mathematical statement involving equalities or inequalities to describe each graph. Tell whether the graph represents a function; if it does, tell the domain and range of the function.



6 During the summer Patty rents an ice cream truck and sells ice cream; she pays $200 per month to rent the truck; she sells ice-cream bars for $1.50 each; and she pays 25 cents for each bar.

a) Write an equation for y_1, Patty's expenses per month, if she sells x ice cream bars per month.

b) Write an equation for y_2, Patty's monthly income (before she pays her expenses) for selling x ice cream bars per month.

c) Graph the two equations in a) and b) and label the coordinates of the intersection of the graphs. Explain how you determine these coordinates and how they relate to the given situation.

d) What is the minimum number of ice cream bars Patty must sell in one month in order for her income to be greater than her expenses. Explain your reasoning.

e) If Patty sells 300 ice cream bars in one month, how much profit will she make? Explain your reasoning.

Connector Student Activity 15.1

NAME _____ **DATE** _____

Period 1—Length of Time in Minutes Between Eruptions of Old Faithful

57, 87, 73, 94, 52, 88, 72, 88, 62, 87, 57, 94, 51, 98, 59

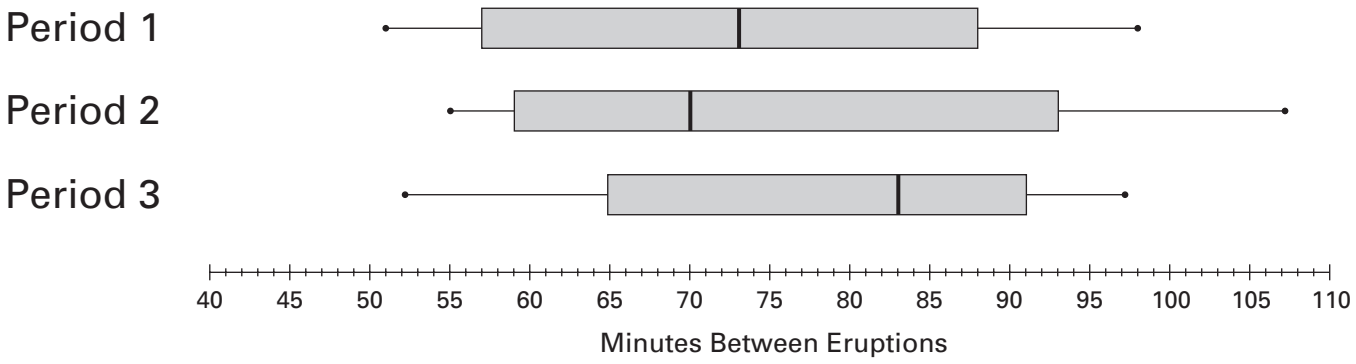
Regarding the above data,

I notice…

I wonder…

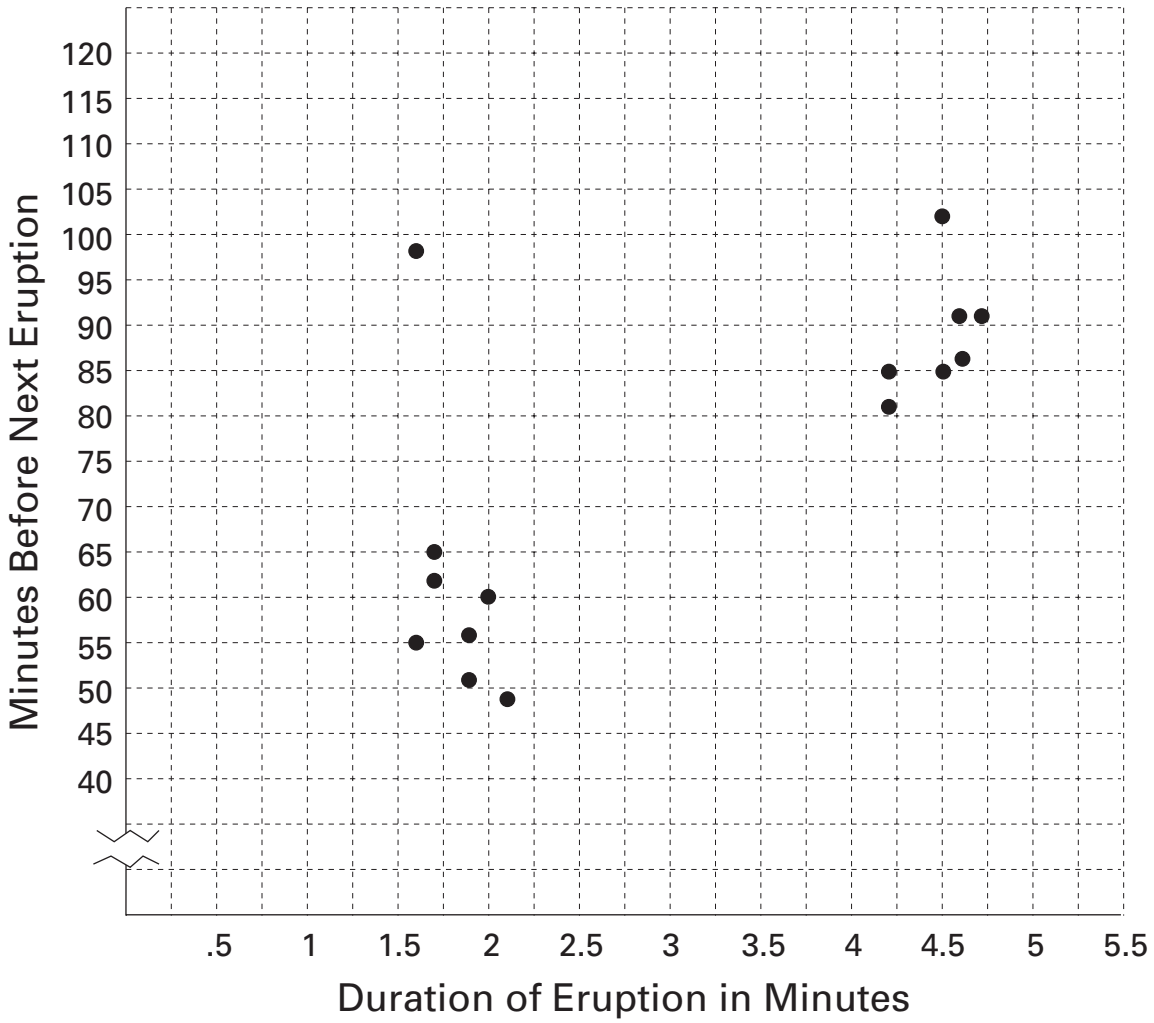
Focus Master A

Old Faithful Data



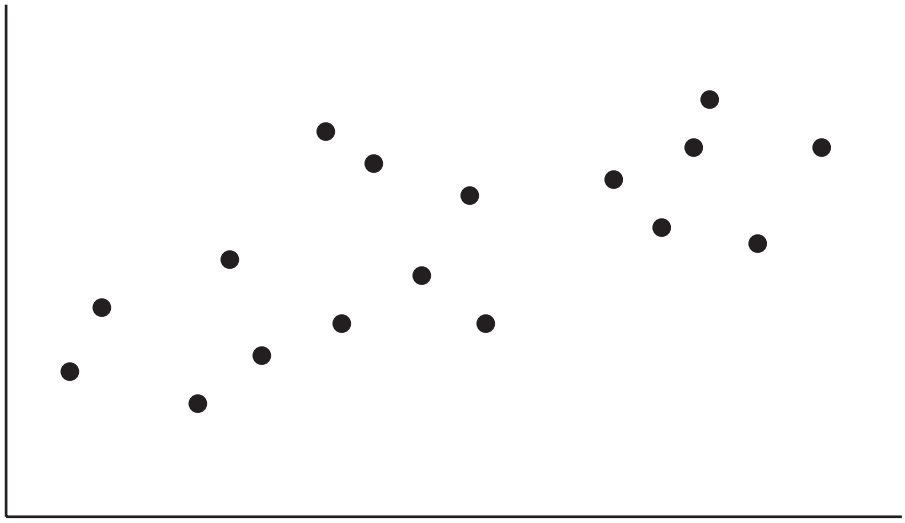
Focus Master B

Eruptions of Old Faithful—April 1997



Focus Master C

Constructing a Median-Fit Line



Focus Master D

U.S. Data*	Percentage of Population Not High School Graduates (1994, to nearest percent)	Average Amount of Money Spent for Education Per Student (1993, in thousands of dollars)	Percentage of Population Below Poverty Level (1993, to tenth of a percent)
Northeast, Mid-Atlantic			
Maine	21	6.1	15.4
New Hampshire	18	5.6	9.9
Vermont	19	6.7	10.0
Massachusetts	20	6.6	10.7
Rhode Island	28	6.6	11.2
Connecticut	21	8.2	8.5
New York	25	8.8	16.4
New Jersey	23	9.5	10.9
Pennsylvania	25	6.9	13.2
Delaware	23	6.4	10.2
Maryland	22	6.4	9.7
District of Columbia	27	8.0	26.4
West Virginia	34	5.7	22.2
Midwest			
Ohio	24	5.3	13.3
Indiana	24	5.4	12.2
Illinois	24	5.4	13.6
Michigan	23	6.4	15.4
Wisconsin	21	6.5	12.6
Minnesota	18	5.6	11.6
Iowa	20	5.2	10.3
Missouri	26	4.5	16.1
North Dakota	23	4.9	11.2
South Dakota	23	4.4	14.2
Nebraska	18	4.9	10.3
Kansas	19	5.5	13.1
South			
Virginia	25	5.3	9.7
North Carolina	30	4.9	14.4
South Carolina	32	4.7	18.7
Georgia	29	4.7	13.5
Florida	26	5.3	17.8
Kentucky	35	4.9	20.4
Tennessee	33	4.0	19.6
Alabama	33	3.8	17.4
Mississippi	36	3.4	24.7
Arkansas	34	3.8	20.0
Louisiana	32	4.3	26.4
Oklahoma	25	4.1	19.9
Texas	28	4.9	17.4
West			
Montana	19	5.5	14.9
Idaho	20	4.0	13.1
Wyoming	17	5.8	13.3
Colorado	16	5.1	9.9
New Mexico	25	4.6	17.4
Arizona	21	4.1	15.4
Utah	15	3.2	10.7
Nevada	21	4.9	9.8
Washington	16	5.5	12.1
Oregon	19	6.1	11.8
California	24	4.6	18.2
Alaska	13	9.3	9.1
Hawaii	20	5.8	8.0

*Statistical Abstract of the United States, 115th ed. (Washington: Bureau of the Census, 1995), pp.159, 168, 461, 482.

