Unit 5 Introduction

The puzzles and problems that make up the Excursions in this unit involve multidigit addition, basic operations with money, and an understanding of place value. As students complete the Adventures in this unit, they will use doubling strategies to solve problems about ski lifts and will extend their work with money as they solve a problem about coins in a cash register. Additionally, students will use algebraic reasoning and their understanding of place value as they complete the Adventures.

### Excursions

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<tr>
<td>B</td>
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<td>C</td>
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<td>D</td>
<td>Sum of Squares Place value; skip-counting by 10s; addition of three-digit whole numbers</td>
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<td>E</td>
<td>All the Single Digits Encore Place value; logical reasoning; odd and even numbers</td>
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<td>F</td>
<td>Maze 1,000 Addition of multiples of ten through 1,000; place value</td>
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### Adventures

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<td>Ski Lift 1 Logical reasoning; doubling Required before Adv. B</td>
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<tr>
<td>B</td>
<td>Ski Lift 2 Logical reasoning; doubling; algebraic reasoning</td>
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<td>C</td>
<td>Nine Coins 2 Coin collections; logical reasoning; skip-counting; addition of two-digit whole numbers</td>
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<tr>
<td>D</td>
<td>Maze Min and Max Addition of multiples of 10 through 1,100; place value</td>
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<td>E</td>
<td>Connected Cubes Doubling; repeated addition or multiplication; algebraic reasoning</td>
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<td>F</td>
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## Unit 5

### Excursions

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<td>B Nine Coins 1</td>
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<td>C Match 1,000</td>
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<tr>
<td>D Sum of Squares</td>
<td></td>
<td></td>
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<tr>
<td>E All the Single Digits Encore</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F Maze 1,000</td>
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### Adventures

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<tr>
<td>A Ski Lift 1 <em>Required before Adv. B</em></td>
<td></td>
<td></td>
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<tr>
<td>B Ski Lift 2</td>
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<td>C Nine Coins 2</td>
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<td>D Maze Min and Max</td>
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<td>E Connected Cubes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F Skyscraper</td>
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</tbody>
</table>
Woodlawn School collected 814 books to send to an elementary school that had its library destroyed by a hurricane. They can send the books in the following ways:

- Large boxes hold 100 books.
- Small boxes hold 10 books.
- Padded envelopes hold 1 book.

Find five different ways that exactly 814 books can be sent. How many large boxes, small boxes, and padded envelopes are needed for each way?
Nine Coins 1

There are 9 coins on the ground outside the grocery store. They are worth $1.00.

a. What coins are on the ground?

b. Can you find another solution?
Match each number in the left column with a number in the right column. When added together, the sum of the two numbers should be 1,000. Write an equation for each match; for example, $342 + 658 = 1,000$.

<table>
<thead>
<tr>
<th>456</th>
<th>514</th>
</tr>
</thead>
<tbody>
<tr>
<td>438</td>
<td>484</td>
</tr>
<tr>
<td>572</td>
<td>544</td>
</tr>
<tr>
<td>486</td>
<td>439</td>
</tr>
<tr>
<td>527</td>
<td>428</td>
</tr>
<tr>
<td>516</td>
<td>562</td>
</tr>
<tr>
<td>561</td>
<td>473</td>
</tr>
</tbody>
</table>

Describe any patterns you see with your equations. Use these patterns to write one more equation with two numbers that sum to 1,000.
Sum of Squares

Below is a portion of a number chart that goes from 201 to 300. Only two squares are filled in so far. Find the sum of the numbers that belong in the shaded squares and write them on the supplement page.

244

257
Sum of Squares

244

257

PREVIEW SAMPLE
All the Single Digits Encore

Some of the riddles below have more than one possible answer. However, the digits 0 through 9 are each used exactly once, and no digit will be used in the answer to more than one riddle. Can you figure out how to use the digits to answer all three riddles?

Riddle 1:

- I am a three-digit, even number.
- The digit in my 10s place is one more than the digit in my 1s place.
- The digit in my 100s place is one more than the digit in my 10s place.

Riddle 2:

- I am the least possible number greater than 1,000 that satisfies the other clues.
- The sum of my digits is 7.
- The digit in my 1s place is four more than the digit in my 100s place.