Focus Student Activity 15.2

NAME _____ **DATE** _____

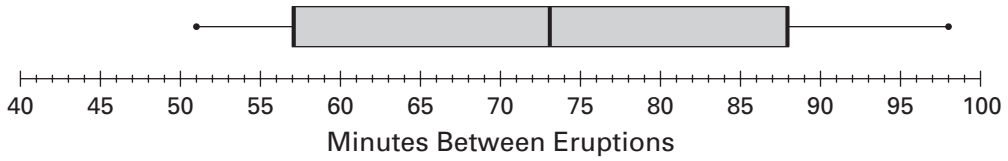
Period 1—Length of Time in Minutes Between Eruptions of Old Faithful

Data in order collected:

57, 87, 73, 94, 52, 88, 72, 88, 62, 87, 57, 94, 51, 98, 59

Data in numerical order:

Box and Whisker Plot



Observations:

Focus Student Activity 15.3

NAME _____ **DATE** _____

Period 1—Length of Time in Minutes Between Eruptions of Old Faithful

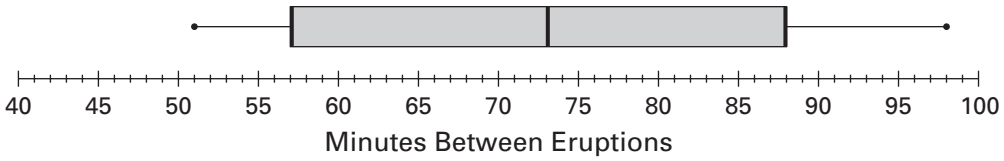
Data in order collected:

57, 87, 73, 94, 52, 88, 72, 88, 62, 87, 57, 94, 51, 98, 59

Data in numerical order:

51, 52, 57, 57, 59, 62, 72, 73, 87, 87, 88, 88, 94, 94, 98

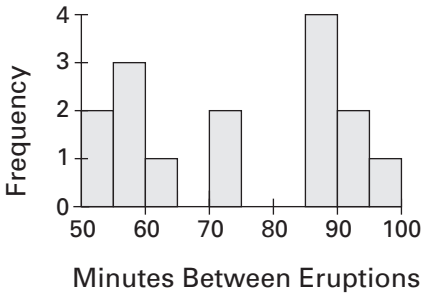
Box and Whisker Plot



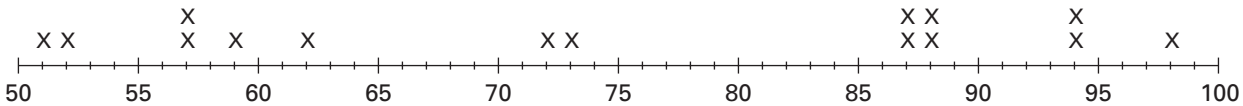
Stem and Leaf Plot

5	1,2,7,7,9
6	2
7	2,3
8	7,7,8,8
9	4,4,8

Histogram



Line Plot



Observations:

Average deviation(s):

Method(s):

Focus Student Activity 15.4

NAME _____ **DATE** _____

Period 2—Length of Time in Minutes Between Eruptions of Old Faithful

Data in order collected:

93, 86, 70, 63, 91, 82, 58, 97, 59, 70, 58, 98, 55, 107, 61

Data in numerical order:

Box and Whisker Plot

Stem and Leaf Plot

Histogram

Line Plot

Observations:

Average deviation(s):

Method(s):

Focus Student Activity 15.5

NAME _____ **DATE** _____

Period 3—Length of Time in Minutes Between Eruptions of Old Faithful

Data in order collected:

82, 91, 65, 97, 52, 94, 60, 94, 63, 91, 83, 84, 71, 83, 70

Data in numerical order:

Box and Whisker Plot

Stem and Leaf Plot

Histogram

Line Plot

Observations:

Average deviation(s):

Method(s):

Focus Student Activity 15.6

NAME _____ **DATE** _____

1 The following data about Old Faithful was recorded, in order, in April 1997. Form a scatter plot of this data by placing the duration of eruptions along the horizontal axis and the time interval before the next eruption along the vertical axis. Record your observations.

Duration of Eruption	Time Interval Before Next Eruption
4 min 24 sec	81 min
2 min 00 sec	60 min
4 min 43 sec	91 min
1 min 55 sec	51 min
4 min 14 sec	85 min
1 min 34 sec	55 min
1 min 34 sec	98 min
2 min 8 sec	49 min
4 min 30 sec	85 min
1 min 43 sec	65 min
4 min 27 sec	102 min
1 min 51 sec	56 min
4 min 35 sec	86 min
1 min 44 sec	62 min
4 min 35 sec	91 min

2 Following are the starting time and durations of 11 different eruptions randomly selected from a 2-day period in May 1997 (not listed in order). Use your scatter plot from Problem 1 to predict the time interval before the next eruption and the time of the next eruption. Explain your methods.

Starting Time	Duration	Predicted Interval Before Next Eruption	Predicted Time of Next Eruption
11:22 am	1 min 50 sec	_____	_____
7:16 am	4 min 19 sec	_____	_____
2:11 pm	1 min 50 sec	_____	_____
8:53 am	1 min 55 sec	_____	_____
12:20 pm	4 min 46 sec	_____	_____
8:30 am	4 min 26 sec	_____	_____
11:18 am	4 min 35 sec	_____	_____
12:40 pm	3 min 46 sec	_____	_____
9:52 am	4 min 35 sec	_____	_____
10:17 am	1 min 45 sec	_____	_____
7:35 am	1 min 55 sec	_____	_____

Focus Student Activity 15.7

NAME _____ **DATE** _____

The predicted starting times of next eruptions of Old Faithful in column (4) below were obtained by the JASON Project in May 1997.

1 Compare the actual starting times of the next eruptions from column (2) in the table below to the JASON Project predicted starting times in column (4). How do you rate the JASON Project predictions? Support your rating with reasoning based on statistical evidence.

2 Complete column (3) of the table below by filling in each of your predicted starting times of next eruptions from Focus Student Activity 15.6 (notice that data in Column (1) is ordered according to its occurence—be sure to enter your data accordingly). Then use at least 2 different statistical methods to compare your predictions in column (3) to those by the JASON Project in column (4) and to the actual starting times of next eruptions in column (2). Use your comparisons as the basis for rating the quality of your predictions, and provide convincing statistical evidence to support your rating.

(1) **Starting Time of Eruption**	(2) **Actual Starting Time of Next Eruption**	(3) **My Predicted Starting Time of Next Eruption** (from Student Activity 15.6)	(4) **JASON Project's Predicted Start of Next Eruption**
7:16 am	8:53 am	_____	8:41 am
8:53 am	9:52 am	_____	9:48 am
9:52 am	11:22 am	_____	11:21 am
11:22 am	12:20 pm	_____	12:15 pm
12:20 pm	1:58 pm	_____	1:52 pm
7:35 am	8:30 am	_____	8:30 am
8:30 am	10:17 am	_____	9:59 am
10:17 am	11:18 am	_____	11:12 am
11:18 am	12:40 pm	_____	12:47 pm
12:40 pm	2:11 pm	_____	1:59 pm
2:11 pm	3:16 pm	_____	3:06 pm

Follow-up Student Activity 15.8

NAME _____ **DATE** _____

Construct all graphs by hand or by a graphing calculator. Show labeled sketches of all graphs you construct.

1 This table shows the starting dates of all eruptions of the Hawaiian volcano, Kilauea, from 1923 to 1963, the duration in days of each eruption, and the time in years before the next eruption. Form a line plot of the eruption dates by year, and form a stem and leaf plot, histogram, and box plot of the Duration in Days data. Next to each plot, write your observations, conjectures, and/or generalizations based on mathematical relationships in the plot. What can you see/conclude from one plot that another doesn't show?

Eruptions of Kilauea on Hawaii from 1923 to 1963

Starting Dates	Duration in Days	Time Interval Before Next Eruption in Years
August 25, 1923	1	.7
May 10, 1924	17	.2
July 19, 1924	11	3.0
July 7, 1927	13	1.6
February 20, 1929	2	.4
July 25, 1929	4	1.4
November 19, 1930	19	1.1
December 23, 1931	14	1.7
September 6, 1934	33	18.6
June 27, 1952	136	1.9
May 31, 1954	3	.7
February 28, 1955	88	4.7
November 14, 1959	36	.2
January 13, 1960	36	1.1
February 24, 1961	1	.2
March 3, 1961	22	.4
July 10, 1961	7	.2
September 22, 1961	3	1.2
December 7, 1962	2	.7
August 21, 1963	2	.1
October 5, 1963	1	1.4

(Continued on back.)

Follow-up Student Activity (cont.)

2 Form a line, stem and leaf, and box plot for the Time Interval Before Next Eruption data. State several observations, conjectures, and/or generalizations. Compare these plots with those from Problem 1. Describe the similarities and differences in the graphs and how these similarities and differences may relate to the data.

3 On ¼" grid paper, form a scatter plot of the eruptions of Kilauea with the Duration of Eruptions along the horizontal axis and the Time Interval Before Next Eruption along the vertical axis. Sketch a median-fit line for this plot. Suppose that in November of 1963 another eruption occurred and it lasted 30 days. Use your plot and median-fit line to predict the length of time before the next eruption of Kilauea.

4 The dates of outbreaks of eruptions at Kilauea from 1964 to 1982 and the duration in days of these eruptions are listed here: March 5, 1964, 10 days; December 14, 1965, < 1 day; November 5, 1967, 251 days; August 22, 1968, 5 days; October 7, 1968, 15 days; February 22, 1969, 6 days; May 24, 1969, 867 days; August 14, 1971, < 1 day; September 24, 1971, 5 days; February 4, 1972, 454 days; May 5, 1973, < 1 day; November 10, 1973, 30 days; December 12, 1973, 203 days; July 19, 1974, 3 days; September 19, 1974, < 1 day; December 31, 1974, < 1 day; November 29, 1975, < 1 day; September 13, 1977, 18 days; November 16, 1979, 1 day; April 31, 1982, < 1 day; September 5, 1982, < 1 day. Note: < stands for "less than."

a) Form a histogram and a box plot for the Eruption Duration in Days data for the years 1964 to 1982. Compare these to the histogram and box plot for Problem 1 and state a few observations.

b) Form a line plot of the dates of eruptions by year for 1964-82. Compare this to the line plot from Problem 1. What do these plots show about the differences in data for these 2 periods?

c) Since January 3, 1983, Kilauea has erupted continuously. This is the longest period of a volcanic eruption in Hawaii in historical times. Comparing the data and graphs from 1923-63 to the data from 1964-82, what clues can you find that might have indicated a drastic change for the years following 1983? Discuss your observations and reasoning.

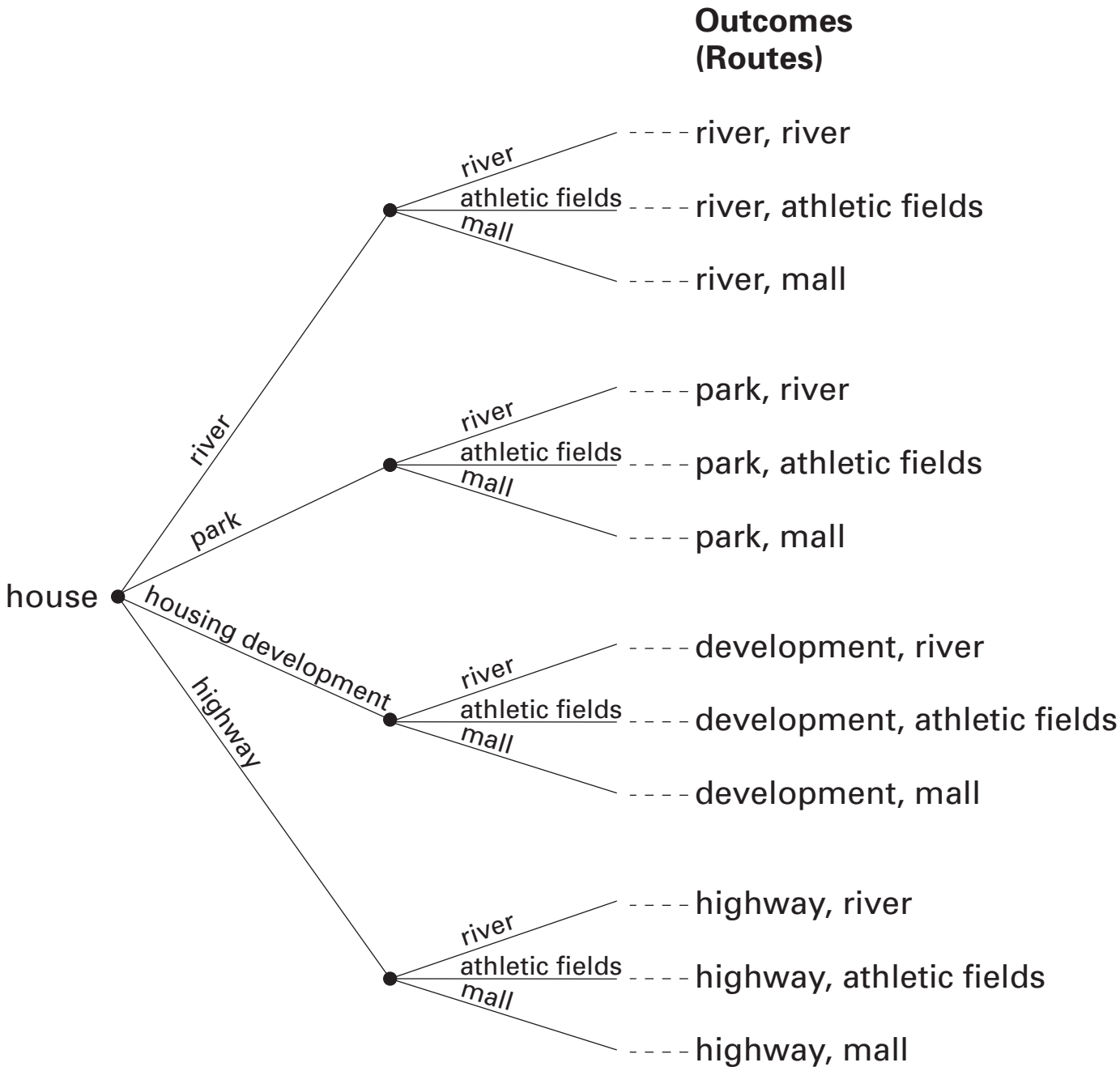
Connector Master A

Marcia's Routes to School

Each morning before school Marcia walks to the post office to mail letters for her parents, and then she continues on to school. Marcia has 4 different routes she can walk from her home to the post office: along the river; through the park; by the new housing development; or on the highway. When she leaves the post office, there are 3 different routes that she can walk to school; along the river; across the athletic fields; or by the mall. How many different routes can Marcia take on her trips to school?

Connector Master B

Tree Diagram



Connector Master C

Routes from home to post office

R	P	D	H

→

Routes from home via post office to school

RR	PR	DR	HR
RF	PF	DF	HF
RM	PM	DM	HM

Connector Student Activity 16.1

NAME _____ **DATE** _____

Solve each problem and show a diagram or write a brief explanation of your reasoning.

1 A house has 3 doors to the outside and 9 windows. In how many ways can a burglar enter the house through a window and leave through a door?

2 A car manufacturer provides 6 different exterior colors, 5 interior colors, and 3 different trims. How many different exterior color/ interior color/trim schemes are available?

3 For breakfast Henry always chooses one of the following drinks: orange juice, tomato juice, or apple juice. For cereal he chooses either corn flakes or bran flakes. For pastry he has either a doughnut, a glazed muffin, or an apple tart. What are all the different possibilities for Henry's breakfasts if he always has cereal, a drink, and a pastry?

4 A teenager posed the following problem: How many different outfits can she wear if she has 2 skirts with different patterns, 3 different colored blouses and 2 different types of shoes?

5 Each student at Athey Creek Middle School has a 4-digit locker number. If the 1st digit of each 7th grader's locker number is a 7 and each of the other 3 digits is one of the digits from 0 through 9, how many different 4-digit numbers are available for the 7th graders? Note: digits can be repeated any number of times in a locker number.

6 A school cafeteria offers a selection of 2 types of meat, 3 types of fish, 4 different vegetables, and 3 desserts. In how many ways can a student select 1 vegetable, 1 dessert, and either 1 type of meat or 1 type of fish?

(Continued on back.)

Connector Student Activity 16.1 (cont.)

7 A building developer may choose from 4 different roofing subcontractors, 5 different electrical subcontractors, 3 different plumbing subcontractors, 2 different carpenters, and 6 different painters. In how many ways can he select 1 of each?

8 The Tigers and the Wildcats are to play a 3-game series in soccer. The first team to win 2 games wins the series. Find all the possible outcomes for the 3 games.

9 The annual Fourth of July raft race in Centerville has 7 entries. If a different prize is offered for each of the following categories; fastest raft, slowest raft, and most original raft, in how many ways is it possible to award the prizes?

10 A store carries 5 styles of backpacks in 4 different sizes. The customer also has a choice of 2 different kinds of material for 3 of the styles, and only 1 material for the other styles. How many different types of backpacks are there to choose from? If Larry is only interested in the 2 largest backpacks, how many different backpacks does he have to choose from?

Focus Student Activity 16.2

NAME _____ **DATE** _____

Show how to use a diagram or model to solve each of the following problems. Note: It is okay to generalize from a partial diagram or model without showing all the parts. If you do so, be sure to show your reasoning.

1 In how many ways can 6 people line up to purchase tickets for a concert?

2 In how many ways can 8 students be seated in a row for a school photo?

3 A sportswriter makes a preseason prediction about the order of the top 5 teams from among 20 teams in the Women's National Basketball Association. How many different possibilities are there?

4 An agricultural scientist wants to test different combinations of 4 types of soybeans with 7 types of fertilizer and 2 types of insecticides. How many experimental plots are needed?

5 A club of 10 people plans to elect 3 people for the offices of president, vice president, and secretary. In how many ways can these 3 offices be filled?

6 Seven cross-country runners are competing for 1st, 2nd, 3rd, and 4th places. How many different possible outcomes are there for these 4 places?

7 How many 3-letter code words can be formed from the alphabet if the code word can not contain a repeated letter?

(Continued on back.)

Focus Student Activity 16.2 (cont.)

8 A contractor wishes to paint 6 houses in a row each with 1 of the colors red, blue, green, and yellow, with the requirement that if 2 houses are side by side they are to have different colors. In how many ways can the houses be painted?

9 Eight members of a fire department are being considered for 3 special awards. In how many ways can 3 of 8 people be selected if no one wins more than 1 award?

10 A school has 683 students. Are there *possibly*/*definitely*/*definitely not* (circle one) any students who have the same pair of initials for their first and last names?

Focus Student Activity 16.3

NAME _____ **DATE** _____

For each of the following problems, communicate your methods and reasoning so that it is clear how and why your methods and answers work.

1 How many different code words with 11 letters can be formed by using the letters in "achievement"?

2 A coin is tossed 5 times and the sequence of heads and tails is recorded for each toss.

a) How many different sequences are possible?

b) In how many ways are 3 heads and 2 tails possible?

3 The 5 starting players of a basketball team are to be introduced before the game. In how many orders can they be introduced?

4 One method of signaling on boats is to arrange 3 flags of different colors vertically on a flagpole. If a boat has flags of 6 different colors, how many different 3-flag signals are possible?

5 For the Lincoln High School jazz band concert, the jazz band will play 5 traditional jazz compositions and 3 original compositions. If the concert begins with any 1 of the traditional compositions, in how many ways can the 8 compositions be arranged on the program?

6 From a group of 12 smokers, a researcher wants to randomly select 8 people for a study. How many different combinations of 8 smokers are possible?

(Continued on back.)

Focus Student Activity 16.3 (cont.)

7 A club of 7 students decides to send 2 of their members to the principal to request a field trip to an art museum.

a) In how many ways can the 2 students be chosen?

b) If the president of the club is to be one of the students, in how many ways can the group of 2 be chosen?

8 A state lottery requires that you pick 6 different numbers from 1 to 99. If you pick all 6 winning numbers, you win $1,000,000.

a) How many ways are there to choose 6 numbers if the order of the numbers is not important? For example, 1,2,3,4,5,6 is the same combination as 2,1,3,4,5,6.

b) How many ways are there to choose 6 numbers if order is important?

9 Two children are allowed to select from 7 baseball cards. The younger child is to select first and can select 2 cards. Then the older child is to select 3 cards from the remaining 5. The older child complains that this is not fair and says that the younger child has more choices. What do you think?

Focus Student Activity 16.4

NAME _____ **DATE** _____

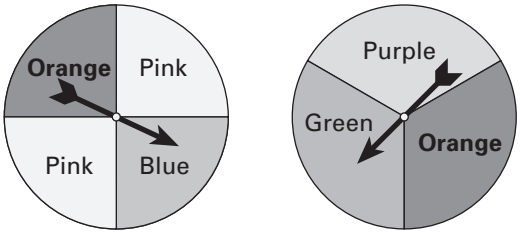
Show your methods and reasoning for each problem.

1 A sack contains 2 blue cubes, 3 yellow cubes, and 5 red cubes. If 1 cube is randomly se1lected and then placed back in the sack and a 2nd cube is selected, what is the probability that both of the cubes will be yellow?

2 Solve Problem 1, but this time assume that the 1st cube selected is not placed back in the sack.

3 There are 2 sacks with cubes and 1 cube will be randomly selected out of each. Sack 1 contains 3 red cubes and 2 blue, and Sack 2 contains 2 yellow, 1 green, and 2 red. What is the probability of selecting 2 red cubes? What is the probability of not selecting 2 red cubes? What is the probability of not selecting any red cubes?

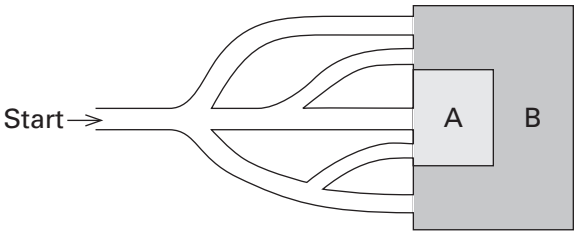
4 The 2 spinners shown at the right are each to be spun once. Determine the probabilities of obtaining the following colors: orange on both spins; pink on one spin and green on the other; blue on one spin and either purple or green on the other.