Riddle 3:

- I am a three-digit number, and all of my digits are odd.
- The sum of my digits is 17.
- My 100s digit is greater than my 10s digit, and my 10s digit is greater than my 1s digit.
Maze 1000

In this maze, the numbers in each of the cells are added as you pass through them. Can you find a path from START to FINISH through which the numbers add to exactly 1,000?

```
START

10 50 70 40 30 20

40 60 30 20 20 30

30 70 10 60 50 40

70 20 50 70 60 10

10 50 30 50 60 40

40 60 20 70 10 30

FINISH
```
Maze 1000

START

10  50  70  40  30  20

40  60  30  20  20  30

30  70  10  60  50  40

70  20  50  70  60  10

10  50  30  50  60  40

40  60  20  70  10  30

FINISH

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Ski Lift 1

Raven is riding one of the ski lifts at a ski resort. The chairs on the ski lift take skiers up to the top of the mountain and then head down to the bottom empty. The chairs are numbered in order, starting with chair 1.

Raven’s family is exactly halfway up the mountain when the ski lift pauses. The empty chair opposite them, heading down, is chair 91. If they are on chair 9, how many chairs are on the ski lift?
Ski Lift 2

Make sure you have completed Ski Lift 1 before you do this task!

Raven rides three more ski lifts at the mountain. On each one, the ski lift pauses exactly halfway up. For each ski lift below, figure out how many chairs there are on the ski lift.

a. When Raven’s family is exactly halfway up, chair 64 is opposite them. They are on chair 2.

b. When Raven’s family is exactly halfway up, chair 17 is opposite them. They are on chair 83.

c. When Raven’s family is exactly halfway up, chair 75 is opposite them. They are on chair 11.

d. Explain how you can figure out how many chairs there are on a ski lift if you know two chair numbers that pass each other at the halfway point.
Nine Coins 2

There are 9 coins in a store’s cash register. There are exactly three different types of coins.

a. What is the most amount of money in coins that could be in the register?

b. What is the least amount of money in coins that could be in the register?
Maze Min and Max

In this maze, the numbers in each of the cells are added as you pass through them.

a. What is the least sum (minimum) you can make going through the maze? Mark your route on the first maze on the supplement page.

b. What is the greatest sum (maximum) you can make going through the maze? Mark your route on the second maze on the supplement page.
<table>
<thead>
<tr>
<th>Maze Min and Max</th>
<th>START</th>
<th>FINISH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10 50 70 40 30 20</td>
<td>40 60 30 20 20 30</td>
</tr>
<tr>
<td></td>
<td>30 70 10 60 50 40</td>
<td>70 20 50 70 60 40</td>
</tr>
<tr>
<td></td>
<td>10 50 30 50 60 40</td>
<td>40 60 20 70 10 30</td>
</tr>
</tbody>
</table>

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Connected Cubes

Below is a picture of eight cubes that have been glued together. Happy face stickers have been placed on all the outer faces of the cubes, including the bottom and the back.

a. How many stickers were used on these eight connected cubes?

b. If there were 5 connected cubes instead of 8, how many stickers would there be?

c. If there were 10 connected cubes, how many stickers would there be?

d. If there were 100 connected cubes, how many stickers would there be?

e. Explain how you can figure out how many stickers there will be on the outside of any number of connected cubes.
Skyscraper

Emily adds to her pattern block building each day.

a. Use pattern blocks to make the building for Day 4 and the building for Day 5.

b. For Day 1 through Day 5, record how many pattern blocks are used to make the building each day.

d. If 71 pattern blocks are used to make a building in this pattern, what Day is it?

e. Is it possible to make a building in this pattern with an even number of pattern blocks? Explain why or why not.
Excursion 5A: Boxing Books

There are many ways to package the books to be sent. A table illustrating a pattern of possibilities is below. For your students’ solutions, be sure to confirm that the total number of large boxes (holding 100 books), small boxes (holding 10 books), and padded envelopes (holding 1 book) accommodates exactly 814 books.