Spinner I Spinner II

5 A family has 4 children. Assuming that the chances of having a boy or a girl on each birth are equally likely, determine the probability that the children were born in *either* the order boy, girl, boy, girl *or* the order girl, boy, girl, boy.

6 Assuming that each branch point in the maze at the right is equally likely to be chosen, determine the probability of entering room A.



(Continued on back.)

Focus Student Activity 16.4 (cont.)

7 A game is to be played in which 1 player holds a sack that contains 5 blue cubes and 5 yellow cubes, and a 2nd player selects 2 cubes from the sack. The player who selects wins the game if 2 cubes of the same color are obtained, and otherwise the player holding the sack wins. Which player has the better chance of winning and what is this player's probability of winning?

Follow-up Student Activity 16.5

NAME _____ **DATE** _____

1 For each of the following problems, communicate your methods and reasoning so that it is clear how and why your methods and answers work.

a) On a sandwich menu, there are 5 choices of bread (French, sourdough, onion, rye, or wheat) and 4 choices of filling (turkey, tuna, egg salad, or beef). How many different sandwiches are possible from these choices?

b) An art gallery has 5 special paintings to display, but space to hang only 4 of them in a row. How many different arrangements of paintings are possible in this row?

c) An area code is a 3-digit number where the 1st digit cannot be 0 or 1. How many different area codes are possible?

d) Five houses in a row are each to be painted with 1 of the colors red, green, blue, or yellow. If no 2 adjacent houses can have the same color, in how many ways can the houses be painted?

e) A little league team is to be formed from 9 children of whom only 1 can be the catcher, only 1 can play 1st base, and only 1 can be the pitcher. The other players can play any of the remaining 6 positions. How many different lineups are possible?

f) A girl dresses each day in a blouse, a skirt, and shoes. She always wears white socks. She wants to wear a different combination on every day of the year. If she has the same number of blouses, skirts, and pairs of shoes, how many of each would she need to have a different combination every day?

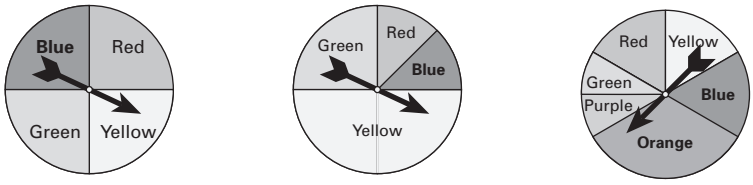
g) How many 5-digit ZIP codes are possible in which the ZIP code is an even number?

h) A Girl Scout troop has 12 members. In how many different ways can the scoutmaster appoint 4 members to clean up the camp?

(Continued on back.)

Follow-up Student Activity (cont.)

2 Use these spinners to determine the probabilities of the outcomes listed below.



Spinner A Spinner B Spinner C

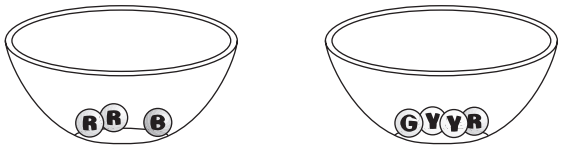
a) blue from Spinner A and red from Spinner B,

b) blue from Spinner B and green from Spinner C,

c) yellow from Spinner A, green from Spinner B, and red from Spinner C,

d) either green or red from Spinner B and either red or yellow from Spinner C.

3 Use these 2 bowls of marbles to determine the probabilities of selecting the following:



Bowl I Bowl II

a) a red from I and a red from II,

b) a blue from I and a yellow from II,

c) 1 marble from each bowl and no red marble,

d) 1 marble from each bowl, no red or green marbles.

4 A gumball machine contains 2 red and 4 white gumballs, and no others. If each gumball has an equal chance of being released and 2 are purchased, what is the probability of getting 2 red gumballs?

5 A coin will be tossed 4 times. What is the probability of obtaining 2 or more heads in a row?

6 Write a letter to an adult explaining the "big ideas" from this lesson about counting concepts and strategies and their role in computing probabilities. Be detailed and include examples that help the adult see why ideas work. Next, have an adult read your letter and write comments about what is clear and unclear. Then edit your letter to clarify as needed. Turn in the original letter with the adult's comments and turn in your edited version.

Connector Master A

Table of Random Numbers

16877	62796	31056	59269	96725	03510	19186	41672	73387	79284	45697	83702
04419	47911	71787	71696	16243	94507	34229	64227	74719	92763	26120	26676
75939	36554	94366	76143	35655	30214	14796	43156	80044	29719	87905	90690
95567	44275	71147	14808	18677	88968	07326	79537	54169	08392	94501	75642
76431	76938	15066	69132	24674	37084	86339	01514	01943	47018	43849	29406
75407	87829	99305	08420	02584	07888	34234	18388	90287	10718	43386	56457
55762	77741	78263	50907	73016	04595	52029	67789	65254	15750	61520	77606
41403	72567	47625	39432	74864	80303	63257	16798	48436	34737	76541	37816
59269	94176	30216	29459	70063	47051	23187	78262	02884	39857	79393	85013
38793	30586	26783	98341	01379	89414	59707	53130	18896	11211	59959	56900
44722	22398	08182	59445	18743	51183	43850	83134	34802	56327	85775	50176
76792	15413	21246	62483	85973	31468	84551	79229	23694	41874	53784	51870
54146	99549	03892	90288	91541	15296	41359	79732	47707	32861	08284	09301
13258	98825	01224	81597	92304	02093	92229	15847	93575	09709	08284	32880
10017	95166	51510	07391	88433	15653	61773	24623	64493	41353	75811	20046
59657	17253	82592	41514	35609	42019	74328	30695	38949	63331	36022	43688
41389	56877	18323	92464	94235	63498	31954	47714	28424	04041	61545	82321
41858	73549	80343	92251	65510	58671	18490	24278	86894	75956	07622	04836
19566	51618	19581	09700	24562	06142	29945	52454	25711	56299	57296	50691
83203	14552	64977	50767	69853	17417	99352	38352	70414	53469	41128	14907
25112	68883	84887	13802	66934	51070	93787	23816	33707	28622	45618	61966
58415	53175	87365	90811	21936	09650	72851	38107	72851	04268	69737	65574
34693	13723	45306	95760	75208	30848	35810	97002	56068	50981	84566	10941
24872	90442	84322	68282	39577	22272	02654	32727	39361	60659	38083	65416
41984	99543	77292	04579	72718	78427	53647	71794	86988	41346	80978	73041
75478	86379	14318	96269	44900	63619	30630	54758	23360	17874	05854	74444
30434	89355	57050	34114	21280	99919	05157	30236	54951	71782	25803	38854
68066	17895	99454	18143	53727	50943	95711	87285	16946	73275	85412	86282
67782	75800	53994	25767	87501	91511	61128	45081	65661	41952	58406	42426
83840	02103	43204	44410	09188	64132	21788	14714	24348	34371	44907	29912
39074	47327	91183	05675	20069	06647	11494	33750	57719	90226	28720	00578
11320	17425	37182	41460	58948	88640	43978	41636	59767	85239	42843	06220
20061	21591	32017	47522	15491	34360	15385	07860	29687	88036	17448	05554
54138	01873	61646	85299	58479	47410	56923	41593	51695	92199	61850	53722
03669	13358	09583	33932	59348	53359	52437	50889	99250	93626	51771	13278
69839	06921	78794	89196	23332	98505	13874	96457	89402	13575	18514	48466
15450	65441	81939	81633	69536	07936	88569	32608	20762	25883	84028	66554
44299	68481	28078	12352	19198	54445	48652	27205	91811	61883	59061	69477
04691	72461	17275	77242	64938	97750	50648	89443	56513	85940	34066	17280
45855	40677	54003	67362	03456	95489	06329	58694	87021	47689	18746	22160
58802	98687	39554	56036	38187	51985	90895	09215	14525	68772	53814	57386
86620	73916	17143	42927	10858	57750	78656	42031	73934	79573	75971	49113
09682	00755	70561	45131	16440	14107	17075	78385	64347	51414	56677	97189
99583	07952	12497	58162	33850	93029	49328	87580	50232	71255	82129	39595
01346	62215	94291	63462	06520	27938	83438	41685	59481	39404	49188	44704

Connector Master B

For each of the following, use a calculator to generate the random numbers that are described. Then record the calculator commands that you enter and the data generated by the commands.

a) Last month Lakeway Theater sold tickets numbered from 372500 to 374000. The theater manager plans to give free passes to the first 5 numbers that are randomly selected from these numbers.

b) To practice multiplication facts, each day Rachelle randomly generates a set of 25 pairs of whole numbers, where each number in a pair is randomly selected from the whole numbers 2 through 12.

c) Some combination locks are designed for use with any 3 numbers from 0 to 45. When these locks are manufactured, the numbers used in each combination are selected at random.

d) In a drawing for 2 CD players, tickets are numbered from 41 through 80. The 1st 2 ticket numbers that are randomly selected and divisible by 3 are the winning numbers.

e) Suzanne's teacher asked her to randomly generate a set of even numbers less than 100 and to select the first 3 that are divisible by 5.

f) A researcher randomly selects 15 numbers from the following set: {1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, … 38, 38.5, 39, 39.5, 40}. Then she computes the mean and the sum of the 15 numbers, and she makes a box plot of 15 numbers.

g) Joey randomly selected 3 odd numbers greater than 100 and less than 140.

Connector Student Activity 17.1

NAME _____ **DATE** _____

1 Discuss possible methods for creating a list of 100 random digits.

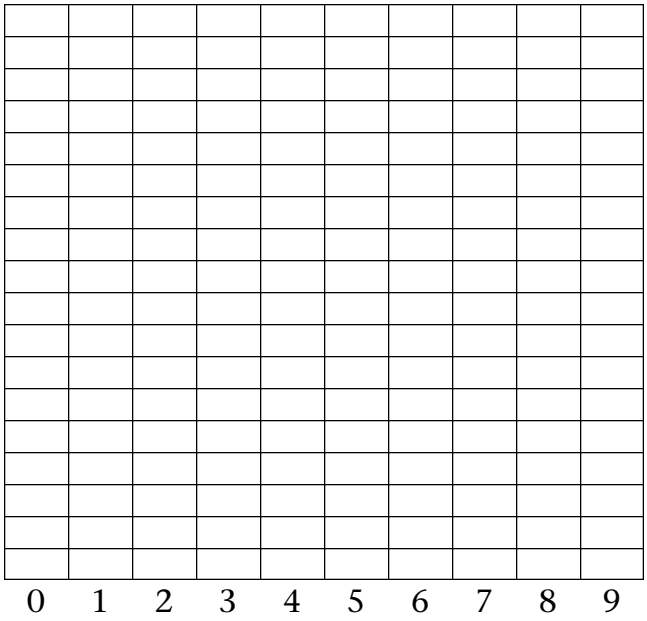
2 Choose a method discussed for Problem 1 to generate a list of 100 random digits.

a) Describe your method of generating 100 random digits:

b) Record your 100 digits, in the order generated, in this table:

—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

c) Make a line plot of your 100 digits here:



3 Do you feel that your set of 100 digits is random? List some conditions that you feel a set of digits should satisfy in order to be considered a set of random digits?

Focus Master A

Blood Type Problem

If, on the average, 2 out of every 5 people have blood type O, and 3 people are randomly selected, what is the probability that exactly 2 of these 3 people will have blood type O?

Cereal Box Problem

As a special promotion, the Crunchy-Crispy Cereal Company includes a baseball trading card in each box of cereal. There are 6 different trading cards, and each cereal box contains exactly 1 of the 6 cards. How many boxes of cereal would you expect to buy in order to collect all 6 trading cards?

Principal Problem

Mount Hood Middle School has 3 classes of 7th graders: 25 students in Class I, 30 in Class II, and 22 in Class III. If 5 of the 77 students are randomly selected to meet with the principal, what is the probability that at least 1 student from each class will be in the meeting?

Focus Master B

Solve the following problems by designing and carrying out simulations.

a) Each box of Pops-a-Lot Popcorn contains 1 of 7 different colored pens. How many boxes of popcorn would you purchase in order to be 90% certain that you would obtain a complete set of all 7 colors?

b) Based on his past archery records, the probability that Eric will hit the bulls-eye of a target is .94. If Eric takes 8 shots at the target, what is the probability he will hit the bulls-eye exactly 7 times?

c) Assume that the probability of a randomly selected person having a birthday in a given month is $1/12$.

i) How many people, on the average, would you need to select to be 90% certain that 2 of them will have a birthday in the same month?

ii) If 8 people are randomly selected, what is the probability that at least 3 will have a birthday in the same month?

d) Two students are playing a coin-tossing game. Each player tosses a coin until obtaining 3 heads or 3 tails in a row. The player who requires the fewest number of tosses wins the game. How many tosses of a coin are required on the average to obtain 3 heads or 3 tails in a row?

e) Hoopersville Hospital uses 2 tests to classify blood. Every blood sample is subjected to both tests. Test A correctly identifies blood type with probability .7 and Test B correctly identifies blood type with probability .5. Determine the probability that at least 1 of the tests correctly determines the blood type.

(Continued on back.)

Focus Master B (cont.)

f) The names of 5 people (all with different names) are placed on 5 separate slips of paper and these slips are placed in a sack. If each person randomly chooses a slip from the sack, on the average, how many people will select their own name?

g) On a quiz show, contestants guess which 1 of 3 envelopes contains a $5000 bill. What is the probability that exactly 4 people out of 8 contestants will select the envelope with $5000?

h) At a certain university it is required that 85% of the students be from within the state. If 6 students are randomly selected from this university's student body, what is the probability that exactly 1 of them will be from outside the state?

i) Assume that the probability of a randomly chosen person having a birthday on a given day of the year is $1/365$. How many people, on the average, would you need select in order to be 90% certain of obtaining exactly 2 people with a birthday on the same day?

Focus Master C

Marker Game

A sack is filled with 5 red game markers and 5 green game markers. One player called the *holder*, holds the sack and the other player called the *drawer*, selects 2 markers at a time. The drawer earns a point if both markers have the same color, and the holder earns a point if the 2 markers have different colors. The drawer continues selecting 2 markers at a time, with the drawer and holder earning points as described above, until all the markers have been selected. The player with the most points wins the game.

Focus Student Activity 17.2

NAME _____ **DATE** _____

Blood Type Problem:

If, on the average, 2 out of every 5 people have blood type O, and 3 people are randomly selected, what is the probability that exactly 2 of these 3 people will have blood type O?

Design and carry out a simulation to answer the above Blood Type Problem. Collect data for *a minimum of 25 trials*, where a trial is the blood type information for a set of 3 people. On separate paper:

a) Describe, in detail, your simulation procedures.

b) Show all of the data that you collect.

c) Show how you organize, graph, and analyze your data to answer the Blood Type Problem. Be sure to support all conclusions with sound mathematical reasoning and a variety of mathematical evidence. If you form graphs or compute statistics on the calculator, sketch and label the results and include them in your written arguments.

d) Challenge. Determine and explain the theoretical probability for the Blood Type Problem. Show/explain your methods and reasoning so that it is clear why your answer is correct.

Focus Student Activity 17.3

NAME _____ **DATE** _____

Cereal Box Problem

As a special promotion, the Crunchy-Crispy Cereal Company includes a baseball trading card in each box of cereal. There are 6 different trading cards, and each cereal box contains exactly 1 of the 6 cards. How many boxes of cereal would you expect to buy in order to collect all 6 trading cards?

Design and carry out 2 simulations to answer the above problem so that one simulation involves the use of a random number function on a graphing calculator or a Table of Random Numbers, and the other simulation does not involve these methods. For each simulation, collect data for a minimum of 20 trials, where one trial consists of selecting until all 6 trading cards are obtained (i.e., the number of selections required for a trial will vary from trial to trial).

a) For each simulation, describe in detail your simulation procedures.

b) For each simulation, show all of the data that you collect.

c) For each simulation, show how you organize, graph, and analyze your data and use the results to answer the Cereal Box Problem. On a scale of 1-100, how confident are you in each solution? Explain.

d) Show how you combine the data from your 2 simulations and analyze the results. Based on this information, now what is your solution to the Cereal Box Problem? On a scale of 1-100, how confident are you now in your solution? Explain.

e) Show how you analyzed the combined data from all groups in your class. Discuss any adjustments this leads you to make in your solution from d). On a scale of 1-100, now how confident are you in your prediction?

Follow-up Student Activity 17.4

NAME _____ **DATE** _____

1 Record your methods and results for each of the following.

a) An electronic lock has digits 0-9, and the code for the lock is 2 3-digit numbers. Randomly generate codes for 5 locks.

b) At a raffle 217 tickets are sold, numbered 1-217. Eight winning numbers are randomly selected. Randomly generate 5 sets of 8 winning numbers.

c) A scientist is studying the directions in which wild animals move and needs to randomly select 12 angles that vary from 0° to 360°. Randomly generate 3 sets of 12 angles.

d) A professional basketball player has a free-throw average of 87% (i.e., 87% of his free throws are successful). Simulate 4 sets of 20 free throws and record his free-throw percentage for each set.

2 Design simulations to solve the following problems. For each problem, explain your simulation procedures, show at least 20 trials, and give statistical evidence to support your answer to the problem.

a) At the school carnival, anyone who correctly predicts 8 or more tosses out of 10 tosses of a coin wins a prize. Luise practiced at home and determined she can predict a coin toss 72% of the time. What is the probability she will win a prize at the school carnival?

b) A baseball player's batting average is the probability of getting a hit each time the player goes to bat. For example, a player with a batting average of .245 has a probability of 24.5% of getting a hit. If a player with a .293 batting average bats 4 times in a game, what is the probability of the player getting 2 or more hits?

c) A newly married couple would like to have a child of each gender. Assuming that the probability of having a girl is 50% and the probability of having a boy is 50%, what is the average number of children the couple must have in order to be 90% certain of having at least 1 girl and 1 boy?

(Continued on back.)

Follow-up Student Activity (cont.)

d) Determine the experimental probability of obtaining a 3 at least twice if a standard die is tossed 5 times.

e) How many times must 2 dice be tossed to be 90% certain of obtaining a sum of 10 or greater?

f) Each box of a certain Kandy Korn contains either a super-hero ring or a super-hero belt buckle. If $\frac{1}{3}$ of the boxes contain a ring and $\frac{2}{3}$ of the boxes contain a belt buckle, what is the probability that a person who buys 3 boxes of Kandy Korn will receive both a ring and a belt buckle?

g) At Kidville Day Care 28% of the children are from 1-child families. How many children must be randomly selected to be 80% certain of obtaining 2 students from 1-child families? to be 85% certain? to be 90% certain?

h) Three 6th graders, two 7th graders, and two 8th graders have been chosen by the student body to receive awards at a school assembly. The principal will randomly select from these awardees to determine the order in which they receive their awards. What is the probability that the first 3 students selected will contain 1 student from each of the 3 grades?

3 Write a letter to Heather, a student from another school, who was absent during all of this lesson. In your letter, explain the following to Heather:

a) the meaning and purpose of a simulation;

b) key points in the design of a simulation;

c) tips for carrying out a simulation;

d) suggestions for analyzing simulation data to solve a problem.

math alive!

VISUAL MATHEMATICS COURSE III
BLACKLINE MASTERS

Grids,
Recording Paper,
and Patterns

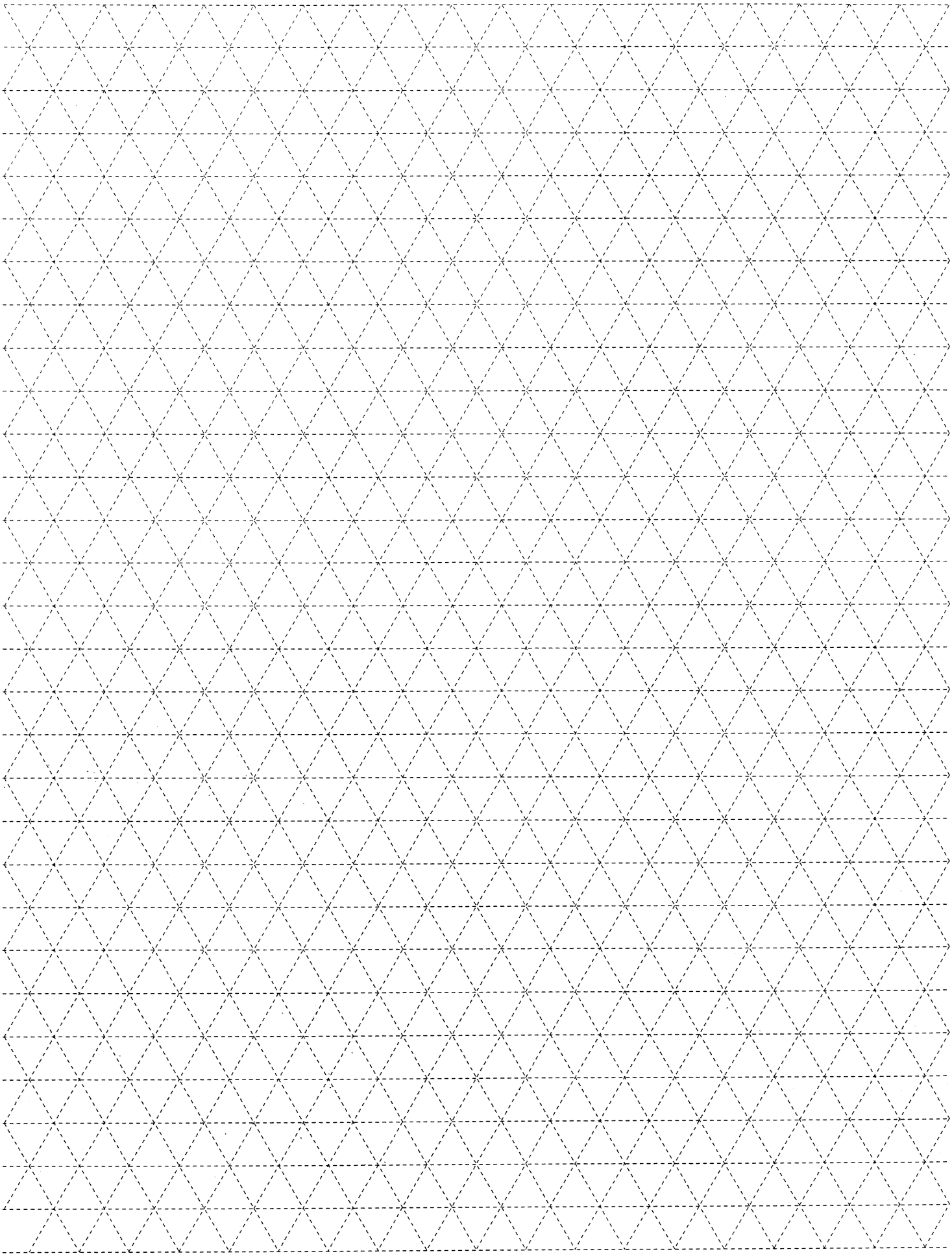
Blackline Masters Grids, Recording Papers, & Patterns