<table>
<thead>
<tr>
<th>Large Boxes (100 books)</th>
<th>Small Boxes (10 books)</th>
<th>Padded Envelopes (1 book)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>7</td>
<td>11</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>14</td>
</tr>
<tr>
<td>7</td>
<td>9</td>
<td>24</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>34</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>44</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>54</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>64</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>74</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>84</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>94</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>104</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>114</td>
</tr>
<tr>
<td>6</td>
<td>21</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>etc., until finally…</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>814</td>
</tr>
</tbody>
</table>
Excursion 5B: Nine Coins 1
There are four possible solutions:
1 quarter, 7 dimes, and 1 nickel
2 quarters, 3 dimes, and 4 nickels
1 half-dollar, 2 dimes, and 6 nickels
1 half-dollar, 1 quarter, 2 dimes, and 5 pennies

Excursion 5C: Match 1000

456 + 544 = 1,000
438 + 562 = 1,000
572 + 428 = 1,000
486 + 514 = 1,000
527 + 473 = 1,000
516 + 484 = 1,000
561 + 439 = 1,000

Students may notice that the digits in the hundreds and tens places sum to 9, while the digits in the ones place sum to 10. This pattern can be used to write other equations, e.g., 817 + 183 = 1,000.
Excursion 5D: Sum of Squares

The sum of these values is 999.

Excursion 5E: All the Single Digits Encore

Riddle 1: The possible three-digit numbers are 876, 654, 432, and 210.

Riddle 2: The possible four-digit numbers that satisfy the second and third clues are 1,024, 1,105, 2,014, and 3,004, but 1,024 is closest to 1,000.

Riddle 3: The possible three-digit numbers are 971 and 953.

Since each digit 0 through 9 can only be used once, the answers must be 876 for Riddle 1, 1,024 for Riddle 2, and 953 for Riddle 3.

Excursion 5F: Maze 1000
**Adventure 5A: Ski Lift 1**

164 chairs.

There are 81 chairs in between chairs 9 and 91 (90 – 9). Because Raven is halfway, there are also 81 chairs in between chairs 91 and 9. This is a total of 162 chairs, but does not include chairs 9 and 91, which are two additional chairs. So, 81 + 81 + 2 = 164.

**Adventure 5B: Ski Lift 2**

a. \((63 - 2) + (63 - 2) + 2 = 124\); or \((63 - 2) \times 2 + 2 = 124\)

b. \((82 - 17) + (82 - 17) + 2 = 132\); or \((82 - 17) \times 2 + 2 = 132\)

c. \((74 - 11) + (74 - 11) + 2 = 128\); or \((74 - 11) \times 2 + 2 = 128\)

d. If you can determine the number of chairs between the two chair numbers given, then that number can be doubled to find the total number of chairs between the two chairs both uphill and downhill. Adding 2 more accounts for the two chairs given in the problem.

*There are several ways that the number of chairs between the two chairs can be found, but this is not a trivial task. Students’ reasoning may not match the reasoning above.*

**Adventure 5C: Nine Coins 2**

If we consider only quarters, dimes, nickels, and pennies:

a. The most money there could be is $1.90 from 7 quarters, 1 dime, and 1 nickel.

b. The least amount of money there could be is 22¢ from 1 dime, 1 nickel, and 7 pennies.

If we consider half-dollars as well, the most money possible changes:

a. The most money there could be is $3.85 from 7 half-dollars, 1 quarter, and 1 dime.

If we consider dollar coins as well, the most money possible changes:

b. The most money there could be is $7.75 from 7 dollar coins, 1 half-dollar, and 1 quarter.
Answer Key

**Adventure 5D: Maze Min and Max**

a. Minimum: 370  

b. Maximum: 1010

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**Adventure 5E: Connected Cubes**

a. There are 34 stickers on the 8 connected cubes  
   \[(4 + 4 + 4 + 4 + 4 + 4 + 4 + 2) \text{ or } 4 \times 8 + 2\].

b. 5 connected cubes: 22 stickers

c. 10 connected cubes: 42 stickers

d. 100 connected cubes: 402 stickers

e. Rule: The number of stickers can be determined by adding 4 stickers for each cube (for the four long faces of the rectangular prism) and adding 2 stickers for the ends.
a. Day 4’s building will consist of 8 squares and 1 triangle. Day 5’s building will consist of 10 squares and 1 triangle.

b. There are 3, 5, 7, 9, and 11 pattern blocks for Days 1 through 5 respectively.

c. Emily will need 21 pattern blocks on Day 10, 101 pattern blocks on Day 50, and 201 pattern blocks on Day 100.

d. It takes 71 pattern blocks on Day 35.

e. No, it is not possible to make a building in this pattern with an even number of blocks. The squares will always be an even number, since they are double the number of the day. Adding the single triangle to this even number will always make the total an odd number.