¼-inch Grid Paper

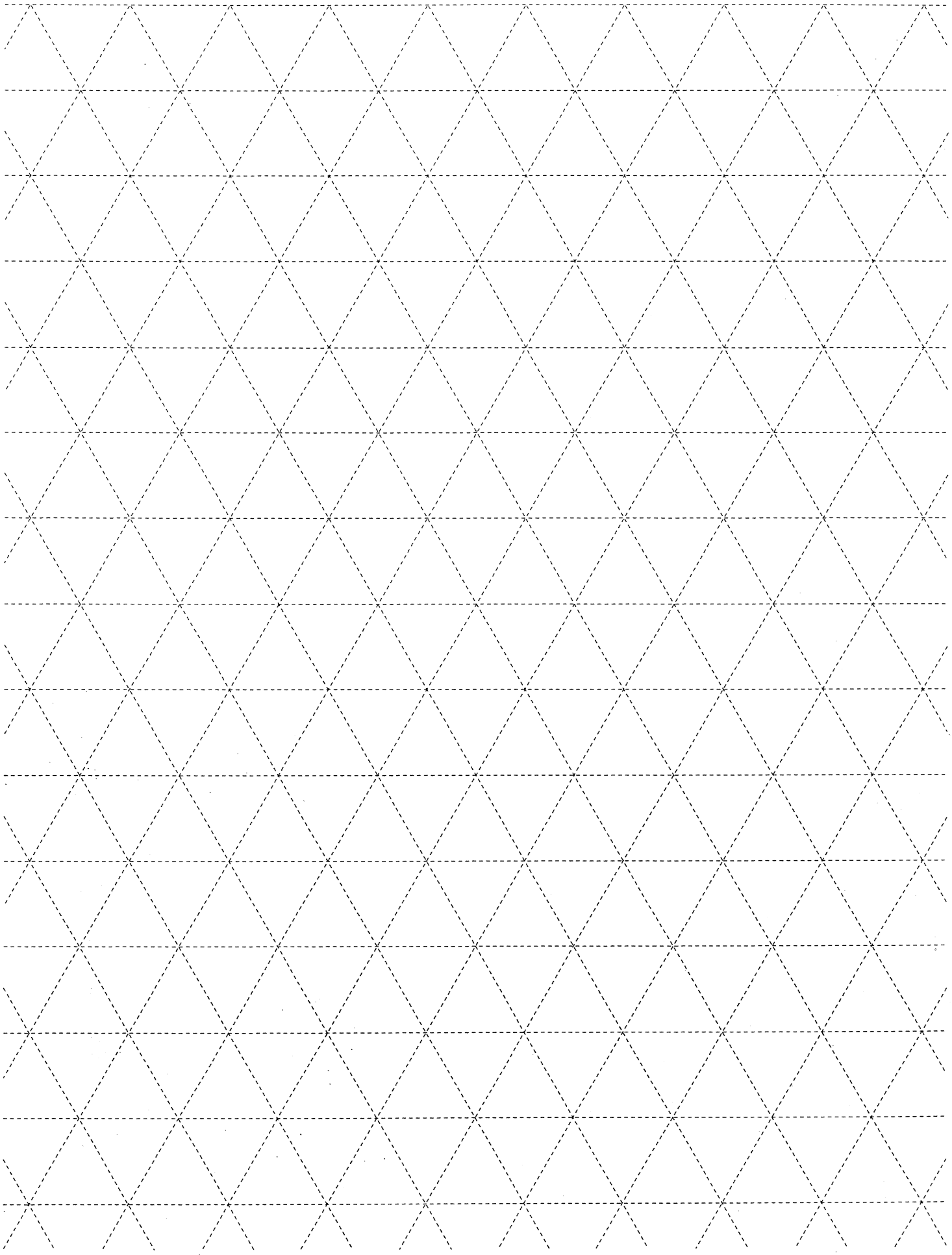
1 cm Grid Paper

2 cm Grid Paper

1-cm Triangular Grid Paper

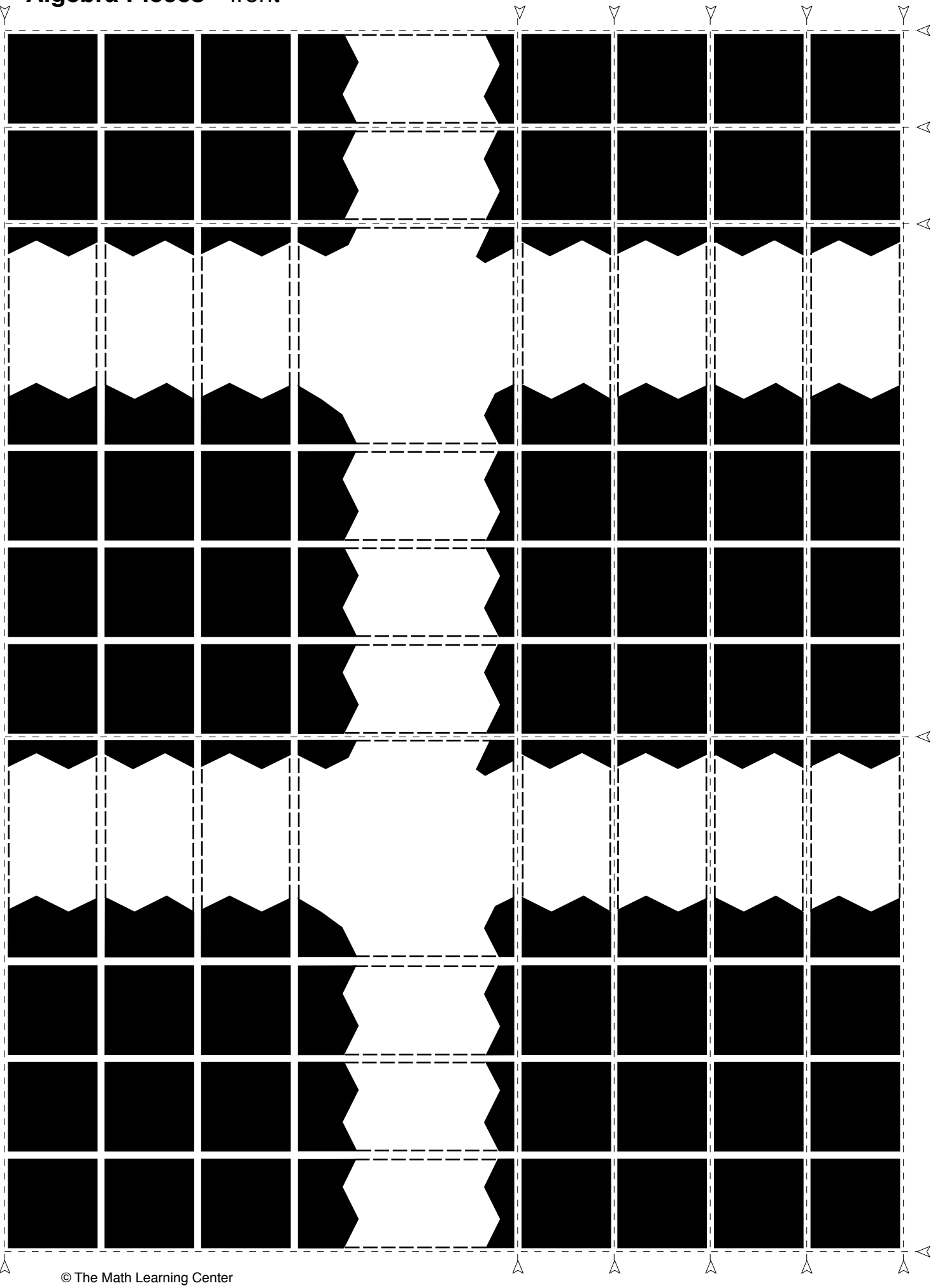


2-cm Triangular Grid Paper



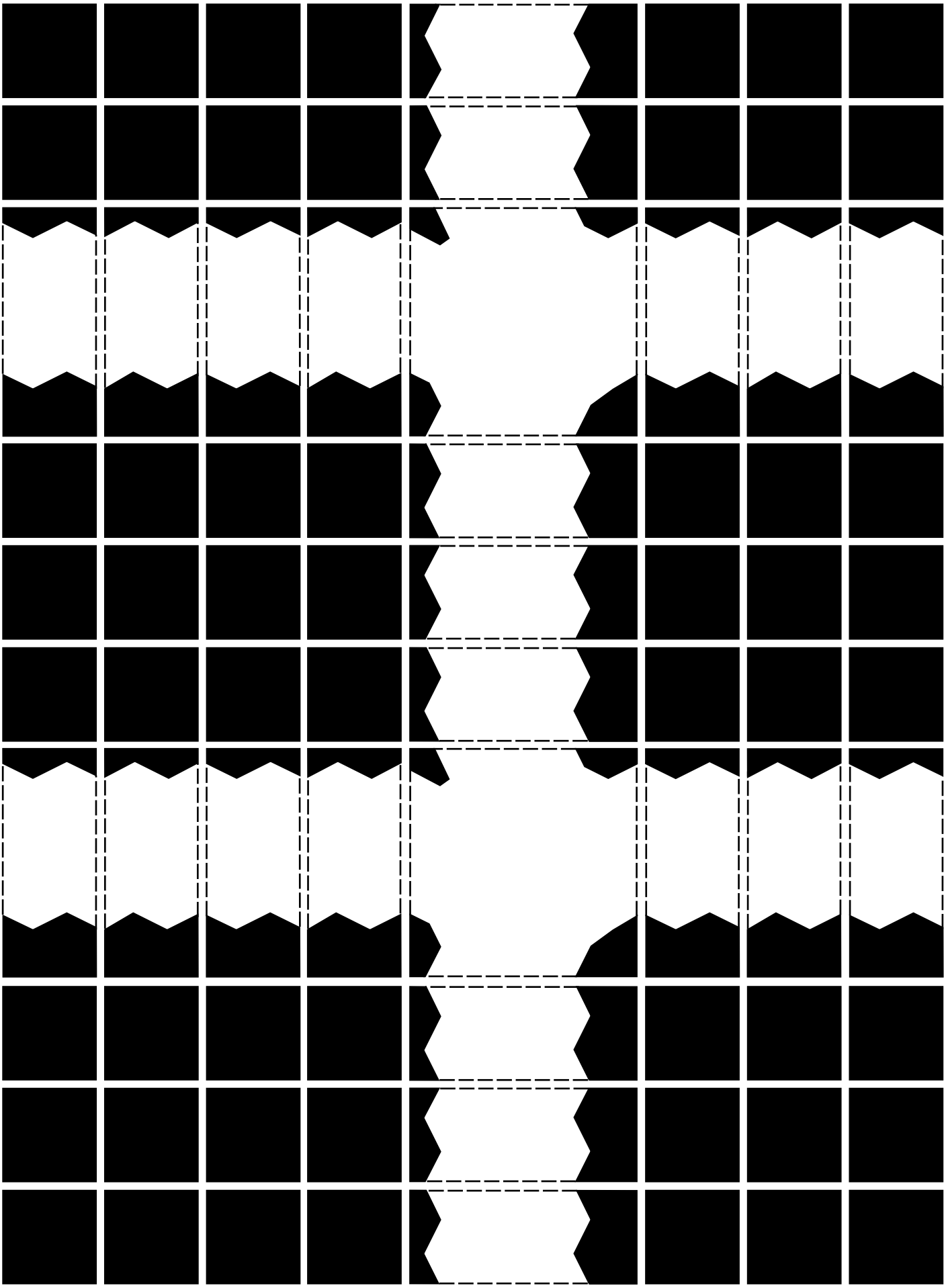
Algebra Pieces—front

(cut on thin dotted lines at arrowheads)



Algebra Pieces—back

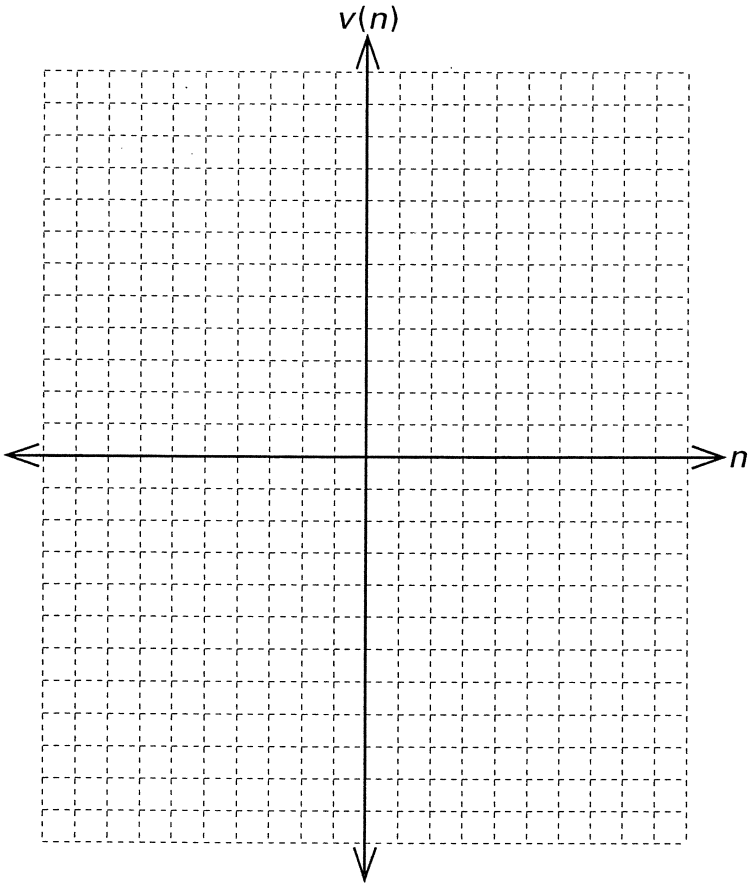
(cut as indicated on front side)

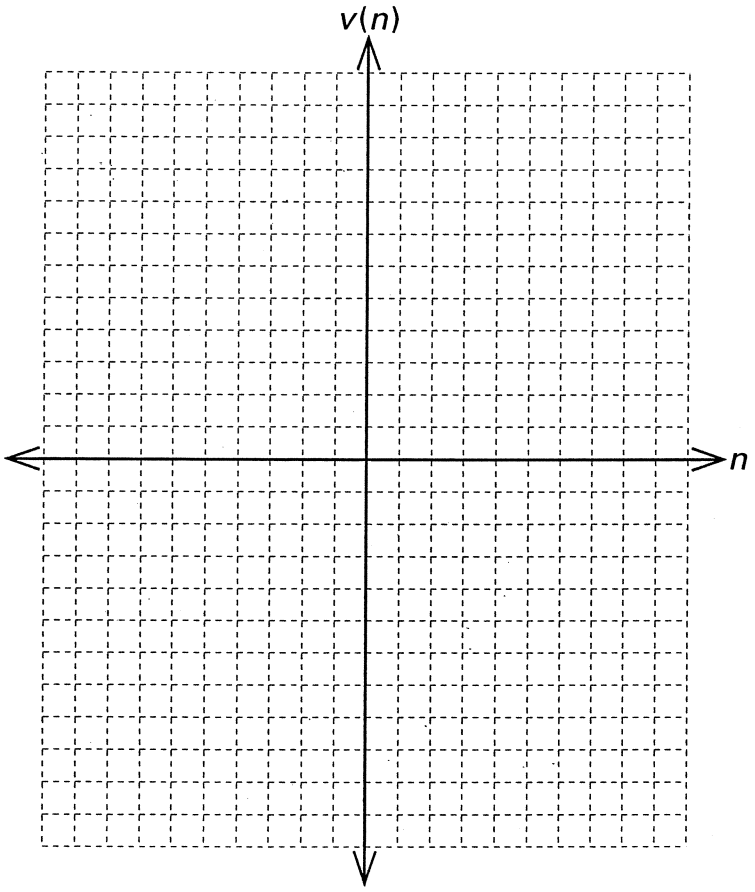


Blank Counting Pieces and N-Strips

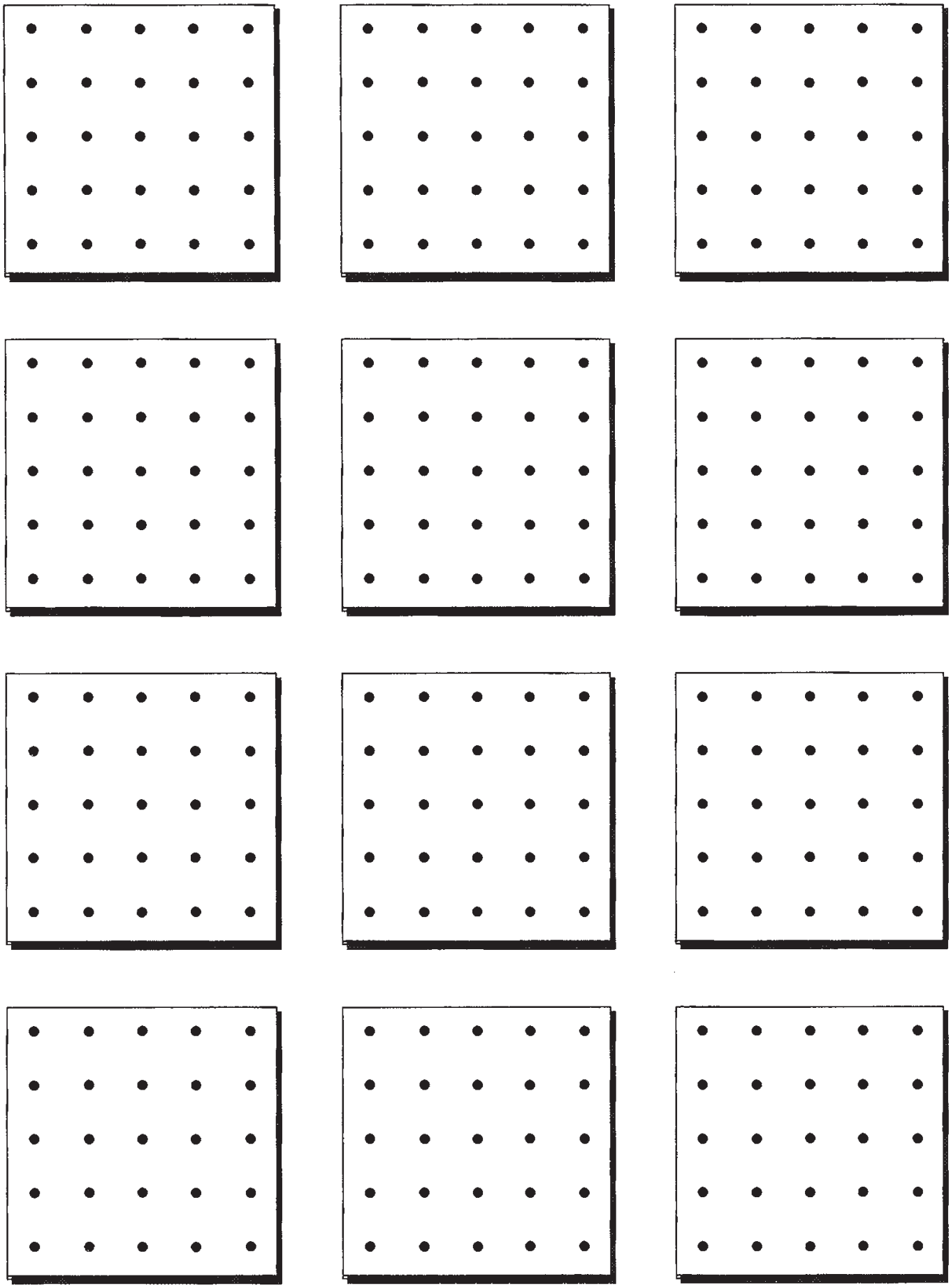
cut on dotted lines

Coordinate Grid Paper





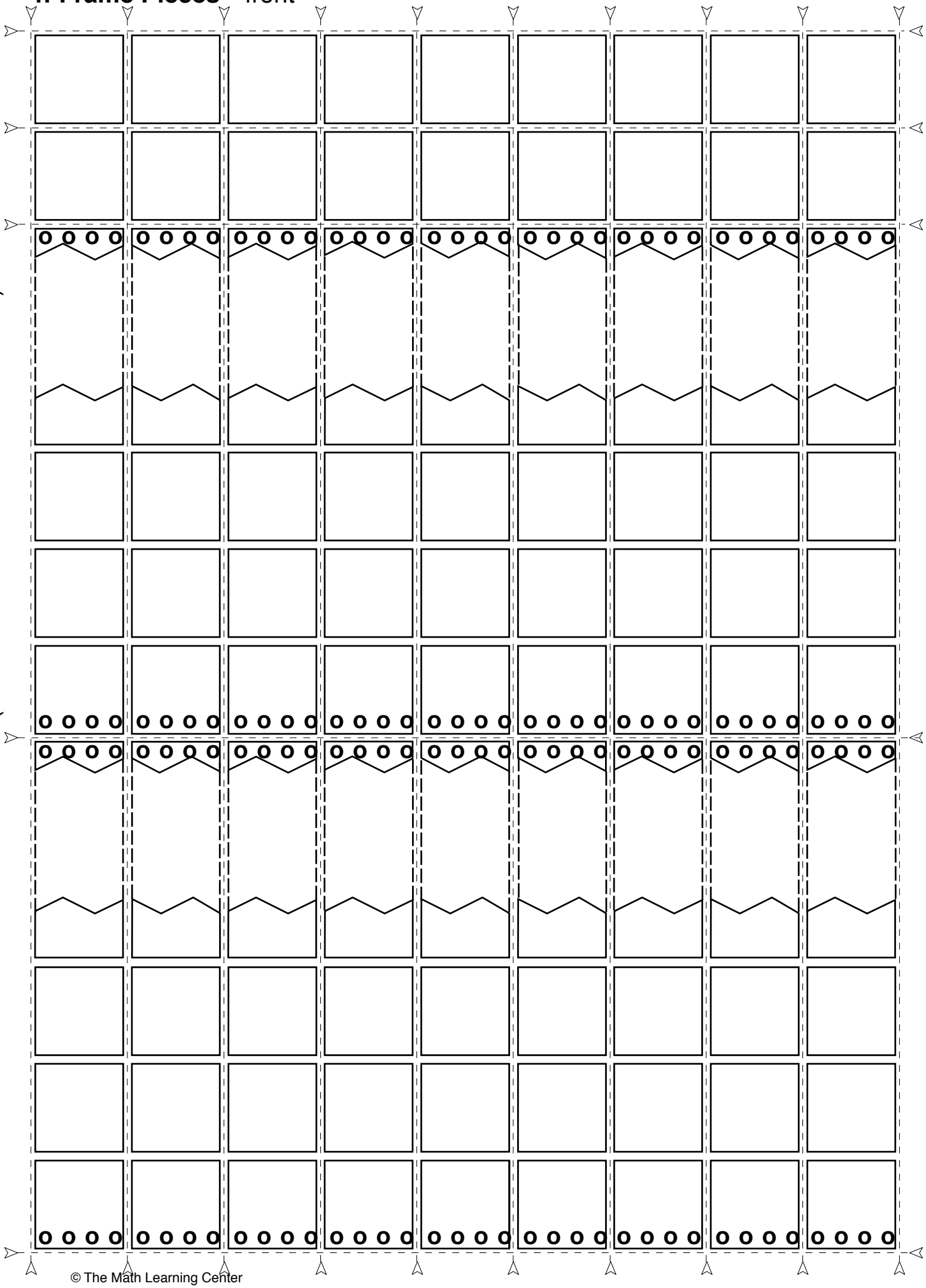
Geoboard Recording Paper



Geoboard Dot Paper

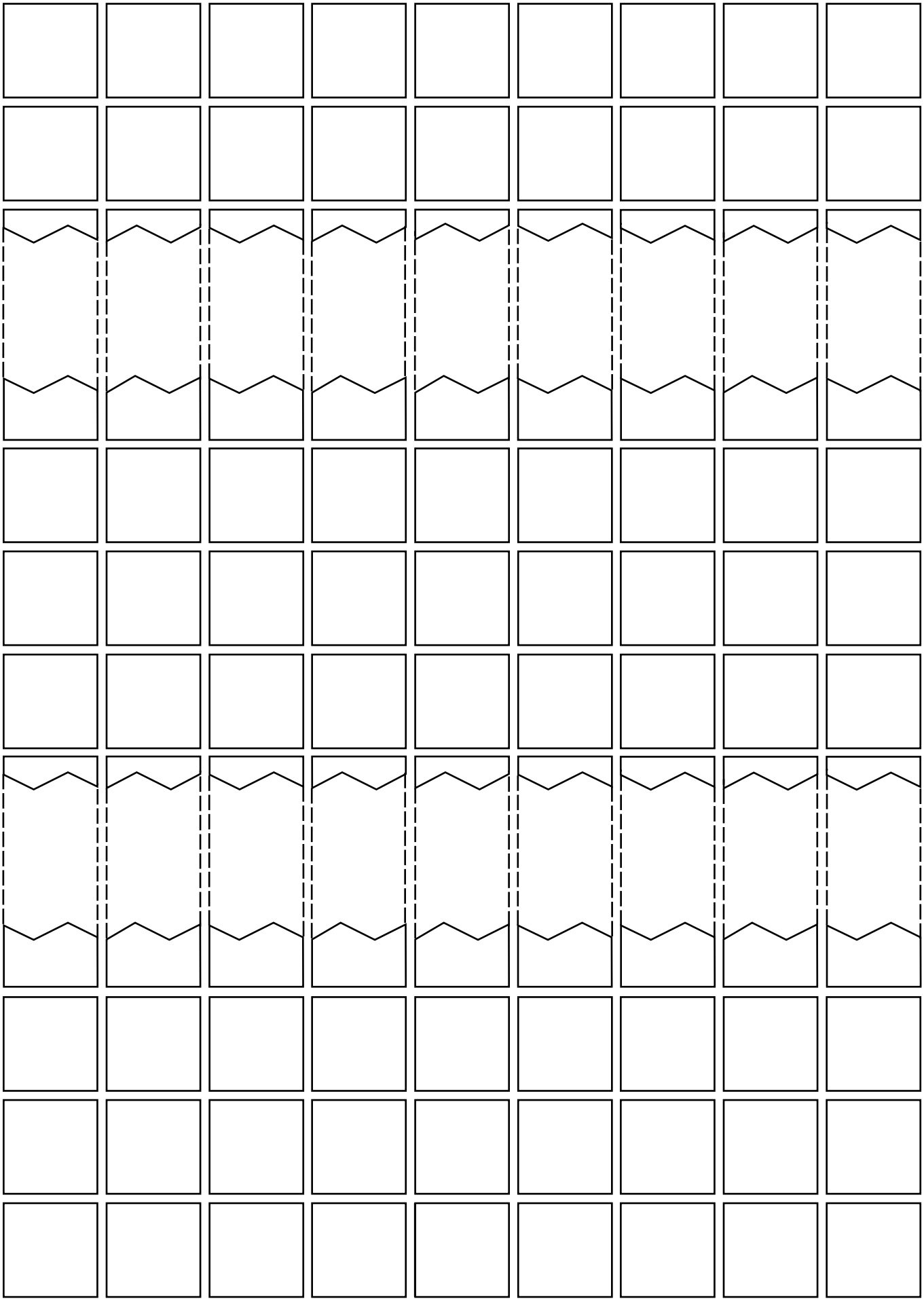
n-Frame Pieces—front

(cut on thin dotted lines at arrowheads)

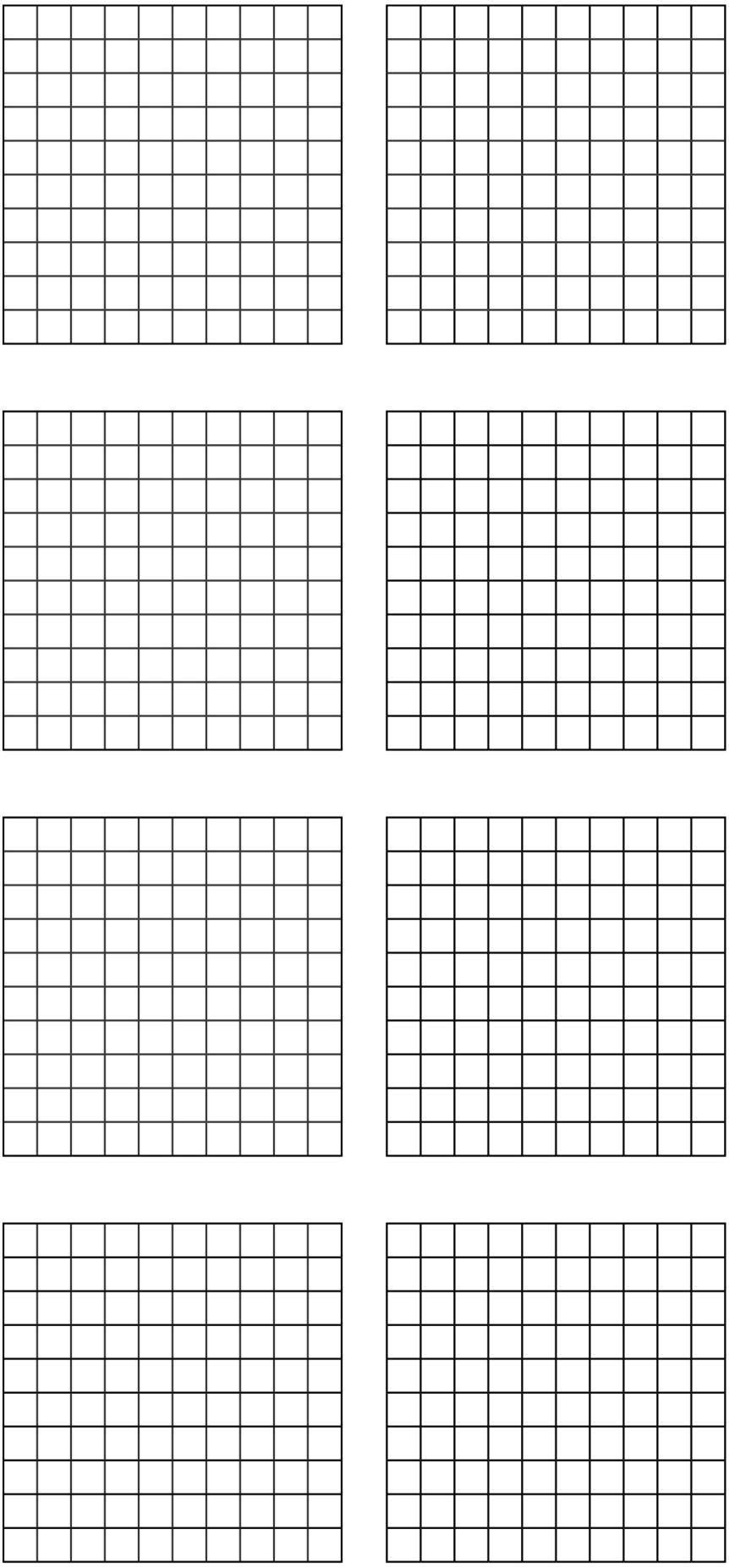


n-Frame Pieces—back

(cut as indicated on front side)



Percent Grids



Protractor